Energy-level displacement of excited *np* states of kaonic deuterium in a Faddeev-equation approach

M. Faber,[1] M. P. Faifman,[2] A. N. Ivanov,[1,*] J. Marton,[3] M. Pitschmann,[1,4,5] and N. I. Troitskaya[6]

[1]*Atominstitut, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Wien, Austria*
[2]*National Research Centre "Kurchatov Institute," Moscow 123182, Russian Federation*
[3]*Stefan Meyer Institut für subatomare Physik, Österreichische Akademie der Wissenschaften, Boltzmanngasse 3, A-1090, Wien, Austria*
[4]*Department of Physics, University of Wisconsin–Madison, 1150 University Avenue, Madison, Wisconsin 53706, USA*
[5]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[6]*State Polytechnic University of St. Petersburg, Polytechnicheskaya 29, 195251 St. Petersburg, Russian Federation*

We calculate the energy-level displacement of the excited *np* state of kaonic deuterium in terms of the *p*-wave scattering length of $K^- d$ scattering. We solve the Faddeev equations for the amplitude of $K^- d$ scattering in the fixed-center approximation and derive the complex *p*-wave scattering length of $K^- d$ scattering in terms of the *s*-wave and *p*-wave scattering lengths of $\bar{K}N$ scattering. The estimated uncertainty of the complex *p*-wave scattering length is of about 15 %. For the calculated width $\Gamma_{2p} = 10.203$ meV of the excited 2*p* state of kaonic deuterium we evaluate the yield $Y_{K^- d} = 0.27\%$ of x rays for the K_α emission line of kaonic deuterium. Using the complex *s*-wave and *p*-wave scattering lengths of $\bar{K}N$ scattering, calculated in B. Borasoy, R. Nißler, and W. Weise [Eur. Phys. J. A **25**, 79 (2005)] and W. Weise and R. Härtle [Nucl. Phys. A **804**, 173 (2008)], we get the width $\Gamma_{2p} = 2.675$ meV of the excited 2*p* state and the yield $Y_{K^- d} = 1.90\%$ of x rays for the K_α emission line of kaonic deuterium. The results obtained in this paper can be used for planning experiments on the measurements of the energy-level displacement of the ground state of kaonic deuterium, caused by strong low-energy interactions.

I. INTRODUCTION

The consistent analysis of the complex *s*-wave scattering length $\tilde{a}^{(0)}_{K^- d}$ of $K^- d$ scattering has been carried out in [1] by means of the solution of the Faddeev equations in the fixed-center approximation. The complex *s*-wave scattering length $\tilde{a}^{(0)}_{K^- d}$ of $K^- d$ scattering has been expressed in terms of the complex *s*-wave scattering lengths of $\bar{K}N$ scattering [1] as

follows:

$$\tilde{a}^{(0)}_{K^- d} = \frac{m_d}{m_K + m_d} \int d^3 x |\Phi_d(\vec{r})|^2 \hat{A}^{(0)}_{K^- d}(r), \qquad (1)$$

where $\Phi_d(\vec{r})$ is the wave function of the deuteron in the ground state [2]. The complex function $\hat{A}^{(0)}_{K^- d}(r)$ is defined by $\hat{A}^{(0)}_{K^- d}(r) = \hat{A}^{(0)}_p(r) + \hat{A}^{(0)}_n(r)$. The functions $\hat{A}^{(0)}_p(r)$ and $\hat{A}^{(0)}_n(r)$ are the solutions of the Faddeev equations in the fixed-center approximation. They are equal to [1]

$$\hat{A}^{(0)}_p(r) = \frac{\hat{a}^{(0)}_p + \dfrac{\hat{a}^{(0)}_p \hat{a}^{0(0)}_n}{r} + \dfrac{\hat{a}^{(0)}_p \hat{a}^{(0)}_n - (\hat{a}^{(0)}_x)^2}{r} + \dfrac{\hat{a}^{(0)}_n [\hat{a}^{(0)}_p \hat{a}^{(0)}_n - (\hat{a}^{(0)}_x)^2]}{r^2}}{1 + \dfrac{\hat{a}^{0(0)}_n}{r} - \dfrac{\hat{a}^{(0)}_p \hat{a}^{(0)}_n}{r^2} - \dfrac{\hat{a}^{(0)}_n [\hat{a}^{(0)}_p \hat{a}^{0(0)}_n - (\hat{a}^{(0)}_x)^2]}{r^3}},$$

$$\hat{A}^{(0)}_n(r) = \hat{a}^{(0)}_n + \frac{\hat{a}^{(0)}_n}{r} \hat{A}^{(0)}_p(r), \qquad \hat{A}^{x(0)}_n(r) = \frac{\hat{a}^{(0)}_x + \dfrac{\hat{a}^{(0)}_x \hat{a}^{(0)}_n}{r} + \dfrac{\hat{a}^{(0)}_x \hat{a}^{(0)}_n}{r^2} \hat{A}^{(0)}_p(r)}{1 + \dfrac{\hat{a}^{0(0)}_n}{r}}, \qquad (2)$$

where the complex *s*-wave scattering lengths of $\bar{K}N$ scattering $\hat{a}^{(0)}_p$, $\hat{a}^{(0)}_n$, $\hat{a}^{(0)}_x$, and $\hat{a}^{0(0)}_n$ are defined by [1]

$$\hat{a}^{(0)}_p = \left(1 + \frac{m_K}{m_N}\right) \tilde{a}_{K^- p}(K^- p), \qquad \hat{a}^{(0)}_n = \left(1 + \frac{m_K}{m_N}\right) \tilde{a}_{K^- n}(K^- n),$$

$$\hat{a}^{(0)}_x = \left(1 + \frac{m_K}{m_N}\right) \tilde{a}_{K^- p}(\bar{K}^0 n), \qquad \hat{a}^{0(0)}_n = \left(1 + \frac{m_K}{m_N}\right) \tilde{a}_{\bar{K}^0 n}(\bar{K}^0 n). \qquad (3)$$

According to [1], the Faddeev equations for the complex *s*-wave scattering length of $K^- d$ scattering length take the form

$$T^{(0)}_p = t^{(0)}_p + t^{(0)}_p G_0 T^{(0)}_n + t^{(0)}_p G_0 T^{x(0)}_n,$$

$$T^{(0)}_n = t^{(0)}_n + t^{(0)}_n G_0 T^{(0)}_p, \qquad (4)$$

$$T^{x(0)}_n = t^{x(0)}_n + t^{0(0)}_n G_0 T^{x(0)}_n + t^{x(0)}_n G_0 T^{(0)}_n.$$

As has been pointed out in [1], these equations describe pure elastic and charge-exchange processes and require as input

*ivanov@kph.tuwien.ac.at

only the amplitudes and propagators, where G_0 is the free kaon propagator, $t_p^{(0)}$ and $t_n^{(0)}$ are the $\mathbb{T}$ matrices for $K^- p$ and $K^- n$ elastic scattering, respectively, and $t_n^{0(0)}$ is the $\mathbb{T}$ matrix of $\bar{K}^0 n$ scattering $\bar{K}^0 n \to \bar{K}^0 n$. For the proton partition $T_p^{(0)}$ there is also a contribution from the charge-exchange channel $K^- p \to \bar{K}^0 n$ with the elementary $\mathbb{T}$ matrices $T_p^{x(0)}$ and $T_n^{x(0)}$, describing $\bar{K}^0 nn \to K^- pn$ transition, including the multiple rescattering in the intermediate inelastic states. In the approximation, proposed in [1], $t_p^{x(0)} = t_n^{x(0)}$ and, correspondingly, $T_n^{x(0)} = T_p^{x(0)}$.

In this paper, following the technique developed in [1], we derive the Faddeev equations for the p-wave amplitude of $K^- d$ scattering. We solve these equations in the fixed-center approximation and calculate the complex p-wave scattering length of $K^- d$ scattering. We express the energy-level displacement of the excited np state of kaonic deuterium in terms of the complex p-wave scattering length of $K^- d$ scattering.

This paper is organized as follows. In Sec. II we derive the Faddeev equations for the p-wave amplitude of $K^- d$ scattering. Solving the Faddeev equations in the fixed-center approximation, we obtain the complex p-wave scattering length of $K^- d$ scattering in terms of the complex s-wave and p-wave scattering lengths of $\bar{K} N$ scattering. The numerical values of the complex s-wave and p-wave scattering lengths of $\bar{K} N$ scattering are given in Sec. III. The calculation of the complex s-wave and p-wave scattering lengths of $\bar{K} N$ scattering is carried out within the SU(3) coupled-channel approach and chiral Lagrangians with derivative meson-baryon couplings invariant under chiral SU(3) $\times$ SU(3) symmetry. In Sec. IV we calculate the numerical values of the complex s-wave and p-wave scattering lengths of $K^- d$ scattering and the energy-level displacement of the kaonic deuterium in the excited np state, caused by strong low-energy interactions. We give the numerical values for the s-wave and p-wave scattering lengths of $K^- d$ scattering and the energy-level displacements of the ground $1s$ state and the excited $2p$ state of kaonic deuterium. The complex s-wave scattering length of $K^- d$ scattering is in reasonable agreement with the results obtained in [1]. In Sec. V, using our prediction for the

width $\Gamma_{2p} = 10.203$ meV of the excited $2p$ state of kaonic deuterium and the quantum-classical Monte Carlo cascade model, developed in [3], we calculate the yield $Y_{K^- d} = 0.27\%$ of x rays for the K_α emission line of kaonic deuterium. In the Conclusion we discuss the obtained results and the estimate of the uncertainty of our solution for the complex p-wave scattering length of $K^- d$ scattering, which is about 15%. We calculate the complex s-wave and p-wave scattering lengths of $K^- d$ scattering and the energy-level displacements of the ground $1s$ and excited $2p$ states of kaonic deuterium for the complex s-wave and p-wave scattering lengths obtained in [4,5]. We get $\Gamma_{2p} = 2.675$ meV for the width of the excited $2p$ state and $Y_{K^- d} = 1.90\%$ for the yield of x rays for the K_α emission line of kaonic deuterium. In Appendix A we give a detailed calculation of the contributions of the single (*impulse*) and double scattering to the complex p-wave scattering length of $K^- d$ scattering. In Appendix B we outline the calculation of the complex s-wave and p-wave scattering lengths of $\bar{K} N$ scattering.

II. FADDEEV EQUATIONS FOR p-WAVE AMPLITUDE OF $K^- d$ SCATTERING AND p-WAVE SCATTERING LENGTH OF $K^- d$ SCATTERING IN THE FIXED-CENTER APPROXIMATION

For the p-wave scattering $\mathbb{T}$ matrices we use the index (1). This defines $t_p^{(1)}$, $t_n^{(1)}$, $t_n^{x(1)}$, $t_n^{0(1)}$, $T_p^{(1)}$, $T_n^{(1)}$, and $T_n^{x(1)}$, respectively. In this notation the Faddeev equations for the p-wave amplitude of $K^- d$ scattering read

$$
\begin{aligned}
T_p^{(1)} &= t_p^{(1)} + t_p^{(1)} G_0 T_n^{(0)} + t_p^{(0)} G_0 T_n^{(1)} + t_p^{x(1)} G_0 T_n^{x(0)} \\
&\quad + t_p^{x(0)} G_0 T_n^{x(1)}, \\
T_n^{(1)} &= t_n^{(1)} + t_n^{(1)} G_0 T_p^{(0)} + t_n^{(0)} G_0 T_p^{(1)}, \\
T_n^{x(1)} &= t_n^{x(1)} + t_n^{0(1)} G_0 T_n^{x(0)} + t_n^{0(0)} G_0 T_n^{x(1)} + t_n^{x(1)} G_0 T_n^{(0)} \\
&\quad + t_n^{x(0)} G_0 T_n^{(1)}.
\end{aligned} \tag{5}
$$

In the fixed-center approximation the Faddeev equations (5) reduce to the system of algebraical equations for the amplitudes $T_p^{(1)} \to A_p^{(1)}(r)$, $T_n^{(1)} \to A_n^{(1)}(r)$, and $T_n^{x(1)} \to A_n^{x(1)}(r)$:

$$
\hat{A}_p^{(1)}(r) = \hat{a}_p^{(1)} + \frac{1}{6}\hat{a}_p^{(1)}\frac{1}{r}\hat{A}_n^{(0)}(r) + \frac{1}{6}\hat{a}_p^{(0)}\frac{1}{r}\hat{A}_n^{(1)}(r) - \frac{1}{6}\hat{a}_x^{(1)}\frac{1}{r}\hat{A}_n^{x(0)}(r) - \frac{1}{6}\hat{a}_x^{(0)}\frac{1}{r}\hat{A}_n^{x(1)}(r),
$$

$$
\hat{A}_n^{(1)}(r) = \hat{a}_n^{(1)} + \frac{1}{6}\hat{a}_n^{(1)}\frac{1}{r}\hat{A}_p^{(0)}(r) + \frac{1}{6}\hat{a}_n^{(0)}\frac{1}{r}\hat{A}_p^{(1)}(r), \tag{6}
$$

$$
\hat{A}_n^{x(1)}(r) = \hat{a}_x^{(1)} - \frac{1}{6}\hat{a}_n^{0(1)}\frac{1}{r}\hat{A}_n^{x(0)}(r) - \frac{1}{6}\hat{a}_n^{0(0)}\frac{1}{r}\hat{A}_n^{x(1)}(r) + \frac{1}{6}\hat{a}_x^{(1)}\frac{1}{r}\hat{A}_n^{(0)}(r) + \frac{1}{6}\hat{a}_x^{(0)}\frac{1}{r}\hat{A}_n^{(1)}(r),
$$

where $\hat{a}_p^{(1)}$, $\hat{a}_n^{(1)}$, and $\hat{a}_x^{(1)}$ are the complex p-wave scattering lengths of $\bar{K} N$ scattering, defined by analogy with $a_p^{(0)}$, $a_n^{(0)}$, and $a_x^{(0)}$ of Eq. (3). The p-wave scattering length of $K^- d$ scattering is equal to

$$
\tilde{a}_{K^- d}^{(1)} = \frac{m_d}{m_K + m_d} \int d^3 x |\Phi_d(\vec{r})|^2 \hat{A}_{K^- d}^{(1)}(r), \tag{7}
$$

where $\hat{A}_{K^-d}^{(1)}(r) = \hat{A}_p^{(1)}(r) + \hat{A}_n^{(1)}(r)$. The amplitudes $\hat{A}_p^{(1)}(r)$ and $\hat{A}_n^{(1)}(r)$ are the solutions of Eq.(6). They are equal to

$$
\hat{A}_p^{(1)}(r)\left(1 + \frac{1}{6}\frac{\hat{a}_n^{0(0)}}{r} - \frac{1}{36}\frac{\hat{a}_n^{(0)}\hat{a}_p^{(0)}}{r^2} - \frac{1}{216}\frac{\hat{a}_n^{(0)}\left[\hat{a}_p^{(0)}\hat{a}_n^{0(0)} - \left(\hat{a}_x^{(0)}\right)^2\right]}{r^3}\right) = \hat{a}_p^{(1)} + \frac{1}{6}\frac{\hat{a}_p^{(1)}\hat{a}_n^{0(0)}}{r} + \frac{1}{6}\frac{\hat{a}_n^{(1)}\hat{a}_p^{(0)}}{r} - \frac{1}{6}\frac{\hat{a}_x^{(1)}\hat{a}_x^{(0)}}{r}
$$

$$
+ \frac{1}{36}\frac{\hat{a}_n^{(1)}\left[\hat{a}_p^{(0)}\hat{a}_n^{0(0)} - \left(\hat{a}_x^{(0)}\right)^2\right]}{r^2} + \frac{1}{6}\frac{\hat{a}_p^{(1)}}{r}\hat{A}_n^{(0)}(r) - \frac{1}{6}\frac{\hat{a}_x^{(1)}}{r}\hat{A}_n^{x(0)}(r) + \frac{1}{36}\frac{\hat{a}_p^{(1)}\hat{a}_n^{0(0)}}{r^2}\hat{A}_n^{(0)}(r) + \frac{1}{36}\frac{\hat{a}_n^{(1)}\hat{a}_p^{(0)}}{r^2}\hat{A}_p^{(0)}(r)
$$

$$
+ \frac{1}{36}\frac{\hat{a}_n^{0(1)}\hat{a}_x^{(0)}}{r^2}\hat{A}_n^{x(0)}(r) - \frac{1}{36}\frac{\hat{a}_x^{(1)}\hat{a}_n^{0(0)}}{r^2}\hat{A}_n^{x(0)}(r) - \frac{1}{36}\frac{\hat{a}_x^{(1)}\hat{a}_x^{(0)}}{r^2}\hat{A}_n^{(0)}(r) + \frac{1}{216}\frac{\hat{a}_n^{(1)}\left[\hat{a}_p^{(0)}\hat{a}_n^{0(0)} - \left(\hat{a}_x^{(0)}\right)^2\right]}{r^3}\hat{A}_p^{(0)}(r),
$$

$$
\hat{A}_n^{(1)}(r) = \hat{a}_n^{(1)} + \frac{1}{6}\frac{\hat{a}_n^{(1)}}{r}\hat{A}_p^{(0)}(r) + \frac{1}{6}\frac{\hat{a}_n^{(0)}}{r}\hat{A}_p^{(1)}(r), \tag{8}
$$

where $\hat{A}_p^{(0)}(r)$, $\hat{A}_n^{(0)}(r)$, and $\hat{A}_n^{x(0)}(r)$ are solutions of the Faddeev equations in the fixed-center approximation for the complex s-wave scattering length of K^-d scattering equation (2) [1].

Expanding the amplitudes $\hat{A}_p^{(1)}(r)$ and $\hat{A}_n^{(1)}(r)$ in powers of $1/r$ and keeping only the terms of order of $1/r$, one arrives at the complex p-wave scattering length of K^-d scattering in the single- and double-scattering approximations:

$$
\tilde{a}_{K^-d}^{(1)} = \frac{m_d}{m_K + m_d}\left[\hat{a}_p^{(1)} + \hat{a}_n^{(1)} + \frac{1}{3}\left(\hat{a}_p^{(1)}\hat{a}_n^{(0)} + \hat{a}_n^{(1)}\hat{a}_p^{(0)}\right.\right.
$$
$$
\left.\left. - \hat{a}_x^{(1)}\hat{a}_x^{(0)}\right)\int\frac{d^3x}{r}|\Phi_d(\vec{r})|^2 + \ldots\right]. \tag{9}
$$

This result is confirmed in Appendix A by a direct calculation in the effective low-energy quantum field theory.

III. COMPLEX s-WAVE AND p-WAVE SCATTERING LENGTHS OF $\bar{K}N$ SCATTERING

For the evaluation of the numerical value of the complex p-wave scattering length of K^-d scattering and the energy-level displacement of the excited np state of kaonic deuterium, we have to calculate the numerical values of the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering. A detailed procedure of the calculation of the complex s-wave and p-wave scattering lengths is given in Appendix B. Their numerical values are equal to

$$
\begin{aligned}
\tilde{a}_{K^-p}^{(0)}(K^-p) &= -0.680 + i0.639 \text{ fm}, & \tilde{a}_{K^-p}^{(1)}(K^-p) &= -0.069 + i0.179 \text{ fm}^3, \\
\tilde{a}_{K^-p}^{(0)}(\bar{K}^0n) &= +0.980 - i0.543 \text{ fm}, & \tilde{a}_{K^-p}^{(1)}(\bar{K}^0n) &= -0.053 + i0.176 \text{ fm}^3, \\
\tilde{a}_{K^-n}^{(0)}(K^-n) &= +0.300 + i0.096 \text{ fm}, & \tilde{a}_{K^-n}^{(1)}(K^-n) &= -0.122 + i0.355 \text{ fm}^3, \\
\tilde{a}_{\bar{K}^0n}^{(0)}(\bar{K}^0n) &= -0.680 + i0.639 \text{ fm}, & \tilde{a}_{\bar{K}^0n}^{(1)}(\bar{K}^0n) &= -0.069 + i0.179 \text{ fm}^3.
\end{aligned} \tag{10}
$$

We have calculated the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering within the SU(3) coupled-channel approach [4], chiral dynamics with chiral SU(3) $\times$ SU(3) invariant low-energy meson-baryon interactions with derivative couplings [4,6,7], and accounting for the contributions of baryon resonances [8] and scalar-meson resonances [9,10]. The chiral Lagrangian of low-energy interactions of the ground-state baryon octet $B(x)$ with the octet of pseudoscalar mesons $P(x)$ invariant under SU(3) $\times$ SU(3) chiral symmetry is [4]

$$
\begin{aligned}
\mathcal{L}(x) &= \langle\bar{B}(x)(i\gamma^\mu\partial_\mu - m_0)B(x)\rangle + \langle\bar{B}(x)i\gamma^\mu[s_\mu(x), B(x)]\rangle - g_A(1-\alpha_D)\langle\bar{B}(x)\gamma^\mu[p_\mu(x), B(x)]\rangle \\
&\quad + \alpha_D\langle\bar{B}(x)\gamma^\mu\{p_\mu(x), B(x)\}\rangle + \frac{1}{4}b_D\langle\bar{B}(x)\{\chi_+(x), B(x)\}\rangle + \frac{1}{4}b_F\langle\bar{B}(x)[\chi_+(x), B(x)]\rangle + \frac{1}{4}b_0\langle\bar{B}(x)\langle\chi_+(x)\rangle B(x)\rangle \\
&\quad + \frac{1}{2}d_1\langle\bar{B}(x)\{p_\mu(x), [p^\mu(x), B(x)]\}\rangle + \frac{1}{2}d_2\langle\bar{B}(x)[p_\mu(x), [p^\mu(x), B(x)]]\rangle + \frac{1}{2}d_3\langle\bar{B}(x)p_\mu(x)\rangle\langle p^\mu(x)B(x)\rangle \\
&\quad + \frac{1}{2}d_4\langle\bar{B}(x)\langle p_\mu(x)p^\mu(x)\rangle B(x)\rangle + \ldots, \\
s_\mu(x) &= \frac{1}{2}[U^\dagger(x), \partial_\mu U(x)], \quad p_\mu(x) = \frac{1}{2i}\{U^\dagger(x)\partial_\mu U(x)\}, \quad \chi_+(x) = 2B_0(U^\dagger(x)\mathcal{M}U^\dagger(x) + U(x)\mathcal{M}U(x)), \tag{11}
\end{aligned}
$$

where $B(x) = (N, \Lambda^0, \Sigma, \Xi)$ is the ground-state baryon octet [11], $U^2(x) = e^{\sqrt{2}i\gamma^5 P(x)/F_\pi}$, $P(x) = (\pi, \eta, K, \bar{K})$ and $F_\pi = 92.4$ MeV are the octets of low-lying pseudoscalar mesons and the partial conservation of axial-vector current (PCAC) constant [12], $\langle \cdots \rangle$ are the traces over the SU(3) indices, $g_A = 1.275$ [13,14], $\alpha_D = 0.635$, and m_0 is the baryon mass for a current quark mass of zero. $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ is a diagonal 3×3 matrix with current quark masses m_q for $q = u, d$, and s, respectively, and $B_0 = -\langle \bar{q}q \rangle / F_\pi^2$, where $\langle \bar{q}q \rangle$ is the quark condensate. The current quark masses and the quark condensate are defined at the normalization scale $\mu = 1$ GeV [15]. The ellipsis denotes the contributions of the derivative couplings of the $\Lambda(1405)$ resonance, the baryon decuplet $\underline{\mathbf{10}} = (\Delta, \Sigma^*, \ldots)$, and other baryon resonances [8] with quantum numbers $J^P = \frac{1}{2}^+$, belonging to octets of SU(3)$_f$ symmetry [11], with the octet of pseudoscalar mesons and the ground-state baryon octet invariant under chiral SU(3) $\times$ SU(3) symmetry and also chiral SU(3) $\times$ SU(3) invariant interactions of the ground-state baryon octet with the nonet of scalar-meson resonances [9,10] (see Appendix B). The parameters b_D and b_F define the mass splitting of the ground-state baryons $m_\Sigma - m_{\Lambda^0} = \frac{4}{3}b_D(m_K^2 - m_\pi^2)$ and $m_\Xi - m_\Sigma = (b_D + b_F)(m_\pi^2 - m_K^2)$. They are equal to $b_D = +0.051$ fm and $b_F = -0.158$ fm. The parameter b_0 determines the $\sigma_{\pi N}$ term of πN scattering $2\sigma_{\pi N} = -m_\pi^2(2b_0 + b_D + b_F)$. It is equal to $b_0 = -0.561$ fm, calculated in terms of the experimental value $\sigma_{\pi N}^{(\text{exp})} = 61$ MeV [16], which agrees well with the theoretical one, $\sigma_{\pi N}^{(\text{th})} = 60$ MeV [17]. The amplitudes M of low-energy $\bar{K}N$ scattering are determined in the SU(3) coupled-channel approach by the matrix equation $M^{-1} = M_0^{-1} - G$, where M_0 are the amplitudes of $\bar{K}N$ scattering, calculated with the chiral Lagrangian (14) and other Lagrangians, adduced in Appendix B, in the tree approximation [4]. Since we are interested in the scattering lengths, we calculate the matrix elements of the diagonal matrix G, given by the meson-baryon loop diagrams, in the nonrelativistic approximation within the dimensional regularization. As a result, the matrix elements are imaginary and proportional to k and k^3 for the s-wave and p-wave $\bar{K}N$ scattering, respectively, for kinematically opened channels, where k is a momentum transfer. The complex s-wave scattering length of $K^- p$ scattering we set equal to the preliminary experimental value by the SIDDHARTA Collaboration [18]. In our approach the imaginary parts of the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering are defined by the contributions of the $\Lambda(1405)$ and $\Sigma(1385)$ resonances. This agrees well with the analysis of low-energy $\bar{K}N$ interactions in the s-wave and p-wave states, carried out for the investigation of the properties of antikaon-nuclear quasibound states in [5]. For the coupling constants $d_j(j = 1, 2, 3, 4)$, which are input parameters, we have obtained the following values: $d_1 = -0.389$ fm, $d_2 = -0.709$ fm, $d_3 = +2.816$ fm, and $d_4 = -0.619$ fm. As has been found, the contribution of the scalar-meson resonances is not essential for reasonable values of coupling constant of the interactions of scalar-meson resonances with the ground-state baryons (see Appendix B).

IV. ENERGY-LEVEL SHIFT AND WIDTH OF EXCITED *np* STATE OF KAONIC DEUTERIUM

Following [8] we define the shift and width of the energy level of the excited *np* state of kaonic deuterium, where *n* is the *principal* quantum number, in terms of the complex p-wave scattering length $\tilde{a}_{K^-d}^{(1)}(K^-d)$ of K^-d scattering. We get

$$\epsilon_{np} = -2\frac{\alpha^5}{n^3}\left(1 - \frac{1}{n^2}\right)\left(\frac{m_K m_d}{m_K + m_d}\right)^4 \text{Re}\tilde{a}_{K^-d}^{(1)},$$

$$\Gamma_{np} = 4\frac{\alpha^5}{n^3}\left(1 - \frac{1}{n^2}\right)\left(\frac{m_K m_d}{m_K + m_d}\right)^4 \text{Im}\tilde{a}_{K^-d}^{(1)}, \quad (12)$$

where $\alpha = 1/137.036$ is the fine-structure constant.

Using the numerical values of the complex s-wave and p-wave scattering lengths, evaluated in Sec. III, we obtain the following numerical values of the s-wave and p-wave scattering lengths of K^-d scattering:

$$\tilde{a}_{K^-d}^{(0)} = -1.273 + i2.435 \text{ fm},$$
$$\tilde{a}_{K^-d}^{(1)} = -0.352 + i0.432 \text{ fm}^3. \quad (13)$$

They give the following energy-level displacements of the ground $1s$ and excited $2p$ states of kaonic deuterium:

$$\epsilon_{1s} = 0.766 \text{ keV}, \quad \Gamma_{1s} = 2.933 \text{ keV},$$
$$\epsilon_{2p} = 4.158 \text{ meV}, \quad \Gamma_{2p} = 10.203 \text{ meV}. \quad (14)$$

The numerical value of the complex s-wave scattering length of K^-d scattering agrees reasonably well with the results obtained in [1]. The uncertainty of the complex p-wave scattering length of K^-d scattering, which is estimated as 15%, will be discussed in the Conclusion.

V. YIELD OF X RAYS FOR THE K_α EMISSION LINE OF KAONIC DEUTERIUM

The results of the calculation of the yields of x rays of the K_α emission lines for kaonic hydrogen and deuterium depend considerably on the values of the widths of the excited $2p$ state of kaonic atoms [19]. Using the calculation scheme based on the quantum-classical Monte Carlo cascade model developed in [3], we obtain the following yields of the K_α emission lines:

$$Y_{K^-p} = 1.80\%, \quad \Gamma_{1p} = 1.979 \text{ meV},$$
$$Y_{K^-d} = 0.27\%, \quad \Gamma_{2p} = 10.203 \text{ meV}, \quad (15)$$

for kaonic hydrogen and deuterium, respectively.

Our result $\Gamma_{2p} = 1.979$ meV for the width of the excited $2p$ state of kaonic hydrogen agrees well with $\Gamma_{2p} = 2$ meV obtained in [8]. The theoretical value $Y_{K^-p} = 1.80\%$ is in good agreement with the experimental one, $Y_{K^-p} = 1.5(5)\%$ [20]. The theoretical value $Y_{K^-d} = 0.27\%$ can be used for planning experiments on the measurement of the energy-level displacement of the ground $1s$ state of kaonic deuterium.

VI. CONCLUSION

We have investigated the properties of exotic atom–kaonic deuterium in the excited *np* state, where *n* is the *principal*

quantum number, relative to strong low-energy interactions, described by chiral Lagrangians with derivative meson-baryon couplings invariant under chiral $SU(3) \times SU(3)$ symmetry. We have calculated the energy-level shift and width of the excited np state in terms of the complex p-wave scattering length of K^-d scattering. Since K^-d scattering is a three-body into three-body reaction, the most appropriate tool for the investigation of the p-wave amplitude of K^-d scattering is the Faddeev equations [1]. Following [1], where the Faddeev equations for the s-wave amplitude of K^-d scattering have been solved in the fixed-center approximation, we have calculated the Faddeev equations for the p-wave amplitude of low-energy K^-d scattering in the fixed-center approximation. In such an approximation the complex s-wave and p-wave scattering lengths of K^-d scattering are expressed in terms of the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering. The calculation of the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering is carried out within the $SU(3)$ coupled-channel approach and chiral Lagrangians with derivative meson-baryon couplings invariant under chiral $SU(3) \times SU(3)$ symmetry. The complex s-wave scattering length of K^-p scattering has been set equal to the recent experimental value measured by the SIDDHARTA Collaboration [18].

We note that our result for the real part of the complex p-wave scattering length of K^-p scattering, obtained in this paper, differs by a sign from that calculated in [8]. Such a discrepancy is caused by different dynamics that are used in [8] and in the present paper. Indeed, for the calculation of the complex p-wave scattering length in the present paper we use chiral Lagrangians with derivative meson-baryon couplings, derived within nonlinear realization of chiral $SU(3) \times SU(3)$ symmetry [6], which contain also additional interactions with the coupling constants d_j for $(j = 1, 2, 3, 4)$ and b_ℓ for $\ell = 0, D, F$ [4]. In [8] the calculation of the complex p-wave scattering length of K^-p scattering has been carried out with chiral Lagrangians, derived within linear realization of chiral $SU(3) \times SU(3)$ symmetry. These chiral Lagrangians do not contain the interactions with the coupling constants d_j for $(j = 1, 2, 3, 4)$ and b_ℓ for $\ell = 0, D, F$, which are specific for the nonlinear realization of chiral symmetry and have no analogy within its linear realization. Since the imaginary part of the complex p-wave scattering length is defined by the dominant contribution of the $\Sigma(1385)$ resonance, the values of the imaginary parts, calculated in the present paper and in [8], agree well.

The numerical value of the complex s-wave scattering length of K^-d scattering equation (13) agrees reasonably well with the results obtained in [1]. We note that the complex s-wave scattering length of K^-d scattering has been also investigated within the effective-field-theory approach [21,22]. In [21] the solution of the Faddeev equations, obtained in the fixed-center approximation, for the complex s-wave scattering length has been confirmed within the effective-field-theory approach. In [22] the effective-field-theory approach has been applied to the calculation of the nucleon-recoil corrections to the double-scattering contribution to the complex s-wave scattering length of K^-d scattering, obtained in the fixed-center approximation. As has been found in [22], the nucleon-recoil corrections make up about 10%–15%.

Now let us discuss the uncertainty of the proposed solution of the Faddeev equations for the complex p-wave scattering length of K^-d scattering, obtained in the fixed-center approximation. As has been pointed out by Gal [23], the uncertainty of the solution of the Faddeev equations for the complex s-wave scattering length of K^-d scattering, calculated in the fixed-center approximation, is about 10%–25%. Such an estimate has been deduced from the comparison of the solution found in [1] with other solutions of the Faddeev equations applied to the calculation of the complex s-wave scattering length of K^-d scattering [24–26]. Of course, the lack of the experimental data on the complex s-wave scattering length of K^-d scattering does not allow us to understand the real uncertainty of theoretical schemes.

Since the calculation of the complex p-wave scattering length of K^-d scattering has not been yet carried out in the literature, we have no possibility to compare our solution with any others. Thus, for the estimate of the theoretical uncertainty of our solution of the Faddeev equations for the complex p-wave scattering length of K^-d scattering we follow the results obtained in [22]. As has been shown in [22], the nucleon-recoil correction to the double-scattering contribution of the static solution of the Faddeev equations makes up about 15%. Since we neglect the nucleon recoil, one can accept 15% as the uncertainty of our static solution of the Faddeev equations.

Such an estimate of the uncertainty, applied to the total solution of the Faddeev equations in the fixed-center approximation, can be supported by a convergence of the expansion of our solution for the complex p-wave scattering length of K^-d scattering in powers of $1/r$. As we have shown in Appendix A, the contribution of the single and double scattering $\tilde{a}^{(1)}_{K^-d} = (\tilde{a}^{(1)}_{K^-d})_{\text{s.sc.}} + (\tilde{a}^{(1)}_{K^-d})_{\text{d.sc.}} = -0.262 + i0.548$ fm^3 dominates in the complex p-wave scattering length of K^-d scattering. The account for the contribution of the triple scattering $\tilde{a}^{(1)}_{K^-d} = (\tilde{a}^{(1)}_{K^-d})_{\text{s.sc.}} + (\tilde{a}^{(1)}_{K^-d})_{\text{d.sc.}} + (\tilde{a}^{(1)}_{K^-d})_{\text{tr.sc.}} = -0.277 + i0.525$ fm^3 obtained from the expansion of the exact solution of the Faddeev equations corroborates only such a dominance. The contributions of higher multiple scattering are equal to $\delta\tilde{a}^{(1)}_{K^-d} = -0.075 - i0.072$ fm^3. They make up about 21% and 17% of the real and imaginary parts of the total complex p-wave scattering length, respectively. Of course, a proof of the convergence by means of the calculation of the contributions of higher n-multiple scattering for $n \geqslant 4$, proportional to the higher powers of $1/r^{(n-1)}$, averaged with the deuteron wave function $|\Phi_d(\vec{r})|^2$ as $\int d^3x |\Phi_d(\vec{r})|^2/r^{(n-1)} = \langle 1/r^{(n-1)} \rangle$, stumbles against the problem of the regularization and renormalization of these averaged values [27]. The solution of this problem goes beyond the scope of this paper. We are planning to carry out this analysis in a future presentation. We note that without truncation the evaluation of the complex p-wave scattering length equation (8), caused by the multiple scattering, does not suffer from divergences at $r \rightarrow 0$. This agrees well with the results obtained in [1].

Using the numerical value of the complex p-wave scattering length of K^-d scattering Eq. (13), we have calculated the width of the excited $2p$ state $\Gamma_{2p} = 10.203$ meV. This result

plays an important role for the theoretical analysis of the yield Y_{K^-d} of x rays of the K_α emission line of kaonic deuterium. Using the quantum-classical Monte Carlo cascade model [3], we have obtained $Y_{K^-d} = 0.27\%$ for the width $\Gamma_{2p} = 10.203$ meV of the excited $2p$ state of kaonic deuterium. For the yield of x rays of the K_α emission line of kaonic hydrogen we have obtained the value $Y_{K^-p} = 1.80\%$, which agrees well with the experimental data $Y_{K^-p} = 1.5(5)\%$ [20].

Concluding this discussion, we would like to note that the complex s-wave scattering length of elastic K^-n scattering $\tilde{a}_{K^-n}^{(0)}(K^-n) = 0.300 + i0.096$ fm or the complex s-wave scattering length of $\bar{K}N$ scattering in the state with isospin $I = 1$, i.e., $\tilde{a}_{I=1} = \tilde{a}_{K^-n}^{(0)}(K^-n)$, calculated in our approach to $\bar{K}N$ scattering (see Appendix B) and given in Eq. (12), possesses a small imaginary part $\text{Im}\,\tilde{a}_{K^-n}^{(0)}(K^-n) = 0.096$ fm. This does not contradict some estimates of the complex s-wave scattering length of $\bar{K}N$ scattering in the state with isospin $I = 1$ obtained in [21] from the complex s-wave scattering length of K^-d scattering. Nevertheless, the theoretical analysis of $\bar{K}N$ scattering carried out in [4,28] shows that the imaginary part of the complex s-wave scattering length of elastic K^-n scattering is commensurable with the imaginary part of the complex s-wave scattering length of elastic K^-p scattering. As has been found in [4], the complex s-wave scattering length of elastic K^-n scattering is equal to $\tilde{a}_{K^-n}^{(0)}(K^-n) = 0.49 + i0.70$ fm, which agrees well with the empirical result $\tilde{a}_{K^-n}^{(0)}(K^-n) = 0.37 + i0.60$ fm obtained in [29] and the theoretical estimates in [28]. Thus, for the completeness of our numerical predictions for the complex p-wave scattering length of K^-d scattering we take into account the complex s-wave scattering lengths of $\bar{K}N$ scattering calculated in [4] and the complex p-wave scattering lengths of $\bar{K}N$ scattering calculated in [5]. We get

$$\begin{aligned} \tilde{a}_{K^-d}^{(0)} &= -1.951 + i0.996 \text{ fm}, \\ \tilde{a}_{K^-d}^{(1)} &= -0.174 + i0.113 \text{ fm}^3. \end{aligned} \quad (16)$$

The calculation is performed for $a_{I=0}^{(0)}(\bar{K}N) = -1.63 + i0.42$ fm and $a_{I=1}^{(0)}(\bar{K}N) = 0.49 + i0.70$ fm [4] and $a_{I=0}^{(1)}(\bar{K}N) = 0$ and $a_{I=1}^{(1)}(\bar{K}N) = -0.114 + i0.098$ fm^3 [5]. The energy-level displacements of the ground $1s$ and excited $2p$ states of kaonic deuterium, calculated in terms of the complex s-wave and p-wave scattering lengths, Eq. (16), are equal to

$$\begin{aligned} \epsilon_{1s} &= 1.175 \text{ keV}, \quad \Gamma_{1s} = 1.200 \text{ keV}, \\ \epsilon_{2p} &= 2.053 \text{ meV}, \quad \Gamma_{2p} = 2.675 \text{ meV}. \end{aligned} \quad (17)$$

In this case the yield of the x rays of the K_α emission line for kaonic deuterium is

$$Y_{K^-d} = 1.90\%, \quad \Gamma_{2p} = 2.675 \text{ meV}. \quad (18)$$

Our results for the yields of x rays of the K_α emission line and the energy-level displacements of kaonic deuterium in the ground $1s$ state, calculated for the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering, obtained in this paper and in [4,5,28], can be used for planning experiments on the measurements of the energy-level displacement of the ground $1s$ state of kaonic deuterium. The results obtained in this paper can be also used by the SIDDHARTA Collaboration, measuring currently the energy-level displacement of the ground $1s$ state of kaonic deuterium [30,31].

APPENDIX A: COMPLEX *p*-WAVE SCATTERING LENGTH OF K^-d SCATTERING: SINGLE- AND DOUBLE-SCATTERING CONTRIBUTIONS

In this Appendix we give a detailed calculation of the complex p-wave scattering length of K^-d scattering, keeping the contributions of the single (impulse) and double scattering only. The p-wave amplitude $M^{(1)}(K^-d \rightarrow K^-d)$ of low-energy elastic K^-d scattering relates to the complex p-wave scattering length $a_{K^-d}^{(1)}$ as follows:

$$M^{(1)}(K^-d \rightarrow K^-d) = 24\pi(m_K + m_d)a_{K^-d}^{(1)}(\vec{k}' \cdot \vec{k}), \quad (A1)$$

where $\vec{k}$ and $\vec{k}'$ are momenta of a relative motion of the K^-d pair in the initial and final states. They are related by $|\vec{k}| = |\vec{k}'|$. In turn, the p-wave amplitude $M^{(1)}(K^-d \rightarrow K^-d)$ is defined in terms of the matrix element of the $\mathbb{T}^{(1)}$ matrix as

$$\begin{aligned} \langle K^-d|\mathbb{T}^{(1)}|K^-d\rangle &= (2\pi)^4\delta^{(4)}(k_d' + k' - k_d + k) \\ &\times M^{(1)}(K^-d \rightarrow K^-d), \end{aligned} \quad (A2)$$

where (k_d, k) and (k_d', k') are four-momenta of the deuteron and K^- meson in the initial and final states, respectively.

For the calculation of the matrix elements of the $\mathbb{T}$ matrix we use the following effective Lagrangian:

$$\mathcal{L}_{\text{int}}(x) = \mathcal{L}_{\text{int}}^{(0)}(x) + \mathcal{L}_{\text{int}}^{(1)}(x), \quad (A3)$$

where the effective Lagrangians $\mathcal{L}_{\text{int}}^{(0)}(x)$ and $\mathcal{L}_{\text{int}}^{(1)}(x)$ define low-energy K^-d interactions in the s-wave and p-wave states. They are given by

$$\begin{aligned} \mathcal{L}_{\text{int}}^{(0)}(x) &= 4\pi\big[\hat{a}_p^{(0)}K^{-\dagger}(x)K^-(x)\bar{p}(x)p(x) + \hat{a}_x^{(0)}\bar{K}^{0\dagger}(x)K^-(x)\bar{n}(x)p(x)\big] + 4\pi\big[\hat{a}_n^{(0)}K^{-\dagger}(x)K^-(x)\bar{n}(x)n(x) \\ &+ \hat{a}_x^{(0)}K^{-\dagger}(x)\bar{K}^0(x)\bar{p}(x)n(x)\big] + 4\pi\big[\hat{a}_n^{0(0)}\bar{K}^{0\dagger}(x)\bar{K}^0(x)\bar{n}(x)n(x)\big] \end{aligned} \quad (A4)$$

and

$$\mathcal{L}_{\text{int}}^{(1)}(x) = 12\pi \left[\hat{a}_p^{(1)} \triangledown K^{-\dagger}(x) \cdot \triangledown K^-(x) \bar{p}(x) p(x) + \hat{a}_x^{(1)} \triangledown \bar{K}^{0\dagger}(x) \cdot \triangledown K^-(x) \bar{n}(x) p(x) \right]$$
$$+ 12\pi \left[\hat{a}_n^{(1)} \triangledown K^{-\dagger}(x) \cdot \triangledown K^-(x) \bar{n}(x) n(x) + \hat{a}_x^{(1)} \triangledown K^{-\dagger}(x) \cdot \triangledown \bar{K}^0(x) \bar{p}(x) n(x) \right]$$
$$+ 12\pi \left[\hat{a}_n^{0(1)} \triangledown \bar{K}^{0\dagger}(x) \cdot \triangledown \bar{K}^0(x) \bar{n}(x) n(x) \right]. \tag{A5}$$

The s-wave and p-wave scattering lengths are determined as shown in Eq. (3). The matrix elements of these Lagrangians define the s-wave and p-wave amplitudes of low-energy $\bar{K}N$ scattering, expressed in terms of the s-wave and p-wave scattering lengths.

Using the effective Lagrangian Eq. (A4), one reproduces the solution of the Faddeev equations in the fixed-center approximation for the complex s-wave scattering length of K^-d scattering, given by Eq. (2) and obtained in [1].

For the calculation of the contributions of the single and double scattering to the complex p-wave scattering length of K^-d scattering we will use both Lagrangians (A4) and (A5). The $\mathbb{T}^{(1)}$ matrix for low-energy K^-d scattering, describing the contributions of the single and double scattering, is defined by

$$\mathbb{T}^{(1)} = \int d^4x \, \mathcal{L}_{\text{int}}^{(0)}(x) + i \int d^4x_1 d^4x_2 T \left[\mathcal{L}_{\text{int}}^{(0)}(x_1) \mathcal{L}_{\text{int}}^{(1)}(x_2) \right] + \dots, \tag{A6}$$

where T is the time-ordering operator and the ellipsis denotes the triple-scattering contributions and so on. The p-wave amplitude of low-energy K^-d scattering, caused by the single and double scattering, is

$$M^{(1)}(K^-d \to K^-d) = M^{(1)}(K^-d \to K^-d)_{\text{s.sc.}} + M^{(1)}(K^-d \to K^-d)_{\text{d.sc.}}, \tag{A7}$$

where $M^{(1)}(K^-d \to K^-d)_{\text{s.sc.}}$ and $M^{(1)}(K^-d \to K^-d)_{\text{d.sc.}}$ are the amplitudes of the single and double scattering, respectively. They are given by

$$M^{(1)}(K^-d \to K^-d)_{\text{s.sc.}} = \langle K^-d | \mathcal{L}_{\text{int}}^{(0)}(0) | K^-d \rangle,$$
$$M^{(1)}(K^-d \to K^-d)_{\text{d.sc.}} = i \int d^4x \langle K^-d | T [\mathcal{L}_{\text{int}}^{(0)}(x) \mathcal{L}_{\text{int}}^{(1)}(0)] | K^-d \rangle. \tag{A8}$$

For the calculation of the matrix element $\langle K^-d | \mathbb{T}^{(1)} | K^-d \rangle$ we use the wave functions of the initial and final states:

$$|K^-d\rangle = c_{K^-}^{\dagger}(\vec{k}) | d(-\vec{k}, \lambda) \rangle,$$
$$\langle K^-d| = \langle d(-\vec{k}', \lambda) | c_{K^-}(\vec{k}'), \tag{A9}$$

where $\vec{k}$ and $\vec{k}'$ are the relative momenta of the K^-d pairs in the initial and final states, respectively, and $c_{K^-}^{\dagger}(\vec{k})$ and $c_{K^-}(\vec{k}')$ are operators of creation and annihilation of the K^- mesons with three-momenta $\vec{k}$ and $\vec{k}'$, respectively. They obey standard relativistic covariant commutation relations [32]. The wave function of the deuteron $|d(-\vec{k}, \lambda)\rangle$ is taken in the momentum and particle number representation. It reads [32]

$$|d(-\vec{k}, \lambda)\rangle = \frac{\sqrt{2E_d(\vec{k})}}{(2\pi)^3} \int \frac{d^3k_p}{\sqrt{2E_N(\vec{k}_p)}} \frac{d^3k_n}{\sqrt{2E_N(\vec{k}_n)}} \delta^{(3)}(\vec{k} + \vec{k}_p + \vec{k}_n) \tilde{\Phi}_d \left(\frac{\vec{k}_p - \vec{k}_n}{2} \right) a_p^{\dagger}(\vec{k}_p, \sigma_p) a_n^{\dagger}(\vec{k}_n, \sigma_n) |0\rangle, \tag{A10}$$

where $E_d(\vec{k})$, $E_N(\vec{k}_p)$, and $E_N(\vec{k})$ are the total energies of the deuteron, proton, and neutron, respectively; $\tilde{\Phi}_d(\vec{q})$ is the wave function of the ground state of the deuteron in the momentum representation; $a_p^{\dagger}(\vec{k}_p, \sigma_p)$ and $a_n^{\dagger}(\vec{k}_n, \sigma_n)$ are the operators of creation of the proton and the neutron with three-momenta $\vec{k}_p$ and $\vec{k}_n$ and polarizations $\sigma_p = \pm\frac{1}{2}$ and $\sigma_n = \pm\frac{1}{2}$, respectively, and they obey standard relativistic covariant anticommutation relations [32]; $\lambda = \sigma_p + \sigma_n$ is the polarization of the deuteron; and $|0\rangle$ is the vacuum wave function. For the deuteron polarization states with $\lambda = \pm1$ and $\lambda = 0$ the product $a_p^{\dagger}(\vec{k}_p, \sigma_p) a_n^{\dagger}(\vec{k}_n, \sigma_n)$ should be replaced by $a_p^{\dagger}(\vec{k}_p, \pm\frac{1}{2}) a_n^{\dagger}(\vec{k}_n, \pm\frac{1}{2})$ and $\frac{1}{\sqrt{2}}[a_p^{\dagger}(\vec{k}_p, \sigma_p) a_n^{\dagger}(\vec{k}_n, -\sigma_p) + a_p^{\dagger}(-\vec{k}_p, \sigma_p) a_n^{\dagger}(\vec{k}_n, \sigma_p)]$, respectively [32].

The p-wave amplitude of the single K^-d scattering is equal to

$$M^{(1)}(K^-d \to K^-d)_{\text{s.sc.}} = \langle K^-d | \mathcal{L}_{\text{int}}^{(0)}(0) | K^-d \rangle = 24\pi m_d \left(\hat{a}_p^{(1)} + \hat{a}_n^{(1)} \right) (\vec{k}' \cdot \vec{k}) \int \frac{d^3q}{(2\pi)^3} \tilde{\Phi}_d^* \left(\vec{q} + \frac{1}{2}\vec{k}' \right) \tilde{\Phi}_d \left(\vec{q} + \frac{1}{2}\vec{k} \right). \tag{A11}$$

The momentum integral defines the form factor $F_d(\vec{Q})$ of the deuteron [1]:

$$\int \frac{d^3q}{(2\pi)^3} \tilde{\Phi}_d^* \left(\vec{q} + \frac{1}{2}\vec{k}' \right) \tilde{\Phi}_d \left(\vec{q} + \frac{1}{2}\vec{k} \right) = \int d^3x |\Phi_d(\vec{r})|^2 e^{i\vec{Q}\cdot\vec{r}} = F_d(\vec{Q}), \tag{A12}$$

where $\vec{Q} = \frac{1}{2}(\vec{k}' - \vec{k})$ is the momentum transfer. Since the form factor of the deuteron is normalized to unity at $\vec{Q} = 0$ [1], the complex p-wave scattering length, calculated in the single-scattering approximation, is equal to

$$\left(\tilde{a}_{K^-d}^{(1)} \right)_{\text{s.sc.}} = \frac{m_d}{m_K + m_d} \left(\hat{a}_p^{(1)} + \hat{a}_n^{(1)} \right) = -0.231 + i0.645 \text{ fm}^3, \tag{A13}$$

where we have used the numerical values of the p-wave scattering lengths of $K^- p$ and $K^- n$ scattering, adduced in Eq. (13).

The amplitude $M^{(1)}(K^- d \to K^- d)_{\text{d.sc.}}$ of the double scattering is defined by the matrix element

$$
M^{(1)}(K^- d \to K^- d)_{\text{d.sc.}} = i \int d^4x \langle K^- d | T(\mathcal{L}_{\text{int}}^{(0)}(x)\mathcal{L}_{\text{int}}^{(1)}(0)) | K^- d \rangle
$$

$$
= 48\pi^2 i \int d^4x \langle K^- d | \{ \hat{a}_p^{(0)}\hat{a}_n^{(1)}[\bar{p}(x)p(x)]T[K^{-\dagger}(x)K^-(x)\nabla K^{-\dagger}(0) \cdot \nabla K^-(0)][\bar{n}(0)n(0)]
$$

$$
+ \hat{a}_n^{(0)}\hat{a}_p^{(1)}[\bar{n}(x)n(x)]T[K^{-\dagger}(x)K^-(x)\nabla K^{-\dagger}(0) \cdot \nabla K^-(0)][\bar{p}(0)p(0)]
$$

$$
+ \hat{a}_x^{(0)}\hat{a}_x^{(1)}[\bar{p}(x)n(x)]T[K^{-\dagger}(x)\bar{K}^0(x)\nabla \bar{K}^{0\dagger}(0) \cdot \nabla K^-(0)][\bar{n}(0)p(0)]
$$

$$
+ \hat{a}_x^{(0)}\hat{a}_x^{(1)}[\bar{n}(x)p(x)]T[\bar{K}^{0\dagger}(x)K^-(x)\nabla K^{-\dagger}(0) \cdot \nabla \bar{K}^0(0)][\bar{p}(0)n(0)] \} | K^- d \rangle. \tag{A14}
$$

Having calculated the matrix element between the K^--meson states, we arrive at the expression

$$
M^{(1)}(K^- d \to K^- d)_{\text{d.sc.}} = 48\pi^2 i \int d^4x \left\{ \hat{a}_p^{(0)}\hat{a}_n^{(1)} \langle d | [\bar{p}(x)p(x)][\bar{n}(0)n(0)] | d \rangle \left(\int \frac{d^4q}{(2\pi)^4 i} \frac{(\vec{k} \cdot \vec{q})e^{-i(q-k')\cdot x}}{m_K^2 - q^2 - i0} \right. \right.
$$

$$
+ \int \frac{d^4q}{(2\pi)^4 i} \frac{(\vec{k}' \cdot \vec{q})e^{+i(q-k)\cdot x}}{m_K^2 - q^2 - i0} \right) + \hat{a}_n^{(0)}\hat{a}_p^{(1)} \langle d | [\bar{n}(x)n(x)][\bar{p}(0)p(0)] | d \rangle \left(\int \frac{d^4q}{(2\pi)^4 i} \frac{(\vec{k} \cdot \vec{q})e^{-i(q-k')\cdot x}}{m_K^2 - q^2 - i0} \right.
$$

$$
\left. + \int \frac{d^4q}{(2\pi)^4 i} \frac{(\vec{k}' \cdot \vec{q})e^{+i(q-k)\cdot x}}{m_K^2 - q^2 - i0} \right) + \hat{a}_x^{(0)}\hat{a}_x^{(1)} \langle d | [\bar{p}(x)n(x)][\bar{n}(0)p(0)] | d \rangle \int \frac{d^4q}{(2\pi)^4 i} \frac{(\vec{k} \cdot \vec{q})e^{-i(q-k')\cdot x}}{m_K^2 - q^2 - i0}
$$

$$
\left. + \hat{a}_x^{(0)}\hat{a}_x^{(1)} \langle d | [\bar{n}(x)p(x)][\bar{p}(0)n(0)] | d \rangle \int \frac{d^4q}{(2\pi)^4 i} \frac{(\vec{k}' \cdot \vec{q})e^{+i(q-k)\cdot x}}{m_K^2 - q^2 - i0} \right\}. \tag{A15}
$$

The matrix elements of the products of the nucleon field operators between the deuteron states, calculated in the nonrelativistic approximation, are equal to

$$
\langle d | [\bar{p}(x)p(x)][\bar{n}(0)n(0)] | d \rangle = \langle d | [\bar{n}(x)n(x)][\bar{p}(0)p(0)] | d \rangle = -\langle d | [\bar{p}(x)n(x)][\bar{n}(0)p(0)] | d \rangle = -\langle d | [\bar{n}(x)p(x)][\bar{p}(0)n(0)] | d \rangle
$$

$$
= 2m_d \int \frac{d^3q'}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \tilde{\Phi}_d^*\left(\vec{q}' + \frac{1}{2}\vec{k}'\right) \tilde{\Phi}_d\left(\vec{q} + \frac{1}{2}\vec{k}\right) e^{-i(\vec{q}'-\vec{q})\cdot\vec{r}} = 2m_d |\Phi_d(\vec{r})|^2 e^{i\frac{1}{2}(\vec{k}'-\vec{k})\cdot\vec{r}}. \tag{A16}
$$

Substituting Eq. (A16) into Eq. (A15) and integrating over time, we transcribe the right-hand side (rhs) of Eq. (A15) into the form

$$
M^{(1)}(K^- d \to K^- d)_{\text{d.sc.}} = 96\pi^2 m_d \int d^3x |\Phi_d(\vec{r})|^2 e^{i\frac{1}{2}(\vec{k}'-\vec{k})\cdot\vec{r}} \left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)} \right)
$$

$$
\times \left(\int \frac{d^4q}{(2\pi)^3} \delta(q_0 - m_K) \frac{(\vec{k} \cdot \vec{q})e^{+i(\vec{q}-\vec{k}')\cdot\vec{r}}}{m_K^2 - q^2 - i0} + \int \frac{d^4q}{(2\pi)^3} \delta(q_0 - m_K) \frac{(\vec{k}' \cdot \vec{q})e^{-i(\vec{q}-\vec{k})\cdot\vec{r}}}{m_K^2 - q^2 - i0} \right), \tag{A17}
$$

where we have neglected the contributions of the kinetic energies of the K^- mesons with respect to their masses in the initial and final states. Having integrated over q_0 we obtain the rhs of Eq. (A17) in the following form:

$$
M^{(1)}(K^- d \to K^- d)_{\text{d.sc.}} = 96\pi^2 m_d \int d^3x |\Phi_d(\vec{r})|^2 \left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)} \right)
$$

$$
\times \left(e^{-i\frac{1}{2}(\vec{k}'+\vec{k})\cdot\vec{r}} \int \frac{d^3q}{(2\pi)^3} \frac{(\vec{k} \cdot \vec{q})}{\vec{q}^{\,2}} e^{+i\vec{q}\cdot\vec{r}} + e^{+i\frac{1}{2}(\vec{k}'+\vec{k})\cdot\vec{r}} \int \frac{d^3q}{(2\pi)^3} \frac{(\vec{k}' \cdot \vec{q})}{\vec{q}^{\,2}} e^{-i\vec{q}\cdot\vec{r}} \right)
$$

$$
= 96\pi^2 m_d \int d^3x |\Phi_d(\vec{r})|^2 \left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)} \right) e^{-i\frac{1}{2}(\vec{k}'+\vec{k})\cdot\vec{r}} (\vec{k}' + \vec{k}) \cdot (-i\nabla) \int \frac{d^3q}{(2\pi)^3} \frac{e^{+i\vec{q}\cdot\vec{r}}}{\vec{q}^{\,2}}
$$

$$
= 24\pi m_d \int d^3x |\Phi_d(\vec{r})|^2 \left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)} \right) e^{-i\frac{1}{2}(\vec{k}'+\vec{k})\cdot\vec{r}} \frac{i(\vec{k}' + \vec{k}) \cdot \vec{r}}{r^3}, \tag{A18}
$$

where we have made a change of variables $\vec{r} \to -\vec{r}$ in the second term and have used $|\Phi_d(-\vec{r})|^2 = |\Phi_d(\vec{r})|^2$.

Expending the exponential $e^{-i\frac{1}{2}(\vec{k}'+\vec{k})\cdot\vec{r}}$ in powers of $\frac{1}{2}(\vec{k}'+\vec{k}) \cdot \vec{r}$, we obtain the contribution of the double scattering to p-wave amplitude of $K^- d$ scattering in the form

$$
M^{(1)}(K^- d \to K^- d)_{\text{d.sc.}} = 24\pi m_d \frac{1}{3} \left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)} \right) (\vec{k}' \cdot \vec{k}) \int \frac{d^3x}{r} |\Phi_d(\vec{r})|^2, \tag{A19}
$$

where we have omitted the terms proportional to $|\vec{k}'|^2$ and $|\vec{k}|^2$, which have no relation to K^-d scattering in the p-wave state. The terms may contribute to the s-wave amplitude of K^-d scattering, defining the effective range of K^-d scattering in the s-wave state, but they vanish in the complex s-wave scattering length of K^-d scattering, calculated at $\vec{k}, \vec{k}' \to 0$.

The contribution of the double scattering to the complex p-wave scattering length of K^-d scattering is

$$\left(\tilde{a}_{K^-d}^{(1)}\right)_{\text{d.sc.}} = \frac{m_d}{m_K + m_d}\frac{1}{3}\left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)}\right)\int\frac{d^3x}{r}|\Phi_d(\vec{r})|^2 = -0.031 - i0.097 \text{ fm}^3, \quad \text{(A20)}$$

where we have used the numerical values of the s-wave and p-wave scattering lengths of $\bar{K}N$ scattering, adduced in Eq. (13). Thus, the complex p-wave scattering length of K^-d scattering, defined by the contributions of the single and double scattering, is equal to

$$\tilde{a}_{K^-d}^{(1)} = \frac{m_d}{m_K + m_d}\left[\hat{a}_p^{(1)} + \hat{a}_n^{(1)} + \frac{1}{3}\left(\hat{a}_p^{(0)}\hat{a}_n^{(1)} + \hat{a}_n^{(0)}\hat{a}_p^{(1)} - \hat{a}_x^{(0)}\hat{a}_x^{(1)}\right)\int\frac{d^3x}{r}|\Phi_d(\vec{r})|^2\right] = -0.262 + i0.548 \text{ fm}^3. \quad \text{(A21)}$$

We obtain the contribution of the triple scattering by using the exact solution. It reads

$$\left(\tilde{a}_{K^-d}^{(1)}\right)_{\text{tr.sc.}} = \frac{m_d}{m_K + m_d}\frac{1}{36}\left(\hat{a}_p^{(1)}\left[7\hat{a}_p^{(0)}\hat{a}_n^{(0)} + (\hat{a}_n^{(0)})^2 - (\hat{a}_n^{0(0)})^2\right] + \hat{a}_n^{(1)}\{7\left[\hat{a}_p^{(0)}\hat{a}_n^{(0)} - (\hat{a}_x^{(0)})^2\right] + \hat{a}_p^{(0)}\left(\hat{a}_n^{(0)} + \hat{a}_n^{0(0)}\right)\right.$$
$$\left. - 2\hat{a}_n^{(0)}\hat{a}_x^{(0)}\} + \hat{a}_x^{(1)}\hat{a}_x^{(0)}\left(\hat{a}_n^{0(0)} - \hat{a}_n^{(0)}\right) + \hat{a}_n^{0(1)}(\hat{a}_x^{(0)})^2\right)\int\frac{d^3x}{r^2}|\Phi_d(\vec{r})|^2 = -0.015 - i0.023 \text{ fm}^3. \quad \text{(A22)}$$

The complex p-wave scattering length of K^-d scattering, accounting for the contributions of the single, double, and triple scattering, is equal to $\tilde{a}_{K^-d}^{(1)} = (\tilde{a}_{K^-d}^{(1)})_{\text{s.sc.}} + (\tilde{a}_{K^-d}^{(1)})_{\text{d.sc.}} + (\tilde{a}_{K^-d}^{(1)})_{\text{tr.sc.}} = -0.277 + i0.525 \text{ fm}^3$. The discrepancy of this value with the complex p-wave scattering length, defined by the solution of the Faddeev equations (8), is $\delta\tilde{a}_{K^-d}^{(1)} = -0.075 - i0.072 \text{ fm}^3$. It makes up about 21% and 17% of the real and imaginary parts of the total complex p-wave scattering length, respectively.

APPENDIX B: COMPLEX s-WAVE AND p-WAVE SCATTERING LENGTH OF $\bar{K}N$ SCATTERING

In this Appendix we outline our procedure for the calculation of the complex s-wave and p-wave scattering lengths of $\bar{K}N$ scattering. Following [4], the amplitude $M_0(\bar{K}N \to PB)$ for the $\bar{K}N \to PB$ reaction, where P is a pseudoscalar meson and B is a ground-state baryon, we calculate in the tree approximation. For this aim we use the chiral Lagrangian (14) and the chiral Lagrangians

$$\mathcal{L}_{\text{int}}[\Lambda^*(x), B(x), P(x)] = g_{\Lambda^*}\bar{\Lambda}^*(x)\gamma^\mu\gamma^5\langle p_\mu(x)B(x)\rangle,$$
$$\mathcal{L}_{\text{int}}[B_j(x), B(x), P(x)] = \langle\bar{B}_j(x)i\gamma^\mu[s_\mu(x), B(x)]\rangle - g_{A_j}(1-\alpha_{D_j})\langle\bar{B}_j(x)\gamma^\mu[p_\mu(x), B(x)]\rangle - g_{A_j}\alpha_{D_j}\langle\bar{B}_j(x)\gamma^\mu\{p_\mu(x), B(x)\}\rangle, \quad \text{(B1)}$$

describing the low-energy interactions invariant under chiral SU(3) × SU(3) symmetry of the $\Lambda(1405)$ resonance, defined by the field operator $\Lambda^*(x)$, and the baryon resonances $B_j(\mathbf{8})$, defined by the field operators $B_j(x)$ for $j = 1, 2$ [8], with the ground-state baryon octet $B(x)$ and the octet $P(x)$ of pseudoscalar mesons.

In addition we take into account the interactions, invariant under chiral SU(3) × SU(3) symmetry, of the $\Delta(1232)$ resonance and $\Sigma(1385)$ resonance, defined by the field operators $D_\mu^{abc}(x)$, with the ground-state baryon octet $B(x)$ and the octet $P(x)$ of pseudoscalar mesons. They are defined by the chiral Lagrangian

$$\mathcal{L}_{\text{int}}[D(x), B(x), P(x)] = \sqrt{2}g_\Delta\bar{D}_\mu^{abc}(x)\Theta^{\mu\nu}\gamma^5(p_\nu(x))_a^d B_b^e(x)\varepsilon_{cde} + \text{H.c.}, \quad \text{(B2)}$$

where the tensor $\Theta^{\mu\nu}$ is given in [33]: $\Theta^{\mu\nu} = g^{\mu\nu} - (Z + 1/2)\gamma^\mu\gamma^\nu$, where the parameter Z is arbitrary. There is no consensus on the exact value of Z. From theoretical point of view $Z = 1/2$ is preferred [33]. Phenomenological studies give only the bound $|Z| \leqslant 1/2$ [34,35]. For the components of the decuplet $D_{abc}(x)$ we use the following definition:

$$D_{111}(x) = \Delta^{++}(x), \quad D_{112}(x) = \frac{1}{\sqrt{3}}\Delta^+(x), \quad D_{122}(x) = \frac{1}{\sqrt{3}}\Delta^0(x), \quad D_{222}(x) = \Delta^-(x),$$

$$D_{113}(x) = \frac{1}{\sqrt{3}}\Sigma^{*+}(x), \quad D_{123}(x) = \frac{1}{\sqrt{6}}\Sigma^{*0}(x), \quad D_{223}(x) = \frac{1}{\sqrt{3}}\Sigma^{*-}(x), \quad \text{(B3)}$$

$$D_{133}(x) = \frac{1}{\sqrt{3}}\Xi^{*0}(x), \quad D_{233}(x) = \frac{1}{\sqrt{3}}\Xi^{*-}(x), \quad D_{333}(x) = \Omega^-(x).$$

According to [11], baryon resonances $B_1(\mathbf{8}) =$ [$N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$] and $B_2(\mathbf{8}) = $ [$N(1710)$, $\Lambda(1810)$, $\Sigma(1880)$] belong to octets of SU(3)$_f$ symmetry [11] with the coupling constants $g_{A_1} = 0.62$, $\alpha_{D_1} = 0.85$ and $g_{A_2} = 0.12$, $\alpha_{D_2} = -1.55$. The experimental value of g_Δ is $g_\Delta^{\text{exp}} = (1.11 \pm 0.04) g_A$ [36], where $g_A = 1.2750$ [13,14]. We take the coupling constant of the $\Lambda(1405)$ resonance equal to $g_{\Lambda^*} = 0.504$. It defines the width $\Gamma_{\Lambda^*} = 40$ MeV, which fits well the imaginary part of the complex s-wave scattering length of $K^- p$ scattering, measured recently by the SIDDHARTA Collaboration [18].

The scalar resonances $f_0(980)$ and $a_0(980)$ with quantum numbers $I(J^P) = 0(0^+)$ and $I(J^P) = 1(0^+)$ [11], respectively, give contributions to the t channels of elastic and inelastic $\bar K N$ scattering. According to Jaffe [9], the scalar mesons $f_0(980)$ and $a_0(980)$ are four-quark states (or $\bar K K$ molecule), which belong to an SU(3)$_{\text{flavor}}$ nonet. According Ecker *et al.* [10], the interaction of the scalar-meson resonances with octets of pseudoscalar mesons with derivative couplings invariant under chiral SU(3) $\times$ SU(3) symmetry takes the form

$$\mathcal{L}_S(x) = 2\sqrt{2} g_S \text{tr}[S(x) \partial_\mu U^\dagger \partial^\mu U(x)]. \tag{B4}$$

Here $S(x)$ is a nonet of scalar $qq\bar q \bar q$ mesons, defined by [9]

$$S_a^b = \begin{pmatrix} \dfrac{a_0^0}{\sqrt{2}} - \dfrac{\varepsilon}{2} & a_0^+ & \kappa^+ \\[2ex] a_0^- & -\dfrac{a_0^0}{\sqrt{2}} - \dfrac{\varepsilon}{2} & \kappa^0 \\[2ex] \kappa^- & \bar\kappa^0 & -\dfrac{f_0}{\sqrt{2}} + \dfrac{\varepsilon}{2} \end{pmatrix}, \tag{B5}$$

where g_S is a phenomenological coupling constant. The components of the nonet Eq. (B5) have the

following quark structures: $\vec a_0 = (a_0^+, a_0^0, a_0^-) = [s\bar s u \bar d,$ $s\bar s (u\bar u - d\bar d)/\sqrt{2}, d\bar u s\bar s]$ is the isotriplet of $a_0(980)$ mesons, $\kappa = (\kappa^+, \kappa^0) = (u\bar s d\bar d, d\bar s u\bar u)$ and $\bar\kappa = (\bar\kappa^0, -\kappa^-) = (s\bar d u\bar u, -s\bar u d\bar d)$ are doublets of strange scalar four-quark states, $f_0 = s\bar s (u\bar u - d\bar d)/\sqrt{2}$ is the $f_0(980)$ meson, and ε is the isoscalar scalar $\varepsilon(700)$ meson with quark structure $\varepsilon = u\bar d d\bar u$ and mass $m_\varepsilon = 700$ MeV [9]. The nonet $S(x)$ is constructed in such a way that the $f_0(980)$ meson decouples from the $\pi\pi$ states, whereas the $\varepsilon(700)$ meson couples to the $\pi\pi$ states but decouples from the $\bar K K$ states [9]. This implies that the $\varepsilon(700)$ meson does not contribute to the amplitude of $K^- p$ scattering. The value of the coupling constant g_S can be defined from the experimental values of the width of the $a_0 \pi \eta$ decay $\Gamma_{a_0}^{\text{exp}} = (50/100)$ MeV and the $\kappa \to K\pi$ decay $\Gamma_\kappa = (290 \pm 21)$ MeV if we identify the scalar-meson resonance κ with the scalar meson $K_0^*(1430)$ having mass $m_{K_0^*} = (1414 \pm 6)$ MeV [11]. We get $g_S = 28$ MeV, which gives the width of the $a_0 \pi \eta$ decay equal to $\Gamma_{a_0} = 59$ MeV. This agrees well with the experimental data $\Gamma_{a_0}^{\text{exp}} = (50/100)$ MeV [11].

We define the interaction of the scalar-meson resonances $S(x)$ with ground-state baryon octets as

$$\mathcal{L}_{SBB}(x) = g_D \text{tr}\{\bar B(x)\{B(x), S(x)\}\} + g_F \text{tr}\{\bar B(x)[B(x), S(x)]\}, \tag{B6}$$

where g_D and g_F are the phenomenological coupling constants of the symmetric and antisymmetric SBB interactions. The coupling constant g_F should be set zero $g_F = 0$ since the $\varepsilon(700)$ meson does not couple to the $\bar N N$ pair [9].

The amplitude of the $\bar K N \to PB$ reaction, calculated in the tree approximation with the Lagrangians (14), (B1), (B2), (B3), (B4), and (B6), takes the form

$$\begin{aligned} M_0(\bar K N \to PB) &= M^{(c)}(\bar K N \to PB) + M^{(b)}(\bar K N \to PB) + M^{(\text{WT})}(\bar K N \to PB) + M_{\Lambda^*}^{(s)}(\bar K N \to PB) \\ &\quad + M_{J^P = \frac{1}{2}^+}^{(s)}(\bar K N \to PB) + M_{J^P = \frac{1}{2}^+}^{(u)}(\bar K N \to PB) + M_{J^P = \frac{3}{2}^+}^{(s)}(\bar K N \to PB) + M_{J^P = \frac{3}{2}^+}^{(u)}(\bar K N \to PB) \\ &\quad + M_{J^P = 0^+}^{(t)}(\bar K N \to PB), \end{aligned} \tag{B7}$$

where the first two amplitudes are defined by the interactions with the coupling constants d_j for $j = 1, 2, 3, 4$ and b_ℓ for $\ell = 0, D, F$, respectively, the third amplitude is caused by the Weinberg-Tomozawa interactions, and the other amplitudes are defined by the exchange of the $\Lambda(1405)$ resonance, the ground-state baryons, the baryon resonances $B_j(\mathbf{8})$ for $j = 1, 2$, the $\Sigma(1385)$ and $\Delta(1232)$ resonances, and the scalar-meson resonances, respectively, in the s, u, and t channels.

Expanding the amplitudes in Eq. (B7) in powers of relative momenta $\vec k$ and $\vec k'$ and keeping only the terms independent of relative momenta and proportional to the scalar product $\vec k' \cdot \vec k$, we define the contributions to the complex s-wave and p-wave scattering lengths of $\bar K N$ scattering. Using the matrix equation

$M^{-1} = M_0^{-1} - G$, we obtain the unitarized amplitudes of the reactions $\bar K N \to PB$ in terms of the complex s-wave scattering lengths of all scattering channels $\bar K N \to PB$ for $PB = \bar K N$ and πY, where $Y = \Sigma$, Λ^0 hyperons. The input parameters of the approach d_j for $j = 1, 2, 3, 4$ and g_D are fitted from the experimental data on the complex s-wave scattering length of the SIDDHARTA Collaboration, the ratios of the cross sections of inelastic $K^- p$ scattering in the s-wave state, measured at threshold of the $K^- p \to \pi Y$ reactions [37,38], and the experimental cross sections of elastic and inelastic $K^- p$ scattering [39–43]. The cross sections for elastic and inelastic $K^- p$ scattering are calculated at the account for the pure Coulomb scattering and the Coulomb interactions for the pairs of

charged particles in the initial and final states. As a result of this fit we get the following numerical values for the input parameters: $d_1 = -0.389$ fm, $d_2 = -0.709$ fm, $d_3 = +2.816$ fm, and $d_4 = -0.619$ fm. As has been found, the contribution of the scalar-meson resonances is not essential for reasonable values of the coupling constant g_D. The calculated complex s-wave scattering lengths of $\bar{K}N$ scattering describe reasonably well the experimental data on elastic and inelastic K^-p scattering in the low-energy region [39–43] not far above the threshold of the production of the $\bar{K}^0 n$ pair, which is equal to $k_0 \simeq 58$ MeV

in the center-of-mass frame. Using the numerical values of the input parameters $d_1 = -0.389$ fm, $d_2 = -0.709$ fm, $d_3 = +2.816$ fm, and $d_4 = -0.619$ fm, we evaluate the complex s-wave scattering lengths and, correspondingly, the complex p-wave scattering lengths of $\bar{K}N$ scattering. The imaginary parts of the s-wave and p-wave scattering lengths are defined by the dominant contributions of the $\Lambda(1405)$ and $\Sigma(1385)$ resonances, which agrees well with the analysis of low-energy $\bar{K}N$ interactions in the s-wave and p-wave states applied to the problem of antikaon-nuclear quasibound states [5].

[1] S. S. Kamalov, E. Oset, and A. Ramos, Nucl. Phys. A **690**, 494 (2001).

[2] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[3] M. P. Faifman and L. I. Men'shikov, Hyperfine Interact. **193**, 141 (2009).

[4] B. Borasoy, R. Nißler, and W. Weise, Eur. Phys. J. A **25**, 79 (2005).

[5] W. Weise and R. Härtle, Nucl. Phys. A **804**, 173 (2008).

[6] B. W. Lee, Phys. Rev. **170**, 1359 (1968).

[7] J. Gasser, V. E. Lyubovitskij, and A. Rusetsky, Phys. Rep. **456**, 167 (2008), and references therein.

[8] A. N. Ivanov et al., Phys. Rev. A **71**, 052508 (2005).

[9] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).

[10] G. Ecker et al., Nucl. Phys. B **321**, 311 (1989).

[11] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[12] S. L. Adler and R. F. Dashen, in *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

[13] H. Abele, Prog. Part. Nucl. Phys. **60**, 1 (2008).

[14] M. Faber et al., Phys. Rev. C **80**, 035503 (2009).

[15] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[16] H.-Ch. Schröder et al., Eur. Phys. J. C **21**, 473 (2001).

[17] A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Phys. Rev. C **59**, 451 (1999).

[18] M. Bazzi et al., Phys. Lett. B **704**, 113 (2011).

[19] T. E. O. Ericson and F. Scheck, Nucl. Phys. B **19**, 450 (1970); T. Koike, T. Harada, and Y. Akaishi, Phys. Rev. C **53**, 79 (1996); T. P. Terada and R. S. Hayano, *ibid.* **55**, 73 (1997).

[20] T. M. Ito et al., Phys. Rev. C **58**, 2366 (1998).

[21] U.-G. Meissner, U. Raha, and A. Rusetsky, Eur. Phys. J. C **47**, 473 (2006).

[22] V. Baru, E. Epelbaum, and A. Rusetsky, Eur. Phys. J. A **42**, 111 (2009).

[23] A. Gal, arXiv:nucl-th/0607067.

[24] G. Toker, A. Gal, and J. M. Eisenberg, Phys. Lett. B **88**, 235 (1979); Nucl. Phys. A **362**, 405 (1981).

[25] M. Torres, R. H. Dalitz, and A. Deloff, Phys. Lett. B **174**, 213 (1986).

[26] A. Bahaoui et al., Nucl. Phys. A **508**, 335c (1990).

[27] M. Pavon Valderrama and E. Ruiz Arriola, arXiv:nucl-th/0605078.

[28] B. Borasoy, U.-G. Meißner, and R. Nißler, Phys. Rev. C **74**, 055201 (2006).

[29] A. D. Martin, Nucl. Phys. B **179**, 33 (1981).

[30] D. Sirghi (unpublished).

[31] C. Curceanu (unpublished).

[32] A. N. Ivanov et al., Eur. Phys. J. A **23**, 79 (2005).

[33] L. M. Nath, B. Etemadi, and J. D. Kimel, Phys. Rev. D **3**, 2153 (1971).

[34] K. Kabir et al., Int. J. Mod. Phys. E **9**, 157 (2000).

[35] T. E. O. Ericson and A. N. Ivanov, Phys. Lett. B **634**, 39 (2006).

[36] T. R. Hemmert, B. R. Holstein, and J. Kambor, J. Phys. G **24**, 1831 (1998).

[37] D. N. Tovee et al., Nucl. Phys. B **33**, 493 (1971).

[38] R. J. Nowak et al., Nucl. Phys. B **139**, 61 (1978).

[39] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).

[40] M. Sakitt et al., Phys. Rev. **139**, B719 (1965).

[41] J. K. Kim, Phys. Rev. Lett. **14**, 29 (1965); **19**, 1074 (1967).

[42] J. Ciborowski et al., J. Phys. G **8**, 13 (1982).

[43] D. Evans et al., J. Phys. G **9**, 885 (1983).