Coulomb breakup of ^{6}Li into $\alpha + d$ in the field of a ^{208}Pb ion

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The triple differential cross section of the ^{208}Pb(^{6}Li,αd)^{208}Pb quasielastic breakup is calculated at a collision energy of 156 MeV and a scattering angle range of 2°–6°. We fit the parameters of the Woods-Saxon potential using the experimental α-d phase shifts for different states to describe the relative motion of the α particle and deuteron. To check the validity of the two particle approach for the α-d system, we apply a potential model to describe the ^{2}H(α,γ)^{6}Li radiative capture. We calculate the Coulomb breakup using the semiclassical method while an estimation of the nuclear breakup is made on the basis of the diffraction theory. A comparison of our calculation with the experimental data of Kiener *et al.* [Phys. Rev. C **44**, 2195 (1991)] gives evidence for the dominance of the Coulomb dissociation mechanism and the contribution of nuclear distortion, but is essentially smaller than the value reported by Hammache *et al.* [Phys. Rev. C **82**, 065803 (2010)]. The results of our calculation for the triple cross sections (contributed by the Coulomb and nuclear mechanisms) of the ^{6}Li breakup hint toward a forward-backward asymmetry in the relative direction of the α particle and deuteron emission, especially at smaller scattering angles, in the ^{6}Li center-of-mass (c.m.) system.

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I. INTRODUCTION

The study of processes, relevant for nuclear astrophysics, by indirect methods gives the possibility to extract the astrophysical S factor at extremely low energies when extraction by direct methods is not possible due to the Coulomb barrier suppression. Among these indirect methods we cite the elastic Coulomb dissociation method suggested by Baur, Bertulani, and Rebel [1–3], the asymptotic normalization coefficient method (ANC) suggested by Mukhamedzhanov and Timofeyuk [4–6], and the Trojan horse method (THM) suggested by Baur and modified by Spitaleri [7,8]. The study of nuclear reactions at high energy is, in general, very complicated owing to the strong nuclear interaction between the colliding nuclei. However, in the peripheral collisions of a light nucleus with heavy target the reaction mechanism becomes simple owing to the negligible contribution of nuclear distortion and excitation becomes purely Coulombic. Electromagnetic excitation is a very powerful tool for the extraction of information concerning radiative capture at extremely low energies when the direct measurements of the radiative capture of nuclei for astrophysical purposes is impossible. The Coulomb dissociation experiments are being performed at different centers around the world (e.g., GSI, Germany [9]; MSU/NSCL, USA [10]; RIKEN, Japan [11]). The value of the astrophysical S factor extracted from the Coulomb breakup experiment is affected by various uncertainties, namely (i) the method of extrapolation of the data to zero energy; (ii) contributions from various electromagnetic multipoles ($E1$, $E2$, $M1$); (iii) assumptions about the nuclear interactions; and (iv) various higher-order effects (see Refs. [12–15] and references therein). The astrophysical S factor at zero energy extracted from the direct radiative capture and the Coulomb breakup reaction should be the same. However, the extrapolation to zero energy from the $d(\alpha,\gamma)^6$Li and ^{208}Pb(^{6}Li,$\alpha\, d$)^{208}Pb reactions gives different values for the astrophysical factor. This is owing to the fact that at αd relative energy below 100 keV the $E1$ dipole cross section becomes larger than the $E2$ quadrupole cross section for the direct capture process, while for the Coulomb breakup the $E1$ cross section is always smaller than the $E2$ one at any given energy. Kiener *et al.* [16] investigated the Coulomb breakup of ^{6}Li in the field of the ^{208}Pb ion and extracted a large value of the astrophysical factor at zero energy from extrapolation in the range $E_{\alpha d} < 400$ keV. Shyam *et al.* [17] underlined that the result of the authors of Ref. [16] was free from the nuclear background, which maybe important. Kiener *et al.* did not take the contribution of the dipole transition to the cross section owing to the isospin selection rule. However, this rule is violated and $E1$ transition can still occur. Recently a new measurement of the Coulomb breakup at extremely high energy of ^{6}Li (150 A MeV) was performed [18] and the authors claimed disclosure of evidence for the large contribution of the Coulomb-nuclear interference. However, the authors did not present the cross sections (the histograms for the counts in Figs. 6 and 7 of Ref. [18] were shown). Figure 10 of Ref. [18] showed the ratio of nuclear to Coulomb differential cross sections for ^{6}Li which is very large compared to the qualitative estimation given earlier in Ref. [19].

The dissociation by the nuclear field of a target can be excluded by the observation of the fragments of the reaction at forward scattering angles. This extremely small angle corresponds to the impact parameter essentially larger than the sum of the nuclear radii of the projectile and the target. For instance, in the experiments [16] the angle was varied in the range 2°–6° corresponding to the impact parameter in the range 20–65 fm. However, the dipole transition in the ^{6}Li Coulomb breakup is suppressed as in the $d(\alpha,\gamma)^6$Li radiative capture, therefore, we can expect that the nuclear dissociation gives a

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relatively large contribution to the ^{6}Li Coulomb breakup cross section.

The problem of the higher-order effects was discussed in many papers (see, e.g., Refs. [3,20], and references therein). In the semiclassical theory these effects are inversely proportional to the impact parameter and the velocity of collision. Therefore we can constrain ourselves by the first-order amplitude for the excitation of a fast nucleus at sufficiently small scattering angles. Among higher-order effects, we also note the three-body Coulomb effects in the final state, which were discussed in Refs. [21,22].

In this paper we consider the dissociation ^{208}Pb(^{6}Li,$\alpha\, d$)^{208}Pb using the time-dependent perturbation theory for the Coulomb breakup, whereas the nuclear breakup is viewed as a diffractive dissociation. Our treatment is used for the complete analysis of the experiments, which were performed with a 156 MeV ^{6}Li beam at the Karlsruhe Isochronous Cyclotron [16]. This reaction is relevant to the $d(\alpha,\gamma)^6$Li radiative capture, which is one of the important nucleosynthesis reactions. When the collision energy is sufficiently large, we can consider the motion of the center of mass of a projectile along a classical trajectory. At small scattering angles of the projectile the value of the impact parameter would be so large that at such distances only the Coulomb interaction between the projectile and the target would be significant. The nuclear breakup does take place, but it mainly occurs near the target surface where the nuclear potential of the interaction falls down quickly. At small scattering angles and high energy collisions, the most appropriate method of estimation of the nuclear breakup is the diffraction theory [23] as also has been mentioned above. Here we mention the experiments concerning the ^{6}Li breakup at energies 60 MeV [24] and 31, 33, 35, and 39 MeV [25], which also confirmed the Coulomb dissociation mechanism of ^{6}Li on ^{208}Pb. However, from our opinion the authors of Ref. [24] made an erroneous conclusion that the second-order Coulomb excitation theory can improve the deviation in the angular distribution between the theory and the experiment. On the basis of the developed method we desire to make the analysis of the data presented in Ref. [16] and to explore the possibility of the extraction of the astrophysical S factor from the ^{208}Pb(^{6}Li,$\alpha\, d$)^{208}Pb reaction by a comparison of our results with the experimental data of Ref. [16].

We use the system of units in which $\hbar = c = 1$.

II. GENERAL FORMALISM

In the semiclassical theory the center-of-mass (c.m.) motion of the projectile is considered classically, whereas the relative motion of the clusters in the projectile is treated completely in a quantum mechanical fashion [3]. In this approach the purely Coulomb breakup cross section of peripheral ^{208}Pb(^{6}Li, αd)^{208}Pb reaction is presented as a product of the Rutherford scattering cross section of ^{6}Li in the field of ^{208}Pb ion and the probability of the $^6\text{Li} \to \alpha + d$ disintegration. We calculate the probability of disintegration in the ^{6}Li frame (projectile frame) because the calculation does not depend on any reference frame. In this frame a heavy ^{208}Pb moves along a straight line with a constant velocity $\mathbf{v}$. If the energy of the αd relative motion is small we can restrict it by using only the low partial waves at $l = 0, 1, 2$ for the description of the αd motion. The $E2$ multipole gives the main contribution to the transition amplitude of the electromagnetic dissociation of the ^{6}Li, however, the $E1$ transition should also be included to the amplitude owing to a violation of the isospin forbidden rule.

The time-dependent perturbation for the Coulomb breakup $A + a \to A + c + b$ is

$$H(t) = \int d^3x \left(\frac{Z_A e}{|\mathbf{x} - \mathbf{R}(t)|} - \frac{Z_A e}{|\mathbf{R}(t)|} \right) \rho(\mathbf{x}), \tag{1}$$

where $Z_A e$ is the charge of a heavy ion A (^{208}Pb target), the projectile a (^{6}Li nucleus) is dissociated into b (deuteron) and c (α particle), $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$ gives the position of the target in the projectile frame, and $\rho(\mathbf{x})$ is the charge density operator, $\mathbf{b}$ is the impact parameter. For the peripheral reaction we can take the charge density operator in the two-body approach as

$$\rho(\mathbf{x}) = Z_b e\delta(\mathbf{x} - \mathbf{r}_b) + Z_c e\delta(\mathbf{x} - \mathbf{r}_c), \tag{2}$$

where $\mathbf{r}_i$ defines the position of particle i in the projectile frame and $Z_i e$ is its charge.

In perturbation theory the amplitude of the transition from the initial state $|i\rangle$ (wave function of the $a = b + c$ system in the ground state) to the final state $|f\rangle$ (wave function of the $b + c$ system in the continuum state) is given as a sum

$$a_{fi} = \delta_{fi} + a_{fi}^{(1)} + a_{fi}^{(2)} + \cdots, \tag{3}$$

of different order contributions. We use the first-order constrain for the amplitude of the transition. Expanding $H(t)$ in multipoles we get

$$a_{fi}^{(1)} = \frac{4\pi Z_A e}{i} \sum_{\lambda,\mu} \frac{(-1)^\mu}{2\lambda + 1} \langle f \mid \mathcal{M}(\lambda, -\mu) \mid i\rangle S_{\lambda\mu}(\omega), \tag{4}$$

where $\mathcal{M}(\lambda, \mu)$ is the electric multipole operator

$$\mathcal{M}(\lambda, \mu) = \int d^3x\rho(\mathbf{x})x^\lambda Y_{\lambda\mu}(\hat{\mathbf{x}}). \tag{5}$$

The semiclassical orbital $S_{\lambda\mu}(\omega)$ integral is given by

$$S_{\lambda\mu}(\omega) = \int_{-\infty}^{+\infty} dt \frac{e^{i\omega t}}{R(t)^{\lambda+1}} Y_{\lambda\mu}[\hat{\mathbf{R}}(t)], \tag{6}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{R}}(t)$ are the unit vectors along the position vectors $\mathbf{x}$ and $\mathbf{R}(t)$, respectively. The integrals $S_{\lambda\mu}(\omega)$ were calculated analytically and the results of the calculations were presented in Ref. [26]. Determining the relative coordinate $\mathbf{r} = \mathbf{r}_b - \mathbf{r}_c$, inserting Eq. (2) into Eq. (5), and performing the integration over variable $\mathbf{x}$ we obtain

$$\mathcal{M}(\lambda, \mu) = \mu_{bc}^\lambda \left[\frac{Z_b e}{m_b^\lambda} + (-1)^\lambda \frac{Z_c e}{m_c^\lambda} \right] r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}), \tag{7}$$

where μ_{bc} is the reduced mass of particles b and c. We see that the dipole transition operator $\mathcal{M}(1, \mu)$ is not equal to zero because the value of the charge-to-mass ratio is slightly different for the α particle and deuteron. Therefore, the $E1$ transition amplitude is not zero.

The triple cross section of the Coulomb breakup from the ground state with angular momentum and parity $J_i^\pi = 1^+$ of ^{6}Li to the final state with relative momentum $\mathbf{k}$ of the α particle and deuteron having reduced mass $\mu_{\alpha d}$ can be expressed in terms of the excitation amplitude a_{fi}

$$\frac{d^3\sigma_C}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}} = \frac{d\sigma_R}{d\Omega_{\rm Li}} \frac{1}{2J_i+1} \sum_{M_i} |a_{fi}|^2 \frac{\mu_{\alpha d} k}{(2\pi)^3}. \qquad (8)$$

The elastic Coulomb cross section $d\sigma_R/d\Omega_{\rm Li}$ is calculated classically for the scattering of the c.m. of the projectile ^{6}Li.

Next we consider the nuclear breakup. As mentioned earlier, the nuclear breakup occurs mostly near the surface of a heavy target nucleus ^{208}Pb. If the scattering angle is small we may apply the diffraction theory assuming that the target nucleus is a completely absorptive "black" sphere of radius $R_{\rm bl}$. The amplitude for the elastic breakup according to the diffraction theory [27] is given by

$$\mathcal{F}(\mathbf{q},\mathbf{k}) = \frac{ik_i}{2\pi} \int d^2b e^{i\mathbf{q}\,\mathbf{b}} \int d^3r \psi^*_{\mathbf{k}}(\mathbf{r}) \omega(\mathbf{b},\mathbf{r}) \psi_0(\mathbf{r}), \qquad (9)$$

where $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$, $\mathbf{k}_i$ is the initial momentum of ^{6}Li and $\mathbf{k}_f$ is the final momentum of the c.m. of the $\alpha\, d$ system after dissociation; $|i\rangle = \psi_0(\mathbf{r})$ and $|f\rangle = \psi_{\mathbf{k}}(\mathbf{r})$ are the wave functions of $\alpha\, d$ in the bound and continuum states, respectively; $\mathbf{b}$ is the impact parameter of the $\alpha\, d$ system; $\omega(\mathbf{b},\mathbf{r})$ is the total profile function for the $\alpha\, d$ system. We may take the vector $\mathbf{q}$ to be orthogonal to the momentum vector $\mathbf{k}_i$ due to high energy collision and small scattering angle ($q \approx k_i \cdot \theta$, where θ is the scattering angle). The total profile function

$$\omega = \omega_\alpha + \omega_d - \omega_\alpha \omega_d \qquad (10)$$

is composed of the profile functions of the fragments. The third term in Eq. (10) describes the double scattering and its contribution to the cross section is much smaller than the contributions of the first two terms. Neglecting the double-scattering term we obtain the cross section of the nuclear breakup as

$$\frac{d^3\sigma_N}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}} = \left| k_i R_{\rm bl} \frac{J_1(qR_{\rm bl})}{q} s(\mathbf{q},\mathbf{k}) \right|^2 \frac{\mu_{\alpha d} k}{(2\pi)^3}, \qquad (11)$$

where $J_1(x)$ is the Bessel function and

$$s(\mathbf{q},\mathbf{k}) = \int d^3r\, e^{i\mathbf{q}\,\mathbf{r}} \psi^*_{\mathbf{k}}(\mathbf{r}) \psi_0(\mathbf{r}). \qquad (12)$$

The total cross section is considered as the sum

$$\frac{d^3\sigma_t}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}} = \frac{d^3\sigma_C}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}} + \frac{d^3\sigma_N}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}}. \qquad (13)$$

In Eq. (13) the Coulomb-nuclear interference term is neglected. We note that the Coulomb and nuclear breakup are calculated using different approaches therefore we cannot calculate the interference term. Even if both the Coulomb and nuclear breakup amplitudes were calculated using the same approach, one expects the interference term to be small (see, for example, the calculation in the distorted wave Born approximation (DWBA) method used by Bertulani and Hussein, Figs. 4 and 5 in Ref. [19]). We further note that at the small scattering angles considered in this calculation, the elastic Coulomb scattering cross section calculated classically is equal to the cross section calculated by the diffraction theory.

III. α-d POTENTIALS

To describe the relative motion of the α particle and deuteron we use the Woods-Saxon potential with the orbital terms

$$V_N(r) = -\left[V_0 - V_{sl}(\mathbf{l}\cdot\mathbf{s}) \frac{1}{m_\pi^2}\frac{1}{r}\frac{d}{dr}\right] \frac{1}{1+\exp[(r-R_N)/a]}, \qquad (14)$$

with the standard value of the diffuseness $a = 0.65$ fm. The parameters V_0 and V_{sl} for different αd states is fitted from the shift phase analysis. We take the nuclear radius of the potential R_N as

$$R_N = r_0 \cdot A^{1/3}, \qquad (15)$$

with the standard value $r_0 = 1.25$ fm and the nuclear mass number $A = 6$ for ^{6}Li. The Coulomb potential is taken as

$$V_C(r) = \begin{cases} \frac{Z_\alpha Z_d e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right), & r < R_C, \\ \frac{Z_1 Z_2 e^2}{r}, & r > R_C, \end{cases} \qquad (16)$$

where $Z_\alpha e$ and $Z_d e$ are the charges of the α particle and deuteron, respectively; $R_C = r_C\, A^{1/3}$ ($r_C = 1.25$ fm).

The parameters of the depth of the potentials V_0 and V_{sl} were fixed by fitting the experimental S, P, and D phase shifts of the elastic α-d scattering [28–32] and the binding energy of the ^{6}Li ground state. We obtained the following values of the depths: $V_0 = 60.73$ MeV for the $^3S_1$ state; $V_0 = 57.0$ MeV, $V_{sl} = 4.0$ MeV for the $^0P_1$, $^1P_1$, and $^2P_1$ states; $V_0 = 55.9$ MeV, $V_{sl} = 4.0$ MeV for the $^1D_1$, $^2D_1$ states; and $V_0 = 55.9$ MeV, $V_{sl} = 5.06$ MeV for the $^3D_1$ state. To describe the 3^+ resonance of ^{6}Li correctly, the depth of the spin-orbital part of the potential for the $^3D_1$ state is taken slightly differently. We note that our fitted parameters of the Woods-Saxon potential are slightly different from the parameters used in Ref. [18]. This difference is obvious as the phase shifts are determined with errors and the resulting fit can give rise to such differences in the parameters of the potential. Note the spin-orbital potential in Eq. (14) contains the dimensional parameter $1/m_\pi^2 = 2.136$ fm^2 while the same potential in Ref. [18] has $\lambda^2 = 4$ fm^2.

It is important to remember that at low energies the αd radiative capture depends on the tail of the ^{6}Li bound state wave function projected on the α-d channel [33,34]. The amplitude of this tail is the asymptotic normalization coefficient (ANC). We note that the range for the ANC obtained by various techniques is wide ($C = 1.51$–3.25 fm$^{-1/2}$) [35]. We calculate the amplitude of the peripheral radiative capture reaction ^{2}H$(\alpha,\gamma)^6$Li at very low collision energies where the ANC for the virtual decay $^6{\rm Li} \to \alpha + d$ from the ground state governs the overall normalization of the peripheral reaction cross section. The value of the ANC (C_0) obtained here using the fitted potential for the $^3S_1$ state of ^{6}Li is 2.7 fm$^{-1/2}$. This value is larger than $C = 2.3$ fm$^{-1/2}$ extracted from the elastic α-d $^3S_1$ experimental phase shift by the analytic extrapolation to the pole of the partial scattering amplitude corresponding to the ^{6}Li ground state [35]. The same value of the ANC was also obtained from the solution of the three body α-p-n equation [36], which was used in Refs. [33,34] and confirmed recently by *ab initio* calculations [37]. To obtain the ANC value of 2.3 fm$^{-1/2}$ we can find the phase-equivalent potential and the corresponding wave function by the method described in Ref. [38] and discussed in Ref. [33]. The phase-equivalent potential does not change the scattering phase shift and the binding energy, but allows one to get the needed value of ANC. The phase-equivalent potential has the form

$$V_{\rm eff}(r) = V_N(r) - 2\frac{d^2}{dr^2} f_{\rm eff}(r), \quad (17)$$

where $f_{\rm eff}(r)$ means

$$f_{\rm eff}(r) = \ln\left[1 + (\lambda - 1)\left(1 - \int_0^r u^2(r)dr\right)\right]. \quad (18)$$

The corresponding new wave function of the bound state is equal to

$$u_{\rm eff}(r) = \lambda^{1/2}\frac{u(r)}{1 + (\lambda - 1)\int_0^r u^2(r)dr}, \quad (19)$$

where $u(r)$ is the wave function obtained from the solution of the Schrödinger equation with the parameters of the Woods-Saxon potential determined from the phase-shift analysis. We note that to decrease the value of the ANC we have to take $\lambda > 1$. For our phase-equivalent potential and the wave function corresponding to $C = 2.3$ fm$^{-1/2}$ the value of λ is 1.38. [This follows from Eq. (19): $C = \lambda^{-1/2}C_0$]. A detailed description of this method and its application in the case of the α-d radiative capture can be found in Ref. [33]. Thus for calculation of the cross sections of the ^{2}H$(\alpha,\gamma)^6$Li radiative capture and the ^{208}Pb(^{6}Li,$\alpha\, d)^{208}$Pb breakup we use the $u_{\rm eff}(r)$ bound wave function having $C = 2.3$ fm$^{-1/2}$.

IV. RADIATIVE CAPTURE REACTION ^{2}H$(\alpha,\gamma)^6$Li

Experimental measurements of the cross section of the ^{2}H$(\alpha,\gamma)^6$Li reaction at extremely low energies are very difficult to carry out because the cross section is the order of a few nanobarns and decreases exponentially if the energy goes to zero. The experimental results for the cross sections of the direct $d + \alpha$ capture were measured by Robertson *et al.* [39] at c.m. energy range 1–3.5 MeV, by Mohr *et al.* [40] at the resonance point of 711 keV, and Cecil *et al.* [41] at an αd c.m. energy of 53 keV. Furthermore, we mention Refs. [42,43] where the analysis of the experimental results and theoretical calculations are given. Nollett *et al.* [43] used a six-body approach for the calculation of the αd capture. From the analysis of the results at the $E_{\alpha d}$ energy close to zero we see an essential difference in the value of the astrophysical S factor depending on the applied value of the ANC. At extremely low energies the initial wave function of the αd system ceases to depend on the parameters of the nuclear potential, which is used to calculate this wave function since the nuclear scattering phase shifts tend to zero. Therefore, we can replace the wave function of the initial state of the purely Coulomb wave if $E_{\alpha d} < 100$ keV. For any radiative capture at low energy, the main contribution comes from $E1$, $E2$, and $M1$ transitions. However, for the ^{2}H$(\alpha,\gamma)^6$Li reaction the main contribution comes from the $E2$ quadrupole transition at the energy larger than 200 keV. The $E1$ transition begins to dominate if the $E_{\alpha d}$ energy becomes less than 100 keV. The $M1$ capture remains negligible for all astrophysical interesting range of energies.

For the calculation of the $d(\alpha,\gamma)^6$Li cross section we use the Woods-Saxon potential with the parameters described in the previous section. The initial wave function of the αd system includes the P and D waves which are solutions of the Schrödinger equation. It is clear that the αd direct capture is a peripheral reaction at the low energy ($E_{\alpha d} < 300$ keV). Accordingly, the cross section is not sensitive to the choice of the parameters of potential describing the continuum states. It is rather strongly dependent on the ANC of the ground state. Such a property was used for the ANC calculation [33,34], where the asymptotic wave function of the ^{6}Li ground state wave function in two-body approximation was applied to find the astrophysical S factor. The main contribution to the matrix elements of the direct reaction at $E_{\alpha d} < 300$ keV comes from the external part of the used wave functions, while the internal part gives a very small contribution. Figure 1(a) shows the radial part of the integrand for transition from $^0P_1$ to $^3S_1$ at the energy $E_{\alpha d} = 0.1$ MeV. We can see that the replacement of

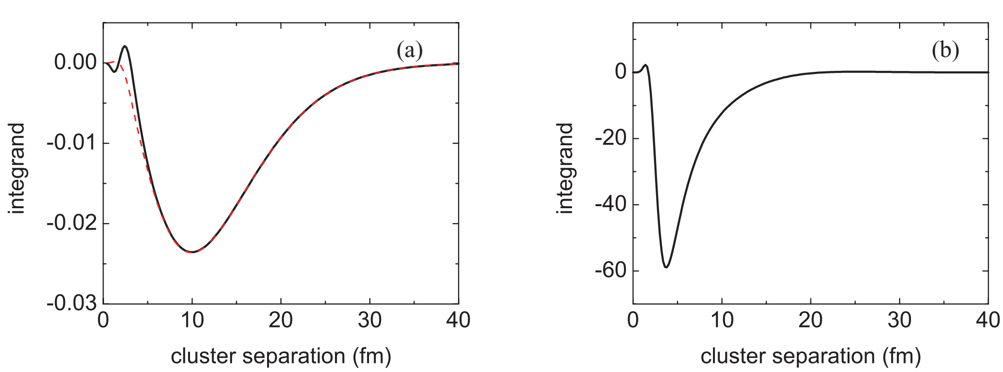

FIG. 1. (Color online) (a) The integrand of the matrix element for transition from $^0P_1$ to $^3S_1$ at energy $E_{\alpha d} = 0.1$ MeV calculated with the continuum wave function which is the solution of the Schrödinger equation (solid curve) and with the regular Coulomb wave function (dashed curve). (b) Same as in (a) for the transition from $^3D_1$ to $^3S_1$ at the resonance energy $E_{\alpha d} = 0.711$ MeV.

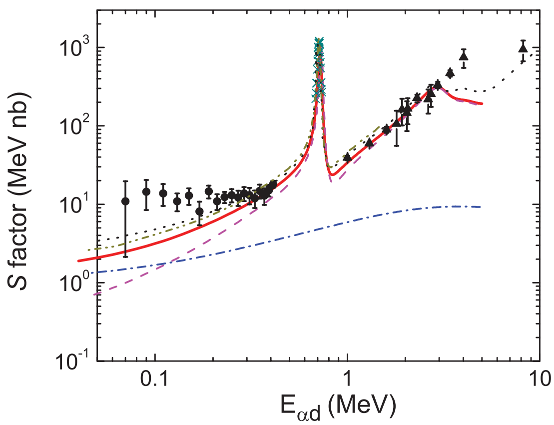

FIG. 2. (Color online) The calculated astrophysical S factor compared with the results of other theoretical calculations and experimental data. The solid line is our result for the total S factor, the dashed and dot-dashed lines are the contributions of $E2$ and $E1$ multipolarities, respectively. The dotted line shows the result of the calculation of the authors of Ref. [43] and dot-dot-dashed line corresponds to Ref. [18]. The experimental data (▲, •, ×) were taken from their graphical presentation in Ref. [43].

the P continuum wave function by the corresponding regular Coulomb wave function leads to almost the same value of the matrix element for the ${}^0P_1 \rightarrow {}^3S_1$ transition. We further note that the same conclusion can be made for the transition from D states at the low energy.

It seems that in the resonance region the complete microscopic model should be used (see Ref. [43] and references therein) because the contribution to the transition amplitude from the internal part of the radial wave functions is also expected. Nevertheless, the two-particle approach can be used to describe the ^{2}H$(\alpha,\gamma)^6$Li resonance reaction and to get a quantitative result for the cross section at the resonance energy region. Such instances may be explained by the suppression of the $E1$ transition. The resonant amplitude is the result of the transition from the state ${}^3D_1$ to the state ${}^3S_1$ and the transition remains peripheral due to the large centrifugal barrier in the ${}^3D_1$ state. The result of our calculations shows that the two-body wave function for the ground state of ^{6}Li gives an acceptable result for the resonance cross section and the internal part of the wave function gives a very small contribution to the transition amplitude at the resonance energy [see Fig. 1 (b)]. The result of the calculations of the astrophysical S factor as a function of energy is shown in Fig. 2 over a wider energy range than shown in Fig. 3 of Ref. [33]. It is clear from Fig. 2 that the two-body approach for the description of the ground and continuum states of the αd system gives perfectly good results at the low energy, including the 3^+ resonance energy region. The value of our calculated astrophysical S factor coincides with the one presented in Ref. [34] for the low energies. At an energy larger than 1 MeV our result definitely agrees well with the experimental data [39], while the results of the authors of Refs. [18,43] clearly overestimated the data. We also see a big disagreement of our result with the astrophysical factor extracted from the Coulomb breakup experiment [16] below 500 keV. We note that the ANC determines the value of the astrophysical S factor at the energy range $E_{\alpha d} < 300$ keV where the reaction has a clear peripheral mechanism. The ratio of the astrophysical S factors (or cross sections) calculated near zero $E_{\alpha d}$ energy using the ground wave functions with a different value of the ANC is equal to the square of the ratio of the corresponding ANCs. As was also mentioned above, the ground state wave function of ^{6}Li used in our calculation for the radiative capture has an ANC equal to 2.3 fm$^{-1/2}$ while the ANC of the bound state wave function used in Refs. [18,43] is 2.7 fm$^{-1/2}$ and 3.2 fm$^{-1/2}$, respectively.[1] The secondary peak of the calculated cross section near $E_{\alpha d} = 3$ MeV appears due to the wide resonance in the ${}^3D_2$ scattering wave at $E_{\alpha d} = 2.838$ MeV. The applied potential describes this resonance. To explain the disagreement of our results with the experimental data for $E_{\alpha d} > 3$ MeV, we refer to the work of Nollett *et al.* [43], where the nature of this disagreement is discussed in detail. Additionally, we note that for the energy $E_{\alpha d} > 3$ MeV the reaction is no longer peripheral. We have not calculated the cross section above 6 MeV because the phase shifts have a large imaginary part due to the open $\alpha + p + n$, ^{5}He + p and ^{5}Li + n channels from the energy $E > 5$ MeV and the restriction by the single αd channel becomes incorrect. From Fig. 2 we see that the $E1$ cross section dominates the $E2$ cross section at energies below 100 keV and $S_{E1}(0) = 1.01$ MeV nb while $S_{E2}(0) = 0.21$ MeV nb. Hence our total calculated value of the astrophysical factor at zero energy is $S(0) = 1.22$ MeV nb.

V. RESULTS FOR THE COULOMB BREAKUP OF ^{6}Li

Using the $E1$ and $E2$ multipole matrix elements calculated to determine the αd capture cross section, we compute the ^{208}Pb(^{6}Li,$\alpha\, d$)^{208}Pb breakup reaction. The calculated triple differential cross section [Eq. (8)] includes $E1$, $E2$, and $E1E2$ terms. The analyzed experimental data of the Coulomb breakup were taken from Ref. [16], where data are presented for the scattering angles $\Theta_{\rm lab} = 2, 3, 4,$ and 6° in the laboratory frame in Tables III through VI and Figs. 7 and 10, respectively. To convert the laboratory cross sections into the αd c.m. cross sections we used the transformation method described in Ref. [44]. Previously, only the data at the scattering angle $\Theta_{\rm lab} = 3^\circ$ were analyzed and the astrophysical factor at zero energy was extracted from that analysis [16]. The experimental data at other scattering angles have not been analyzed so far to the best of our knowledge. We chose the impact parameter corresponding to the selected scattering angles when we calculate the Coulomb breakup contribution. In spite of the fact that the $E1$ cross section becomes larger than the $E2$ cross section for the direct radiative capture at energies $E_{\alpha d} < 100$ keV, the $E1$ triple cross section is always less than the $E2$ one for the Coulomb breakup at any energy including

[1]The authors of Ref. [43] quoted that the two-cluster αd distribution function had the dimensionless ANC $C_0 = 2.26 \pm 0.05$ which gave the dimension ANC equal to 1.77 ± 0.04 fm$^{-1/2}$. This ANC must lead to $S(0) = 0.72$ MeV nb, but from Fig. 8 of Ref. [43] we see that $S(0) \approx 2.5$ MeV nb. Therefore we reestimated the value of the ANC.

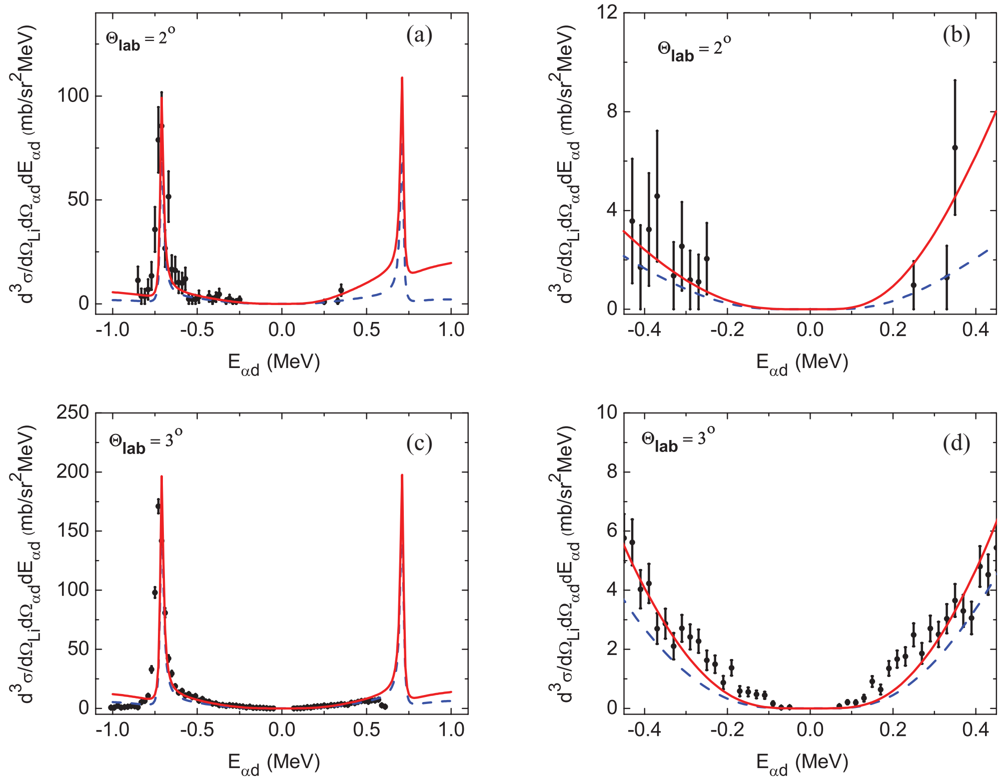

FIG. 3. (Color online) (a) Energy distribution of the total triple differential cross section for the ^{6}Li breakup as a function of the relative energy $E_{\alpha d}$ at an angle of 2° (solid line). The distribution for the purely Coulomb breakup is shown with a dashed line. The experimental data were taken from Ref. [16]. (b) Same as in (a) but for energy in the close vicinity of zero. Negative and positive $E_{\alpha d}$ energies denote backward and forward emission, respectively, of the α particle in the ^{6}Li c.m. frame. (c) and (d) like to (a) and (b), respectively, but at the scattering angle 3°.

the range $E_{\alpha d} < 100$ keV. In the Coulomb breakup, like the radiative capture reaction, the contribution of $E1$ transition cross section decreases more slowly than $E2$ cross section with reduction of energy. For instance, at an αd energy of 100 keV the $E1$ contribution is less than the $E2$ one by a factor 70, and at $E_{\alpha d} = 10$ keV their ratio is equal to 1/24. This means that the Coulomb breakup cross section is mostly defined by the $E2$ quadrupole transition at the extremely low energy as well. There is a contribution of the $E1E2$ interference term, but the contribution of the $E1$ term to the cross section is very small as compared to the $E1E2$ interference term.

When we calculate the nuclear breakup cross section according to Eq. (11), the radius R_{bl} of the "black" nuclei ^{208}Pb is fixed by means of χ^2 so that the sum of the Coulomb and nuclear cross sections is close to the experimental data in the energy region 0.4 MeV $< E_{\alpha d} < 0.73$ MeV. For the calculation of χ^2 we take into account the region where deuteron emission occurs in the forward direction in the ^{6}Li c.m. frame. We do not include the region, $E_{\alpha d} < 0.4$ MeV because in this region higher-order effects, such as the three-body Coulomb effects [21,22], can give a significant contribution. In the region $E_{\alpha d} > 0.73$ MeV the behavior of the experimental cross section is not clear for the experimental scattering angles and the experimental error is comparable with the value of the cross section (see Tables III through VI of Ref. [16]). The value of χ^2 has a minimum when R_{bl} is taken equal to 6.30, 6.94, 6.34, and 6.25 fm corresponding to angles $\Theta_{lab} = 2°, 3°, 4°$, and 6°, respectively. We see that the values of R_{bl} are close to the radius of ^{208}Pb.

The results of the calculation of the triple cross sections are shown in Figs. 3 and 4. A comparison of our calculated triple cross sections with the experimental data shows that the Coulomb breakup gives the main contribution into disintegration process in the energy range 0.4 MeV $< E_{\alpha d} < 0.8$ MeV. A discrepancy exists with the experimental data at the energies near zero and larger than 0.8 MeV for all scattering angles (particularly in the case $\Theta_{lab} = 3°$). As mentioned above, the purely Coulomb wave function for the αd continuum state may be applied when the relative αd energy is close to zero. From analyzing the results done by the authors of Refs. [16,24,25] at small values of the $E_{\alpha d}$ relative energy, it is clear that the purely Coulomb breakup cannot explain the behavior of the cross section near the threshold. However, the results of the authors of Ref. [18] showed a huge contribution of the nuclear breakup at all αd energy regions. Such a phenomenon may be explained by the wide angular region of the scattering angle of the c.m. of ^{6}Li (0°–5°) while the grazing angle at this experiment is $\sim$ 2.5°–3°. One expects the purely Rutherford scattering only below $\sim$ 2° (see also Figs. 7 and 10 of Ref. [18]). At the grazing angle the projectile ^{6}Li moves along the trajectory tangential to the surface of the target ^{208}Pb where the nuclear breakup mostly occurs. We

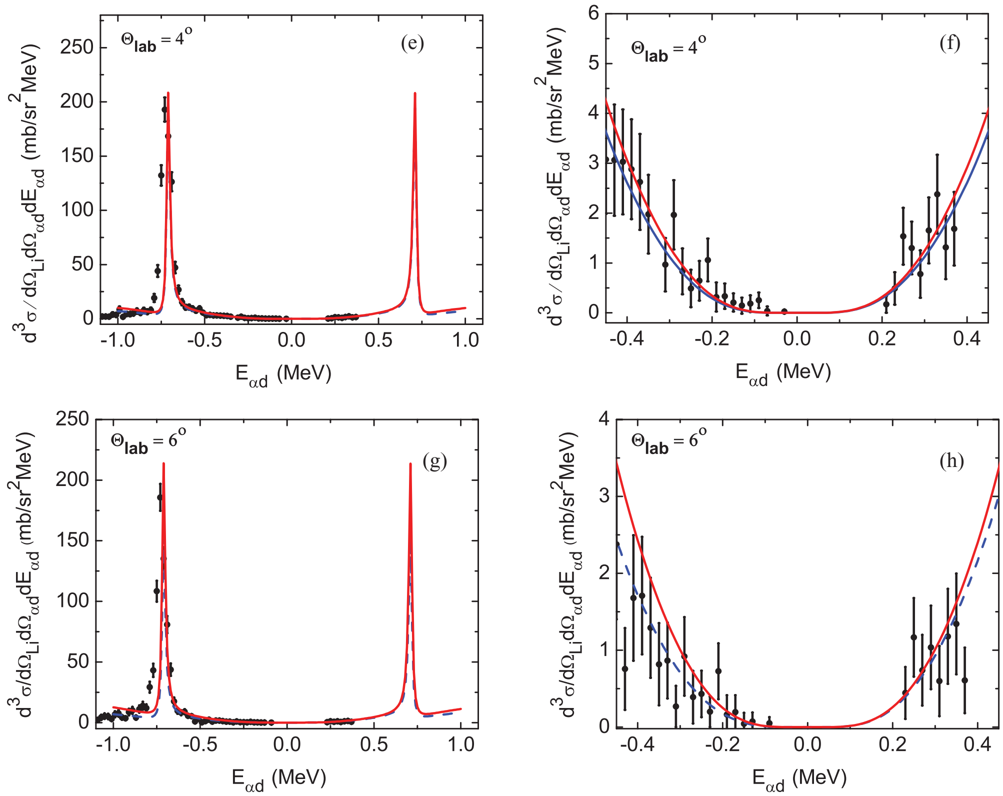

FIG. 4. (Color online) Same as in Fig. 3. Panels (e) and (f) show results at the scattering angle 4°, while panels (g) and (h) show results at the scattering angle 6°.

note that an analysis of the Kiener *et al.* results concerning angular dependence $\Theta_{\rm lab}$ at very low $E_{\alpha d}$ energy shows a large divergence with the theoretical results following from the purely Coulomb disintegration. The contribution of the nuclear breakup increases the total cross section at most by a factor of 2 in the region $E_{\alpha d} > 0.8$ MeV. A great disagreement still exists between the theoretical and experimental results near zero αd relative energy. Figure 5 demonstrates the dependence of the total cross section and its Coulomb part on the scattering angle at three values of $E_{\alpha d}$. Figure 5(a) shows that the addition of the nuclear disintegration cannot decrease the disagreement with the experimental data at the extremely low energies. At still higher energy the inclusion of the nuclear breakup into the theory might assist in achieving reasonable agreement with the experimental data [Figs. 5(b) and 5(c)]. Accounting for the higher-order effects (as mentioned in Sec. I) may lead to better agreement with the experimental data at the extremely low energies. In the Kiener *et al.* work the grazing angle was $\sim 13°$ while the measurement was performed at the scattering angle range 2°–6°. Therefore the contribution of the nuclear breakup to the differential cross section was small. The same conclusion follows from the results of our calculations presented in Table I. Note that similar behavior is observed for other scattering angles. When the energy $E_{\alpha d}$ becomes larger than the resonant energy, the nuclear and Coulomb cross sections become comparable to each other for the scattering angles 2° and 3°. However, at scattering angles 4° and 6°, the Coulomb breakup cross section remains larger than the nuclear breakup cross section. Figure 3 of Ref. [24] shows how the Coulomb cross section approaches the experimental data when the relative energy $E_{\alpha d}$ increases.

Figures 3 and 4 of our paper also depict the existence of the backward-forward asymmetry for deuteron emission in the ^{6}Li c.m frame, especially at smaller scattering angles. Such

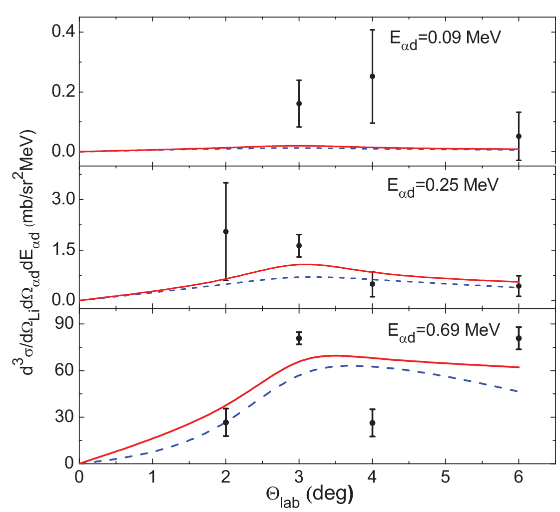

FIG. 5. (Color online) Triple differential cross section for the ^{6}Li breakup as a function of the scattering angle $\Theta_{\rm lab}$ at selected values of the relative $E_{\alpha d}$ energy. Solid line is the sum of the Coulomb and nuclear contributions, while dashed line is the purely Coulomb one. The experimental data were taken from Ref. [16].

TABLE I. Dependence of the ratio of the nuclear ($\sigma_N = \frac{d^3\sigma_N}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}}$) and the Coulomb ($\sigma_C = \frac{d^3\sigma_C}{d\Omega_{\alpha d} d\Omega_{\rm Li} dE_{\alpha d}}$) triple cross sections for the ^{208}Pb(^{6}Li,$\alpha\, d$)^{208}Pb breakup on $E_{\alpha d}$ energy. The scattering angle $\Theta_{\rm lab}$ is 3°.

$E_{\alpha d}$ (MeV)	σ_N (mb/MeV/sr^2)	σ_C (mb/MeV/sr^2)	σ_N/σ_C
1.00	6.774	6.316	1.072
0.95	6.372	5.921	1.076
0.90	5.932	5.376	1.104
0.85	5.459	4.641	1.176
0.80	4.981	3.809	1.308
0.77	4.806	3.968	1.211
0.76	4.847	4.708	1.030
0.75	5.048	6.632	0.761
0.74	5.663	11.73	0.483
0.73	7.555	26.98	0.280
0.72	13.97	79.58	0.175
0.71	24.61	172.8	0.142
0.70	16.56	111.9	0.148
0.65	5.043	18.10	0.279
0.60	3.757	10.49	0.358
0.55	3.007	7.621	0.394
0.50	2.395	5.854	0.409
0.45	1.856	4.503	0.412
0.40	1.378	3.372	0.409
0.35	0.964	2.400	0.402
0.30	0.621	1.581	0.393
0.25	0.354	0.927	0.382
0.20	0.168	0.453	0.371
0.15	0.058	0.162	0.360
0.10	1.07×10^{-2}	3.060×10^{-2}	0.349
0.05	3.05×10^{-4}	9.047×10^{-4}	0.337

a type of asymmetry was also observed in the Kiener *et al.* experiment [16]. This asymmetry appears due to the change of sign, both in the Coulomb and nuclear dipole transition amplitudes, when the direction of deuteron emission is changed from forward to backward in the ^{6}Li c.m frame.

Thus, the results of our calculations confirm that the nuclear contribution exists at the considered scattering angles and the αd energies, but it is not as significant as that concluded by the authors of Ref. [18]. Such a huge contribution also contradicts the qualitative estimation given in Ref. [19].

VI. CONCLUSION

Using the simple two-body approach and the effective potential we have described the cross section and the astrophysical S factor of the $\alpha + d \to {}^6\text{Li} + \gamma$ radiative capture. The results are in good agreement with the known experimental data for the range $0.4 < E_{\alpha d} < 3.0$ MeV. For radiative capture the contribution of the $E1$ transition to the cross section becomes larger than the $E2$ one at an energy less than 100 keV. The calculated total value of the astrophysical S factor is equal to $S(0) = 1.22$ MeV nb, while $S_{E1}(0) = 1.01$ MeV nb (83% of the total S factor) and $S_{E2}(0) = 0.21$ MeV nb (17% of the total S factor). The results for the purely Coulomb breakup are in good agreement with the known experimental data for the range $0.4 < E_{\alpha d} < 0.8$ MeV. This shows the validity of the semiclassical method of the calculation for the Coulomb breakup at high energy collision. The contribution of the $E1$ transition in the Coulomb breakup is always less than the $E2$ one for all energy regions. The nuclear disintegration is analyzed by a diffraction method which can be applied at small scattering angles. The radius of "black" target ^{208}Pb is taken as a fit parameter and the application of the χ^2 method gives reasonable fit to the radius of the target. Our calculated nuclear distortion is not large and suggests an overestimation of the contribution of nuclear distortion made by Hammache *et al.* [18]. A comparison of our calculation of the triple cross section, consisting of the Coulomb and nuclear parts, with the experimental data, shows that disagreement still exists for $E_{\alpha d}$ near zero. Taking into account the higher-order effects may reduce this discrepancy at the low αd relative energy ($E_{\alpha d} < 0.3$ MeV). For instance, the three-body Coulomb effects are known to strengthen with decreasing αd relative energy [21,22]. Due to the existence of the nuclear breakup and the small contribution of the $E1$ Coulomb disintegration to the total cross section, it is impossible to extract the correct value of the astrophysical S factor for $\alpha + d \to {}^6\text{Li} + \gamma$ radiative capture at low $E_{\alpha d}$ energy. Extrapolation to zero energy can give the value of the $E2$ component, if one is able to separate the contributions of the nuclear disintegration and the higher-order effects from experimental data. To us, the simplest way to get an accurate value of the astrophysical S factor of peripheral reactions is to measure the ANC of the wave function in the bound state with a high accuracy which governs the overall normalization of the peripheral reaction cross section near zero energy.

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[1] G. Baur, C. A. Bertulani, and H. Rebel, Nucl. Phys. A **458**, 188 (1986).
[2] C. A. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
[3] G. Baur, K. Hencken, and D. Trautmann, Prog. Part. Nucl. Phys. **51**, 487 (2003).
[4] A. M. Mukhamedzhanov and N. K. Timofeyuk, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 247 (1990) [JETP Lett. **51**, 282 (1990)].
[5] A. M. Mukhamedzhanov and N. K. Timofeyuk, Yad. Fiz. **51**, 679 (1990) [Sov. J. Nucl. Phys. **51**, 431 (1990)].
[6] H. M. Xu, C. A. Gagliardi, R. E. Tribble, A. M. Mukhamedzhanov, and N. K. Timofeyuk, Phys. Rev. Lett. **73**, 2027 (1994).
[7] G. Baur, Phys. Lett. B **178**, 135 (1986).
[8] C. Spitaleri *et al.*, Phys. Rev. C **63**, 055801 (2001).

[9] H. H. Gutbrod *et al.*, editors, An International Accelerator Facility for Beams of Ions and antiprotons, Gesellschaft für Schwerionenforschung, Darmstadt, 2001.
[10] See webpage at [http://www.nscl.msu.edu/].
[11] See webpage at [http://www.rarf.riken.go.jp/ribf/].
[12] H. Esbensen, G. F. Bertsch, and C. A. Bertulani, Nucl. Phys. A **581**, 107 (1995).
[13] C. A. Bertulani, Nucl. Phys. A **587**, 318 (1995).
[14] S. Typel and G. Baur, Phys. Rev. C **64**, 024601 (2001).
[15] P. Banerjee, G. Baur, K. Hencken, R. Shyam, and D. Trautmann, Phys. Rev. C **65**, 064602 (2002).
[16] J. Kiener *et al.*, Phys. Rev. C **44**, 2195 (1991).
[17] R. Shyam, G. Baur, and P. Banerjee, Phys. Rev. C **44**, 915 (1991).
[18] F. Hammache *et al.*, Phys. Rev. C **82**, 065803 (2010).
[19] C. A. Bertulani and M. S. Hussein, Nucl. Phys. A **524**, 306 (1991).
[20] C. A. Bertulani, arXiv:0908.4307.
[21] E. O. Alt, B. F. Irgaziev, and A. M. Mukhamedzhanov, Phys. Rev. Lett. **90**, 122701 (2003).
[22] E. O. Alt, B. F. Irgaziev, and A. M. Mukhamedzhanov, Phys. Rev. C **71**, 024605 (2005).
[23] A. I. Akhieser and A. G. Sitenko, Phys. Rev. **106**, 1236 (1957).
[24] J. Hesselbarth and K. T. Knöpfle, Phys. Rev. Lett. **67**, 2773 (1991).
[25] M. Mazzocco *et al.*, Eur. Phys. J. A **18**, 583 (2003).
[26] H. Esbensen and C. A. Bertulani, Phys. Rev. C **65**, 024605 (2002).
[27] H. Rebel and D. K. Srivastava, *Mechanisms of Li-Projectile Break-Up* (Institut für Kernphysik, Karlsruhe, Germany, 1990), KfK 4761.
[28] L. C. McIntyre and W. Haeberli, Nucl. Phys. A **91**, 382 (1967).
[29] P. A. Schmelzbach, W. Grüeber, V. König, and P. Marmier, Nucl. Phys. A **184**, 193 (1972).
[30] W. Grüebler, P. A. Schmelzbach, V. König, P. Risler, and D. Boerma, Nucl. Phys. A **242**, 265 (1975).
[31] B. Jenny, W. Grüebler, V. König, P. A. Schmelzbach, and C. Schweizer, Nucl. Phys. A **397**, 61 (1983).
[32] V. M. Krasnopol'sky, V. I. Kukulin, E. V. Kuznetsova, J. Horacek, and N. M. Queen, Phys. Rev. C **43**, 822 (1991).
[33] A. M. Mukhamedzhanov, L. D. Blokhintsev, and B. F. Irgaziev, Phys. Rev. C **83**, 055805 (2011).
[34] A. M. Mukhamedzhanov, R. P. Schmitt, R. E. Tribble, and A. Sattarov, Phys. Rev. C **52**, 3483 (1995).
[35] L. D. Blokhintsev, V. I. Kukulin, A. A. Sakharuk, D. A. Savin, and E. V. Kuznetsova, Phys. Rev. C **48**, 2390 (1993).
[36] V. I. Kukulin *et al.*, Nucl. Phys. A **417**, 128 (1984); **483**, 365 (1986).
[37] P. Navrátil and S. Quaglioni, Phys. Rev. C **83**, 044609 (2011).
[38] R. G. Newton, *Scattering Theory of Waves and Particles*, 2nd ed. (Springer-Verlag, New York, 1982).
[39] R. G. H. Robertson *et al.*, Phys. Rev. Lett. **47**, 1867 (1981).
[40] P. Mohr *et al.*, Phys. Rev. C **50**, 1543 (1994).
[41] F. E. Cecil, J. Yan, and C. S. Galovich, Phys. Rev. C **53**, 1967 (1996).
[42] K. M. Nollett, M. Lemoine, and D. N. Schramm, Phys. Rev. C **56**, 1144 (1997).
[43] K. M. Nollett, R. B. Wiringa, and R. Schiavilla, Phys. Rev. C **63**, 024003 (2001).
[44] H. Fuchs, Nucl. Instrum. Methods, **200**, 361 (1982).