Solving the relativistic Rankine-Hugoniot condition in the presence of a magnetic field in the astrophysical scenario of a neutron star

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The Rankine-Hugoniot condition has been solved to study phase transition in an astrophysical scenario mainly in the case of phase transition from a neutron star (NS) to a quark star (QS). The equations of state and temperature play a huge role in determining the nature of the front propagation, which brings about the phase transition in a NS. The shock jump conditions can be solved analytically, but the situation changes drastically by the inclusion of the magnetic field. High magnetic fields, which are always associated with a NS play a huge role in determining the structure and evolution of a NS. So, a magnetic field has been introduced in the shock jump condition in the de Hoffmann-Teller frame. The modified conservation condition for the perpendicular and oblique shocks is obtained in this frame. Numerical solution of the perpendicular shock has been obtained, which shows considerable deviation from the nonmagnetic case. The results show that the magnetic field helps in shock generation. It also indirectly hints at the instability of the matter and thereby the NS for very high magnetic field, implying that NSs can only support a magnetic field of some finite strength.

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I. INTRODUCTION

When the velocity of a fluid in motion becomes comparable with or exceeds that of sound, the effect due to compressibility of the fluid becomes of prime importance. For a wave propagating in a nonconducting gas, when the amplitude is so small that linear theory applies, the disturbance propagates as a sound wave. If the gas has a uniform pressure and density, the speed of propagation of sound and the wave profile maintains a fixed shape, since each part of the wave moves with the same speed. However, when the wave possesses a finite amplitude, so that nonlinear terms in the equation become important, the crest of the sound wave moves faster than its leading or trailing edge. This causes a progressive steepening of the front portion of the wave as the crest catches up and, ultimately, the gradient of pressure, density, temperature, and velocity becomes so large that dissipative processes, such as viscosity or thermal conduction, are no longer negligible. Then a steady wave shape is attained, called a shock wave. The shock wave moves at a speed in excess of the speed of sound, so the information cannot be propagated ahead to signal its imminent arrival, since such information would travel at sound speed relative to the undisturbed medium ahead of the shock. The dissipation inside the shock front leads to a gradual conversion of the energy being carried by the wave into heat. Thus, the effects of the passage of a shock wave are to convert ordered (flow) energy into random (thermal) energy through particle collisions and also to compress and heat the gas. The shock front itself is in reality a very thin transition region. Its width is typically only a few mean-free paths, with particle collisions establishing the new uniform state behind the shock.

The relativistic shock propagates into a medium with a changing equation of state. Therefore, a simple analysis of the jump condition for a polytropic or perfect fluid is not adequate and a deep understanding of this problem calls for the full theoretical description of the relativistic shock in a medium with arbitrary equation of state. Further complication might arise if there is the presence of a significant magnetic field. In fact, one can show that the relative importance of a magnetic field can grow during a collapse. In the field of nuclear physics, high-energy collisions among heavy ions can be modeled by using fluid dynamical concepts. Also, some current models under investigation predict that relativistic shocks (or relativistic detonation and deflagration) might be related to the phase transition from nuclear matter to quark matter.

Relativistic shock waves have been the subject of early investigation in relativistic fluid dynamics and magnetofluid dynamics. In relativistic fluid dynamics the pioneering work is that of Taub [1] where the relativistic form of jump condition is established. A detailed analysis of the thermodynamic treatment of classical shock waves was done by Thorne [2]. Explicit solutions of the jump condition have been obtained for special equation of states. De Hoffmann and Teller [3] presented a relativistic magnetohydrodynamic (MHD) treatment of shocks, eliminating the electric field by transforming to a frame where the flow velocities are parallel to the magnetic field vector (called the de Hoffmann- Teller frame). Shock waves in relativistic magnetofluid have been investigated extensively and in a rigorous mathematical way by Lichnerowicz [4]. Detonation and deflagration waves in relativistic magnetofluid dynamics for nuclear physics and cosmology have been investigated in Refs. [5,6]. Recently it has been shown [7] that the normal of the front can be a timelike four-vector. This is frequently the case for rapid energetic processes such as in supernovas or hadronization and freeze-out in ultrarelativistic heavy-ion collisions. This possibility was overlooked in Taub's original publication and in several subsequent works and was corrected by Csernai [7,8]. With a homogeneous scalar background field on one side of the front (quark gluon plasma with MIT bag model equation

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of state) was solved in general thereafter [9,10]. The detailed method to solve such problems was recently described [11].

In the astrophysical scenario shock plays a very important role in determining the outcome of compact stars. In the case of massive stars, in the range between 8–100 solar masses, which are thought to be the progenitors of type II supernovas, one of the most viable mechanisms for producing an explosion is gravitational collapse and bounce [12]. In this case a shock is formed outside the inner core, which propagates outwards reaching relativistic speeds. Shock waves are also responsible for phase transition and gamma ray bursts (GRBs) in compact stars.

In this paper I will mostly concentrate on the phase transition scenario in a compact star. Witten [13] conjectured of strange quark matter (SQM), consisting of approximately equal numbers of up (u), down (d), and strange (s) quarks, is believed to be the ground state of strong interaction. This was supported by model calculations for certain ranges of values for strange quark mass and strong coupling constant [14]. After that there have been constant efforts at confirming the existence of SQM, though transiently, in ultrarelativistic collisions. On the other hand, SQM could naturally occur in the cores of compact stars, where central densities are expected to be an order of magnitude higher than the nuclear matter saturation density. Thus, neutron stars that have sufficiently high central densities might convert to strange stars, or at least hybrid (a star with a quark core) stars. These transitions may lead to various observable signatures in the form of a jump in the breaking index and gamma ray bursts [15,16], and a full quark star (QS) might help in explaining the phenomena of observed quasiperiodic oscillations [17].

There may be several scenarios by which neutron stars could convert to quark stars. It could happen through a seed of external SQM [18], or be triggered by the rise in the central density due to a sudden spin-down in older neutron stars [19]. Several authors have studied the conversion of nuclear matter to strange matter under different assumptions [20–30]. They have been summarized in a recent work of mine [31] and for the constraint of space, I do not repeat them here.

After the discovery of magnetars, some compact stars were found to have very high surface magnetic fields. The magnetic fields in such stars are so high that it is believed the evolution of the star is hugely influenced by the magnetic field. The nature of the magnetic field in the interior of the star is hidden from direct observation and is still a matter of much debate. The nuclear matter of the neutron star (NS) is charge neutral on the whole but locally it is not. So in the interior, locally, it may be considered as a plasma. So it may be considered that the magnetic fields are frozen to the plasma and the liquid can move freely along the magnetic lines of force without affecting them. Therefore the simple relativistic Rankine-Hugoniot condition is not sufficient for such cases of stars with high magnetic field. To have a full understanding of the properties of a NS and its phase transition to QS, such conditions should be examined in the presence of high magnetic fields. In this paper I intend to carry out such a basic calculation. The paper is organized as follows. First I will discuss the general Rankine-Hugoniot condition as a discontinuity in the conversion front. In Sec. III, I will introduce magnetic field and study the set of modified conservation equations in the de Hoffmann-Teller frame. Then in Sec. IV, I show my results and, finally, I conclude in Sec. V by discussing and summarizing them.

II. GENERAL RANKINE-HUGONIOT CONDITION

In this section I will discuss the general Rankine-Hugoniot condition as the conservation equations that balance the conversion of neutron-proton (n-p) matter to two-flavor quark matter, consisting of u and d quarks along with electrons for ensuring charge neutrality. I heuristically assume the existence of a combustive phase transition front. Using the macroscopic conservation conditions, I examine the range of densities for which such a combustion front exists. It should be mentioned here that I am following the work of Taub, that is only considering the spacelike four-vector and ignoring the timelike four-vector. This is therefore not the most general solution of the problem but is quite useful for the astrophysical case. The normal to the front is said to be spacelike or timelike if the normalization of the unit vector is 1 or -1. The spacelike surface corresponds to the ordinary surface in three-dimensional coordinate space, which travels with a velocity smaller than that of light, while the timelike surface appears when an instantaneous bulk phase transition of plasma occurs. Both kinds of shocks satisfy the same shock equation, however the physical consequence are quite different. To have a more detailed study of such shock one should read the pioneering work by Csernai [7,8] and its application in nuclear collision [9,32]. However most of such analysis is confined to the discussion of heavy-ion collision and the timelike four-vector analysis is most important in the case where the interior region of the plasma is sufficiently supercooled and an instantaneous global phase transition to a superheated dense hadron gas may take place through a timelike surface. This may also be important for implosion shocks in a NS preceding a supernova. Therefore a general (both spacelike and timelike) approach is advisable. As this introduces a magnetic field in the Rankine-Hugoniot condition in the astrophysical scenario, I am assuming the simplest case, which will give an insight into the problem, and the general problem would be my next goal.

Let us consider the physical situation where a combustion front has been generated in the core of the NS. This front propagates outward through the neutron star with a certain velocity, leaving behind a u-d matter. In this case my assumptions are that the front is infinitesimal, the matter flow velocities are perpendicular to the front, and I am in the rest frame of the conversion front. In the following, I denote all the physical quantities in the hadronic sector by subscript 1 and those in the quark sector by subscript 2. The conservation condition for energy momentum and baryon number relates the quantities on the opposite sides of the front. In the rest frame of the combustion front, these conservation conditions is given by [30,33,34]

$$\omega_1 v_1^2 \gamma_1^2 + p_1 = \omega_2 v_2^2 \gamma_2^2 + p_2, \tag{1}$$

$$\omega_1 v_1 \gamma_1^2 = \omega_2 v_2 \gamma_2^2, \tag{2}$$

and

$$n_1 v_1 \gamma_1 = n_2 v_2 \gamma_2. \tag{3}$$

In the above three conditions v_i $(i = 1, 2)$ is the velocity, p_i is the pressure, $\gamma_i = \frac{1}{\sqrt{1-v_i^2}}$ is the Lorentz factor, $\omega_i = \epsilon_i + p_i$ is the specific enthalpy, and ϵ_i is the energy density of the respective phases. The velocities of the matter in the two phases, given by Eqs. (1)–(3), can be solved, such that [33]

$$v_1^2 = \frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)}, \tag{4}$$

and

$$v_2^2 = \frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}. \tag{5}$$

It is possible to classify the various conversion mechanisms by comparing the velocities of the respective phases with the corresponding velocities of sound, denoted by c_{si}, in these phases. These conditions are summarized in Ref. [35].

For the conversion to be physically possible, velocities should satisfy an additional condition, namely, $0 \leqslant v_i^2 \leqslant 1$. Here I find that the velocity condition puts severe constraint on the allowed equations of state.

III. MAGNETIC FIELD INCLUSION

The NS as a whole may be charge neutral but locally it is not, as a huge number of neutron, protons, and electrons are in motion within the star. So, for the infinitesimal shock front I treat it as a plasma. The magnetic fields are thought to be frozen in plasma, so that the particles move along the magnetic field vector. In order to limit myself to the simplest possible case I assume that the conductivity is infinite. The assumption also indicates that the electric field vanishes in a coordinate system that is at rest in the liquid. That is, I am moving in the de Hoffmann-Teller frame.

In a conducting gas, a magnetic field can interact strongly with the flow. The analysis of the shock waves therefore becomes more complex, but the basic principles remain the same. A set of jump conditions can again be derived, but they are considerably more complicated than the pure hydrodynamic shock case. The extra complexity arises both from the presence of an extra variable, namely the magnetic field strength, and also from the fact that the magnetic field and the matter velocities may be inclined with the shock normal.

A shock propagating through a magnetic fluid produces a significant difference in matter properties on either side of the shock front. The thickness of the front is determined by a balance between convective and dissipative effects. However, dissipative effects at high temperature are only comparable to convective effects when the spatial gradients in matter variables become extremely large. Hence, shocks in such matter tend to be extremely narrow, and are well approximated as discontinuity. The hydrodynamical equations, and Maxwell's equations, can be integrated across a shock to give a set of jump conditions that relate matter properties on each side of the shock front. If the shock is sufficiently narrow then these relations become independent of its detailed structure.

In the rest frame of the shock, it propagates in the x direction, that is, the shock front coincides with the y-z plane. Furthermore, the regions of the plasma upstream and downstream of the shock, which are termed regions 1 and 2, respectively, are spatially uniform and non-time-varying. It follows that $\partial/\partial t = \partial/\partial y = \partial/\partial z = 0$. Moreover, $\partial/\partial x = 0$, except in the immediate vicinity of the shock. Finally, let the velocity and magnetic fields upstream and downstream of the shock all lie in the x-y plane. The magnetic field is given by B_i for the respective phases.

The first nontrivial shock is called perpendicular shock in which both the upstream and downstream plasma flows are perpendicular to the magnetic field, as well as the shock front. Due to infinite conductivity, the electric fields vanishes, and by definition of the perpendicular shock only the y component of the magnetic field is present. As only the y component survives, I denote $B_{1,y} = B1$ and $B_{2,y} = B2$. B_1 and B_2 are related by virtue of the fact that the magnetic field lines are frozen to the plasma. Therefore

$$\frac{B_1}{n_1} = \frac{B_2}{n_2}. \tag{6}$$

As the number density of the particles are still conserved, I have

$$n_1 v_1 \gamma_1 = n_2 v_2 \gamma_2. \tag{7}$$

Therefore I can write

$$B_1 v_1 \gamma_1 = B_2 v_2 \gamma_2. \tag{8}$$

Following similarly Ref. [3], the conservation equation for the momenta can be written as

$$\omega_1 v_1^2 \gamma_1^2 + p_1 + \frac{{B_1}^2}{8\pi} = \omega_2 v_2^2 \gamma_2^2 + p_2 + \frac{{B_2}^2}{8\pi}, \tag{9}$$

and the conservation equation for the energy is expressed as

$$\omega_1 v_1 \gamma_1^2 + v_1 \gamma_1 \frac{{B_1}^2}{4\pi} = \omega_2 v_2 \gamma_2^2 + v_2 \gamma_2 \frac{{B_2}^2}{4\pi}. \tag{10}$$

The last three equations can be reduced to two equations, given by

$$\omega_1 v_1^2 \gamma_1^2 + p_1 + \frac{{B_1}^2}{8\pi} = \omega_2 v_2^2 \gamma_2^2 + p_2 + \frac{{B_1}^2}{8\pi}\left(\frac{v_1\gamma_1}{v_2\gamma_2}\right)^2, \tag{11}$$

$$\omega_1 v_1 \gamma_1^2 + v_1 \gamma_1 \frac{{B_1}^2}{4\pi} = \omega_2 v_2 \gamma_2^2 + v_2 \gamma_2 \frac{{B_1}^2}{4\pi} \frac{(v_1\gamma_1)^2}{v_2\gamma_2}, \tag{12}$$

and can be solved numerically to obtain v_1 and v_2.

The most general shock is the oblique shock in which the plasma velocities and the magnetic fields on each side of the shock are neither parallel nor perpendicular to the shock front. The conservation conditions for such shocks are given by

$$\omega_1 v_{1x}^2 \gamma_1^2 + p_1 + \frac{{B_1}^2}{8\pi} - \frac{{B_{1x}}^2}{4\pi} = \omega_2 v_{2x}^2 \gamma_2^2 + p_2 + \frac{{B_2}^2}{8\pi} - \frac{{B_{2x}}^2}{4\pi}, \tag{13}$$

$$\omega_1 v_{1x} v_{1y} \gamma_1^2 + v_{1x}\gamma_1 \frac{B_{1x}B_{1y}}{4\pi} = \omega_2 v_{2x} v_{2y}\gamma_2^2 + v_{2x}\gamma_2 \frac{B_{2x}B_{2y}}{4\pi}, \tag{14}$$

$$\omega_1 v_{1x}\gamma_1^2 + v_{1x}\gamma_1 \frac{{B_1}^2}{4\pi} - \frac{B_{1x}(B_1 v_1)\gamma_1}{4\pi} = \omega_2 v_{2x}\gamma_2^2 + v_{2x}\gamma_2\frac{{B_2}^2}{4\pi} - \frac{B_{2x}(B_2 v_2)\gamma_2}{4\pi}, \tag{15}$$

$$B_{1y} v_{1x}\gamma_1 - v_{1y} B_{1x}\gamma_1 = B_{2y} v_{2x}\gamma_2 - v_{2y} B_{2x}\gamma_2 \tag{16}$$

$$B_{1x} = B_{2x} \tag{17}$$

and

$$n_1 v_{1x}\gamma_1 = n_2 v_{2x}\gamma_2. \tag{18}$$

There may be two easier cases for the above complicated equations.

Case 1. $v_{1y} = 0 = v_{2y}$

Then the first four equations would simplify to

$$\omega_1 v_{1x}^2\gamma_1^2 + p_1 + \frac{{B_1}^2}{8\pi} = \omega_2 v_{2x}^2\gamma_2^2 + p_2 + \frac{{B_2}^2}{8\pi}, \tag{19}$$

$$\omega_1 v_1\gamma_1^2 + v_1\gamma_1\frac{{B_1}^2}{8\pi} - \frac{B_{1x}(B_1 v_1)\gamma_1}{4\pi} = \omega_2 v_2\gamma_2^2 + v_2\gamma_2\frac{{B_2}^2}{8\pi} - \frac{B_{1x}(B_2 v_2)\gamma_2}{4\pi}, \tag{20}$$

$$B_{2y} = \frac{v_1\gamma_1}{v_2\gamma_2} B_{1y} \tag{21}$$

Case 2. $B_{1y} = 0 = B_{2y}$

$$\omega_1 v_{1x}^2\gamma_1^2 + p_1 = \omega_2 v_{2x}^2\gamma_2^2 + p_2, \tag{22}$$

$$\omega_1 v_{1x}\gamma_1^2 + v_{1x}\gamma_1\frac{{B_1}^2}{8\pi} - \frac{B_{1x}(B_1 v_1)\gamma_1}{4\pi} = \omega_2 v_{2x}\gamma_2^2 + v_{2x}\gamma_2\frac{{B_2}^2}{8\pi} - \frac{B_{1x}(B_2 v_2)\gamma_2}{4\pi}, \tag{23}$$

$$\omega_1 v_{1x} v_{1y} {\gamma_1}^2 = \omega_2 v_{2x} v_{2y} {\gamma_2}^2 \tag{24}$$

IV. RESULTS

I started my calculations by using the nuclear matter equations of state (EOS) obtained through the nonlinear Walecka model [36]. In the present paper, I consider the conversion of nuclear matter, consisting of only neutrons, protons, and electrons, to a two-flavor quark matter. The final composition of the quark matter is determined from the nuclear matter EOS by enforcing the baryon number conservation during the conversion process. While describing the state of matter for the quark phase, I consider a range of values for the bag constant. Nuclear matter EOS is calculated at zero temperature, whereas the two-flavor quark matter EOS is obtained both at zero temperature as well as at finite temperatures, because the propagation of the shock may heat up the matter.

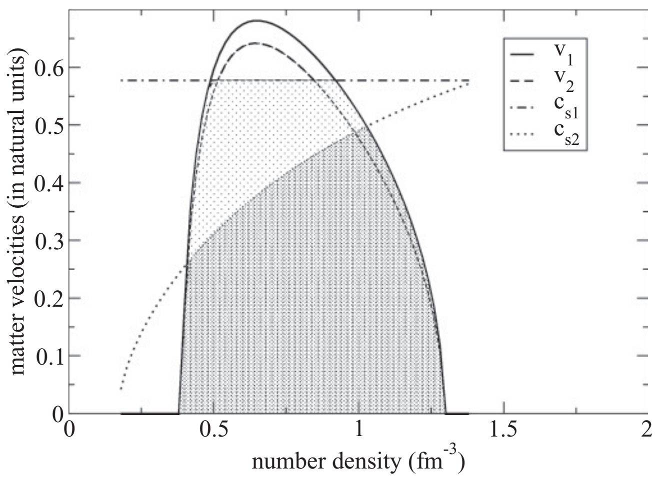

FIG. 1. Variation of different matter velocities with baryon number density for $T = 0$ MeV, ${B_G}^{1/4} = 160$ MeV and strange quark mass $m_s = 200$ MeV. The dark-shaded region corresponds to deflagration, the light-shaded region corresponds to detonation, and the unshaded region corresponds to supersonic conversion processes.

To examine the nature of the hydrodynamical front arising from the neutron to two-flavor quark matter conversion, I plot, in Fig. 1, the quantities v_1, v_2, c_{s1}, and c_{s2} as functions of the baryon number density (n_B). As mentioned earlier, the u and d quark content in the quark phase is kept the same as the one corresponding to the quark content of the nucleons in the hadronic phase. With these fixed densities of the massless u and d quarks and electrons, the EOS of the two-flavor matter has been evaluated using the bag model prescription.

I find that the velocity condition (${v_i}^2 > 0$) is satisfied only for a small window of $\approx\pm 5.0$ MeV around the bag pressure ${B_G}^{1/4} = 160$ MeV. The constraint imposed by the above conditions results in the possibility of deflagration, detonation, or supersonic front as shown in the Fig. 1.

In Fig. 1, I considered both the phases to be at zero temperature. A possibility, however, exists that a part of the internal energy is converted to heat energy, thereby increasing the temperature of the two-flavor quark matter during the exothermic combustive conversion process. In Fig. 2, I plot the variation of velocities with density at $T = 50$ MeV, for which significant change is noticed. This figure shows that the range of values of baryon density, for which the flow velocities are physical, increases with temperature. In the present paper I have considered only the zero-temperature nuclear matter EOS. On the other hand, equations of state of quark matter has a finite temperature dependence and hence the difference between v_1 and v_2, varies with temperature.

The variation of the velocities with temperature is due to the fact that higher temperature means higher energy. As the energy increases the particles becomes more energetic, which means the matter becomes more excited and compressible. As matter becomes more compressible due to increase in temperature, there is now a chance for shock formation, which was previously not possible.

Until now, figures had been plotted solving the Rankine-Hugoniot condition. The change in temperature in the EOS accounted for the change in the nature of the graph. Next I

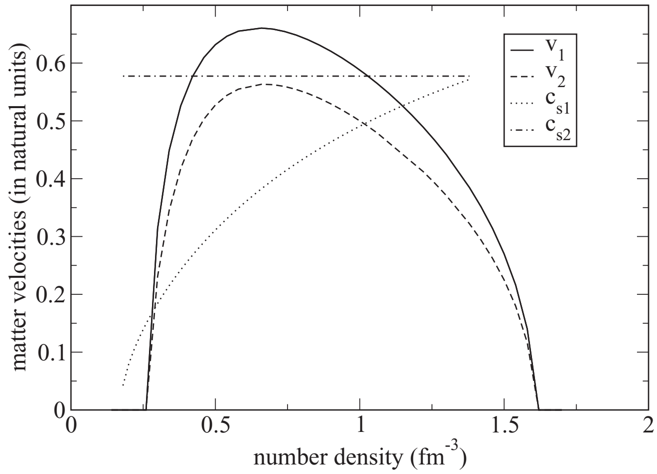

FIG. 2. Variation of velocities with baryon number density for $T = 50$ MeV, ${B_G}^{1/4} = 160$ MeV and strange quark mass $m_s = 200$ MeV.

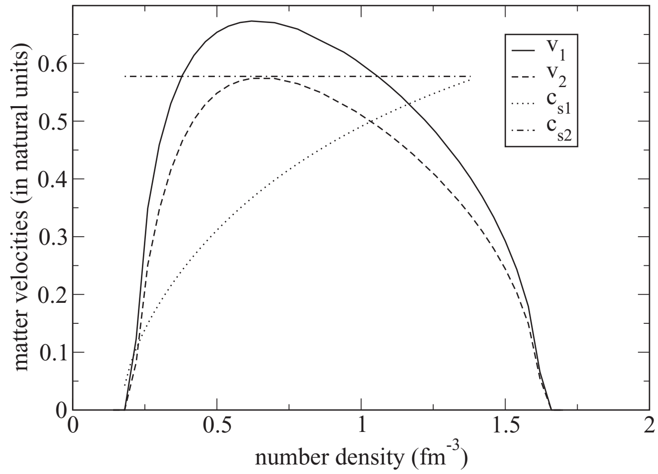

FIG. 4. Variation of different matter velocities with baryon number density for $T = 50$ MeV and magnetic field strength of $B = 5 \times 10^{15}$ G.

plot graphs with a magnetic field included in the conservation condition for the perpendicular shock. In Fig. 3, I have plotted curves for different flow velocities varying with baryon density for the zero-temperature case. The magnetic field used for the generation of the curve is $B = 5 \times 10^{15}$ G. I find that due to introduction of the magnetic field, the range of values of baryon density, for which the flow velocities are physical, increases. Its nature is quite similar to that for the case of temperature. Due to the introduction of the magnetic field the pressure increases (pressure due to magnetic field is $B^2/8\pi$) for the same value of baryon number density. The magnetic field in the respective phases adjusts in such a way that the resultant energy and pressure of the two phases gets modified to make the range of baryon number density increase. For both higher and lower values of baryon density, previously there was less chance of shock formation, but now due to the introduction of the magnetic field there is a greater chance of shock formation. And therefore the range of baryon density, for which the flow velocities are physical, gets wider.

Next (Fig. 4), I plot curves for different velocities for the finite temperature case $T = 50$ MeV. For same value of the magnetic field $B = 5 \times 10^{15}$ G, the range of baryon density gets much wider. This is due to the fact that now both the temperature and the magnetic field work hand in hand to increase the chances of shock generation. Both processes ensure in their own way that the matter parameters adjust themselves in such a way that there is a greater probability of shock generation.

In Fig. 5 I have plotted exclusively v_1 and v_2 for two cases, one without magnetic field and the other with magnetic field. The nature of the curve remains the same, which is v_1 is always greater than v_2, meaning the shock front propagates outward. The range of baryon density, for which the matter velocities are finite, increases with the magnetic field. I have plotted this for the zero-temperature case. In Fig. 6 I have plotted the same

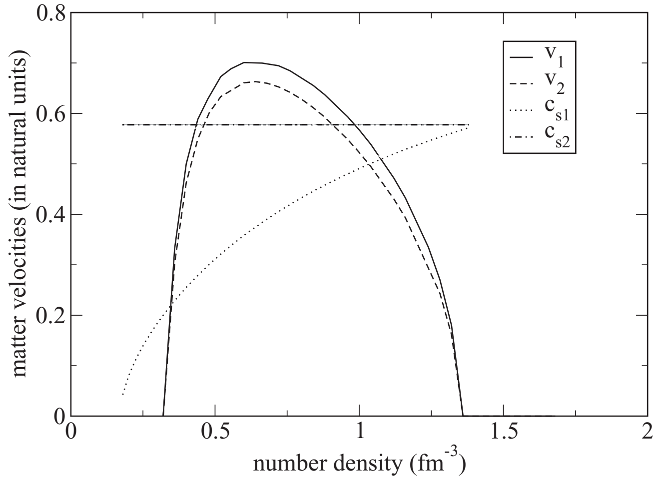

FIG. 3. Variation of different matter velocities with baryon number density for $T = 0$ MeV and magnetic field strength of $B = 5 \times 10^{15}$ G.

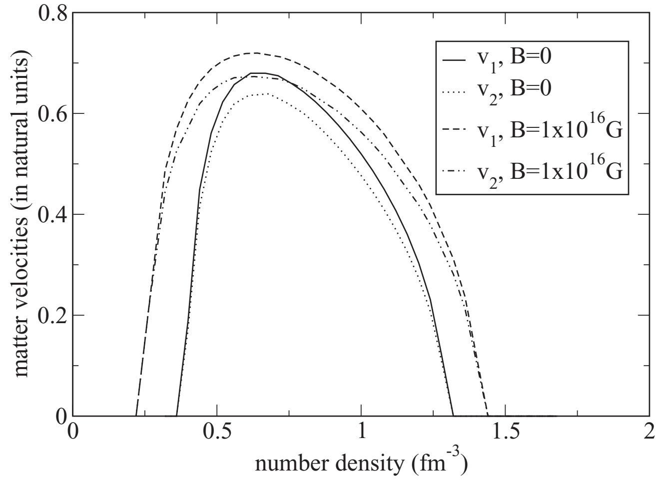

FIG. 5. Comparison of v_n and v_s with baryon number density for $T = 0$ MeV, for two values of magnetic field strength $B = 0$ and $B = 1 \times 10^{16}$ G.

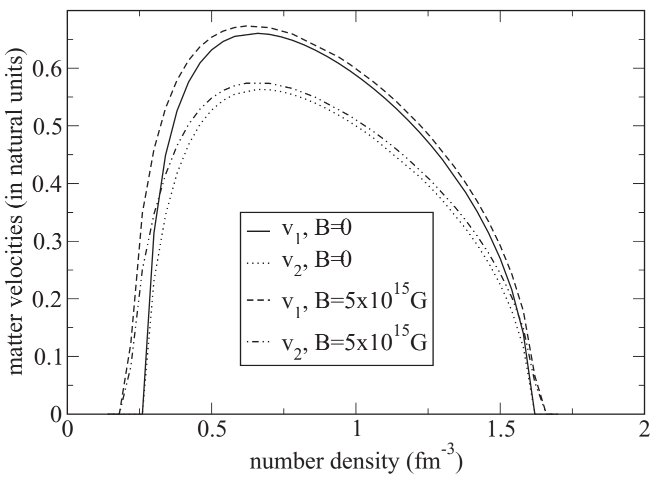

FIG. 6. Comparison of v_n and v_s with baryon number density for $T = 50$ MeV, for two values of magnetic field strength $B = 0$ and $B = 5 \times 10^{15}$ G.

for the finite-temperature case and find that for lesser value of magnetic field, the same increase in range of baryon number density is seen, and the reason is the same as discussed earlier.

In Fig. 7, I have plotted v_1 for different values of magnetic field. I find that as the value of the magnetic field increases the range of baryon number density increases, which is what was expected (and discussed previously). But as I go on increasing the magnetic field v_1 does not comes down on the lower half of the curve. This is due to the fact that, at such high value of the magnetic field the matter becomes unstable. At the other half where matter is at much higher density, it can support such field strength. But if I further increase the magnetic field the matter cannot support such high fields whatever the density might be. The maximum value of magnetic field that matter can support is a few times 10^{17} G (in my case the cutoff value is 2×10^{17} G). So I find that there is a cutoff value for the magnetic field, which the matter can support. It seems from this analysis that a NS (the analysis is being done for the phase

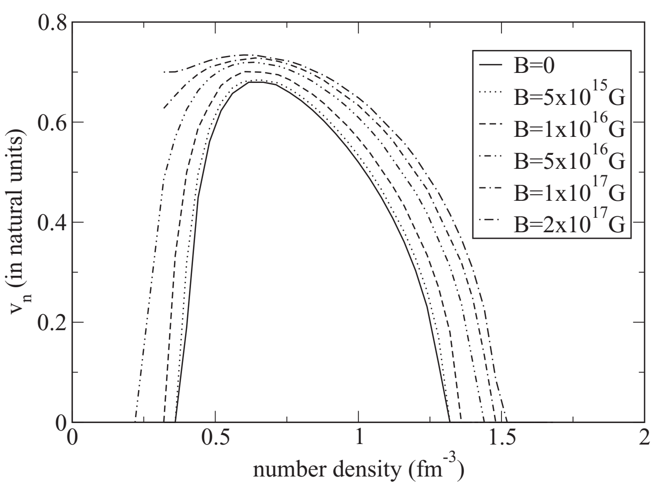

FIG. 7. Comparison of v_n with baryon number density for $T = 0$ MeV with different values of magnetic field strength.

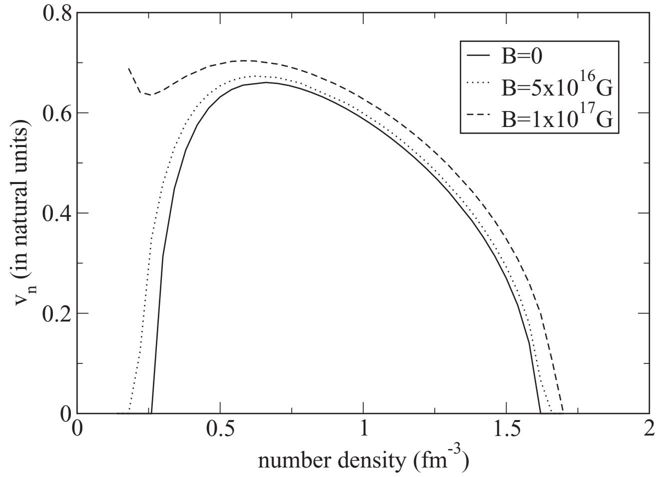

FIG. 8. Comparison of v_n with baryon number density for $T = 50$ MeV with different values of magnetic field strength.

transition of nuclear matter to two-flavor quark matter in a NS) also can support up to a finite value of magnetic field. This curve is plotted for zero temperature, and in Fig. 8 I have plotted the same for finite temperature. Qualitatively the nature of the graph remains the same; only the quantitative value of the magnetic field changes. It shows that hotter matter can support a lesser value of magnetic field than colder matter.

V. CONCLUSION

Finally in this section I summarize my results. I find that the Rankine-Hugoniot condition can be solved to determine the condition for different types of wave generation in a neutron star. It also determines the mode of the propagation of the wave front. The temperature (finite temperature of the matter) helps in the generation of the front and as the temperature rises the shock front can generate both at much lower and at much higher baryon densities. Next I write down the modified conservation conditions in the presence of magnetic field. To do such an analysis I go the de Hoffmann-Teller frame, with matter having infinite conductivity. The matter is also field frozen. With such assumptions I have written down the conservation conditions for both the perpendicular and oblique shock waves. The inclusion of the magnetic field introduces not only extra conditions but also the earlier existing conditions get modified. I have numerically solved for the velocity of the matter of the two phases for the perpendicular shock. I have matched my nonmagnetic numerical results with the analytically solvable nonmagnetic solutions. The oblique wave equation gets very much complicated and the solution of the simultaneous equations does not converge for different values of the baryon densities. So I have not plotted the results for the oblique waves.

Solving the perpendicular wave for finite magnetic field I found that the range of baryon density, for which the flow velocities of matter are physical, increases with magnetic field strength. This is because by the introduction of the magnetic field the resultant pressure and energy redistribute in such a way that, for the same baryon density, the probability for shock generation increases. From the results it seems that there is a cutoff in the magnetic field strength that matter can support, beyond which the matter becomes unstable and the flow velocities become imaginary. This indirectly, on the other hand, suggests that a NS cannot support a magnetic field up to certain field strength. This may be a very significant result in the sense that it provides an upper limit of magnetic field strength in a NS. Observations suggest that the maximum surface magnetic field in NS can be of the order of a few times 10^{15} G. The interior of the star is hidden from direct observation, so various models for magnetic field structure at the core of the star have been proposed. Some such models predict that the field strength at the interior may be abnormally high (much higher than 10^{18} G. My analysis gives an upper limit to the magnetic field at the interior of the star, which may discard such abnormally high magnetic field models. At this point I should mention the fact that one of the reasons for having imaginary velocities might be the noninclusion of the timelike four-vector. That has been the case for ultrarelativistic heavy-ion collision where considering only the spacelike four-vector has given rise to imaginary charges. Therefore the inclusion of the timelike four-vector may set a cutoff for the magnetic field. The shocks with timelike four-vector is much more important for the implosion shocks in a NS preceding a supernova explosion, but it may also set a cutoff for my case. Therefore final say for such a cutoff of magnetic field value has to be done after we have done a more general solution of the problem. However, this is the simplest nontrivial solution with magnetic field, and I expect correction in the magnetic field value (cutoff) with more complicated models (such as oblique shocks and with timelike four-vector). To finally summarize my results, I must mention that this is a treatment of modified conservation conditions done in the presence of magnetic field in the astrophysical phase transition scenario. This provides interesting results, which indirectly hint at the upper limit of magnetic field strength that may be present in a NS. More interesting results are anticipated with the full general solution (having both spacelike and timelike four-vector and also solving for the oblique waves). Again the jump conditions are solved in the special de Hoffmann-Teller frame, and more complicated solutions are expected for a general frame without such assumptions. Recently more general jump conditions have been solved mainly in the heavy-ion collision scenario and a possible focus for the future is to solve them in the astrophysical scenarios of a NS.

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