Spin excitonic and diffusive modes in superfluid Fermi liquids

E. E. Kolomeitsev[1] and D. N. Voskresensky[2,3]

[1]*Department of Physics and Centre of Science and Research, University of Matej Bel, SK-97401 Banska Bystrica, Slovakia*
[2]*National Research Nuclear University "MEPhI," Kashirskoe sh. 31, 115409 Moscow, Russia*
[3]*GSI, Helmholtzzentrum für Schwerionenforschung GmbH, Planckstrasse 1, D-64291 Darmstadt, Germany*

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The role of the particle-particle p-wave spin interaction in Fermi liquids with an s-wave pairing is studied. Depending on the sign of the interaction, there arises either a new exciton collective mode below the pair-breaking threshold or a diffusive excitation mode above the threshold. The Landau parameters that control the interaction strength are evaluated for various systems: dilute fermion gases, a degenerate electron liquid, metals, atomic nuclei, and neutron matter. The interaction removes also the square-root singularity in the phase space of pair-breaking processes. How these effects influence the neutrino emissivity in neutron Cooper-pair recombinations in neutron stars is shown.

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Processes with recombinations of Cooper pairs provide important information about interparticle interactions and pairing mechanisms in different fermionic systems: ordinary superconductors [1], liquid ^{3}He and ^{3}He-^{4}He mixtures [2], cold atomic gases [3], atomic nuclei [4], neutron stars [5,6], and other systems. In superconductors, they are studied by absorption of infrared radiation or by Raman scattering [7]. In the cold fermion-atom gas one can use the Stokes scattering to detect the onset of the pairing [8]. Inverse pair breaking and formation (PBF) reactions constitute an important mechanism of the neutron-star cooling [6,9]. In these processes the energy is released in the form of neutrino-antineutrino pairs radiated off the star. The superburst ignition depth is sensitive to the value of the PBF emissivity in the inner neutron star crust [10]. The PBF processes are important [11] for the description of the recently observed rapid cooling of the young neutron star in Cassiopeia A.

It was shown [12] that the residual interaction of single-particle excitations, which does not contribute to pairing, can bind them in a state orthogonal to the Cooper pair, generating collective excitation modes in superconductors. For superfluid ^{3}He a similar mechanism was studied in Ref. [13]. A collective mode originated by f-wave interactions in the pairing channel was detected in superfluid ^{3}He-B [14]. Interactions in the same spin channel in which the pairing occurs have been studied so far. The influence of the interaction in one spin channel on pairing in the other spin channel has not yet been considered.

In this Brief Report we study the effects of a p-wave interaction in the spin-one channel on excitations in a Fermi system with spin-zero pairing. We calculate a response induced by the external spin- and helicity-density sources and show that, depending on the sign of the effective interaction, either a new exciton mode or a diffusive excitation mode appears. Then, we evaluate the strength of this effective interaction for different Fermi systems and, as an example, calculate the neutrino emissivity in the PBF processes for a neutron star with the $^1S_0$ neutron pairing, taking into account the effects of the new collective modes and correlations.

We use the Fermi liquid theory extended to systems with pairing by Larkin and Migdal and by Leggett [15]. For processes induced by weak nucleon interactions this approach was adopted in Ref. [16]. In strongly interacting systems interactions in the particle-particle (ξ) and particle-hole (ω) channels are essentially different. The interaction amplitude of two fermions with momenta $\vec{p} = p_{\rm F}\vec{n}$ and $\vec{p}\,' = p_{\rm F}\vec{n}\,'$ before and after the interaction in the ξ channel is parameterized as $\widehat{\Gamma}^{\xi} = \Gamma_0^{\xi}(\vec{n},\vec{n}\,')(i\sigma_2')(i\sigma_2) + \Gamma_1^{\xi}(\vec{n},\vec{n}\,')(i\sigma_2'\vec{\sigma}\,')(\vec{\sigma}\,i\sigma_2)$, and in the ω channel it is parameterized as $\widehat{\Gamma}^{\omega} = \Gamma_0^{\omega}(\vec{n},\vec{n}\,')1'\,1 + \Gamma_1^{\omega}(\vec{n},\vec{n}\,')\vec{\sigma}\,'\vec{\sigma}$. Here $p_{\rm F}$ stands for the Fermi momentum; $\vec{n}$ and $\vec{n}\,'$ are the unit vectors. The unit matrices 1 and 1′ and the Pauli matrices $\vec{\sigma}$ and $\vec{\sigma}\,'$ act in the nucleon spin space. A superscript ω indicates that the amplitude in this channel is taken for $|\vec{q}\,\vec{v}_{\rm F}| \ll \omega \ll \epsilon_{\rm F}$, $v_{\rm F}$ is the Fermi velocity, $\epsilon_{\rm F}$ is the Fermi energy, and $q = (\omega, \vec{q})$ is the transferred four-momentum. Coefficients of the harmonic expansion of the scalar $\Gamma_0^{\xi,\omega}$ and spin $\Gamma_1^{\xi,\omega}$ amplitudes (Landau parameters) should be either evaluated microscopically or extracted from the analysis of experimental data [4].

The singlet pairing in a Fermi liquid occurs owing to the attractive interaction, $a^2\rho\,\Gamma_0^{\xi} = f_0^{\xi} < 0$. At zero temperature a paring gap Δ follows from $-1/f_0^{\xi} = A_0/(a^2\,\rho) = \ln(2\,\epsilon_{\rm F}/\Delta)$, where a is the residue of a pole in the quasiparticle Green's function and ρ is the density of states at the Fermi surface. This expression is naturally generalized for finite temperature T; cf. Eq. (5) in the second paper in Ref. [16]. Since $g_0^{\xi} \equiv 0$ for the scattering of identical fermions, the spin interaction in the ξ channel simplifies as $a^2\rho\Gamma_1^{\xi}(\vec{n},\vec{n}') = g_1^{\xi}(\vec{n}\,\vec{n}')$. It is usually assumed that the higher Legendre harmonics are much smaller [15]. Since we focus on the spin channel, the interaction Γ_0^{ω} decouples and can be dropped. In the ω channel, $a^2\rho\Gamma_1^{\omega} = g_0^{\omega} + g_1^{\omega}(\vec{n}'\vec{n})$. Contributions from the zeroth harmonics g_0^{ω} are accompanied by the factor $v_{\rm F}^2$ (see Ref. [16]) and can be dropped for the nonrelativistic Fermi liquids under consideration ($v_{\rm F}^2 \ll 1$). Thus we remain with only three relevant Landau parameters, $f_0^{\xi} < 0$, g_1^{ξ}, and g_1^{ω}. Let us first set g_1^{ω} to zero and demonstrate the influence of the interaction in the spin-one ξ channel, g_1^{ξ}, on the s-wave pairing effects. Then, we recover the dependence on g_1^{ω}.

Consider now an external perturbation, which couples to the spin-density operator $\vec{s}(x) = \psi^\dagger(x)\vec{\sigma}\psi(x)$ and the helicity-density operator $h(x) = \psi^\dagger(x)(\vec{\sigma}\hat{\vec{p}} + \hat{\vec{p}}\vec{\sigma})\psi(x)/(2m)$, where ψ is the spinor of a nonrelativistic fermion, $\hat{\vec{p}}$ is the momentum operator, and m is the mass of the free fermion. From these quantities one can build the axial (A) fermion current $\mathcal{J}^\mu = (h, \vec{s})$. The Fourier transform of its bare components after the Fermi liquid renormalization becomes $J^{\omega,\mu}(\vec{n}, q) = (\vec{\sigma}\vec{\tau}^{\omega}_{A,1}, \vec{\sigma}\tau^{\omega}_{A,0})$. Here $\tau^{\omega}_{A,0} = e_A/a$ and $\vec{\tau}^{\omega}_{A,1} = e_A v_F \vec{n}/a$ are the bare ω vertices; e_A is an effective charge of the quasiparticle. For $\Gamma^{\omega}_1 = 0$, which we now exploit, the in-medium vertices are $\tau_{A,0}(\vec{n}, q) = \tau^{\omega}_{A,0}$ and $\vec{\tau}_{A,1}(\vec{n}, q) = \vec{\tau}^{\omega}_{A,1}(\vec{n}, q)$. For $\Gamma^{\omega} \neq 0$ these vertices are modified [16]. Additionally, in a system with pairing new vertices responsible for the PBF processes arise: $\vec{\sigma}\tilde{\tau}_{A,0}(\vec{n}, q)$ and $\vec{\sigma}\tilde{\vec{\tau}}_{A,1}(\vec{n}, q)$. They follow from the solution of the Larkin-Migdal equations [15,16],

$$\tilde{\tau}_{A,0}(\vec{n}, q) = -\frac{g_1^\xi}{a^2\rho}\Big(\langle(\vec{n}\,\vec{n}')\,[N(\vec{n}', q) + A_0]\,\tilde{\tau}_{A,0}(\vec{n}', q)\rangle_{\vec{n}'} + \big\langle(\vec{n}\,\vec{n}')\,O(\vec{n}', q; -1)\,\tau^{\omega}_{A,0}\big\rangle_{\vec{n}'}\Big), \tag{1a}$$

$$\tilde{\vec{\tau}}_{A,1}(\vec{n}, q) = -\frac{g_1^\xi}{a^2\rho}\Big(\langle(\vec{n}\,\vec{n}')\,[N(\vec{n}', q) + A_0]\,\tilde{\vec{\tau}}_{A,1}(\vec{n}', q)\rangle_{\vec{n}'} + \big\langle(\vec{n}\,\vec{n}')\,O(\vec{n}', q; +1)\,\vec{\tau}^{\omega}_{A,1}(\vec{n}')\big\rangle_{\vec{n}'}\Big). \tag{1b}$$

The angle brackets indicate the angular averaging $\langle\cdots\rangle_{\vec{n}} = \int\frac{d\Omega_{\vec{n}}}{4\pi}(\cdots)$. The loop functions $O(\vec{n}, q; \pm 1) = \frac{1}{2}a^2\rho\,(z_+ \pm z_-)g_T(\vec{n}, \omega, \vec{q}\,)$ and $N(\vec{n}, q) = a^2\rho\, z_+ z_-\, g_T(\vec{n}, \omega, \vec{q}\,)$ with $z_\pm = (\omega \pm \vec{v}\,\vec{q}\,)/(2\,\Delta)$, and the master function

$$g_T(\vec{n}, \omega, \vec{q}\,) = \Delta^2 \int_{-\infty}^{+\infty} \frac{d\xi_p}{\epsilon_+\,\epsilon_-}\left[\frac{E_-\,F_-}{\omega^2 - E_-^2} - \frac{E_+\,(1 - F_+)}{\omega^2 - E_+^2}\right],$$

where $E_\pm = \epsilon_+ \pm \epsilon_-$, $F_\pm = f(\epsilon_-) - f(\epsilon_+)$, $f(x) = 1/[\exp(x/T) + 1]$, and $\epsilon_\pm = [(\xi_p \pm \vec{v}\vec{q}\,)^2 + \Delta^2]^{1/2}$. The solution of Eq. (1a) is

$$\tilde{\tau}_{A,0} = -\frac{(\vec{v}\,\vec{q}\,)}{2\,\Delta}\,\tau^{\omega}_{A,0}\,\gamma^{\xi}_{\parallel}\,\langle g_T(\vec{n}')\,(\vec{n}_q\,\vec{n}')^2\rangle_{\vec{n}'}\,,$$
$$\tilde{\vec{\tau}}_{A,1} = \frac{\omega}{q}\vec{n}_q\tilde{\tau}_{A,0} - \frac{\omega\,\tau^{\omega}_{A,1}}{2\,\Delta}\gamma^{\xi}_{\perp}\langle g_T(\vec{n}')\frac{1}{2}[1 - (\vec{n}_q\,\vec{n}')^2]\rangle_{\vec{n}'}\,\vec{P}_\perp\,, \tag{2}$$

where $\vec{P}_\perp = \vec{n} - \vec{n}_q\,(\vec{n}\,\vec{n}_q)$, $\vec{n}_q = \vec{q}/|\vec{q}|$, and the correlation factors

$$[\gamma^{\xi}_{\perp}]^{-1} = \frac{1}{3}\,C_0 + \left\langle\frac{\omega^2 - (\vec{v}\,\vec{q}\,)^2}{4\,\Delta^2}g_T(\vec{n})\frac{1}{2}[1 - (\vec{n}\vec{n}_q)^2]\right\rangle_{\vec{n}}\,,$$
$$[\gamma^{\xi}_{\parallel}]^{-1} = \frac{1}{3}\,C_0 + \left\langle\frac{\omega^2 - (\vec{v}\,\vec{q}\,)^2}{4\,\Delta^2}g_T(\vec{n})(\vec{n}\vec{n}_q)^2\right\rangle_{\vec{n}} \tag{3}$$

are controlled by one effective interaction parameter,

$$C_0 = 3/g_1^\xi - 1/f_0^\xi. \tag{4}$$

The singlet pairing occurs for $f_0^\xi < 0$ and $3f_0^\xi < g_1^\xi$. Then, if $g_1^\xi < 0$, we have $C_0 < 0$; otherwise, the p-wave pairing is preferable. For $g_1^\xi > 0$ we have $C_0 > 0$.

A response of the Fermi system to the excitation (A), is determined by the symmetrical current-current correlator $\Pi^{\mu\nu}(q) = \frac{1}{2}\langle\mathrm{Tr}\{J^{\omega,\mu}(\vec{n}, q)\,J^{\nu}(\vec{n}, q)\}\rangle_{\vec{n}}$ where the in-medium current $J^{\mu}(\vec{n}, q) = [\vec{\sigma}\,\vec{\chi}_{A,1}(\vec{n}, q), \vec{\sigma}\,\chi_{A,0}(\vec{n}, q)]$ is expressed via the reduced current correlators, is [16]

$$\chi_{A,0}(\vec{n}, q) = L(\vec{n}, q; -1)\,\tau_{A,0}(\vec{n}, q) + M(\vec{n}, q)\,\tilde{\tau}_{A,0}(\vec{n}, q),$$
$$\vec{\chi}_{A,1}(\vec{n}, q) = L(\vec{n}, q; +1)\,\vec{\tau}_{A,1}(\vec{n}, q) + M(\vec{n}, q)\,\tilde{\vec{\tau}}_{A,1}(\vec{n}, q),$$

where $M(\vec{n}, q) = -\,a^2\,\rho\,z_+\,g_T(\vec{n}, \omega, \vec{q}\,)$ and $\frac{L(\vec{n}, q; \pm 1)}{a^2\,\rho} = (\frac{z_\pm}{z_-} - 1)g_T(\vec{n}, (\vec{v}\vec{q}), \vec{q}) - (\frac{z_\pm}{z_-} - \frac{1\mp 1}{2})\,g_T(\vec{n}, \omega, \vec{q}\,)$. The temporal and spatial components of the tensor are $\Pi^{00} = \langle\vec{\tau}^{\omega}_{A,1}\,\vec{\chi}_{A,1}(\vec{n}, q)\rangle_{\vec{n}}$ and $\Pi^{ij} = \delta^{ij}\langle\tau^{\omega}_{A,0}\,\chi_{A,0}(\vec{n}, q)\rangle_{\vec{n}}$ with

$$\frac{1}{3}\sum_i \Pi^{ii} = e_A^2\rho\left\langle\frac{(\vec{v}\,\vec{q})}{\omega - \vec{v}\,\vec{q}}[g_T(\vec{n}, (\vec{v}\vec{q}), \vec{q}) - g_T(\vec{n}, \omega, \vec{q})]\right\rangle_{\vec{n}} + e_A^2\rho\,\frac{v_F^2\,\vec{q}^{\,2}}{4\,\Delta^2}\gamma^{\xi}_{\parallel}\,\langle(\vec{n}_q\,\vec{n})^2\,g_T(\vec{n}, \omega, \vec{q})\rangle^2_{\vec{n}}\,,$$
$$\Pi^{00} = v_F^2\frac{1}{3}\sum_i \Pi^{ii} + e_A^2\,\rho v_F^2\langle g_T(\vec{n}, \omega, \vec{q})\rangle_{\vec{n}} + e_A^2\,\rho\,v_F^2\,\frac{\omega^2}{2\,\Delta^2}\,\gamma^{\xi}_{\perp}\left\langle g_T(\vec{n}, \omega, \vec{q})\frac{1}{2}[1 - (\vec{n}_q\,\vec{n})^2]\right\rangle^2_{\vec{n}} + e_A^2\,\rho\,v_F^2\,\frac{\omega^2 - v_F^2\,q^2}{4\,\Delta^2}\,\gamma^{\xi}_{\parallel}\,\langle g_T(\vec{n}, \omega, \vec{q})\,(\vec{n}_q\,\vec{n})^2\rangle^2_{\vec{n}}. \tag{5}$$

The mixed components are $\Pi^{i0} = \Pi^{0i} = \vec{n}^{\,i}_q\,\frac{\omega}{3|\vec{q}|}\sum_j \Pi^{jj}$.

From Eq. (2) we see that the external perturbation can induce a singular response in the PBF amplitudes at the values ω and $\vec{q}$, corresponding to the poles of the functions $\gamma^{\xi}_{\perp}$ and $\gamma^{\xi}_{\parallel}$. These poles determine new transverse and longitudinal collective modes (spin excitons). For $\vec{q} = 0$, the longitudinal and transverse modes coincide, and their frequency ω follows from the condition

$$C_0 + y^2\Re\tilde{g}_T(y) = 0\,, \quad y = \omega/(2\Delta), \tag{6}$$

where $\tilde{g}_T(y) \equiv g_T(0, 2\Delta y - i\,0, 0)$.

Although the full inclusion of the g_1^ω dependence is rather tedious, the modification of Eq. (6) is simply given by the replacement $\Re\tilde{g}_T(y) \to \Re\tilde{g}_T(y)/[1 + \frac{1}{3}\,g_1^\omega\,\Re\tilde{g}_T(y)]$. For $|C_0| \gg 1$ this induces the shift

$$C_0 \to C = C_0/\big(1 + C_0 g_1^\omega/3\big). \tag{7}$$

This relation interpolates between the limits $|C_0| \ll 3/|g_1^\omega|$ when $C \approx C_0$ and $|C_0| \gg 3/|g_1^\omega|$ when $C \simeq (3/g_1^\omega)[1 - 3/(g_1^\omega C_0)]$. So parameter C controls the effects of residual interactions on the PBF processes.

In the long-wavelength limit (for $\omega > |\vec{q}|$) from Eq. (5) we get $\Im\Pi^{ij}(q) = \frac{\delta^{ij}}{3}\frac{\vec{q}^{\,2}}{\omega^2}\Im\Pi^{00}(\omega)$. The response function, which, for $y \sim 1$, has the form

$$R(y, C) \equiv \frac{\Im\Pi^{00}}{e_A^2\rho v_F^2} = \frac{C^2\Im\tilde{g}_T(y)}{[C + y^2\Re\tilde{g}_T(y)]^2 + [y^2\Im\tilde{g}_T(y)]^2} + \pi\frac{C^2}{y^2}\delta[C + y^2\Re\tilde{g}_T(y)],$$
$$\Im\tilde{g}_T(y) = \frac{-\pi\,\tanh\left(\frac{y\Delta}{2T}\right)\theta(y)}{2y\sqrt{y^2 - 1}}, \tag{8}$$

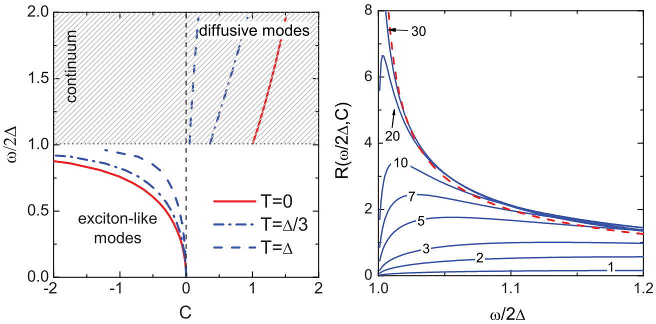

FIG. 1. (Color online) (left) Energies of collective modes (for $\vec{q}=0$) as functions of the parameter C for various values of T/Δ. (right) The response function $R(y)$ for $T=0$ and $y>1$ given by Eq. (8) for various values of the parameter C (solid lines). Dashed line shows the function $R(y, C\to\infty)$.

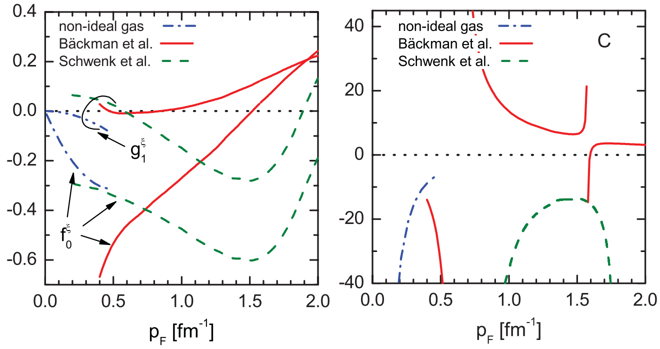

FIG. 2. (Color online) Parameters (left) f_0^ξ and g_1^ξ and (right) C for the neutron matter reconstructed with the Landau ω parameters calculated in Refs. [18,22] using Eq. (9) as functions of the Fermi momentum.

determines the probability of PBF processes. The cross section of the excitation scattering in matter is determined by this response function R.

Solutions of Eq. (6) are shown in Fig. 1 (left) as a function of C. For $C<0$, solutions with $y<1$ correspond to the undamped spin-exciton branch at $\omega<2\Delta$ since $\Im\tilde{g}_T(y<1)=0$. For $C>0$, solutions exist only if $C>-\Re\tilde{g}_T(1+0)$ and $\omega>2\Delta$. Since here $\Im\tilde{g}_T(y)\neq 0$, they constitute the diffusive spin mode. The frequencies of the excitonic and diffusive modes increase with an increase of T. The response function $R(y)$ at $T=0$ and $y>1$ is plotted in Fig. 1 (right) for various values of the parameter C. For $y>1$ the function R only weakly depends on the sign of C; therefore we show it only for $C>0$. The function $R(y, C\to\infty)=\Im\tilde{g}_T(y)$ is shown by the dashed curve. It has a square-root divergence at $y\to 1+0$, which is smeared out for finite values of C. Thus a finite value of C leads to a reduction of a spin response of a Fermi liquid close to the threshold for $\omega>2\Delta$. A similar effect was discussed in Ref. [7] for the Raman scattering on metallic superconductors.

To estimate the value of our key parameter C we need to know Landau parameters g_1^ξ, f_0^ξ, and g_1^ω. For a dilute Fermi gas we can use quasiparticle scattering amplitudes derived in Ref. [17] up to second order in the parameter $\zeta=2a_{\rm eff}\,p_{\rm F}/\pi$, where $a_{\rm eff}$ is the effective scattering length. We derive $f_0^\xi=\zeta+\zeta^2(2\ln 2+1)/3$, $g_1^\xi=3\zeta^2(1-2\ln 2)/5$, $g_1^\omega=-2\zeta^2(\ln 2+2)/5$ and obtain $C\approx -5.7/(a_{\rm eff}\,p_{\rm F})^2$. For the neutron gas the vacuum scattering length is very large, $a\simeq 20$ fm, but the effective scattering length is much shorter [18], being determined, e.g., by the $V_{{\rm low}-k}$ potential as $a_{\rm eff}\simeq 2$ fm.

For more complex systems parameters in the ξ channel can be estimated with the help of the Landau ω parameters in the s-p approximation [19] as

$$f_0^\xi=\sum_{l=0}^{\infty}(-1)^l\frac{A_l^s-3A_l^a}{4},\quad g_1^\xi=\sum_{l=0}^{\infty}(-1)^l\frac{A_l^s+A_l^a}{4},\qquad (9)$$

where $A_l^s=f_l^\omega/(1+\frac{f_l^\omega}{2l+1})$ and $A_l^a=g_l^\omega/(1+\frac{g_l^\omega}{2l+1})$.

The Fermi-liquid approach was applied to the degenerate electron liquid in Ref. [20]. Using Tables I and II of Ref. [20], we find $C=-2.54$ for a small value of the parameter $a_{\rm B}\,p_{\rm F}=0.032$, where $a_{\rm B}$ is the Bohr radius.

For alkali metals at zero pressure the first three ω harmonics are calculated in Ref. [21]. Applying (9), we then find for sodium $f_0^\xi({\rm Na})=-0.11$, $g_1^\xi({\rm Na})=-0.38$, and $g_1^\omega({\rm Na})=-0.075$. Here the p-wave paring is realized since $C_0>0$, but the value $|C_0|$ is very small. Bearing in mind large uncertainties in estimates of the Landau ω parameters, one cannot exclude that $C<0$ at $|C|\ll 1$. In the latter case we would deal with very pronounced effects of the spin-exciton mode. This case can also be realized if one allows a variation of the pressure. Thus the presence or absence of the new exciton mode could indicate the kind of pairing in the given system. For potassium $f_0^\xi({\rm K})=-0.56$, $g_1^\xi({\rm K})=-0.89$, and using $g_1^\omega({\rm K})=-0.12$, we obtain $C=-1.48$.

For the nucleon matter several harmonics of the Landau ω parameters were evaluated in many works (e.g., see Refs. [18,22]). The parameter f_0^ξ related to the ${}^1S_0$ pairing was also calculated; see Ref. [6]. On the other hand, the g_1^ξ parameter is poorly known. Using results from Refs. [18,22], we reconstruct g_1^ξ and f_0^ξ with the help of Eq. (9) and evaluate parameters C_0 and C. For the neutron matter the results are shown in Fig. 2 as a function of the Fermi momentum. We see that estimations of C are very uncertain due to the discrepancy in different estimates of the Landau ω parameters. The results show that $|C|$ might be less than 10–20 at some densities in the range of the ${}^1S_0$ paring, and C might even cross zero. The existence of regions where $C<0$ implies the possibility to observe the effects of the exciton modes.

Using the values of the Landau ω parameters and their density dependence extracted from the atomic nucleus properties [4,23], we obtain $C\sim -10$ for $p_{\rm F}\gtrsim 1$ fm^{-1}. Thus the exciton mode could manifest itself in the nuclear surface phenomena.

Now we apply Eq. (5) to calculate the neutrino emissivity in the neutron-star matter in the region of the ${}^1S_0$ pairing. It is mainly determined by the neutron PBF process induced by the axial-vector current $\propto\mathcal{J}^\mu$ [16]; the vector current contribution is $O(v_{\rm F}^4)$ and can be neglected [16,24]. For one type of neutrino the emissivity then is given by [16]

$$\varepsilon_{\nu\bar{\nu}} = G^2 g_A^2 \int_0^\infty d\omega\omega \int_0^\omega \frac{d|\vec{q}\,|\vec{q}\,^2}{48\,\pi^4} \frac{(q_\mu q_\nu - g_{\mu\nu})\,\Im\Pi^{\mu\nu}(q)}{\exp(\omega/T)-1},$$

where G and g_A are the weak-interaction and axial-vector coupling constants. Integration over $|\vec{q}\,|$ yields

$$\varepsilon_{\nu\bar{\nu}} \simeq \frac{8}{35\,\pi^3} G^2 g_A^2 e_A^2 \rho\, v_{\rm F}^2 \Delta^7 \int_1^\infty \frac{dy\, y^6\, R(y,C)}{\exp(2y\Delta/T)+1}, \qquad (10)$$

where, according to Eq. (8), there can be two contributions to $\varepsilon_{\nu\bar{\nu}}$: one, for arbitrary C, from the pair-breaking continuum with the diffusive modes at $\omega > 2\,\Delta$ and the other one, for negative C, from the spin-exciton mode with the frequency $\omega(\vec{q}\,)$ at $0 < \omega(\vec{q} = 0) < 2\,\Delta$. The later contribution is associated with the processes of breaking and formation of spin excitons. In the limit $|C| \to \infty$ the collective mode contribution vanishes as $\propto 1/|C|$, and we recover the result [16], $\varepsilon^{(0)}_{\nu\bar{\nu}}$, which follows from (10) after the replacement $R(y, C) \to R(y, C \to \infty) = \Im\tilde{g}_T(y)$.

Effects of the finite value of C on the neutrino emissivity in the neutron PBF process are illustrated in Fig. 3, where we plot the ratio $\varepsilon_{\nu\bar{\nu}}/\varepsilon^{(0)}_{\nu\bar{\nu}}$ taking into account the standard temperature dependence of the $^1S_0$ pairing gap $\Delta(T) \simeq 3.1\,T_c\,(1 - T/T_c)^{1/2}$, with T_c being the critical temperature. For $|C| \sim 5$–10 [cf. Fig. 2 (right)], the effect becomes pronounced for $T/T_c \lesssim 0.5$, yielding a suppression for $C > 0$ and an enhancement for $C < 0$. Thus in different density regions there may arise either an enhancement or a suppression of the PBF emissivity. Effect becomes even more pronounced for smaller values of $|C|$.

In conclusion, we found that the spin p-wave interaction in the particle-particle channel can produce new spin excitonic and diffusive modes in Fermi systems with singlet paring. This interaction leads also to smearing out the threshold singularity in Cooper-pair–breaking reactions. We calculated the relevant coupling parameters for several Fermi systems. Spin excitons may exist in superconducting potassium, in rare fermion gases, and in the neutron matter. In atomic nuclei the new spin-exciton mode may be present in a surface layer. Modifications of the neutrino emissivity due to the presence of the spin excitonic and diffusive modes may have an impact on neutron-star cooling.

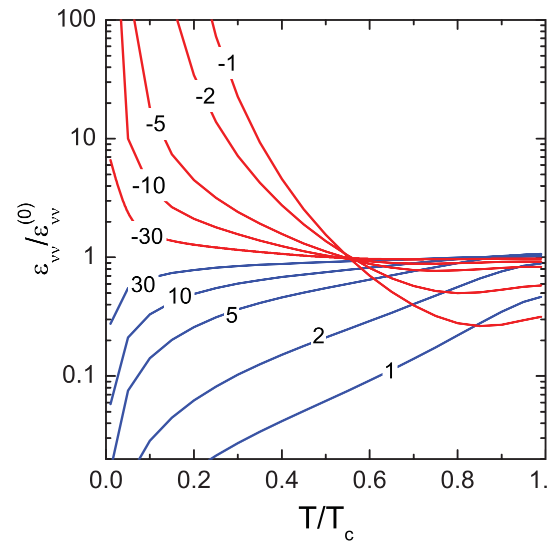

FIG. 3. (Color online) The ratio $\varepsilon_{\nu\bar{\nu}}/\varepsilon^{(0)}_{\nu\bar{\nu}}$ as a function of T/T_c for various values of C (see curve labels).

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[1] P. C. Martin, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969), Vol. I, p. 371.
[2] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor, London, 1990); G. Baym and C. J. Pethick, *Landau Fermi-Liquid Theory: Concepts and Applications* (Wiley, New York, 1991).
[3] S. Giorgini *et al.*, Rev. Mod. Phys. **80**, 1215 (2008).
[4] A. B. Migdal, *Theory of Finite Fermi Systems and Properties of Atomic Nuclei* (Wiley, New York, 1967).
[5] A. B. Migdal *et al.*, Phys. Rep. **192**, 179 (1990).
[6] D. G. Yakovlev *et al.*, Phys. Rep. **354**, 1 (2001).
[7] H. Monien and A. Zawadowski, Phys. Rev. B **41**, 8798 (1990).
[8] G. M. Bruun and G. Baym, Phys. Rev. Lett. **93**, 150403 (2004).
[9] G. Flowers *et al.*, Astrophys. J. **205**, 541 (1976); D. N. Voskresensky and A. V. Senatorov, Sov. J. Nucl. Phys. **45**, 411 (1987).
[10] A. Cumming *et al.*, Astrophys. J. **646**, 429 (2006); S. Gupta *et al.*, *ibid.* **662**, 1188 (2007).
[11] D. Page *et al.*, Phys. Rev. Lett. **106**, 081101 (2011); D. G. Yakovlev *et al.*, Mon. Not. R. Astron. Soc. **411**, 1977 (2011); D. Blaschke *et al.*, arXiv:1108.4125.
[12] A. Bardasis and J. R. Schrieffer, Phys. Rev. **121**, 1050 (1961); V. G. Vaks *et al.*, Sov. Phys. JETP **14**, 1177 (1962); A. I. Larkin, *ibid.* **19**, 1478 (1964); P. Fulde and S. Strassler, Phys. Rev. **140**, A519 (1965).
[13] M. Baldo *et al.*, Phys. Lett. A **65**, 418 (1978).
[14] J. P. Davis *et al.*, Phys. Rev. Lett. **97**, 115301 (2006).
[15] A. I. Larkin and A. B. Migdal, Sov. Phys. JETP **17**, 1146 (1963); A. J. Leggett, Phys. Rev. **140**, A1869 (1965); **147**, 119 (1966).
[16] E. E. Kolomeitsev and D. N. Voskresensky, Phys. Rev. C **77**, 065808 (2008); **81**, 065801 (2010).
[17] A. A. Abrikosov and I. M. Khalatnikov, Sov. Phys. JETP **6**, 888 (1958); H. H. Fu and Ch. J. Pethick, Phys. Rev. B **14**, 3837 (1976); M. Yu. Kagan and A. V. Chubukov, JETP Lett. **47**, 614 (1988).
[18] A. Schwenk *et al.*, Nucl. Phys. A **713**, 191 (2003).
[19] B. R. Patton and A. Zaringhalam, Phys. Lett. A **55**, 95 (1975).
[20] S. Küchenhoff and S. Schiller, Phys. Rev. B **43**, 10310 (1991).
[21] J. A. Leiro, Solid State Commun. **93**, 953 (1995).
[22] S.-O. Bäckman *et al.*, Phys. Lett. B **43**, 263 (1973).
[23] I. N. Borzov *et al.*, Z. Phys. A **355**, 117 (1996).
[24] L. B. Leinson and A. Perez, Phys. Lett. B **638**, 114 (2006).