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Compton scattering from nuclei and photo-absorption sum rules

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We revisit the photo-absorption sum rule for real Compton scattering from the proton and from nuclear targets. In analogy with the Thomas-Reiche-Kuhn sum rule appropriate at low energies, we propose a new "constituent quark model" sum rule that relates the integrated strength of hadronic resonances to the scattering amplitude on constituent quarks. We study the constituent quark model sum rule for several nuclear targets. In addition, we extract the $\alpha = 0$ pole contribution for both proton and nuclei. Using the modern high-energy proton data, we find that the $\alpha = 0$ pole contribution differs significantly from the Thomson term, in contrast with the original findings by Damashek and Gilman.

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I. INTRODUCTION

Compton scattering on composite objects has served as a valuable tool for studying internal structure of nuclei and nucleon. At very low photon energy, electromagnetic waves are scattered without absorption and solely probe the macroscopic properties of the target as a whole, e.g., its mass and electric charge, and the scattering amplitude is determined by the classical Thomson limit. As the photon energy ν is increased above the absorption threshold, the internal structure of the target is revealed. The atomic, nuclear, and hadronic physics domains roughly correspond to keV, MeV, and GeV photon energies, respectively, and the three orders of magnitude difference between neighboring domains indicates that the dynamics of nuclei are well separated from those of quarks so that each can be clearly identified.

In this work, we compare photon scattering from nuclei with photon scattering off the individual nucleons. At energies beyond the nuclear absorption range, i.e., of the order of tens of MeV's, the interaction time between the photon and the target is much shorter than that between individual nucleons, and in this regime the scattering amplitude is determined by Thomson scattering on independent nucleons. By applying the optical theorem, this relation can be made quantitative, and this leads to a sum rule relating the low (MeV range) and medium energy (tens of MeV's) scattering amplitudes on a nucleus to the total photo-absorption cross section [1] in this energy range. Following the success of the constituent quark model of low-energy hadron structure, one would expect that this nuclear sum rule might be extended to cover the energy range between pion production threshold (roughly 100 MeV) to above the range of nucleon resonances (a few GeV). Such an extended "constituent quark model" sum rule would therefore relate the photo-absorption cross section on a nuclear target to the difference between the low-energy scattering amplitude on the nucleus and the scattering amplitude describing photon interactions with individual constituent quarks. In this paper, we will derive a finite energy sum rule involving constituent quarks. We will investigate its validity by including nuclear photo-absorption data above the pion production threshold, which is now available for a wide range of nuclear targets.

As the energy range is further increased, we expect that the QCD structure of the constituent quarks will be resolved, eventually revealing scattering on point-like quarks. Thus, at asymptotically high energies, point-like interactions involving photons can contribute an energy-independent constant to the amplitude, which corresponds to a Regge pole at $\alpha = 0$ [2–10], as is shown schematically in Fig. 1. In the presence of other poles in the right-half of the angular momentum plane, the $\alpha = 0$ pole[1] produces a subleading contribution to the scattering amplitude. Nevertheless, since contributions from leading poles with Re$\alpha > 0$ can be determined by fitting a Regge-type amplitude to the high-energy data, it might be possible in principle to extract the residual $\alpha = 0$ pole. In the past, this procedure has been carried out for the proton [5,11–14] and the deuteron [15]. In this work we reexamine the procedure for extracting the $\alpha = 0$ pole by including in our analysis data at very high energies that were not available when the original analysis was performed in 1969. We will show that, with this new data, one can unambiguously extract the $\alpha = 0$ pole. We also examine possible $\alpha = 0$ pole contributions to Compton scattering on heavier nuclei.

Our paper is organized as follows. In the next section, we focus on the energy range up to the pion production threshold and we summarize the derivation of the nuclear photo-absorption sum rule, also referred to as the Thomas-Reiche-Kuhn (TRK) sum-rule [1,16–18]. In Sec. III, we generalize the TRK sum rule to cover energies beyond the pion threshold, where we include hadronic resonances and we test the validity of a new finite-energy sum rule based on a constituent quark picture. Finally, we consider energies above the GeV range. We discuss the implications of scattering on

[1]The difference between the notation $\alpha = 0$ pole and $J = 0$ pole, more commonly used in the literature, is a subtle one. J refers to the spin of an exchanged particle, whereas α refers to a parameter describing the high-energy behavior of a scattering amplitude, the intercept of a Regge trajectory. For mesonic trajectories, $\alpha = 0, 1, 2, \ldots$ corresponds to physical particles of integer spin J. Instead, the $J = 0$ pole cannot be identified with such an exchange, therefore, we opt for the notation $\alpha = 0$ pole as more suitable.

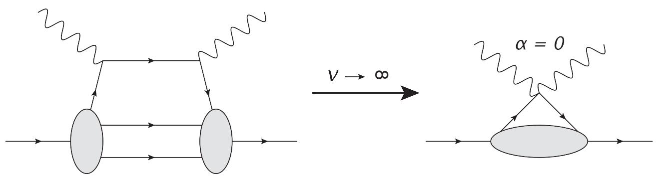

FIG. 1. The fixed-pole contribution to the Compton amplitude may arise due to an effective local two-photon coupling to elementary constituents within the proton.

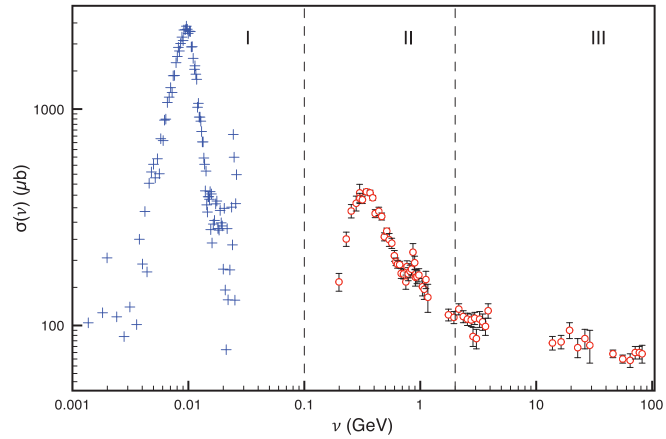

FIG. 2. (Color online) Photoabsorption cross-section data for a ^{207}Pb target. Data in the nuclear range $\nu \leqslant 27$ MeV (crosses) are from Ref. [19]; data in the hadronic and high-energy range 0.2 GeV$\leqslant \nu \leqslant$100 GeV are from Refs. [20–23]. Nuclear deformations are responsible for the giant resonance that saturates the cross section for $\nu \lesssim 100$ MeV (region I). Excitations of individual nucleons are responsible for the hadronic resonances (region II) in the energy range between pion production threshold and $\mathcal{O}$ (2–3 GeV). Finally, for energies above a few GeV (region III), the smooth cross section is the result of partonic scattering via Regge exchanges.

QCD partons and we extract the $\alpha = 0$ pole contribution to scattering at asymptotic energies for various nuclear targets. Our summary and conclusions are presented in Sec. IV.

II. NUCLEAR PHOTO-ABSORPTION AT LOW ENERGIES

The spin-averaged forward Compton scattering amplitude $T(\nu)$ satisfies a once-subtracted dispersion relation where the subtraction constant at $\nu = 0$ is determined by the classical Thomson limit,

$$\mathrm{Re}T(\nu) = -\frac{Z^2}{A^2}\frac{\alpha}{M_N} + \frac{\nu^2}{\pi}\int_0^\infty \frac{d\nu'^2}{\nu'^2(\nu'^2-\nu^2)}\mathrm{Im}T(\nu'), \tag{1}$$

where the integral in Eq. (1) is understood in terms of its principal value. To facilitate easier comparison between different nuclei we have normalized $T(\nu)$ by dividing it by A, the number of nucleons. The nuclear Thomson term, i.e., the constant on the r.h.s. of Eq. (1) is given in terms of the fine structure constant α, the net charge Z of the target, and the mass of the nucleus given by A times the nucleon mass, M_N (in the following we ignore isospin breaking terms). The optical theorem relates the imaginary part of the Compton amplitude to the total photoabsorption cross section per nucleon $\sigma(\nu)$,

$$\mathrm{Im}T(\nu) = \frac{\nu}{4\pi}\sigma(\nu), \tag{2}$$

so that the dispersion relation takes the form

$$\mathrm{Re}T(\nu) = -\frac{Z^2}{A^2}\frac{\alpha}{M_N} + \frac{\nu^2}{2\pi^2}\int_0^\infty \frac{d\nu'}{\nu'^2-\nu^2}\sigma(\nu'). \tag{3}$$

To evaluate the dispersive integral, strictly speaking the photoabsorption cross section should be included all the way up to infinite energy; however, the scale separation between the nuclear and hadronic domains allows us to approximate the integral by using a limited range of nuclear photoabsorption data. As shown in Fig. 2, for a typical target nuclear resonances saturate the photoabsorption cross section for energies below $E_{\mathrm{max}} \approx 30$ MeV. The dominant feature of nuclear photoabsorption in the MeV range is the giant dipole resonance (GDR) (cf. Ref. [24] for a comprehensive review of GDR data and theory). As an example, the ^{207}Pb data in the nuclear range are plotted along with the higher energy data in Fig. 2, in which the GDR is seen as a sharp peak with width $\Gamma_{\mathrm{GDR}} \approx 7$ MeV. We evaluate the dispersion relation at $\nu_{\mathrm{max}} \lesssim 100$ MeV, which roughly demarcates the scale of hadronic physics where single-nucleon resonances begin contributing to the cross section,

$$\mathrm{Re}T(\nu_{\mathrm{max}}) \approx -\frac{Z^2}{A^2}\frac{\alpha}{M_N} - \frac{1}{2\pi^2}\int_0^{E_{\mathrm{max}}} d\nu'\sigma(\nu'). \tag{4}$$

For an energy that is low compared to the hadronic scale, the scattering amplitude can be approximated by the sum of contributions describing photon interactions with point-like nucleons, i.e., it is given by a sum of Thomson terms on Z protons,

$$\mathrm{Re}T(\nu_{\mathrm{max}}) \approx -\frac{Z}{A}\frac{\alpha}{M_N}. \tag{5}$$

Combining Eqs. (4) and (5) leads to the Thomas-Reiche-Kuhn sum rule [1] (with $\alpha/M_N \approx 3.03$ mb MeV),

$$\int_0^{E_{\mathrm{max}}} d\nu\sigma(\nu) = 2\pi^2\frac{NZ}{A^2}\frac{\alpha}{M_N} \approx 60\frac{NZ}{A^2}\mathrm{mbMeV}. \tag{6}$$

Furthermore, adopting a Breit-Wigner form for the GDR cross section,

$$\sigma(\nu) \approx \sigma_{\mathrm{GDR}}(\nu) = \frac{M_{\mathrm{GDR}}^2\Gamma_{\mathrm{GDR}}^2\sigma_{\mathrm{GDR}}}{\left(\nu^2 - M_{\mathrm{GDR}}^2\right)^2 + M_{\mathrm{GDR}}^2\Gamma_{\mathrm{GDR}}^2}, \tag{7}$$

the integral over the resonance photoabsorption cross section gives $\pi\sigma_{\mathrm{GDR}}\Gamma_{\mathrm{GDR}}/2$, and the TRK sum rule leads to the relation

$$\sigma_{\mathrm{GDR}}\Gamma_{\mathrm{GDR}} \approx 12\pi\frac{NZ}{A^2}\ \mathrm{mb\ MeV}. \tag{8}$$

In Eq. (8), σ_{GDR} is the value of the photoabsorption cross section at the peak of the GDR resonance, and Γ_{GDR} is the resonance half-width. This sum rule has been confronted with experimental data on a vast number of nuclear targets and is found to be satisfied to within $\sim$30%. This level of agreement demonstrates that the physics of nuclear excitations is correctly described by a model assuming quasifree nucleons within the nucleus, which leads to Eq. (5). However, several model assumptions were used to equate the (model-dependent) r.h.s. of Eq. (1) to the integral over the total photoabsorption cross section. First, one assumes that the integral in the l.h.s. of Eq. (6) converges if the integration is extended to infinite energy, and in any case is dominated by the nuclear spectrum at $\nu \lesssim E_{\max}$. Second, to eliminate the ν-dependence, one assumes that in the expansion

$$\nu^2 \int_0^{E_{\max}} d\nu' \frac{\sigma(\nu')}{\nu^2 - \nu'^2} = \left[1 + \frac{\langle \nu^2 \rangle}{\nu^2} + \dots\right] \int_0^{E_{\max}} d\nu' \sigma(\nu'), \quad (9)$$

the second term in the bracket, proportional to the mean squared energy averaged over the nuclear spectrum, satisfies

$$\langle \nu^2 \rangle = \frac{\int_0^{E_{\max}} d\nu' \nu'^2 \sigma(\nu')}{\int_0^{E_{\max}} d\nu' \sigma(\nu')} \ll 1 \,. \quad (10)$$

For typical values, $M_{\mathrm{GDR}} \sim 15$ MeV, $\Gamma \sim 7$ MeV, $E_{\max} = 30$ MeV, and $\nu = \nu_{\max} = 100$ MeV, one finds that $\langle \nu^2 \rangle / \nu^2_{\max}$ amounts to a 10–15% difference between the dispersive integrals in Eqs. (3) and (4). With increasing $\nu_{\max}$, the correction term between the two integrals becomes smaller; however, the proximity of the pion production threshold and the nucleon excitation spectrum induces an important systematic error that cannot be accounted for within the framework of the TRK sum rule. These issues are addressed in the following section.

III. NUCLEAR PHOTOABSORPTION IN THE RANGE OF NUCLEON RESONANCES: A “CONSTITUENT QUARK MODEL” SUM RULE

We now extend the arguments that lead to the TRK sum rule to energies in the nucleon excitation region, which we will define as the energy range between the threshold for pion production on a free nucleon, i.e., ∼100 MeV and a few GeV. For energies above $\nu_{\max} = 2$–3 GeV, the cross-section is smooth and does not exhibit resonance behavior. Above the resonance range, we expect the cross section to be described by scattering on individual constituents of the nucleons, i.e., constituent quarks. In analogy to Eq. (5), we thus assume

$$\mathrm{Re}T_{\mathrm{CQM}} \equiv \mathrm{Re}T(\nu_{\max}) \approx -\frac{1}{A} \sum_{q \in A} \frac{\alpha}{m_q} e_q^2 = -\frac{3Z + 2N}{A} \frac{\alpha}{M_N}. \quad (11)$$

Following the derivation of the TRK sum rule we want to identify $\mathrm{ReT_{CQM}}$ with the sum of the nuclear Thomson term and the photoabsorption cross-section integrated up to some energy above the nucleon resonance region. This is complicated by the fact that above the resonance region, the photoabsorption cross section does not fall off with energy but instead increases until it is close to the Froissart bound [25]. This increase with energy occurs because in QCD the photon does not interact with a fixed number of hadron constituents (e.g., the nucleon, pion, constituent quarks), but as the beam energy increases gluon showers build up between the photon and the target. Phenomenologically one describes this energy region in terms of Pomeron exchange. Furthermore, at intermediate energies before the universal Pomeron scattering takes over, Reggeon exchanges, i.e., parton showers dominated by exchange of quarks, contribute a significant background to hadron resonance production. Since it is only the hadron resonances that can be associated with constituent quark degrees of freedom, a sum rule involving constituent quarks must involve cross sections with both the Reggeon and the Pomeron contributions subtracted.

The Regge and Pomeron contributions to the cross section per nucleon are conventionally parametrized by

$$\sigma^{R+P}(\nu) = \sigma_T^R + \sigma_T^P = \sum_{i=R,P} c_i \left(\frac{\nu}{\mathrm{GeV}}\right)^{\alpha_i(0)-1}, \quad (12)$$

where for the Regge and Pomeron contributions we use the intercepts $\alpha_R(0) = 1/2$ and $\alpha_P(0) = 1.097$, respectively [27]. This corresponds to an amplitude given by

$$T^{R+P}(\nu) = T^R + T^P = -\sum_{i=R,P} \frac{c_i}{4\pi} \frac{1 + e^{-i\pi\alpha_i(0)}}{\sin \pi\alpha_i(0)} \nu^{\alpha_i(0)} = \frac{\nu^2}{2\pi^2} \int_0^\infty \frac{d\nu'}{\nu'^2 - \nu^2} \sigma^{R+P}(\nu'). \quad (13)$$

Equation (1) can now be rewritten by adding and subtracting the asymptotic contributions given by Eqs. (12) and (13) to obtain

$$\mathrm{Re}T(\nu) = -\frac{Z^2}{A^2} \frac{\alpha}{M_N} + \frac{\nu^2}{2\pi^2} \int_0^\infty d\nu' \frac{\sigma(\nu') - \sigma^{R+P}(\nu')}{\nu'^2 - \nu^2} + \mathrm{Re}T^{R+P}(\nu) \,. \quad (14)$$

In Eq. (14), the integrand on the r.h.s. vanishes asymptotically and we can take the limit $\nu \to \infty$ to obtain

$$\lim_{\nu \to \infty} \frac{\nu^2}{2\pi^2} \int_0^\infty d\nu' \frac{\sigma(\nu') - \sigma^{R+P}(\nu')}{\nu'^2 - \nu^2} = -\frac{1}{2\pi^2} \int_0^E d\nu' \sigma(\nu') + \sum_{i=R,P} \frac{c_i \,\mathrm{GeV}}{2\pi^2 \alpha_i(0)} \left(\frac{E}{\mathrm{GeV}}\right)^{\alpha_i(0)}. \quad (15)$$

In Eq. (15), E is the energy above which we can neglect the difference between the data $\sigma(\nu)$ and the high-energy asymptotic form $\sigma^{R+P}(\nu)$. We used $E = 2$ GeV in our calculations. To extend the TRK sum rule to energies above the nucleon resonance region, we postulate that the contribution to the r.h.s. of Eq. (14) from the photoabsorption cross section in the nucleon resonance region, reduced by the Regge plus Pomeron background, can be represented by Thomson scattering on the constituent quarks. This leads to a phenomenological finite-energy sum rule

$$\mathrm{Re}T_{\mathrm{CQM}} = -\frac{Z^2}{A^2} \frac{\alpha}{M_N} - \frac{1}{2\pi^2} \int_0^E d\nu' \sigma(\nu') + \frac{c_R \,\mathrm{GeV}}{2\pi^2 \alpha_R(0)} \left(\frac{E}{\mathrm{GeV}}\right)^{\alpha_R(0)} \quad (16)$$

or

$$-\left(2+\frac{ZN}{A^2}\right)\frac{\alpha}{M_N} = -\frac{1}{2\pi^2}\int_0^E d\nu'\sigma(\nu') + \frac{c_R\,\mathrm{GeV}}{2\pi^2\alpha_R(0)}\left(\frac{E}{\mathrm{GeV}}\right)^{\alpha_R(0)}. \tag{17}$$

In addition to the phenomenological constituent quark model (CQM) sum rule, the dispersion relation in Eq. (14) can also be used to calculate the value of the subleading energy-independent contribution to the photonuclear Compton amplitude, i.e., the $\alpha = 0$ pole discussed in Sec. I.

$$\begin{aligned}\mathrm{Re}T^{\alpha=0} &\equiv \lim_{\nu\to\infty}[\mathrm{Re}T(\nu) - \mathrm{Re}T^{R+P}(\nu)]\\ &= -\frac{Z^2}{A^2}\frac{\alpha}{M_N} - \frac{1}{2\pi^2}\int_0^E d\nu'\sigma(\nu')\\ &\quad + \sum_{i=R,P}\frac{c_i\,\mathrm{GeV}}{2\pi^2\alpha_i(0)}\left(\frac{E}{\mathrm{GeV}}\right)^{\alpha_i(0)}.\end{aligned} \tag{18}$$

From Eq. (18), we see that the $\alpha = 0$ pole contribution is given by the difference between the full scattering amplitude and the contribution to the scattering amplitude from Regge plus Pomeron terms.

A. Numerical results

To compute the integral over the photoabsorption cross section, we parametrize the hadronic cross-section by a sum of up to 6 Breit-Wigner resonances plus a smooth background:

$$\sigma(\nu) = \sum_{i=1}^{6}\frac{M_i^2}{s}\frac{\sigma_i M_i^2\Gamma_{\mathrm{tot},i}\Gamma_{\gamma,i}}{\left(s-M_i^2\right)^2 + M_i^2\Gamma_{\mathrm{tot},i}^2} + \sigma^{\mathrm{Back}}(\nu). \tag{19}$$

In Eq. (19), $s = M^2 + 2M\nu$ is the square of the c.m. energy. In order of increasing mass, we account for the following resonances: $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1665)$, $F_{15}(1680)$, and $F_{37}(1950)$. We use energy-dependent widths,

$$\begin{aligned}\Gamma_{\gamma,i} &= \Gamma_i\left[\frac{1+X^2/K_i^2}{1+X^2/K^2}\right]^{J_\gamma}\\ \Gamma_{\mathrm{tot},i} &= \Gamma_i\frac{q}{q_i}\left[\frac{1+X^2/q_i^2}{1+X^2/q^2}\right]^{l},\end{aligned} \tag{20}$$

where K and q are, respectively, the momenta of the photon and single pion decay in the c.m. frame, and K_i and q_i refer to their values at the resonance position $\sqrt{s} = M_i$. J_γ is the spin of the resonance, l is the angular momentum of the decay products, and Γ_i is the intrinsic width of the ith resonance. The damping parameter X was set to $X = 0.15$ GeV for the $P_{33}(1232)$ and we chose $X = 0.35$ GeV for all other resonances.

The background $\sigma^{\mathrm{Back}}(\nu)$ in Eq. (19) is chosen so that it explicitly matches onto the Regge plus pomeron cross section $\sigma^{R+P}(\nu)$ of Eq. (12),

$$\sigma^{\mathrm{Back}}(\nu) = \left[1 - e^{-\frac{2(\nu-\nu_\pi)}{M}}\right]\sigma^{R+P}(\nu). \tag{21}$$

The threshold prefactor introduced above ensures that the background cross section vanishes at pion production threshold, and we use the form proposed in Ref. [23]. It can be seen that with this form, for $\nu \geqslant 2$ GeV,

$$\frac{\sigma(\nu) - \sigma^{R+P}(\nu)}{\sigma(\nu)} \ll 2\%, \tag{22}$$

and within the experimental errors is negligibly small. This justifies the choice of the parameter $E = 2$ GeV in Eqs. (15), (17), and (18).

Rather than using the Regge intercepts as free parameters, we take those intercepts from fits to photoabsorption data on the proton [27]. This is partly motivated by the limited energy range over which nuclear data are available. In Fig. 3 we display the results of the cross section fits using Eq. (19) (solid lines) along with the Regge + Pomeron background (dashed lines). The parameters of the background cross sections are listed in Table I for each nuclear target.

We note that, in general, due to nuclear effects such as resonance broadening and Fermi motion, the division of the cross section into resonance plus background becomes somewhat ambiguous. Examining Fig. 3, we notice that for heavier nuclei, as opposed to the proton and to some degree the deuteron, there is no clear resonance structure around and above $\nu = 1$ GeV. For heavier nuclei, the effect of broadening and overlapping of resonances can alternatively be reproduced by enhancing the Reggeon strength c_R; thus, the strength of the Regge background is sensitively correlated with the choice of the resonance parameters.

Uncertainties in identification of the Pomeron and Reggeon background for nuclear targets produces error bars for those parameters that are significantly larger than those parameters for the proton. In particular, for ^{27}Al the highest energy data available are as low as 10 GeV and the corresponding fit does not allow for a precise determination of the background parameters, and this is reflected by the large errors in Table I. Our fits provide a reduced χ^2 per degree of freedom of order one, except for the Aluminum target, where it was greater than two.

In Table II we list the numerical values of the various contributions to the constituent quark sum rule in Eq. (17) and the $\alpha = 0$ pole contribution given in Eq. (18). A comparison of theoretical and experimental contributions to the finite energy sum rules and the $\alpha = 0$ pole contribution are displayed in Fig. 4. The upper panel shows the comparison of data in the nuclear energy range with the TRK sum rule for four nuclear targets (we plot the fraction of the TRK sum rule for each nucleus); the middle panel shows the comparison of data in the nuclear and hadronic energy region with the predictions from our new constituent quark model or CQM sum rule (this is given by the fourth and fifth rows of Table II); finally, the lower panel shows the predictions of dispersion relations for the value of the $\alpha = 0$ pole for the six nuclear targets in comparison with the corresponding Thomson term values (the final and second-to-last row in Table II, respectively).

We observe that the CQM sum rule is better obeyed for heavier nuclei than for the proton or deuteron. One possible explanation could be that this sum rule amounts to counting the effective number of quarks within the target;

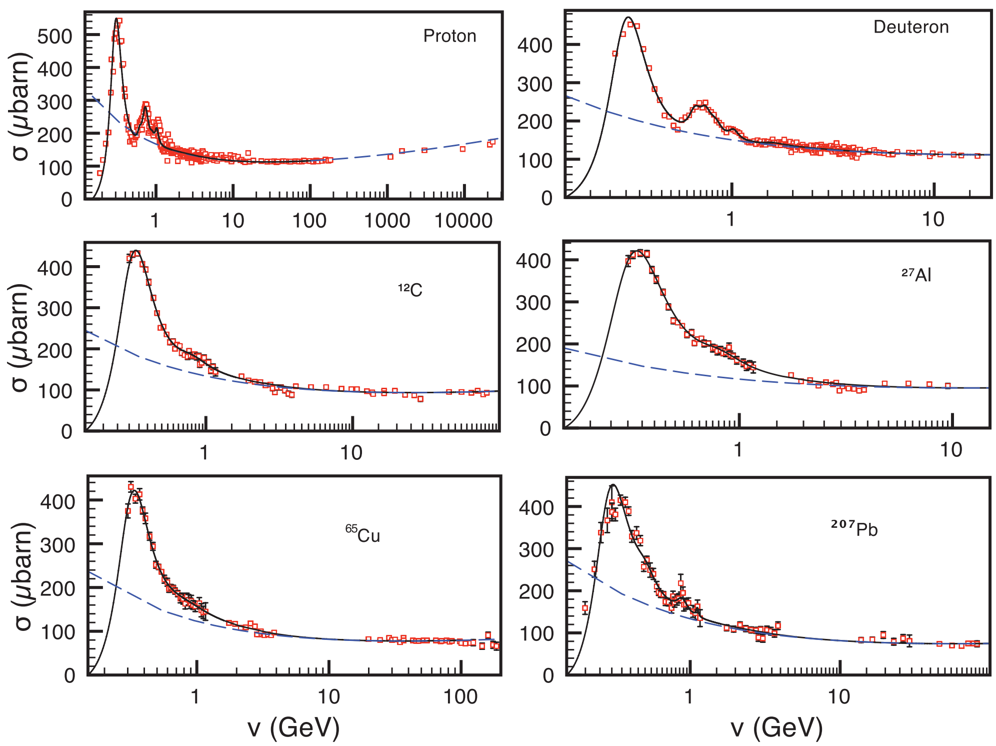

FIG. 3. (Color online) High energy photoabsorption cross sections per nucleon for six nuclear targets compared to the fit results (solid lines) using the Breit-Wigner resonance plus background pametrization of Eq. (19). Data are from Ref. [26] for the proton and the deuteron, and from Refs. [21–23] for heavier nuclei. The Regge plus Pomeron curves are shown by dashed lines. The background fit parameters are given in Table I.

this relies on a mean-field approach to the target, which we would expect to become more accurate as the number of target nucleons increases. For the $\alpha = 0$ pole contribution, our new result for the proton is significantly different from the Thomson term, which is at variance with the original result of Damashek and Gilman [5]. This discrepancy is due to our use of the very high energy photoabsorption data that has become available only recently [27]. As a result, instead of the high-energy parametrization used in Ref. [5],

$$\sigma^{R+P}(\nu) \approx \left(96.6 + 70.2\sqrt{\frac{1\,\text{GeV}}{\nu}}\right)\mu\text{b}, \qquad (23)$$

we find

$$\sigma^{R+P}(\nu) \approx \left[68.0\left(\frac{\nu}{1\,\text{GeV}}\right)^{0.097} + 99.0\sqrt{\frac{1\,\text{GeV}}{\nu}}\right]\mu\text{b}. \qquad (24)$$

At an energy $\nu = 1$ GeV, both formulas give almost identical results, but at high energies they differ dramatically. At the same time, the data in the resonance region have not changed much, so this leads to our new value for the $\alpha = 0$ contribution to photoabsorption on the proton.

For heavier nuclei, however, the bottom panel of Fig. 4 and the final row of Table II show that the $\alpha = 0$ contribution appears to be consistent with the Thomson term. This result is due to an interplay of various nuclear effects in the resonance region that affect the value of the integrated photoabsorption cross section and also shadowing at medium-to-high energies. Shadowing at energies below $\nu = 200$ GeV causes the value of c_P to decrease from 68 μb for the proton to approximately 43 μb for lead, respectively. On the other hand, the Pomeron is a QCD phenomenon that is due to the interaction of quarks and gluons and should be the leading mechanism of photoabsorption at extremely high energies. It can be expected that at asymptotic energies nuclear effects should be negligible, and the strength of the Pomeron should be the same for both the proton and heavier nuclei. If in the future nuclear photoabsorption data above $\nu = 200$ GeV becomes available, they could shed more light on the asymptotic behavior of

TABLE I. Reggeon and Pomeron parameters in μb

	Proton	Deuteron	$^{12}_{6}$C	$^{27}_{13}$Al	$^{65}_{29}$Cu	$^{207}_{82}$Pb
c_P (μb)	68.0 ± 0.2	70.08 ± 1.26	57.24 ± 1.13	62.70 ± 6.0	45.88 ± 0.57	42.08 ± 1.96
c_R (μb)	99.0 ± 1.15	80.50 ± 2.27	76.49 ± 4.40	53.53 ± 11.6	76.95 ± 3.60	91.43 ± 9.14

TABLE II. Contributions to the finite energy sum rule for selected targets in units of GeV·μb. The entries in the second row are taken from a review on nuclear data in Ref. [24].

	Proton	Deuteron	$^{12}_{6}$C	$^{27}_{13}$Al	$^{65}_{29}$Cu	$^{207}_{82}$Pb
$\frac{1}{2\pi^2 A}\sigma_{\rm int}^{\rm had}$	18.60 ± 0.31	17.46 ± 0.51	16.80 ± 0.62	16.54 ± 1.50	16.16 ± 0.57	16.57 ± 1.02
$\frac{1}{2\pi^2 A}\sigma_{\rm int}^{\rm nucl}$	–	–	0.197	0.30	0.480	0.69
$\frac{1}{2\pi^2}c_R\frac{(E/GeV)^{1/2}}{1/2}$	14.19 ± 0.16	11.54 ± 0.39	10.96 ± 0.63	7.67 ± 1.66	11.03 ± 0.52	13.10 ± 1.31
r.h.s. of Eq. (17)	-4.21 ± 0.35	-5.92 ± 0.65	-6.04 ± 0.88	-9.17 ± 2.24	-5.61 ± 0.77	-4.16 ± 1.66
$-\left(2+\frac{ZN}{A^2}\right)\frac{\alpha}{M}$	-6.06	-6.82	-6.82	-6.82	-6.81	-6.78
$\frac{1}{2\pi^2}c_P(E/GeV)$	6.72 ± 0.02	6.92 ± 0.12	5.65 ± 0.11	6.19 ± 0.59	4.53 ± 0.06	4.16 ± 0.25
$-\frac{Z^2}{A^2}\frac{\alpha}{M}$	-3.03	-0.76	-0.76	-0.70	-0.60	-0.48
${\rm Re}T^{\alpha=0}$	-0.72 ± 0.35	0.25 ± 0.65	-1.14 ± 0.89	-3.68 ± 2.31	-1.71 ± 0.77	-0.48 ± 1.68

the forward nuclear Compton amplitude and could remove uncertainties regarding the strength of the Pomeron, Reggeon, and $\alpha = 0$ pole contributions.

Finally, in addition to the paper by Damashek and Gilman [5], there have been other evaluations of the $\alpha = 0$ pole for forward Compton scattering. Dominguez, Ferro Fontan, and Suaya [11] and Shibasaki, Minamikawa, and Watanabe [12] used a similar approach to that of Ref. [5] and independently arrived at a qualitatively similar result,

$${\rm Re}T_p^{\alpha=0} = (-3 \pm 2)\mu{\rm b\,GeV}, \tag{25}$$

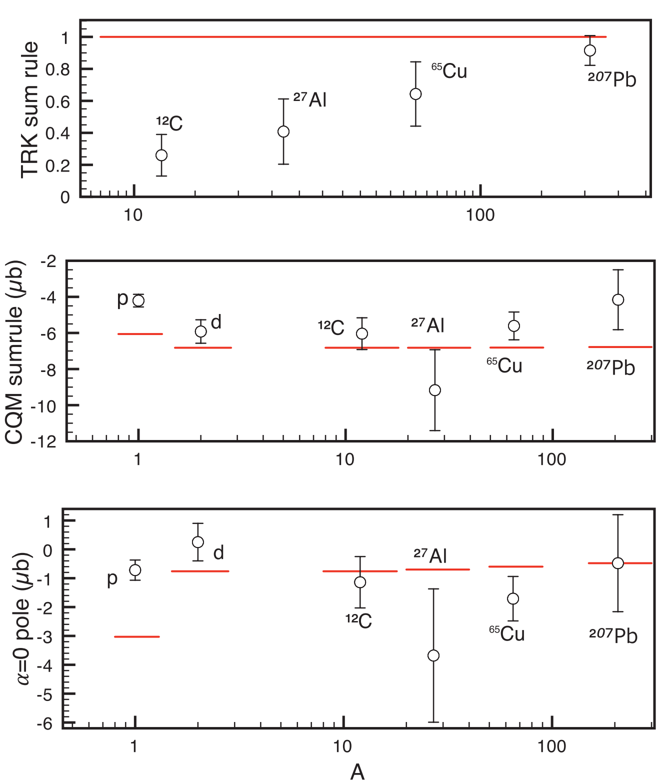

FIG. 4. (Color online) Upper panel: the fraction of the TRK sum rule for nuclear targets ^{12}C, ^{27}Al, ^{65}Cu, and ^{207}Pb; middle panel: experimental values (data points) vs. theoretical expectation (dotted line) for our new constituent quark model (CQM) sum rule for the proton, deuteron, ^{12}C, ^{27}Al, ^{65}Cu, and ^{207}Pb, in units of μb; lower panel: results for the $\alpha = 0$ pole for all targets considered, in μb.

where the uncertainty is dominated by the parameters of the high-energy fit, reflecting the limited range of high-energy data available at that time.

In Ref. [15], Dominguez, Gunion, and Suaya extended this analysis by including the deuteron photoabsorption data. They employed a model for nuclear effects to extract parameters of the neutron from deuteron and proton data and evaluated the finite energy sum rules (FESR) for both nucleons. Their conclusions were that the $\alpha = 0$ pole is consistent with the respective Thomson term for both,

$$\begin{aligned}{\rm Re}T_n^{\alpha=0} &= (0 \pm 1.5)\mu{\rm b\,GeV},\\ {\rm Re}T_p^{\alpha=0} &= (-3 \pm 0.8)\mu{\rm b\,GeV},\end{aligned} \tag{26}$$

where ${\rm Re}T_{p(n)}^{\alpha=0}$ refers to the proton (neutron), respectively. Tait and White in Ref. [14] re-analyzed the FESR using a more recent data set and obtained a much more conservative estimate:

$${\rm Re}T_p^{\alpha=0} = \left(-3^{+4}_{-5}\right)\mu{\rm b\,GeV}. \tag{27}$$

Based on the recent proton data on photoabsorption at very high energies [27] and the analysis of Tait and White [14], we conclude that the errors in Eq. (26) were significantly underestimated.

IV. SUMMARY AND CONCLUSIONS

In summary, we revisited the finite energy sum rules for forward real Compton scattering on the proton and heavier nuclei. As the photon energy increases and its wavelength decreases, the Compton amplitude becomes sensitive to progressively smaller features of a nuclear target. At the lowest energies, the Compton amplitude is determined by scattering on the target as a whole, whereas in the high-energy limit it is expected to be determined by scattering on elementary target constituents.

Finite energy sum rules provide a qualitative comparison between the high-energy and low-energy limits of the scattering amplitude. For nuclei, the Thomas-Reiche-Kuhn sum rule relates the strength of the giant dipole resonance to the difference between the nuclear Thomson term and the incoherent sum of Thomson terms of protons residing in the nucleus. In a similar fashion, we have proposed a new sum rule that describes the integrated strength of the nucleon resonances as a difference between the nuclear Thomson term and the incoherent sum of Thomson terms from constituent quarks residing in the target. In this process it is crucial to separate the Reggeon and Pomeron high-energy contributions to the FESR, and we call this new sum rule the "constituent quark model" or CQM sum rule.

We analyzed the TRK and CQM sum rules for the proton, deuteron, ^{12}C, ^{27}Al, ^{65}Cu, and ^{207}Pb targets. All nuclear data are consistent with the CQM sum rule; however, for the proton the comparison is not as favorable. This may be explained by the fact that in a nucleus, the errors due to various systematic effects are averaged over a large number of constituent quarks and thus may be statistically less significant than for the proton.

Theoretical arguments suggest that Compton scattering amplitudes at high energies should contain an energy-independent constant that corresponds to a Regge pole at $\alpha = 0$ [2–10]. Previous attempts to extract this constant obtained results consistent with this amplitude being approximately equal to the value at low energies, i.e., the Thomson term.

We were able to demonstrate that high-energy photoabsorption data on the proton confirms that the $\alpha = 0$ pole and the Thomson term, $\mathrm{Re}T_p(0) = -3.03\,\mu$b GeV are significantly different:

$$\mathrm{Re}T_p^{\alpha=0} = (-0.72 \pm 0.35)\,\mu\mathrm{b\,GeV}. \tag{28}$$

The difference between our result and the value consistent with the Thomson term from previous analyses [5,11,12,14] is due to the recent high-energy photoabsorption data [27], which changes the Regge plus Pomeron contribution from Eq. (23), which was used in Ref. [5], to our background form, Eq. (24). With this form for the background amplitude, we extract the new value for the $\alpha = 0$ pole that differs significantly from the value of the Thomson term.

We extended this analysis to a number of nuclear targets. In the case of the sum rule for the $\alpha = 0$ pole, only the proton result is unambiguously distinct from the Thomson term, whereas for other targets the result was consistent with the Thomson term within the experimental errors. Our results are relevant to the question of the A-dependence of the Pomeron contribution. The Pomeron in QCD is isospin-independent, and for asymptotically high energies, one generally expects that the Pomeron contribution from a free proton should equal the average nucleon Pomeron contribution in a nucleus. Current nuclear data only extend up to ~200 GeV, and at these energies shadowing effects are responsible for suppressing the Pomeron contribution by some 30% relative to the proton value. It is an open question whether future nuclear photoabsorption data at higher energies similar to those currently available for the proton will tend to bring the Pomeron strength per nucleon up to the free proton value.

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