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Multiparticle-multihole states in ^{31}Mg and ^{33}Mg: A critical evaluation

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The experiments that provide information about the level structure of the "island of inversion" isotopes ^{31}Mg and ^{33}Mg are reviewed. Since the model-independent measurement of their ground-state spins was done, much experimental data can be reinterpreted, and spins and parities can be assigned to their excited states. Both experimental level schemes are found in very good agreement with calculations based on antisymmetrized molecular dynamics combined with the generator coordinate method. These calculations predict that both ground states are dominated by $2\hbar\omega$ neutron excitations (more than 85% of the wave function). In the case of ^{33}Mg, the energy of the $1\hbar\omega$ and $3\hbar\omega$ levels are calculated about 400 keV too high with respect to the ground state, while in ^{31}Mg the $1\hbar\omega$ levels are calculated only 200 keV too high. New key experiments are suggested.

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I. INTRODUCTION

For several decades now, the "island of inversion" around ^{32}Mg has been the subject of many experimental and theoretical studies. The nuclei in this region of the nuclear chart are characterized by a ground-state wave function that is dominated by neutron excitations from the sd shell into the pf shell, across the magic $N = 20$ shell gap. States dominated by particle-hole excitations across a magic shell gap exist in many regions of the nuclear chart at low excitation energy (e.g., in the neutron midshell regions around $Z = 50$ and $Z = 82$ [1,2]). In the island of inversion however, these "intruder" configurations are found to become the ground state. The understanding of this inversion of "normal" and intruder configurations has been the subject of many theoretical studies [3–9], yet the predictive power of most theories is not good enough to firmly establish the low-energy structure of the isotopes in this region. Normal $0\hbar\omega$ configurations (without excitations across the magic shell gap) coexist at low energy with $1\hbar\omega$ and $2\hbar\omega$ intruder configurations (with, respectively, one or two neutrons excited across the $N = 20$ shell gap) and compete for being the ground state [8,9]. Indications for this came from Coulomb excitation experiments on the even-even Mg isotopes, which revealed a very large transition probability to the low-lying first excited 2^+ level in ^{32}Mg [10,11]. Comparing the measured $B(E2,0^+ \rightarrow 2^+)$ transition probability to shell model calculations with and without excitations of neutrons to the pf shell suggested that these levels are dominated by $2\hbar\omega$ configurations. The transition probability in the ^{30}Mg isotope could be interpreted as due to normal $0\hbar\omega$ configurations [12], illustrating that ^{31}Mg is a transitional nucleus. The coexistence of normal and intruder configurations in these even-even isotopes was established recently by observing a normal 0_2^+ state in ^{32}Mg around 1 MeV [13] and an intruder 0_2^+ state in ^{30}Mg around 1.8 MeV [14].

The low-energy properties of the odd-mass Mg isotopes are, however, the most sensitive probes to study the coexistence of normal and intruder configurations at low excitation energy. Normal $0\hbar\omega$ configurations as well as intruder $2\hbar\omega$ excitations have a positive parity in ^{31}Mg$_{19}$ and a negative parity in ^{33}Mg$_{21}$. The $1\hbar\omega$ and $3\hbar\omega$ intruder excitations have negative parity in ^{31}Mg and positive parity in ^{33}Mg. Thus assigning spins and parities to their levels provides a means to study the coexistence of normal and intruder configurations.

II. REVIEW OF EXPERIMENTAL OBSERVABLES FOR ^{31}Mg

As ^{31}Mg lies between the normal ^{30}Mg and the intruder ^{32}Mg, its low-energy structure is expected to be dominated by positive parity $0\hbar\omega$ and $2\hbar\omega$ states, as well as negative parity $1\hbar\omega$ and $3\hbar\omega$ excitations. An overview of the conclusions from different experiments is given in Table I, and the revised level scheme is presented in Fig. 1.

The starting point for making spin-parity assignments to levels in ^{31}Mg has been the direct measurement of the ^{31}Mg ground-state spin. It has been determined in a model-independent way by combining the results from a g-factor measurement (via the β-nuclear magnetic resonance (β-NMR) method) with the magnetic hyperfine splitting measured in collinear laser spectroscopy [15]. The hyperfine splitting yields the magnetic moment, and the ratio of both observables yields the nuclear spin ($I = \mu/g$), firmly establishing $I = 1/2$ for the ^{31}Mg ground state. The parity is determined by comparing the measured magnetic moment (with sign) to calculated values for the lowest positive and negative parity $I = 1/2$ levels using different shell model effective interactions as well as in a Nilsson model approach [15]. In all models, good agreement is observed for the $1/2^+$ state magnetic moment, while the $1/2^-$ level has the wrong sign and magnitude of μ. Excellent agreement is found if the magnetic moment is calculated in the sd-pf model space with two neutrons blocked in the pf shell. Thus the ^{31}Mg $1/2^+$ ground state (g.s.) has a nearly pure $2p3h$ ($2\hbar\omega$) neutron configuration. This is consistent with the fact that the observed magnetic moment [$\mu = -0.88355(15)\mu_N$] is close to that calculated for the first excited $1/2^+$ state in ^{29}Mg ($\mu = -0.83\mu_N$) using the universal-sd (USD) interaction [16]. The ^{31}Mg ground state therefore has a three-neutron-hole wave function as in ^{29}Mg, with the two neutrons in the pf shell acting as spectators (thus having their spins coupled to zero).

TABLE I. Overview of experimental data on ^{31}Mg.

Method	Observables	Year	Reference
β decay: $^{27-34}$Na	Level scheme	1984	[18]
β decay: ^{31}Na ($3/2^+$) to ^{31}Mg to ^{31}Al ($5/2^+$)	Ground state (g.s.) spin/parity assigned $(3/2)^+$	1993	[19]
	Strongly populated $E_x = 50$ keV, $t_{1/2} = 16(3)$ ns: $\pi = +$		
	Strongest populated levels: $E_x = 50$, 673, 2244 keV		
	Weakly populated levels: $E_x = 221$, 945, 1029 keV		
^{32}Na ($\pi = -$) β-delayed neutron decay	Populated levels in ^{31}Mg: $E_x = 461$, 945, 1390 keV	1993	[19]
Laser and β-NMR spectroscopy	Measured ground-state spin and g factor, $I^\pi = 1/2^+$	2005	[15,17]
	Parity based on sign of g factor		
	Assigned spin/parities to 0-, 50-, 221-, and 461-keV levels		
β decay: ^{31}Mg ($1/2^+$) to ^{31}Al ($5/2^+$)	Lifetime and g.s. feeding revised	2005	[21]
Fast timing measurement	Lifetime 461 keV, $I^\pi = 7/2^-$ assigned, $\pi = -$ for 1154 keV	2005	[26]
^{32}Na β-delayed neutron decay	$E_x = 461$, 945, 1154, 1390 keV populated	2007	[27]
Neutron knockout from ^{32}Mg	221, 461 keV strongly populated	2008	[29]
	673, 945, 1154, 2243 keV less populated		
Proton knockout from ^{32}Al(1^+)	$E_x = 673$, 2015 keV populated; $3/2^+$, $5/2^+$ assigned	2009	[23]
Low-energy Coulomb excitation of ^{31}Mg	Spin/parity assignments to 673, 945 keV ($3/2^+$, $5/2^+$)	2011	[28]

The measured g.s. spin of ^{31}Mg is not in agreement with the earlier observed 13(6)% g.s. feeding in the ^{31}Mg to ^{31}Al β decay [19]. Because the ground-state spin/parity of ^{31}Al is $5/2^+$ (established in a g factor measurement [20]), such high ground-state feeding does not agree with a second forbidden $1/2^+ \rightarrow 5/2^+$ decay. A new ^{31}Mg β-decay study by Marechal *et al.* revealed less than 1% feeding to the ^{31}Al($5/2^+$) g.s. [21], with a $\log ft > 6.8$, in agreement with a second forbidden decay.

With this established g.s. spin/parity, assignments to other levels can be made using all other available experimental data. In the left part of Fig. 1, the levels that are strongly populated in the ^{31}Na β decay [19] are indicated by thick black lines. The ^{31}Na g.s. spin 3/2 has been measured by combining magnetic resonance and hyperfine structure data [22]. The parity of this ground state is positive due to the three protons in the sd shell. Therefore, the 50-, 673-, and 2243-keV levels in ^{31}Mg, most probably have a positive parity, which is confirmed by other experiments for the 50- and 673-keV levels. Indeed, the half-life of 16.0(2.8) ns for the 50-keV level suggests an $M1$ decay to the g.s. (see Ref. [19] for details), thus a $3/2^+$ spin/parity is assigned to the 50-keV level. The level at 673 keV as well as a level at 2015 keV have been populated in the proton knockout reaction on ^{32}Al (dashed blue lines) [23]. The ^{32}Al ground state has a normal sd-shell structure (established through g factor and quadrupole moment measurements [24,25]), and therefore $0\hbar\omega$ neutron-hole configurations are populated in this knockout reaction. Miller *et al.* suggests spins/parities $3/2^+$ and $5/2^+$ for these levels, with a leading $\nu d_{3/2}^{-1}$ configuration [23].

The 221-keV level has a large $\log ft$ and was therefore proposed to have negative parity. The observed multipolarity $L = 1$, for its decay to the ground state and to the 50-keV level, supports $I^\pi = 3/2^-$ [15,19]. Weakly populated levels in the ^{31}Na decay appear at 945 and 1029 keV (thin black lines). The levels at 461, 945, 1154, and 1390 keV are populated via the β-delayed neutron decay from ^{32}Na [19,26,27] (dashed red lines). The 461-, 1154-, and 1390-keV levels are not populated in the ^{31}Na decay, and therefore have either negative parity and/or spin $I = 7/2$. The lifetime of the 461-keV level was measured by Mach *et al.* [26] and is consistent with an $E2$ decay to the $3/2^-$ 221-keV level, supporting a $7/2^-$ assignment.

Coulomb excitation of ^{31}Mg mainly populates the 945-keV level [28] (squares next to levels seen in this work). The authors demonstrated that it is populated directly via an $E2$ transition from the $1/2^+$ ground state, thus a spin/parity $5/2^+$ was assigned to it.

Finally, in a neutron knockout reaction from ^{32}Mg [29], the 221- and 461-keV levels are strongly populated, with cross sections of respectively, 23(3) and 19(6) mb. The higher lying levels at 673, 945, 1154, and 2244 keV are also directly populated in this reaction (stars in Fig. 1), with respective cross sections between 2 and 6 mb. Assuming that ^{32}Mg has a ground state dominated by a $2p2h$ neutron configuration, this reaction will preferably populate $1p2h$ ($3/2^-$, $7/2^-$, $1/2^-$) and $2p3h$ ($1/2^+$,$3/2^+$,$5/2^+$,$7/2^+$) levels (illustrated in Fig. 2). The momentum distribution of the 945-keV decay corresponds to an $l = 0$ or $l = 1$ momentum transfer, suggesting this state is dominated by a positive parity $s_{1/2}$ or negative parity $p_{3/2}$ configuration. Considering this level has been excited strongly in the Coulomb excitation experiment, it has most likely a similar $2p3h$ structure as the ground state [thus dominated by a $(s_{1/2}d_{3/2}^2)_{5/2^+}$ configuration].

With these assigned spins and parities to all levels below 1 MeV, it is possible to compare the level scheme to a calculated scheme by Kimura [9] using antisymmetrized molecular dynamics plus the generator coordinate method with the Gogny D1S force (left side of Fig. 1). The calculated leading configurations are given next to each level, and a correspondence is suggested with the experimentally observed levels (full lines if the correspondence is supported by experimental evidence, dashed lines if the correspondence is tentative). The agreement between the calculated and experimental levels is remarkably good up to 1 MeV. The

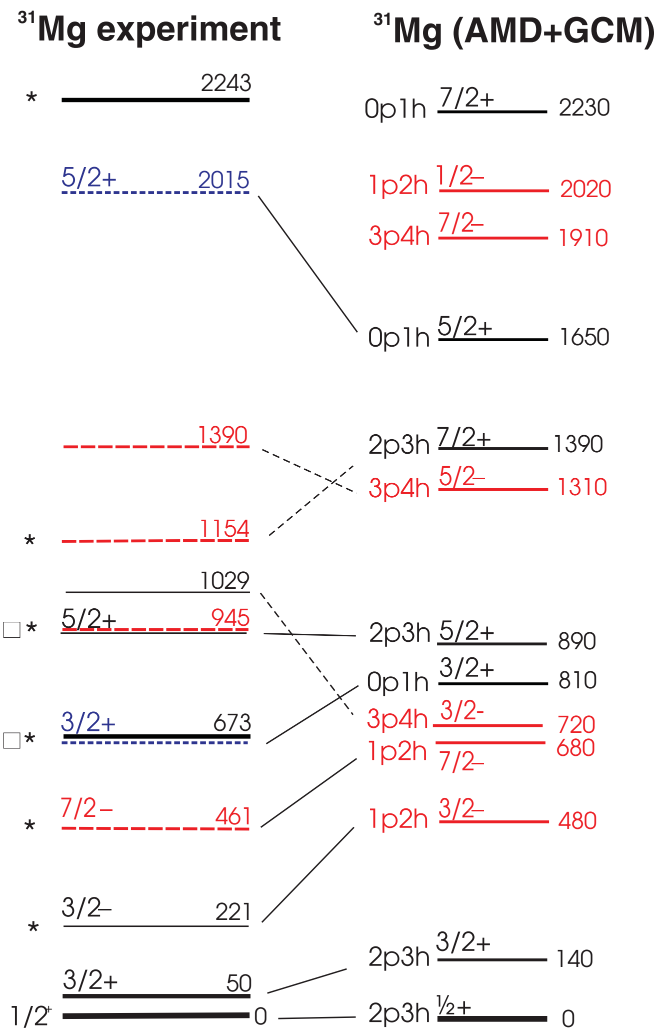

FIG. 1. (Color online) Revised level scheme of ^{31}Mg, with spin/parity assignments based on combining the results from different experiments and compared to an antisymmetrized-molecular-dynamics plus generator-coordinate-method (AMD + GCM) calculation from Kimura [9]. Calculated negative parity levels are red, positive parity levels are black. Codes in the experimental scheme are given in the text.

ground state has a nearly pure (95%) $2p3h$ structure, with a magnetic moment $\mu_{\text{AMD}} = -0.91\mu_N$ in good agreement with the experimental value $\mu_{\text{exp}} = -0.88355(15)\mu_N$. The next level has a more mixed structure, but is still dominated (61%) by a $2p3h$ configuration as well. Above the lowest

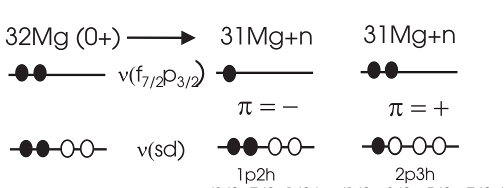

FIG. 2. Configurations most likely populated in a $1n$-knockout reaction from ^{32}Mg. The lowest spins per excitation are taken from the AMD + GCM calculations in Fig. 1

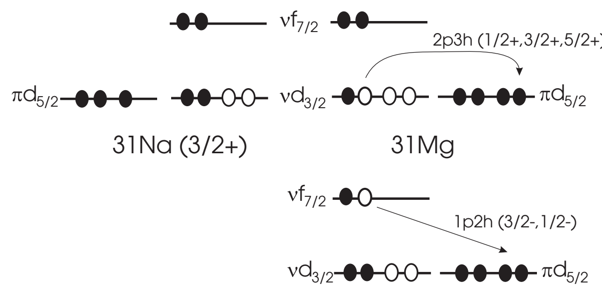

FIG. 3. Allowed (top) and first-forbidden (bottom) β-decay channels.

$2\hbar\omega$ levels, there appear two levels with a dominant $1p2h$ structure, though for the $3/2^-$ level it is strongly mixed with the $3p4h$ configuration. These levels correspond to the observed negative parity levels at 461 and 221 keV. The second $3/2^-$ state with a dominant $3p4h$ configuration appears only 250 keV higher in the calculations, but the lowest experimental level that might correspond to this state appears at 1029 keV. The first level with a normal $0p1h$ structure appears experimentally around 700 keV (800 keV in the calculation). For the other levels, no firm correspondence can be made with the calculated levels, except for the second $0p1h$ state which appears at 2 MeV (and is calculated about 350 keV lower). The structure of the 2243-keV level remains puzzling. This level is populated in the ^{32}Mg n-knockout reaction (populating mostly $1p2h$ and $2p3h$ configurations according to Fig. 2). It is however also the strongest populated level in the ^{31}Na β decay (feeding mainly levels shown in Fig. 3, through allowed Gamow-Teller decay). Therefore it is unlikely that this level corresponds to the calculated intruder $1/2^-$ level, which would correspond to a first-forbidden decay being favored over allowed Gamow-Teller (GT) decay. Note that in ^{29}Mg a similar large feeding is observed at more than 2 MeV in the β decay of ^{29}Na to ^{29}Mg. Therefore the 2243 KeV might correspond to a $0p1h$ neutron configuration, and be related to the allowed GT decay from a $\nu d_{3/2}$ to a $\pi d_{3/2}$.

III. OVERVIEW OF EXPERIMENTS RELATED TO ^{33}Mg

Of all isotopes in the island of inversion, ^{33}Mg is one of the most studied ones; however it is also a controversial one. Many different experiments have been performed to study its properties and to try unravel its structure (see Table II), yet the structure and level scheme of ^{33}Mg remained uncertain till now.

The direct measurement of the ^{33}Mg ground-state spin $I = 3/2$ in 2007 [30], using the same method as applied for ^{31}Mg, provides a crucial starting point for interpreting all previously measured data.

The parity of the ^{33}Mg ground state is presently being debated in literature. From the measured magnetic moment (and in particular its negative sign), a pure $3p2h$ negative parity ground-state configuration is invoked [30,31]. It is impossible to explain a negative magnetic moment with a positive parity $2p1h$ or $4p3h$ configuration, as suggested in [33]. A negative magnetic moment has indeed been observed for the positive

TABLE II. Overview of experiments related to ^{33}Mg.

Method	Observables	Year	Reference
β decay of $^{27-34}$Na	Half-life, P_n and P_{2n}, 6 γ's	1984	[18]
β decay: ^{33}Na ($\pi = +$) to ^{33}Mg	(β,n,2n); 705 keV level strongest populated, 1243 keV weakly populated (3/2)$^+$ assigned to g.s., strong transition at 546 keV not linked to others Forbidden transition to 484-keV level (log ft > 6.6)	2001	[32]
^{33}Mg from double fragmentation	Levels at 490 (484 keV) and 1250 (1234 keV) populated (yrast)	2001	[38]
^{33}Mg coulex in fragmentation	E2-transition probability to 484-keV level, same parity as ground state	2002	[37]
β decay: ^{33}Mg to ^{33}Al, ^{32}Al	Delayed neutron branches	2006	[41]
Proton inelastic scatter ^{33}Mg(p, p')	484 keV level populated; $\beta_M = 0.47(8)$, $\beta_n = 0.46$, and $\beta_p = 0.52$ deduced	2006	[39]
1n knockout from ^{34}Mg	484 and 546-keV levels populated, suggested opposite parity	2006	[39]
Laser spectroscopy and β NMR	$I = 3/2$ measured, sign of g factor suggests negative parity	2007	[30]
β decay: ^{33}Mg to ^{33}Al	large g.s. feeding to ^{33}Al (5/2$^+$), positive parity assigned	2008	[33]
Comment about g.s. parity	^{33}Mg g.s. spin and magnetic moment reinterpreted Nilsson model and shell model both exclude positive parity for a spin 3/2 g.s.	2010	[31]
1n removal from ^{33}Mg	$\nu p_{3/2}$ dominant component in g.s. wave function	2010	[42]

parity ^{31}Mg g.s. $2p3h$ configuration, which would correspond in ^{33}Mg to a $4p3h$ positive parity configuration. However, only if the spin is $I = 1/2$, will this configuration have a negative magnetic moment, while for $I = 3/2$ this configuration gives a positive magnetic moment, as shown in [30,31] both with shell model and Nilsson model calculations. On the other hand, β-decay studies of ^{33}Na to ^{33}Mg [32] and of ^{33}Mg to ^{33}Al [33] conclude that the ^{33}Mg ground state must have positive parity, based on strong ground-state feedings in both decay schemes.

In trying to resolve this puzzle, all spectroscopy experiments that have provided information about the structure of ^{33}Mg are reviewed here in chronological order. For each experiment the conclusions deduced from the measured observables are reconsidered, baring in mind that we now know that its ground-state spin is $I = 3/2$.

A. Results from ^{33}Na to ^{33}Mg β decay

The first low-energy level scheme of ^{33}Mg was presented in 2001, deduced from the β decay of ^{33}Na [32]. $\beta - \gamma$, $\beta - \gamma - \gamma$, and $\beta - n - \gamma$ coincidences were measured. Five γ-ray transitions were placed in the ^{33}Mg level scheme, based on coincidence data or sums of γ energies. Thus excited states at 484, 705, and 1243 keV were established. A very strong transition at 546 keV, with a similar decay time as ^{33}Na, could not be linked to any of the other five transitions, but was firmly assigned to the ^{33}Mg decay scheme. Three scenarios are possible to put this transition into the ^{33}Mg level scheme (Fig. 4). First, it de-excites a level at 546 keV toward the ^{33}Mg ground state. Second, it de-excites from a level at 546 $+\Delta$ or 546 $-\Delta$ into a low-lying long-lived state at $\pm\Delta$ with respect to the present ground state (the excitation energy should be less than the γ observation window, i.e., below 40 keV). Third, the transition de-excites one of the established levels, with a subsequent 159 keV isomeric γ transition outside the observation time window ($t_{1/2} > 500$ ns). The latter option 3 is adopted in [32], but there are no strong arguments to exclude options 1 or 2, so we keep those options as possible scenarios for further discussion. Note that no γ-decaying isomer was reported in isomer studies of isotopes in the island of inversion [34]. Option 2 has not been considered in [32], but we consider it here as a possible explanation for the conflicting g factor and β-decay data. Indeed, two near-degenerate β-decaying states, populated in different ratios in fragmentation and ISOL production, might explain this discrepancy. However, only if their energy difference is below a few keV, could the β decay possibly compete with an $E1$ γ transition between both 3/2 states.

Absolute γ-intensities and P_n values were measured for the ^{33}Na decay, from which β branchings and log ft values are deduced. A large feeding of 20(10)% was attributed to the ^{33}Mg ground state, with a log $ft = 5.27(26)$. Considering

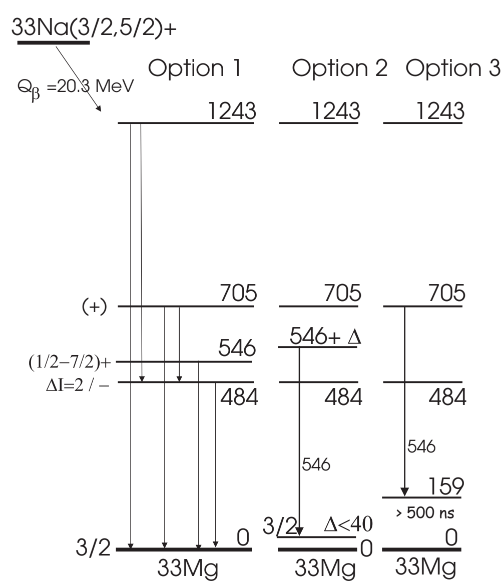

FIG. 4. The transitions and levels observed in the ^{33}Na β decay [32], with three possible options to place the 546-keV transition in the level scheme. In option 2 a β-decaying near-degenerate isomeric level with spin 3/2 is assumed to account for the conflicting magnetic moment and β-decay results. In option 3, adopted in [32], a γ-decaying isomer is proposed with a lifetime larger than 500 ns. The ground-state spin is measured to be 3/2 [30].

that ^{33}Na has a positive parity ground state (three protons in the *sd* shell), this led the authors to conclude that ^{33}Mg has a positive parity ground state as well. Note however, that more than 20 MeV is available for the decay of ^{33}Na to ^{33}Mg; thus the pandemonium effect cannot be neglected, and the g.s. feeding, as well as the feeding to other levels, might be wrongly estimated [35,36]. A similar effect has been seen in the ^{31}Mg to ^{31}Al decay, which has a decay window of 11.7 MeV. Initially a g.s. feeding of 13(6)% was determined by Klotz *et al.* [19] leading to a $\log ft = 4.9$, suggesting allowed decay. A remeasurement of this decay in 2005 showed that the g.s. branch accounts for less than 1% of the decay [21] with a $\log ft > 6.8$, consistent with a second forbidden decay from the ^{31}Mg, $1/2^+$ to the ^{31}Al, $5/2^+$ ground state. Thus the feeding from the ^{33}Na to the ^{33}Mg g.s. could easily be overestimated in a similar way, and therefore the $\log ft$ could be significantly larger than the observed 5.26(26), preventing a conclusive parity assignment for the ^{33}Mg g.s.

In Fig. 4 the possible spin/parity assignments for this β-decay study are summarized, along with the firm g.s. spin measured in [30]. The 484-keV level has a $\log ft > 6.6$ firmly establishing a forbidden decay to this level. The strong intensity of the 546-keV transition suggests it decays from a level fed by an allowed transition, thus having positive parity and spins between $1/2^+$ and $7/2^+$ (the ground-state spin of ^{33}Na is 3/2 or 5/2). The $\log ft$ of the 705-keV level [5.20(22)] is similar to that of the g.s. level, but considering that this is the most intense γ transition in the spectrum (apart from the 546 keV transition), we tentatively assign a positive parity to this level. The $\log ft = 5.97(25)$ of the 1243 keV suggests possibly a forbidden decay to this level.

B. Conclusion from intermediate energy Coulomb excitation

In the intermediate-energy Coulomb excitation of ^{33}Mg, a γ line at 478(5) keV is observed with a cross section of 81(25) mb [37]. It is proposed that this corresponds to the 484-keV level seen in the ^{33}Na decay [32]. The large cross section suggests either a very strong $E1$ transition or a strong $E2$ transition. A strong $E2$ decay is characteristic for a rotational excitation built on the ground state, with a spin one unit higher than that of the ground state, thus $I = 5/2$ (since a two-unit excitation at such low energy would give an unphysically large moment of inertia).

Because the 484-keV level was fed by a forbidden decay from ^{33}Na, it must have a negative parity. An $E2$ decay from this $I^\pi = 5/2^-$ level to the ground state leads to a negative parity for the ground state. A very strong $E1$ transition from a $5/2^-$ level could also populate a level with spin/parity $3/2^+$, which would be consistent with the presence of a near-degenerate isomeric level (option 2 in Fig. 4). However, in this scenario, the 484-keV level would have to decay by two branches [484 and 478(5) keV], which has not been observed in the β-decay study. Therefore this scenario is excluded.

Finally, if the ground state would have positive parity, then the $E2$ Coulomb excitation to the 484-keV level results in a spin/parity $5/2^+$ for this level. That is inconsistent with the β-decay data of Nummela *et al.* [32], which firmly establish a forbidden decay branch between the $(3/2,5/2)^+$ ^{33}Na ground state and the 484-keV level. Thus, from these considerations, a positive-parity ground state for ^{33}Mg can be excluded. This leaves options 1 and 3 as still possible, with spin/parity for the g.s. and 484-keV level as given in Fig. 5, where the results from all experiments are combined.

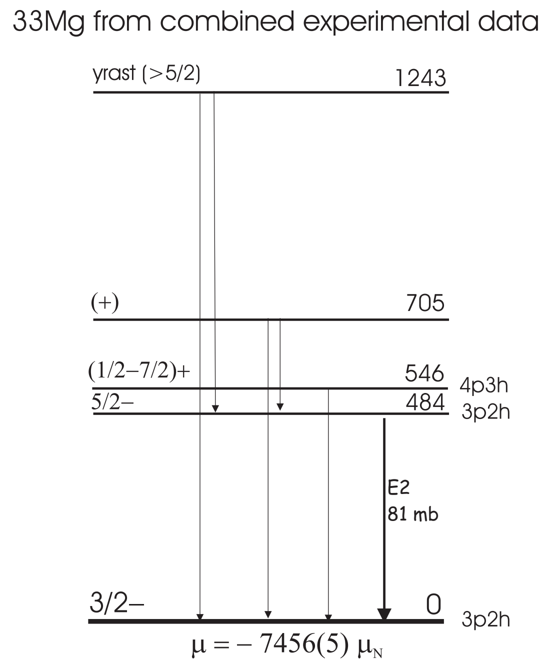

FIG. 5. Spin/parity assignments for ^{33}Mg based on the combined results from all experimental observables. Option 2 from Fig. 4 is excluded by the Coulomb excitation experiment in combination with the measured ground-state spin and the β-decay data. Option 3 is excluded by the ^{34}Mg $1n$-knockout experiment. See text for details.

C. In-beam γ spectroscopy in fragmentation

Excited states of ^{33}Mg were produced in the fragmentation reaction of a secondary ^{36}Si beam on a ^{9}Be target [38]. Prompt γ rays were observed by a NaI detector array placed around the target, in coincidence with particle identification performed at forward angles. Figure 2 of Ref. [38] shows the prompt γ spectrum for the ^{33}Mg fragments. Transitions at 490 and 1250 keV correspond to those observed in the ^{33}Na β decay, depopulating levels at 484 and 1243 keV, respectively. Because in fragmentation reactions, excited states along the yrast line are favorably populated, this limits the possible spin assignments for the 1243-keV level (Fig. 5).

D. Conclusion from a g factor measurement

The g factor of ^{33}Mg has been measured by the β-NMR method on a laser-polarized ^{33}Mg beam produced at ISOLDE-CERN [30]. If a long-lived $3/2^+$ β-decaying isomer would be present in ^{33}Mg (with $t_{1/2} > 10$ ms to survive the transport to the setup), the hyperfine structure of the optically polarized ^{33}Mg beam would have revealed the presence of two long-lived states. No evidence has been observed for such an isomer. A shorter-lived β- or γ-decaying isomer cannot be excluded in this study.

By comparing the measured g factor to predictions by various nuclear theories, the composition of the ground-state wave function can be determined.

In [30] the g factor [$g_{\exp} = -0.4971(4)$] is compared to large-scale shell model calculations with different effective interactions for the sd-pf shell. Unmixed calculations were performed for the $0\hbar\omega$ ($3/2^-$), $1\hbar\omega$ ($3/2^+$), and $2\hbar\omega$ ($3/2^-$) configurations. Both negative parity states have a negative magnetic moment, in agreement with experiment. The magnetic moment of the $3/2^+$ state is positive, in disagreement with experiment. This is a very strong indication that the parity of the ^{33}Mg ground state is negative, in agreement with the combined conclusion from the β-decay and Coulomb excitation data. Furthermore, the good agreement of the experimental magnetic moment with that calculated for the pure $2\hbar\omega$ configuration ($\mu_{2\hbar\omega} = -0.45$) supports the $3p2h$ character of the ^{33}Mg ground state. Thus, like the other isotopes in the island of inversion, ^{33}Mg has a ground state with two neutrons excited across the $N = 20$ gap.

The measured magnetic moment was also interpreted in terms of the deformed shell model by Hamamoto *et al.* ([7] and [31]). With a Woods-Saxon potential, the $3/2^-[321]$ orbital at a deformation $\beta_2 \approx 0.5$ leads to a $3/2^-$ ground state with a magnetic moment in good agreement with the experimental value. Slightly better agreement is observed with the magnetic moment of a $3/2^-$ state that becomes the ground state at a deformation $\beta_2 \approx 0.3$, and is built on the $1/2^-[330]$ orbital. Also, in this model the magnetic moments of the $3/2^+$ states are positive, in disagreement with experiment.

E. Results from proton inelastic scattering and *n*-knockout experiments

In an experiment performed at RIKEN on beams of ^{33,34}Mg [39], the ^{33}Mg isotope has been excited and investigated in two different reactions.

In the proton inelastic scattering of a ^{33}Mg beam on a H target, the ^{33}Mg isotope is produced in its 484-keV level. The production cross section in the (p, p') reaction is 30(10) mb, which allowed physicists to determine a ^{33}Mg mass deformation of $\beta_M = 0.47(8)$. This value is deduced assuming a $\Delta I = 1$ decay, and it depends very little on the assumed spins (here a $7/2^+$ to $5/2^+$ transition was assumed). No conclusion on the spin/parity of the 484-keV level is made.

In the neutron knockout from a ^{34}Mg beam on the H target, again the 484(20)-keV transition is seen. However, the most intense γ line has an energy of 561(17) keV, and it is suggested to correspond to the 546-keV transition, which is also the most intense line in the ^{33}Na β decay (decaying from a level with positive parity) [32]. Population of a positive parity state by a neutron knockout from ^{34}Mg corresponds to the knockout of a neutron from the sd orbits, while a negative parity level is populated by the $1n$ knockout from the pf orbits. Considering that the ^{34}Mg ground state has most likely four neutrons in the fp shell and two holes in the sd shell [11,40] (Fig. 6), it means that the 546-keV transition decays from a $4p3h$ positive parity

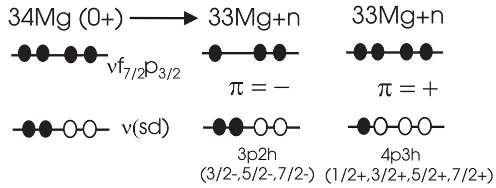

FIG. 6. Configurations most likely populated in a $1n$-knockout reaction from ^{34}Mg. For each configuration the lowest calculated spins are taken from theory (AMD + GCM [43] or unmixed shell model [32]).

level, while the 484-keV transition decays from a negative parity $3p2h$ level (parities based on the ^{33}Na β-decay results from Fig. 4). Such a configuration for the 484-keV level is in agreement with the Coulomb excitation and g-factor data, which are consistent with a strong $E2$ transition between the $3p2h$ $3/2^-$ g.s. and a $5/2^-$ excited state belonging to the same structure (summarized in Fig. 5).

It was proposed by Nummela *et al.* [32] that the 546-keV transition de-excites the level at 705 keV (option 3 in Fig. 4). This level has a 705-keV decay branch directly to the g.s., with an intensity about one-half of the 546-keV intensity [32]. This 705-keV line is not seen in the knockout reaction. Therefore it seems unlikely that the 546-keV transition decays from the 705-keV level, and therefore the proposed option 3 for the ^{33}Mg level scheme (Fig. 4) is not very probable. The fact that no microsecond isomeric state at 159 keV has been reported in isomer studies in this region [34] supports the rejection of option 3 for the ^{33}Mg level scheme. Thus only option 1 remains compatible with all experimental data, and the 546-keV γ line decays from a positive parity level at 546 keV directly to the ground state (Fig. 5).

F. β decay of ^{33}Mg to ^{33}Al

The β decay of ^{33}Mg, produced by projectile fragmentation, has been studied at GANIL [41] and at MSU [33]. The published decay schemes as well as the conclusions are somewhat different. In both studies, only levels above 1.6 MeV have been populated in the $N = 20$ isotope ^{33}Al. The major difference between both decay schemes is the strong feeding of a level at 1647 keV in [41], while most of this intensity is included in the 37(8)% missing intensity, attributed to the g.s. feeding in [33]. As the g.s. of ^{33}Al is known to be $5/2^+$ (established through a g-factor measurement [24]), Tripathi *et al.* conclude, based on the large g.s. feeding, that the parity of the ^{33}Mg g.s. is positive. Note that if a β-decaying isomeric state would be present in ^{33}Mg, it is likely also populated in the fragmentation reaction, and it could be this level that leads to this strong g.s. feeding. A detailed measurement of the half-life in the different γ lines after the ^{33}Mg β decay would provide evidence for such a β-decaying isomer, but in neither of the studies was this investigated. Another possibility is that the large Q_β value of 13.4 MeV opens a wide window for β decay to excited levels up to 5.5 MeV (the neutron separation energy), which have not been observed. This missing (probably fragmented) γ intensity could be wrongly attributed to the g.s. branch (the well-known Pandemonium effect; see, e.g., [35,36]).

In conclusion, a firm ^{33}Mg g.s. parity assignment based on the possible nonobservation of the full decay intensity (leading to a large feeding in ^{33}Al) seems difficult. A more detailed study of the absolute β-decay branches with a direct counting of the incoming and outgoing intensities (eventually also the subsequent feeding of levels in the granddaughter) is needed to account for the possible Pandemonium effect. Additionally, a detailed study of the β-γ lifetimes would be useful to establish or rule out the presence of a β-decaying isomer in ^{33}Mg.

G. ^{33}Mg 1*n* removal

The one-neutron removal cross section and the longitudinal momentum distribution of ^{32}Mg fragments produced by the reaction of a ^{33}Mg beam on a C target were measured at GSI [42]. The narrow momentum distribution is a direct indication that the $\nu p_{3/2}$ orbital is significantly occupied in the ^{33}Mg g.s. wave function. The data were compared to calculations assuming, respectively, a $3/2^+$ or a $3/2^-$ ground state for ^{33}Mg, with mixed configurations of ^{32}Mg in some excited states coupled to an odd neutron in the sd-pf shell. For both parity assumptions, a good agreement is obtained with the data by fitting the spectroscopic factors for the different configuration mixings, and therefore no firm conclusion is made about the ground-state parity. The occupation of the $\nu p_{3/2}$ orbit is found higher than that of the $\nu f_{7/2}$ orbit, independent of the assumed g.s. parity. When comparing the data to a calculation with the wave function composition from a Monte Carlo Shell Model (MCSM) calculation [with the (SDPF-M) [44] interaction], agreement is improved by lowering the $\nu p_{3/2}$ level by 1 MeV, thus resulting in a larger $\nu p_{3/2}$ contribution in the g.s. wave function. The calculated magnetic moment of the $3/2^-$ level is not much changed by this operation, and the value remains in agreement with the observed g.s. magnetic moment.

IV. PROPOSED ^{33}Mg LEVEL SCHEME

The summarized experimental information in Fig. 5, showing only the unambiguous results from all experiments, can now be used to make further spin/parity assignments to the higher levels, using the information from the observed γ-decay branches from [32]. The 546-keV level has a positive parity (allowed GT decay from ^{33}Na), and its spin is most likely $I = 1/2$. Only this spin assignment can explain why there is no decay observed to the $5/2^-$ level. Indeed, a $1/2^+$ to $5/2^-$ $M2$ decay would not compete very strongly with the $E1$ decay to the $3/2^-$ ground state (see Fig. 7). The 705-keV level, being the second most strongly populated level in the decay from ^{33}Na, is then most likely the $(3/2^+)$ level. No feeding to the $(1/2)^+$ level is seen in [32]; instead this level decays to the $5/2^-$ and $3/2^-$ levels with about equal intensities. Assuming $I^\pi = 3/2^+$ for the 705 level, both are $E1$ transitions, which could easily compete with the $E2$ decay from the $(3/2^+)$ to the $(1/2)^+$ state. The tentative spins and parities assigned to these levels could easily be verified experimentally by, e.g.,

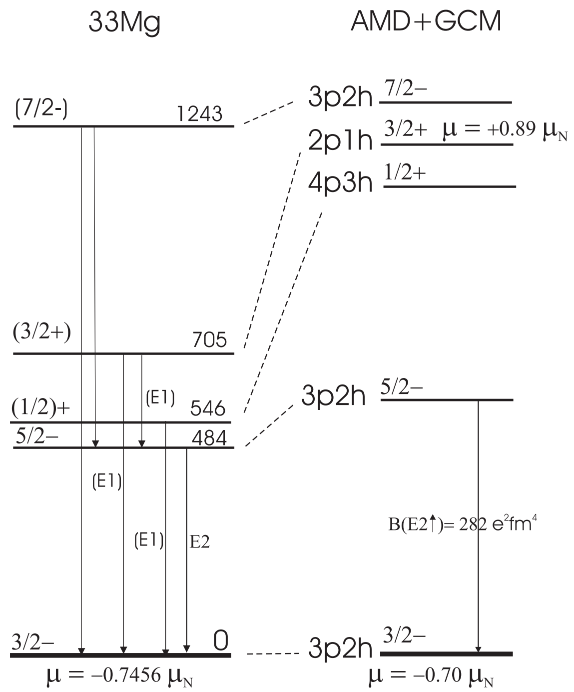

FIG. 7. Comparison of the proposed ^{33}Mg level scheme to the results from an AMD + GCM calculation performed by Kimura [43].

angular distribution measurements after a 1n or 2n transfer reaction on ^{32}Mg or ^{31}Mg.

The proposed ^{33}Mg level scheme in Fig. 7 is compared to the results from the AMD + GCM calculation by Kimura [43], who was very successful in reproducing the ^{31}Mg level scheme up to 1 MeV (Fig. 1). These calculations predict a $3/2^-$ ground state (with an 87% $3p2h$ configuration as well) and a first excited $5/2^-$ state that belongs to the $K^\pi = 3/2^-$ ground band dominated by $3p2h$ configurations as well. The calculated $E2$ transition probability to the $5/2^-$ level is $282e^2 fm^4$. This value can be compared to the experimental B($E2$) value deduced from the observed Coulomb excitation cross section of 81(25) mb [37], which was determined assuming a $5/2^+$ ground state, yielding a charge deformation $\beta_C = 0.52(12)$ and $B(E2) = 232(150)e^2 fm^4$. Assuming weak coupling between the ^{32}Mg deformed core and the odd neutron determining the spin/parity $3/2^-$, we can expect that the B($E2$) of ^{33}Mg is largely dominated by the deformation of the ^{32}Mg core and would not very much depend on the assumed spin. Therefore we can compare this B($E2$) value to the calculated one, and it is in good agreement with it.

The next two excited states in the calculation, as well as in the proposed level scheme, have positive parity. From the experimental data discussed in Sec. III E it was concluded that the 546-keV level has a dominating $4p3h$ configuration, so the suggested $(1/2)^+$ spin/parity as well as the suggested configuration are in agreement with the results from the calculation. The calculated excitation energy of this $3\hbar\omega$ intruder level is however overestimated by about 400 keV. The next calculated level is dominated by a $1\hbar\omega$ configuration and has spin/parity $3/2^+$, in agreement as well with the proposed spin/parity of the 705-keV level. The fact that both positive parity levels are dominated by different neutron configurations might also explain why the $E2$ transition between these levels is not competing with the $E1$ decay to the lower-lying $2\hbar\omega$ levels.

V. CONCLUSIONS

By combining the results from different experiments performed to investigate the level structure of the ^{31}Mg and ^{33}Mg isotopes, a coherent interpretation of all data is achieved. For both isotopes, spin/parities could be assigned to all known levels below 1 MeV. Most of these assignments are based on the results from at least two different types of measurements, yielding not only tentative values but allowing in some cases firm assignments.

For both isotopes it has been demonstrated that the ground-state wave function is dominated by the excitation of two neutrons into the pf orbits. For ^{31}Mg, with only these two neutrons in the pf shell, the pf neutrons act as "spectators" with their spins coupled to zero. That is demonstrated through the measured g factor, which corresponds to that of the normal $1/2^+$ state in ^{29}Mg. For ^{33}Mg, this $3p2h$ configuration has three neutrons in the pf shell. In this case, the sd neutrons act as spectators: the experimental g factor, $g_{\rm exp}(^{33}{\rm Mg}) = -0.4971(4)$, is very similar to that of its isotone, $g_{\rm exp}(^{35}{\rm Si}) = (-)0.468(1)$ [45], which has a normal $\nu f_{7/2}$ g.s. wave function. As the g factor of the $\nu f^n_{7/2}$ configuration does not depend on the number of nucleons occupying the orbital, one might conclude from this similarity that the three neutrons in ^{33}Mg reside mostly in the $\nu f_{7/2}$ orbital. However, from the measured longitudinal momentum distribution after a $1n$-removal reaction on ^{33}Mg, it was concluded that the $\nu p_{3/2}$ orbital must also be significantly populated in the ^{33}Mg ground state [42]. These data were well reproduced by calculations with a modified SDPF-M interaction (by lowering the $\nu p_{3/2}$ orbital with 1 MeV), which showed that the magnetic moment for the $3/2^-$ level is not changed a lot by this modification and remains in agreement with the observed value. This illustrates that in this particular case the magnetic moment is not very sensitive to the amount of mixing of $\nu p^n_{3/2}$ components in the wave function. To further investigate the composition of the g.s. wave function, its quadrupole moment could be measured.

VI. OUTLOOK

For several of the levels in ^{31}Mg, experiments have been performed to determine the leading configuration in the wave function. Similar experiments should be performed to further test the proposed level scheme of ^{33}Mg. For example, a low-energy Coulomb excitation experiment on a postaccelerated ^{33}Mg beam would preferentially populate the $5/2^-$ and the suggested $7/2^-$ levels. Such an experiment would thus confirm the proposed spin/parity for the 1243-keV level. Other key experiments in this respect are $1n$- and $2n$-transfer reaction studies with postaccelerated ^{31,32}Mg beams or a proton knockout reaction from a relativistic ^{34}Al beam. The ^{34}Al isotope has a mixed $1p0h$ and $3p2h$ ground-state structure (about 50% each), as determined in a g-factor measurement [46]. Thus, mostly such neutron configurations would be populated in ^{33}Mg, when produced by the proton knockout on a ^{34}Al beam. It would allow us to search for the lowest $1p0h$ state in ^{33}Mg, which has not yet been observed, and establish the nature of the proposed $(7/2^-)$ level at 1243 keV. However, the presence of a long-lived β-decaying 1^+ $1\hbar\omega$ isomer in ^{34}Al, predicted in shell model calculations in [46] and observed recently at GANIL [47], will also populate $2p1h$ levels, thus allowing us to confirm the suggested structure of the 705-keV level.

The only data that remain inconsistent with the proposed level scheme of ^{33}Mg are the large ground-state feedings observed in the $^{33}{\rm Na} \rightarrow {}^{33}{\rm Mg} \rightarrow {}^{33}{\rm Al}$ β decay. However, as it was shown for the $^{31}{\rm Mg} \rightarrow {}^{31}{\rm Al}$ β decay, where a similar large ground-state feeding was reported initially [19], a careful remeasurement of the decay could solve such discrepancies [21]. For the case of ^{33}Mg, such measurement should preferably be done using a total absoption spectrometer in order not to miss any of the expected very high-energy γ transitions which might be present due to the pandemonium effect [37], and applied to an ultra pure ^{33}Mg beam. Ultrapure beams are best obtained at ISOL facilities using a laser ion source and using additional purification. This is possible with a Penning trap (trap assisted decay spectroscopy as applied recently at Jyvaskyla [48]) or by using collinear resonance ionization which is being developed at ISOLDE (laser assisted decay spectroscopy [49]). Additionally, the loss of feeding due to the presence of isomeric states (both in ^{33}Mg and ^{33}Na) should be verified by a dedicated isomer search in these isotopes (in a wide lifetime range).

In summary, the present work suggests several new experiments which can be performed to further test the proposed ^{33}Mg level scheme, and will give input to nuclear theorists for further refining their calculations.

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