Nuclear electron capture in Li-like ions

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The nuclear electron capture (EC) process in lithium-like ions is studied. We find that the daughter helium-like ions are created mostly in the $2\,^1S_0$ and $2\,^3S_1$ excited states with probabilities denoted as P_0 and P_1, respectively. The ratio of probabilities depends on spin I of a mother nucleus and the type of the EC transition. For allowed EC transitions $I \to I \pm 1$ the ratio has a simple form $\frac{P_0}{P_1} = \frac{2I+1}{2(I\pm1)+1}$. Additionally, we found the simple relation between probabilities of EC decay for lithium- and hydrogen-like ions $P_{Li} = P_H(\frac{2(I\pm1/2)+1}{(2I+1)} + \frac{\rho^{2s}(Z)}{2\rho^{1s}(Z)})(1-\frac{q}{Z})^3$, where $q = 0.464$. We also discuss applications of excited states formed in helium-like ions, especially the parity nonconservation effect.

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I. INTRODUCTION

Recently, a few experiments on orbital electron capture (EC) in helium- and hydrogen-like ions have been performed at GSI-Dramstadt [1,2]. A striking result has been observed in hydrogen-like ions of ^{142}Pm and ^{140}Pr where the probability of orbital EC in the allowed transitions $1 \to 0$ is about 50% higher than in helium-like ions. In the present paper we study the orbital EC properties of lithium-like ions. The ions decay into ground or excited states of helium-like ions. We derived the relation between EC probabilities for lithium- and hydrogen-like species. The relation is similar to the probability ratio investigated for helium- and hydrogen-like ions [3,4].

Helium-like ions are the simplest multielectron systems in nature. Investigation of these species attracts the attention of theoreticians and experimentalists for some time. Especially, the formation of excited states in helium-like ions is the subject of many experimental and theoretical studies [5–7]. Up to now, the excited states were created applying three experimental methods: electron capture by hydrogen-like ions [8,9], the excitation of helium-like ions [10] and the ionization of lithium-like ions [11–13]. Here we propose to form $2\,^1S_0$ and $2\,^3S_1$ excited states by nuclear electron capture in lithium-like ions. We calculated the relative ratio of probabilities to reach the $2\,^1S_0$ and $2\,^3S_1$ excited states in helium-like ion. Recently, based on a rigorous QED approach the transition probabilities from these states to the 1^1S_0 ground state have been studied [14].

In the $2\,^1S_0$ state of helium-like ions, the parity-nonconservation (PNC) phenomenon could be investigated [15]. The PNC phenomenon is investigated in atoms theoretically and experimentally since a few decades [16–20]. The PNC effect has been studied in neutral Cs, Tl, Pb and Bi atoms. The most precise measurement has been performed for the S and P mixed states in Cs atoms [18]. However, the main uncertainty in a theoretical analysis of the PNC effect for many-electron atoms is connected with the electron-electron interaction. Precise calculations were performed only for simple systems with one or two electrons [14]. Therefore the PNC effect should be studied in the $2\,^1S_0$ excited state of helium-like ions.

II. WAVE FUNCTIONS

In this section we construct and discuss the wave functions for lithium-, helium-, and hydrogen-like ions. The wave functions with the proper orbital momentum are built from nuclear and electronic parts.

The electronic ground state wave function of lithium-like ion with spin -1/2 could be expressed as the antisymmetrized product of $1s$ and $2s$ normalized relativistic Dirac spinors $\chi^i_{ns}(k)$ [21–23], each with spin -1/2,

$$|1/2,1/2\rangle_{3e} = \frac{\mathcal{A}[\chi^+_{1s}(1)\chi^-_{1s}(2)\chi^+_{2s}(3)]}{\sqrt{6}}. \tag{1}$$

Spinor indexes i and k denote the sign of the spin projection and the electron ordering number, respectively. The antisymmetrization operator was denoted by $\mathcal{A}$. However, Eq. (1) can be rewritten in the following form:

$$\begin{aligned}|1/2,1/2\rangle_{3e} &= \frac{1}{\sqrt{3}}|1^1S_0,0\rangle_{1,2}\chi^+_{2s}(3) - \frac{1}{\sqrt{6}}|2\,^1S_0,0\rangle_{1,2}\chi^+_{1s}(3)\\ &\quad + \frac{1}{\sqrt{6}}|2\,^3S_1,0\rangle_{1,2}\chi^+_{1s}(3) - \frac{1}{\sqrt{3}}|2\,^3S_1,1\rangle_{1,2}\chi^-_{1s}(3),\end{aligned} \tag{2}$$

where the wave functions of helium-like ion and the electron enumerated by the index 3 were separated. The helium-like wave functions are defined as

$$\begin{aligned}|1^1S_0,0\rangle_{1,2} &= \frac{\mathcal{A}[\chi^+_{1s}(1)\chi^-_{1s}(2)]}{\sqrt{2}},\\ |2\,^1S_0,0\rangle_{1,2} &= \frac{\mathcal{A}[\chi^+_{1s}(1)\chi^-_{2s}(2) - \chi^-_{1s}(1)\chi^+_{2s}(2)]}{2},\\ |2\,^3S_1,0\rangle_{1,2} &= \frac{\mathcal{A}[\chi^+_{1s}(1)\chi^-_{2s}(2) + \chi^-_{1s}(1)\chi^+_{2s}(2)]}{2},\\ |2\,^3S_1,\pm1\rangle_{1,2} &= \frac{\mathcal{A}[\chi^\pm_{1s}(1)\chi^\pm_{2s}(2)]}{\sqrt{2}}.\end{aligned} \tag{3}$$

A total wave function of a lithium-like ion is constructed from the electronic part, given by Eq. (1), and the nuclear part with orbital momentum equal to I. Both parts are coupled to the total orbital momentum $I \pm 1/2$:

$$|I \pm 1/2, M\rangle_{Li} = \sum_{i=-1/2}^{1/2} (1/2, i, I, M-i|I \pm 1/2, M) \times |1/2, i\rangle_{3e}|I, M-i\rangle_N, \quad (4)$$

where the expression $(1/2, i, I, M-i|I \pm 1/2, M)$ denotes the Clebsch-Gordan coefficient.

In the EC process, a lithium-like ion having Z protons decays into a helium-like ion with $Z-1$ protons and emits a neutrino with spin -1/2. A helium-like ion is created at the ground state or at the $2\,^3S_1$ or $2\,^1S_0$ excited states. The final wave function of the $2\,^1S_0$ excited state of helium-like ion and the neutrino has the form

$$|I \pm 1/2, M, 2\,^1S_0\rangle_{\mathrm{He}} = \sum_{n=-1/2}^{1/2} (1/2, n, I \pm 1, M-n|I \pm 1/2, M) \times |2\,^1S_0, 0\rangle_{1,2}|1/2, n\rangle_\nu|I \pm 1, M-n\rangle_N. \quad (5)$$

However, the excited state $2\,^3S_1$ could be coupled together with neutrino to the spin 1/2 or 3/2 (denoted later as S) and the final wave function is then given by the formula

$$|I \pm 1/2, M, 2\,^3S_1, S\rangle_{\mathrm{He}} = \sum_{n=-1/2}^{1/2}\sum_{i=-S}^{S} (S, i, I \pm 1, M-i|I \pm 1/2, M) \times (1, i-n, 1/2, n|S, i) \times |2\,^3S_1, i-n\rangle_{1,2}|1/2, n\rangle_\nu|I \pm 1, M-i\rangle_N. \quad (6)$$

The initial (final) wave function of the hydrogen-like ion contains the nuclear part of the mother (daughter) nucleus with spin I ($I \pm 1$), coupled with an orbital electron (neutrino) to spin $I \pm \frac{1}{2}$:

$$\left|I \pm \frac{1}{2}, M\right\rangle_X = \sum_{k=-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{1}{2}, k, I \pm 1, M-k|I \pm \frac{1}{2}, M\right) \times |I \pm 1, M-k\rangle_N \left|\frac{1}{2}, k\right\rangle_l, \quad (7)$$

where two indexes (X) and (l) denote a H-like system (H) with one electron (e) in the initial state or a bare nucleus (B) with emitted neutrino (ν) in the final state.

III. MATRIX ELEMENTS

The weak interaction operator $\hat{O}$, responsible for the EC decay [24] acts on nuclear and leptonic variables involved in the process and has nonzero matrix elements only between states with identical total orbital momentum and its projection. Additionally, the square of its expectation value does not depend on the projection of the total momentum.

The EC-decay probability per time unit P is given by Fermi's formula [25]

$$P = \frac{2\pi}{\hbar}|\langle f|\hat{O}|i\rangle|^2\rho_f, \quad (8)$$

where ρ_f is the density of the neutrino final states per energy unit, i and f describe the initial and final states.

We consider allowed orbital EC decay of a mother nucleus with spin I into a parent one with spin $I \pm 1$. We assume that lithium-like ion decays from a state with total spin $I \pm 1/2$. The ground state of a lithium-like ion has total spin $I - 1/2$ ($I + 1/2$) when the nuclear magnetic moment is positive (negative).

The probability P_0 that a lithium-like ion in the EC process decays into the $2\,^1S_0$ excited helium-like ion is expressed by the matrix element of hydrogen-like ion and is proportional to

$$P_0 \propto 3_{\mathrm{He}}\langle I \pm 1/2, M, 2\,^1S_0|\hat{O}|I \pm 1/2, M\rangle^2_{\mathrm{Li}} = \frac{1}{2}\left(1 - \frac{q}{Z}\right)^3 {}_B\left\langle I \pm \frac{1}{2}, M\middle|\hat{O}\middle|I \pm \frac{1}{2}, M\right\rangle^2_{\mathrm{H}}, \quad (9)$$

where the initial and the final wave functions are defined by Eqs. (4) and (5). The influence of the electron-electron interaction in a Li-like ion we estimated in the nonrelativistic approach. The wave function we approximated by a product of $1s$, $1s$, and $2s$ hydrogen-like wave functions with the effective charge as a free parameter. The effective charge we found to be equal $Z - q$ with $q = 0.464$.

In a similar way can be estimated the probability P_1 to reach the state $2\,^3S_1$ in a helium-like ion

$$P_1 \propto 3_{\mathrm{He}}\langle I \pm 1/2, M, 2\,^3S_1, 1/2|\hat{O}|I \pm 1/2, M\rangle^2_{\mathrm{Li}} + 3_{\mathrm{He}}\langle I \pm 1/2, M, {}^3S_1, 3/2|\hat{O}|I \pm 1/2, M\rangle^2_{\mathrm{Li}} = \frac{2(I \pm 1)+1}{2(2I+1)}\left(1 - \frac{q}{Z}\right)^3 {}_B\left\langle I \pm \frac{1}{2}, M\middle|\hat{O}\middle|I \pm \frac{1}{2}, M\right\rangle^2_{\mathrm{H}}. \quad (10)$$

The factor 3 in the front of Eqs. (9) and (10) reflects three possibilities of electron capture in a lithium-like ion. We assumed that states $1\,^1S_0$, $2\,^1S_0$, and $2\,^3S_1$ in the Coulomb fields of Z and $Z-1$ protons [see Eq. (3)] are orthogonal. However, the matrix elements

$${}_{Z-1}\langle 2s|1s\rangle_Z \equiv \int_0^\infty [f_{2s,Z-1}(r)f_{1s,Z}(r) + g_{2s,Z-1}(r)g_{1s,Z}(r)]r^2dr, \quad (11)$$

calculated for $Z = 5$, 60, and 90 are 0.105, 0.011, and 0.009, respectively. The matrix elements ${}_{Z-1}\langle 2s|1s\rangle_Z$ are plotted in Fig. 1 as a function of Z.

Finally, the ratio of probabilities P_0 and P_1 is given by the simple expression

$$\frac{P_0}{P_1} = \frac{2I+1}{2(I \pm 1)+1}. \quad (12)$$

A few examples of the $\frac{P_0}{P_1}$ ratio are calculated in Table I. The most effective way to reach the $2\,^1S_0$ state in a helium-like ion (see Table I) is orbital EC with the spin transition $1 \rightarrow 0$.

The probability to reach the ground state can be calculated similarly as the probability P_0. However, in that case the electron is captured from the orbital $2s$ instead of $1s$. Therefore, the ratio of probabilities to reach the states $1\,^1S_0$ or $2\,^1S_0$ equals to the ratio of the electron densities $\rho_e^{2s}(Z)/\rho_e^{1s}(Z)$ at a nucleus. The electron density averaged over the nuclear

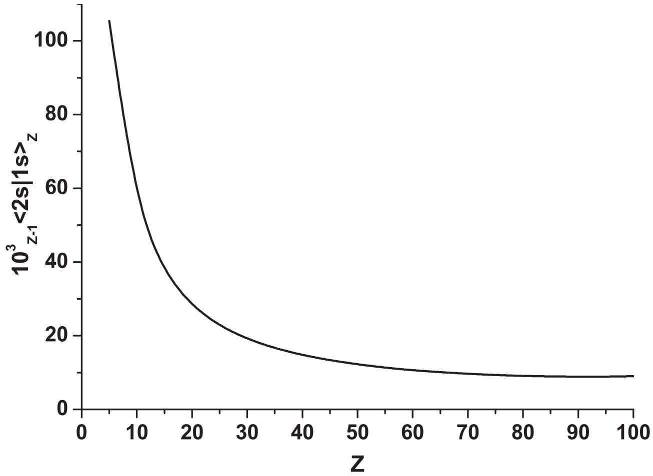

FIG. 1. The matrix elements $_{Z-1}\langle 2s|1s\rangle_Z$ calculated for two relativistic spinors 1s and 2s as a function of the charge of a nucleus Z (multiplied by 1000).

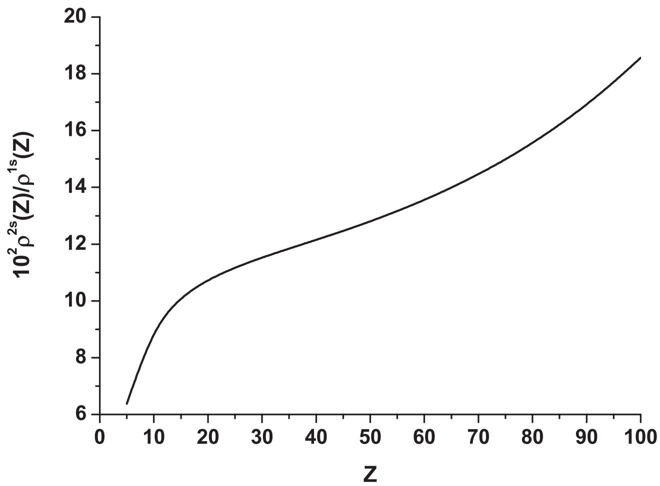

FIG. 2. The ratio of two densities $\rho^{2s}(Z)$ and $\rho^{1s}(Z)$ for 2s and 1s hydrogen-like relativistic states in the Coulomb field with charge Z is plotted. The average electron density at a nucleus is calculated according Eq. (12). In the nonrelativistic limit, the ratio has the constant value 0.125. The plotted ratio is multiplied by the factor 100.

volume with radius $R_a = 1.24A^{1/3}$ fm is given by

$$\rho_e^{ns}(Z) \equiv \frac{3}{R_a^3}\int_0^{R_a}[f_{ns}(r,Z)^2 + g_{ns}(r,Z)^2]r^2dr, \quad (13)$$

where $f_{ns}(r,Z)$ and $g_{ns}(r,Z)$ denote two components of the ns relativistic spinor. The ratio $\rho_e^{2s}(Z)/\rho_e^{1s}(Z)$ is plotted in Fig. 2. Observe that for light elements the probability to form the ground state $1\,^1S_0$ is almost 16 times smaller then to form the excited state $2\,^1S_0$. For the heaviest elements, the ratio approaches the value 0.18. In the nonrelativistic limit the ratio is constant and equals to 0.125.

We denote by P_{Li} and P_H the orbital EC probabilities for allowed transitions $I \to I+1$ ($I \to I-1$) in lithium- and hydrogen-like ions and we assume that lithium-like ions initially are in the states with the spin $I \to I+1/2$ ($I \to I-1/2$), respectively. Then the probability P_{Li} equals the sum of probabilities that lithium-like ion decays into the 1^1S_0 ground state or into the $2\,^1S_0$, $2\,^3S_1$ excited states. The EC probability into the ground state is proportional to $P_0\frac{\rho^{2s}(Z)}{\rho^{1s}(Z)}$. Adding three probabilities P_0 and P_1, given by Eqs. (9) and (10), and the probability to reach the ground state we obtain

TABLE I. The ratio of probabilities P_0/P_1 that the lithium-like ion decays into the excited states $2\,^1S_0$ and $2\,^3S_1$ of the helium-like ion. The ratio is calculated for two types of EC decays $I \to I \pm 1$.

I_i	$\frac{P_0}{P_1}$ $I \to I-1$	$\frac{P_0}{P_1}$ $I \to I+1$
0		$\frac{1}{3}$
$\frac{1}{2}$		$\frac{1}{2}$
1	3	$\frac{3}{5}$
$\frac{3}{2}$	2	$\frac{2}{3}$
...	...	...
∞	1	1

the simple relation

$$P_{\text{Li}} = \left(\frac{2(I \pm 1/2)+1}{(2I+1)} + \frac{\rho^{2s}(Z)}{2\rho^{1s}(Z)}\right)\left(1-\frac{q}{Z}\right)^3 P_{\text{H}}, \quad (14)$$

where we put $P_H \propto {}_B\langle I \pm \frac{1}{2}, M|\hat{O}|I \pm \frac{1}{2}, M\rangle_H^2$. The derived formula is similar to the relation between orbital EC probabilities in helium- and hydrogen-like ions [3,4]. The last term $\rho^{2s}(Z)/\rho^{1s}(Z)/2$ varies from 0.03 for the light nuclei up to 0.10 for the heaviest ones (see Fig. 2).

In the allowed EC transition $I \to I+1$ ($I \to I-1$), due to the conservation of the total orbital momentum hydrogen-like ion decays only from a state with spin equal to $I+1/2$ ($I-1/2$), respectively. However, a lithium-like ion, contrary to the case of a hydrogen-like ion, can also decay from the state $I-1/2$ ($I+1/2$). This is allowed, because the $2\,^3S_1$ helium excited state couples with a neutrino to spin 3/2, and both states can couple with the spin of a daughter nucleus $I+1$ ($I-1$) to total spin $I-1/2$ ($I+1/2$), respectively.

The probability that in the allowed EC transition $I \to I+1$ ($I \to I-1$) a lithium-like ion decays from the state $I-1/2$ ($I+1/2$) we denote as P'_{Li} and it can be expressed by the decay probability of a hydrogen-like ion P_{H}

$$P'_{\text{Li}} = \frac{2(I \pm 1/2)+1}{(2I+1)}\left(1-\frac{q}{Z}\right)^3 P_{\text{H}}. \quad (15)$$

The relation is similar to Eq. (14) however, it does not depend on the electron density.

IV. EXPERIMENTAL APPLICATIONS

As a first example, we shortly discuss the parity nonconservation effect in helium-like ions. The excited state $2\,^1S'_0$ is a linear combination of the $2\,^1S_0$ and $2\,^3P_0$ states both with zero angular momentum and different parity. The states are mixed by the weak interaction and the resulting state $2\,^1S'_0$ has the form

$$|2\,^1S_0'\rangle = |2\,^1S_0\rangle + \delta_w|2\,^3P_0\rangle, \quad (16)$$

where the coefficient δ_w calculated in the perturbation theory [26,32] has the form

$$\delta_w = \frac{\langle 2\,^1S_0|\frac{G_F}{2\sqrt{2}}(1-4\sin^2\vartheta_W - N/Z)\rho\gamma_5|2^3P_0\rangle}{E_{2\,^1S_0} - E_{2\,^3P_0}}. \quad (17)$$

In the latter equation G_F denotes Fermi's constant, N the neutron number, Z the proton number, ρ the nuclear density normalized to Z, and ϑ_W the Weinberg angle. The separation energy $E_{2\,^1S_0} - E_{2\,^3P_0}$ between two states has minimal value in the vicinity of nuclei with $Z = 62$ or $Z = 90$ [26–29]. The mixing parameter δ_w is of order 10^{-6} [30,31]. The nuclear electron capture process responsible for formation of the excited state $2\,^1S_0'$ occurs around $Z = 62$ in the light isotopes of Pr, Nd, Pm, Sm, Eu, or Gd.

The simplest experimental method to measure the quantity δ_w would be to determine the ϵ_2 polarization and the k_2 wave vector of one of the two photons and averaging over all directions and polarizations of the second one. The probability of the two-photon decay of the $2\,^1S_0'$ excited state is given by the formula [32,33]

$$\frac{dw_{2\gamma}}{dk_1} = A + \delta_w B\hat{\mathbf{k}}_\mathbf{1}\cdot\hat{\varepsilon}_\mathbf{1}^*\times\hat{\varepsilon}_\mathbf{1}, \quad (18)$$

where the average over all directions of the wave vector k_2 and the polarization ϵ_2 of the second photon has been found. A detailed discussion of the coefficients A and B can be found in the paper [32].

As a second example of applications we discuss the possibility of spin measurement in a mother nucleus of a decaying lithium-like ion. This is especially important for short-lived neutron-deficient exotic nuclei, in majority, with unknown spin value at the ground state. Assuming the type of the orbital EC transition ($I \rightarrow I \pm 1$) and measuring the ratio P_0/P_1, the spin I in mother nucleus can be experimentally determined from Eq. (12).

As a last application one could measure the ratio of relativistic $2s$ and $1s$ densities at a nucleus. From the experimentally determined ratio $P_{\mathrm{Li}}/P_{\mathrm{H}}$ together with additional knowledge of the type of orbital EC transition the density ratio could be determined from Eq. (14).

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