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Existence of two-solar-mass neutron star constrains gravitational constant G_N at strong field

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In general relativity, there is a maximum mass allowed for neutron stars that, if exceeded, entails collapse into a black hole. Its precise value depends on details of the nuclear matter equation of state, a subject where much progress has been accomplished thanks to low energy effective theories. The discovery of a two-solar-mass neutron star, near that maximum mass, when analyzed with modern equations of state, implies that Newton's gravitational constant in the star cannot exceed its value on Earth by more than 12% at the 95% confidence level. This significantly extends the gravitational field intensity at which the constant has been constrained at the 10% level.

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The gravitational attraction force between two bodies of mass m_1 and m_2 at distance r is approximately given by the renowned law of Newton:

$$\mathbf{F} = -G_N \frac{m_1 m_2}{r^2}\hat{\mathbf{r}}. \tag{1}$$

The Newtonian constant G_N, first measured by Cavendish, also features in the more precise field equations of Einstein's general relativity,

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = 8\pi G_N T_{\mu\nu}, \tag{2}$$

which needs to be used under intense gravitational fields with Einstein's curvature tensor $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ and matter source $T_{\mu\nu}$ or otherwise if high precision is expected. This constant has been carefully measured on numerous occasions [1,2] and is currently taken to be $6.6738(8)\,10^{-11}\ \mathrm{N(m/kg)^2}$.

The study of the orbital evolution of binary pulsars [3] has also allowed researchers to establish the validity of general relativity (and indirectly provided a measurement of G_N) under stronger gravity conditions. While on the Earth's surface $g \simeq 9.8\ \mathrm{m/s^2}$, for the binary pulsar J0737-3039 the relevant acceleration is $g \simeq 270\ \mathrm{m/s^2}$, and for PSR B1913 + 16 it is $330\ \mathrm{m/s^2}$. The assessments of G_N in those systems have precisions of 0.05% and 0.2% respectively (from measurements of corrections to Kepler's law, particularly a parameter called s that is proportional to $G_N^{-1/3}$).

Cavendish's constant has also been constrained at a white dwarf [4], where gravitational accelerations are of order $g = 2 \times 10^6\ \mathrm{m/s^2}$.

We here point out that the new discovery [5] of a neutron star with a mass equal to 1.97(4) solar masses (a convenient unit weighing about 2×10^{30} kg), confirming previous claims of neutron stars in this mass range [6], is so close to the maximum mass that such an object can have [7], by nuclear physics considerations, that it significantly constrains the value that the gravitational constant can take in its interior. The reason is that there is an equilibrium between gravitational attraction and interneutron repulsion at short distances that cannot be maintained somewhat above two solar masses, and heavier objects collapse into black holes. Thus, an increase of the gravitational constant G_N that diminishes the maximum mass attainable is excluded by the discovery of these superheavy neutron stars (many of the neutron stars known to date had masses near 1.4 times that of the Sun and provide only a weaker constraint on G_N at the level of 40%).

The situation is described in Fig. 1. From left to right, we show the laboratory value (precision 1×10^{-4}); the astronomical value as inferred from double pulsars J0737-3039 and Hulse-Taylor PSR B 1913 + 16, at acceleration near $300\ \mathrm{m/s^2}$ (respective precisions of 5×10^{-4} and 2×10^{-3}); the constraint of Hut from white dwarf studies [4]; and our constraint from the existence of a neutron star with mass equal to 1.4 and 1.97(4) solar masses (G_N cannot exceed its earthly value by more than 12% at the 95% confidence level for a mass of 1.97 solar masses).

To obtain the bound, we employ the equation of hydrostatic equilibrium of Tolman-Oppenheimer-Volkoff [8,9], a consequence of Eq. (2). This equation governs the variation of the pressure P inside a spherically symmetric, static star at radial distance r from its center, inside which a mass $M(r)$ due to the mass-energy density $\varepsilon(r)$, has accumulated:

$$\frac{dP}{dr} = -\frac{G_N}{r^2}\frac{[\varepsilon(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{1 - \frac{2G_N M(r)}{r}}. \tag{3}$$

This equation can be integrated numerically from the inside of the star ($r = 0$) to the outside by a standard Runge-Kutta computer algorithm. The initial condition is supplied as a value of the pressure in the star's center, and the equation is considered solved at the distance R where the pressure drops to zero. R is then interpreted as the star radius. The mass function $M(r)$ and total star's mass $M(R)$ are obtained by adopting an equation of state that relates the total energy density $\varepsilon(r)$ to the pressure $P(r)$. Here is where steady progress in nuclear theory allows us to have reasonable confidence in the equation of state input, shown as the solid red line in Fig. 2.

The plot also shows, for comparison, the equation of state for a pure neutron Fermi gas [13]. This is much less "stiff"

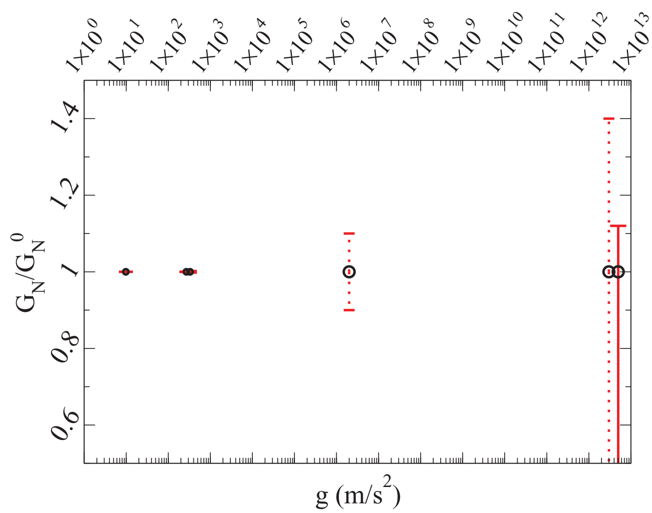

FIG. 1. (Color online) The gravitational constant remains (so far) a constant. The Newton-Cavendish constant is normalized by its accepted value, 6.6738(8) 10^{-11} N(m/kg)^2. From left to right: laboratory on Earth, orbital determinations of binary pulsars, white dwarf structure, neutron stars with 1.4 solar masses, and neutron star with 1.97(4) solar masses. At the intense gravitational field in such a neutron star, G_N cannot exceed 12% of its value on Earth at the 95% confidence level.

(lower pressure at given energy density) since it does not include the interneutron interaction, which is repulsive. Further, we show an independent computation of the equation of state at low energy densities that is in good agreement [11] with our own computation. A slight discrepancy can be ascribed to our employing pure neutron matter for simplicity, while those authors include a small amount of protons in dynamical β equilibrium with the neutrons. Other equations shown are variations of the many-body potential equations proposed by Pandharipande and collaborators [14,15]. We have chosen three more equations that include β equilibrium, one a parametrization by Heiselberg and Hjorth-Jensen [10] and two (with or without the electron pressure) from Manuel and Tolos [12].

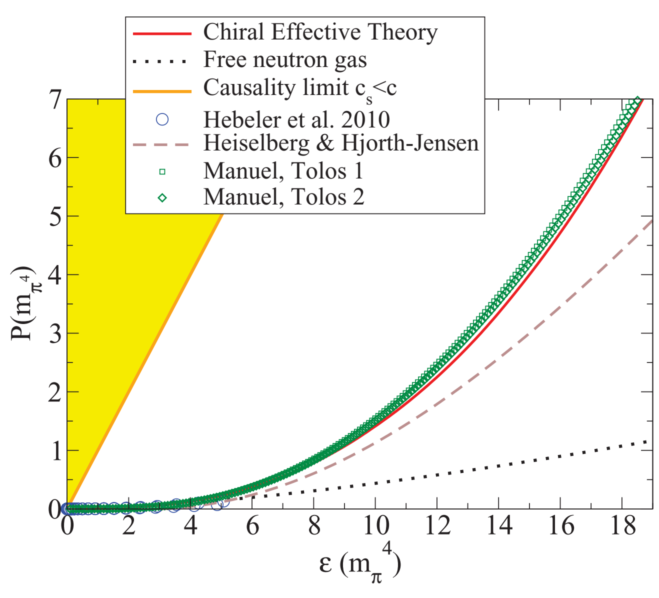

FIG. 2. (Color online) The equation of state (pressure as function of density) for pure neutron matter (solid red line) compared with the free neutron gas (dashed black line) is much stiffer because of repulsive interactions (nuclear matter is barely compressible). We give other independent determinations of the equation of state [10–12]. Also shown is the causality limit, given by the condition that sound propagates slower than light, $c_s^2 = \partial P/\partial \rho < c$.

Our improvements concern the use of a recently developed chiral effective-field theory (EFT) for nuclear matter [16–19] based on a new power counting. The latter is able to single out the set of Feynman diagrams, including infinite strings of them, that are required to calculate order by order in the chiral expansion for (asymmetric) nuclear matter. This is a novelty since previous calculations employ standard many-body methods not based in the new paradigm of EFT. The chiral power counting [16] considers multinucleon forces both from pion exchanges as well as from short-range contributions. It also takes into account the important infrared enhancement affecting nucleon propagators in the multinucleon reducible loops. This allows to control the size of many-body (three-body, four-body, etc.) forces and decide, systematically and to the precision desired, of what medium effects are to be kept at each stage of the calculation.

Further details on the equation of state that we employ are documented in Fig. 3, which presents the pressure and energy densities as functions of the Fermi momentum for the neutron gas as well as the speed of sound $\sqrt{\partial P/\partial \rho}$.

It should be noted that while the equation of state was obtained with sophisticated modern EFT treatments [16], it is in broad agreement with vintage nuclear theory treatments based on phenomenological potentials [15].

Once the equation of state has been fixed and the integration of the Tolman-Oppenheimer-Volkoff equilibrium equation (3) has proceeded, one obtains the standard mass-radius plot shown in Fig. 4.

As has been known since the early work of Oppenheimer and Volkoff, a pure neutron gas supports no star with mass above approximately 0.7 solar masses, providing a check of our computer program (dashed red line). The full calculation including interactions can elevate the maximum mass above 2 solar masses. We discontinue the solid black line at a point where the effective theory breaks down as manifested by reaching the causality limit[1] $c_s = c$ (the Fermi momentum at that point, about 600 MeV, is also quite high). Stars above 2.3 solar masses are not supported. For a star with mass of about 2 solar masses, near the maximum possible, we give in Fig. 5 the profiles of pressure and intensity of gravity from the center of the star. The acceleration of gravity does not grow uniformly from the center to the edge as in Newtonian mechanics (as seen easily from Gauss's law) due to the pressure contribution in the relativistic expression for the potential Φ

[1]An additional higher order computation of the sound velocity within the EFT [16] is planned since the excess over the casuality limit from the thermodynamical formula $c_s^2 = \partial P/\partial \rho$ is only at the 10% level. This has no impact in our current results.

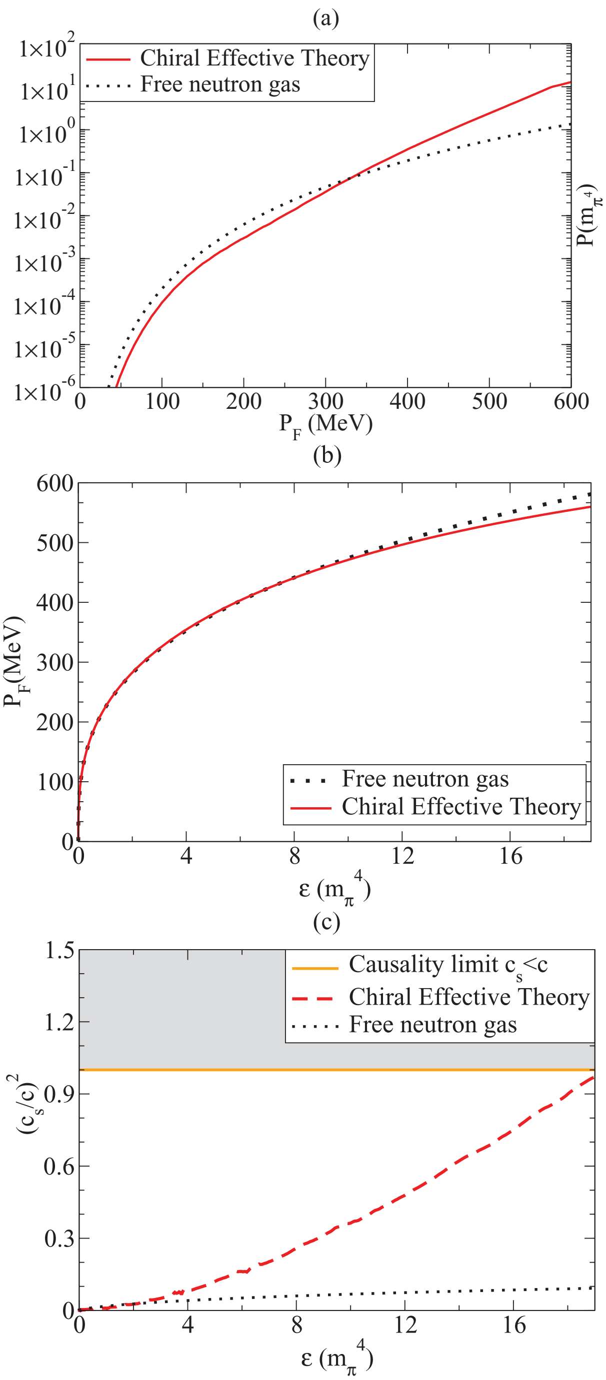

FIG. 3. (Color online) Equation of state for pure neutron matter in effective theory (red solid line) vs the free neutron gas (black dashed line). (a) Pressure against Fermi momentum. (b) Energy density as function of the Fermi momentum. (c) Speed of sound showing the causality limit where the effective theory description is expected to break down.

(given in geometrodynamic units $c = G_N = 1$)

$$g = \frac{d\Phi}{dr} = \frac{M(r) + 4\pi r^3 P(r)}{r[r - 2M(r)]}. \tag{4}$$

The order of magnitude of g in the full calculation can be understood from simple Newtonian considerations as $G_N M(R)/R^2$ at the star's surface, and if corrected by the relativistic denominator $[1 - 2G_N M(R)/c^2]^{-1} \simeq 1.4$ one obtains[2] about $g \simeq 2 \times 10^{12}$ m/s^2.

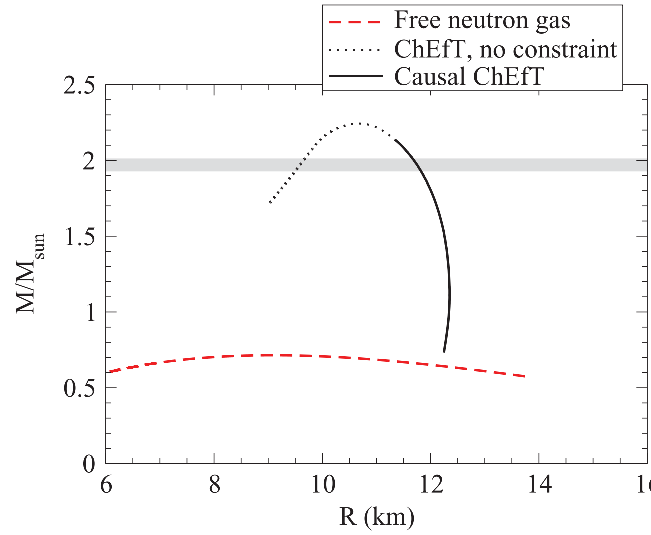

FIG. 4. (Color online) Mass-radius plot for the neutron star solutions of the Tolman-Oppenheimer-Volkoff equation of hydrostatic equilibrium. As is well known, the free neutron gas equation of state cannot reach masses beyond about 0.7 solar masses. However, the interacting equation of state, being more repulsive, supports stars slightly above 2 solar masses against gravitational collapse, in agreement with the observation of a star with 1.97(4) solar masses.

Thus, our constrain on G_N pushes the 10% variation limit to the 10^{12} m/s^2 order of magnitude (where only much looser bounds could be stated to date). This can be of use to constrain modified theories of gravity, motivated by string theory and by cosmology, that suggest that for largely different values of the curvature R (or acceleration field g), gravity separates from its Einsteinian formulation.

We complete our analysis by returning to Eq. (3) and varying G_N. Since reducing it simply delays gravitational collapse and eventually allows for arbitrarily heavy stars, no constraint is put in smaller than physical G_N values, as reflected in Fig. 1. However, increasing G_N rapidly reduces the maximum possible mass of the neutron star.

To control the systematic uncertainty at very high energy densities, where other phenomena might arise (activation of the strangeness degree of freedom, transition to a different phase of nuclear matter not accessible from the nucleon effective theory, etc.), and since we are interested in imposing *an upper bound* on G_N, we substitute our equation of state with the one most stiff allowed by causality,[3] such that $c_s = c$, yielding $P = c^2(\rho - \rho_{\rm max}) + P_{\rm max}$, above a maximum Fermi momentum of either $k_F = 600$ or 450 MeV. Shown in Fig. 6

[2]Since the pulsar's period is measured to be about 3.15 ms, the maximum centripetal acceleration at the equator is two orders of magnitude smaller than gravity, and we therefore neglect the (naturally) very small oblateness of the star.

[3]Introducing additional, possibly exotic, degrees of freedom cannot stiffen the equation of state beyond causality.

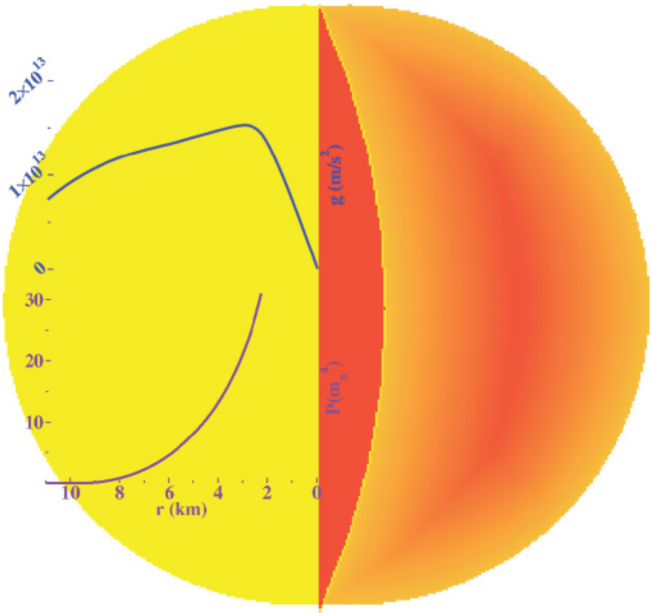

FIG. 5. (Color online) For a neutron star with mass near the maximum allowed by hydrostatic equilibrium, we show the pressure profile and the acceleration of gravity (note its non-Newtonian behavior due to the relativistic pressure term).

is the mass/radius plot that adopts the first value, allowing the Cavendish constant to vary. From this calculation, we derive the bound on a 12% variation of G_N.

Should one adopt the second value due to putative errors that we may have not identified in our equation of state at higher energy density, the constrain on G_N is somewhat relaxed, but remains meaningful, excluding a variation of 25% at the 2σ level.

On the opposite, low density limit neglect corrections due to the neutron star skin containing several atomic sheets [20] as well as ordinary nuclear (not neutron) matter, for it is known

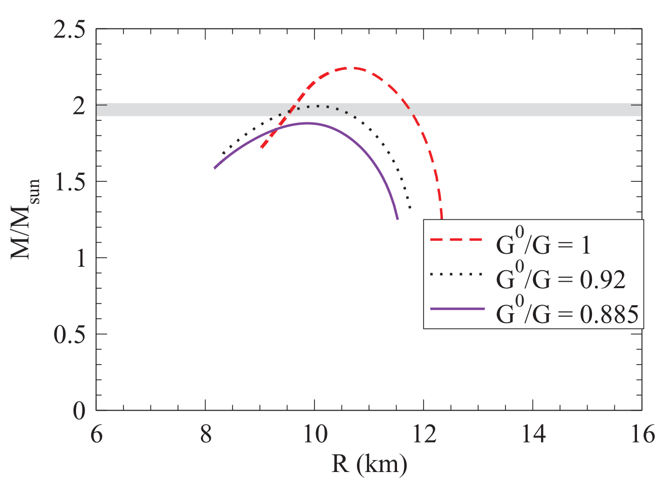

FIG. 6. (Color online) Mass-radius plot as in Fig. 4 but varying the gravitational Newton-Cavendish constant. As this grows (or, as shown, the ratio of the constant in Earth to that constant at high field decreases), the star becomes more prone to gravitational collapse, and thus the maximum reachable mass drops below the requisite 2 solar masses. Thus, increases of G_N are now constrained.

that its total contribution to the star's mass is rather small [11,21].

We have repeated our computation for three other equations of state taken from the vast literature and find similar results. First, we test the simple parametrization in Ref. [10], with energy density ϵ given in terms of $u = n/n_0$, the ratio of baryon density to nuclear saturation density, and proton fraction x_p

$$\epsilon = \epsilon_0 u \frac{u - 2 - \delta}{1 + u\delta} + S_0 u^\gamma (1 - 2x_p)^2 \,. \tag{5}$$

The first term is a compression term with constants $\epsilon_0 \simeq -15.8$ MeV and $\delta = 0.2$. The second is a symmetry term with $S_0 = 32$ MeV and $\gamma = 0.6$. The nuclear saturation density adopted is $n_0 = 0.16$ fm^{-3}. This variant seems somewhat softer than our own and produces a maximum mass slightly below 1.97(4) solar masses. An increase on the order of 3% of Cavendish's constant is sufficient to bring this mass below the 95% confidence exclusion level, thus not contradicting our results.

We also employed numerical data based on [12]. Two equations are provided, both including protons in β equilibrium with the neutron matter. One includes, in addition, the pressure due to electrons. At the highest densities (slightly out of Fig. 2), we extrapolate them linearly with $\rho = 3P$. These equations are slightly stiffer than our own, but a variation of G_N of 12% also brings the maximum neutron star mass below 1.89 solar masses.

In conclusion, we believe that we have made a relevant contribution in employing the two-solar-mass neutron star to constrain the running of the gravitational constant G_N [22] with field intensity (gravitational acceleration), making use of the nuclear matter equation of state, as opposed to attempting to constrain the equation of state and possibly exotic forms of nuclear matter about which abundant literature exists [23–27].

Other authors [11] have already pointed out that nuclear physics is precise enough to constrain the radius of the star given its mass. The discovery of higher-mass neutron stars allows us to establish meaningful limits on allowed variations of gravity itself in a hitherto unexplored regime.

Not long ago, a two-solar-mass neutron star was thought unlikely [28]. Although we now know that nuclear physics can accommodate it, the margin is narrow and allows constraints on gravity.

Future work may include an examination of modified theories of gravity (for the case of scalar modifications of the action in $f(R)$ theories, the Tolman-Oppenheimer-Volkoff equations are already available in the literature [29]).

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