

α -cluster-model description of nuclei

S. A. Sofianos*

Physics Department, University of South Africa, P.O. Box 392, Pretoria 0001, South Africa

R. M. Adam†

South African Nuclear Energy Corporation, P.O. Box 582, Pretoria 0001, South Africa

V. B. Belyaev‡

Joint Institute for Nuclear Research, Dubna RU-141980, Russia

(Received 31 May 2011; revised manuscript received 9 October 2011; published 2 December 2011)

The description of nuclei as a system of α particles is considered using a two-variable integrodifferential equation describing A -boson systems. The method is based on the assumption that two-body forces are the dominant ones within the system. This allows the expansion of the A -body wave function in Faddeev components which in turn can be expanded in potential harmonics that result either in a coupled system of differential equations in the hyper-radius r or, when projected on the r_{ij} space, in a single two-variable, integrodifferential equation that includes the two-body correlations exactly. The formalism can be readily applied to systems of up to $A \sim 20$. Going beyond this number one encounters increasingly difficult numerical problems stemming mainly from the structure of the kernel in the integral. However, these problems can be eliminated by transforming the equation, when $A \rightarrow \infty$, into a new one having a kernel which has a simple analytical form and is easy to use in calculations. We employed the transformed equation to investigate the possibility of describing nuclei consisting of A α particles. It was found that for the Ali-Bodmer potential the $A = 5$ system, i.e., the ^{20}Ne , is the most stable while the $A = 10$ system, i.e., the ^{40}Ca , the binding energy has a maximum. Various aspects concerning the formation of $A\alpha$ nuclei are discussed.

DOI: [10.1103/PhysRevC.84.064304](https://doi.org/10.1103/PhysRevC.84.064304)

PACS number(s): 21.60.Gx, 24.10.Cn, 21.30.-x

I. INTRODUCTION

Of all possible cluster formations inside nuclei, the α cluster is the most favored due to its symmetry and the large binding energy that makes it stable in the nuclear environment. Therefore, it comes as no surprise that a vast number of nuclear structure calculations and experiments have been carried out in the past based on the α -cluster formation and its role in nuclear reactions (see, for example, Ref. [1]). As a consequence, a range of $\alpha - \alpha$ and α -nucleus interactions have been constructed and a plethora of models and methods have been employed in these studies.

Many investigations were also carried out based on the assumption that the nucleus is composed entirely of α particles. For light nuclei, one can even apply accurate methods employed in few-body system studies. We mention here, as examples, the variational calculations for the ground state of ^{12}C [2], the α -cluster description of the $^{12}\text{C}(0_2^+)$ resonance using a hyperspherical harmonics type formalism [3], the practically exact calculations based on the Alt-Grassberger-Sandhas integral equations [4] for the ^{12}C [5] and for the ^{16}O [6] systems, and the calculations for light nuclei within the framework of the generator coordinate approach [7]. More recently, Ronen *et al.* [8] employed the hyperspherical formalism to study the spectrum of the ^{16}O assuming that

it is a 4α particle system and showed that the use of two- and three-body forces reproducing the $\alpha - \alpha$ phase shifts, the ^8Be resonance, and the first and second O^+ states of ^{12}C are not sufficient to describe the low-lying experimental spectrum of ^{16}O . This was achieved by using an additional four-body force. The condensation of α cluster in nuclei was also considered with increasing interest and numerous investigations concerning this problem were carried out while the open questions in the field have been summarized by Funaki *et al.* in Ref. [9].

In the present work we are concerned with the application of a formalism suitable to study systems consisting of A bosons. This formalism is based on the assumption that the wave function of the A -body system can be expanded in Faddeev components which in turn are expanded in terms of potential harmonics (PH) [10] to obtain a set of differential equations in the hyper-radius r . Alternatively, by projecting on the r_{ij} space, with $\mathbf{r}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ in terms of the particle coordinates \mathbf{x}_i , one can obtain a single integrodifferential equation (IDE) for the wave function components which depend on two variables only, namely, the hyper-radius r and an angular variable z , while the corresponding kernel is expressed in terms of Jacobi polynomials $P_K^{\alpha, \beta}(z)$ where $\alpha = (3A - 8)/2$, $\beta = 1/2 + \ell$. For pairs in an S state ($\ell = 0$) this equation is known as the S -projected integrodifferential equation (SIDE) [11,12] and it has been successfully used in few-body system studies in the past.

The IDE can be easily applied to bosonic systems (up to $A \sim 20$). For larger systems numerical problems creep

*sofiasa@science.unisa.ac.za

†rob.adam@necsa.co.za

‡belyaev@theor.jinr.ru

in due to the large number of oscillations of the Jacobi polynomials $P_K^{\alpha,\beta}(z)$ as well as to the difficulties in using the so-called weight function $W(z) \equiv (1-z)^\alpha(1+z)^\beta$ involved in the calculations since the $W(z)$ has a δ -function-like peak for $z \sim -1$ as $A \rightarrow \infty$, i.e., as $\alpha \rightarrow \infty$, which is difficult to control numerically. However, as shown in Ref. [14], the IDE can be transformed, for large A to a much simpler form in which the kernel has a simple analytical form that does not depend on any polynomial, is independent from α , and only depends linearly on the number of particles A .

It is noted here that the SIDE, and consequently the SIDEL, formalism takes into account the two-body correlations exactly while the Coulomb interactions can also be included. This has been discussed in the past by Fabre and collaborators (see, for example, Ref. [13]). This formalism has been successfully applied to Bose-Einstein condensation of ^{87}Rb atoms confined by an externally applied trapping potential $V_{\text{trap}}(r)$ [14] and the results obtained were in excellent agreement with those obtained by the potential harmonics expansion method (PHEM) [15] and the diffusion Monte Carlo (DMC) [16] method.

In Sec. II, we briefly outline the basic ideas and derivations of our formalism. Our results are presented in Sec. III followed by a summary of our conclusions in Sec. IV.

II. FORMALISM

Let us briefly summarize our formalism and discuss its basic characteristics. We start from the assumption that the system consists of A identical bosons and that the interboson interaction can be expressed as a sum of pairwise acting central potentials so that the Hamiltonian of the system has the form

$$H = T + V = T + \sum_{i < j \leq A} V_{ij}. \quad (1)$$

The inclusion of higher order correlations in the Hamiltonian and the expansion of the corresponding A -body wave function in a systematic way has been described by Barnea and Viviani [17]. The Hamiltonian (1) implies that the wave function can be written as a product of a harmonic polynomial $H_{[L]}(\mathbf{x})$ of minimal degree L , characterized by the quantum numbers $[L]$ describing the state, and a sum of two-body amplitudes

$$\Psi(\mathbf{x}) = H_{[L]}(\mathbf{x}) \sum_{i < j \leq A} F(\mathbf{r}_{ij}, r), \quad (2)$$

where \mathbf{x} is the coordinate vector $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_A)$, $\mathbf{r}_{ij} = \mathbf{x}_i - \mathbf{x}_j$, in terms of the particle coordinates \mathbf{x}_i , and r is the hyper-radius, $r = [2/A \sum_{i < j \leq A} r_{ij}^2]^{1/2}$. The two-body amplitudes $F(\mathbf{r}_{ij}, r)$ obey the A -body Faddeev-type equation

$$\begin{aligned} [T - E] H_{[L]}(\mathbf{x}) F(\mathbf{r}_{ij}, r) \\ = -V(r_{ij}) H_{[L]}(\mathbf{x}) \sum_{k < l \leq A} F(\mathbf{r}_{kl}, r). \end{aligned} \quad (3)$$

In the present work we consider pairs in an S state with $L = 0$ in which case the amplitude F is a function of $r_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$, i.e., the amplitudes are now written as $F(r_{ij}, r)$. Letting

$F(r_{ij}, r) = P(z, r)/r^{\mathcal{L}+1}$ where now $\mathcal{L} = (D-3)/2$ [D is the dimensionality of the A -boson system, $D = 3(A-1)$] and $z = 2r_{ij}^2/r^2 - 1$, and projecting on the r_{ij} space one gets the SIDE equation describing the A -particle system [11,12]

$$\begin{aligned} \frac{\hbar^2}{m} \left[\hat{T}(r) + \frac{4}{r^2} \hat{T}(z) - E \right] P(z, r) \\ = -V(r_{ij}) \left[P(z, r) + \int_{-1}^{+1} \mathcal{K}(z, z') P(z', r) dz' \right], \end{aligned} \quad (4)$$

where $\hat{T}(r)$ and $\hat{T}(z)$ are given by

$$\begin{aligned} \hat{T}(r) &= -\frac{\partial^2}{\partial r^2} + \frac{\mathcal{L}(\mathcal{L}+1)}{r^2}, \\ \hat{T}(z) &= -\frac{1}{W(z)} \frac{\partial}{\partial z} (1-z^2) W(z) \frac{\partial}{\partial z} \end{aligned} \quad (5)$$

and $W(z)$ is the weight function which, for bosonic systems, is given by

$$W(z) = (1-z)^\alpha(1+z)^\beta, \quad (6)$$

where $\alpha = (D-5)/2$ and $\beta = 1/2$. The kernel $\mathcal{K}(z, z')$ is the projection function which is expressed in terms of the Jacobi polynomials $P_K^{\alpha,\beta}(z)$,

$$\mathcal{K}(z, z') = W(z') \sum_K \frac{(f_K^2 - 1)}{h_K} P_K^{\alpha,\beta}(z) P_K^{\alpha,\beta}(z'). \quad (7)$$

The h_K is a normalization constant given by

$$h_K = \int_{-1}^{+1} [P_K^{\alpha,\beta}(z)]^2 W(z) dz \quad (8)$$

and f_K^2 by [11,14]

$$\begin{aligned} f_K^2 &= 1 \\ &+ \frac{2(A-2)P_K^{\alpha,\beta}(-1/2) + [(A-2)(A-3)/2]P_K^{\alpha,\beta}(-1)}{P_K^{\alpha,\beta}(+1)}. \end{aligned} \quad (9)$$

When the number of particles A is large the evaluation of the Jacobi polynomials $P_K^{\alpha,\beta}(z)$ and of the weight function $W(z)$ becomes problematic since the polynomials are highly oscillatory for large α , while the behavior of the weight function for $z \rightarrow -1$ peaks sharply, resulting in intractable numerical problems. To avoid this we consider first the transformation [14]

$$r_{ij} = r\zeta/\sqrt{\alpha} \quad (10)$$

with $z = 2\zeta^2/\alpha - 1$. Then for large A the Jacobi polynomials $P_K^{\alpha,\beta}$ can be expressed in terms of Laguerre polynomials $L_K^{1/2}$,

$$\begin{aligned} P_K^{\alpha,\beta}(2r_{ij}^2/r^2 - 1) &\xrightarrow{\alpha \rightarrow \infty} (-1)^K L_K^{1/2}(\alpha r_{ij}^2/r^2) \\ &\equiv (-1)^K L_K^{1/2}(\zeta^2), \end{aligned} \quad (11)$$

which are independent of α , while the troublesome term $(1-z)^\alpha$ is removed from the weight function which now becomes

$$W(z) = C_W \zeta e^{-\zeta^2}, \quad (12)$$

where C_W is a normalization constant. By considering a further transformation

$$P(\zeta, r) = \frac{e^{\zeta^2/2}}{\zeta} Q(\zeta, r), \quad (13)$$

one obtains the integrodifferential equation for large A (SIDEL) [14]

$$\begin{aligned} \frac{\hbar^2}{m} \left\{ \hat{T}(r) + \frac{4}{r^2} \hat{T}(\zeta) - E \right\} Q(\zeta, r) \\ = -V(r_{ij}) \left[Q(\zeta, r) + \int_0^{\sqrt{\alpha}} \mathcal{K}_I(\zeta, \zeta') Q(\zeta', r) d\zeta' \right], \quad (14) \end{aligned}$$

where now the $\hat{T}(\zeta)$ is given by

$$\hat{T}(\zeta) = \frac{\alpha}{4} \left[-\frac{\partial^2}{\partial \zeta^2} + \zeta^2 - 3 \right]. \quad (15)$$

The new form of the kernel \mathcal{K}_I is

$$\begin{aligned} \mathcal{K}_I(\zeta, \zeta') = \frac{2(A-2)}{\sqrt{3}} \left\{ \left[A - 3 - \frac{2}{3} \left(\zeta^2 - \frac{3}{2} \right) \left(\zeta'^2 - \frac{3}{2} \right) \right] \right. \\ \times \zeta \zeta' e^{-(\zeta^2 + \zeta'^2)/2} + \frac{4}{\sqrt{3}} \left[e^{-[5(\zeta - \zeta') + 2\zeta \zeta']/6} \right. \\ \left. \left. - e^{-[5(\zeta + \zeta') - 2\zeta \zeta']/6} \right] \right\} \quad (16) \end{aligned}$$

which can be easily computed for any value of A .

We stress once more that the above formalism takes the two-body correlations exactly into account while the inclusion of the Coulomb potential $V_C(r_{ij})$ is trivial. Higher-order correlations stemming from other partial waves can be included, albeit in an approximate way, using the hypercentral potential as discussed in Refs. [11,14].

Equation (14) can be solved using the adiabatic approximation which involves first the solution of the eigenequation,

$$\begin{aligned} \frac{\hbar^2}{m} \left[\frac{4}{r^2} \hat{T}(\zeta) + U_\lambda(r) \right] Q_\lambda(\zeta, r) \\ = -V \left(\frac{r}{\sqrt{\alpha}} \zeta \right) \left[Q_\lambda(\zeta, r) + \int_0^{\sqrt{\alpha}} \mathcal{K}_I(\zeta, \zeta') Q_\lambda(\zeta', r) d\zeta' \right], \quad (17) \end{aligned}$$

to obtain the eigenpotentials $U_\lambda(r)$ for each value of r having an eigenvalue k_λ^2 . The later is obtained by solving

$$u_\lambda''(r) + [k_\lambda^2 + V_{\text{eff}}(r)] u_\lambda(r) = 0, \quad (18)$$

TABLE I. The ground state binding energies E_A (in MeV), the E_A/A , and the average size of the condensate r_{av} (in fm) obtained with the variant $\alpha - \alpha$ Ali-Bodmer potential AB1 (the rms of the α -particle is not added to r_{av}).

A	3	4	6	8	10	13	15	17	20	25
E_A (MeV)	1.13	2.33	3.99	5.00	5.42	5.27	4.81	4.17	2.96	0.56
E_A/A	0.38	0.58	0.67	0.63	0.54	0.41	0.32	0.25	0.15	0.002
r_{av}	2.70	2.84	3.12	3.32	3.47	3.63	3.72	3.793	3.89	4.07

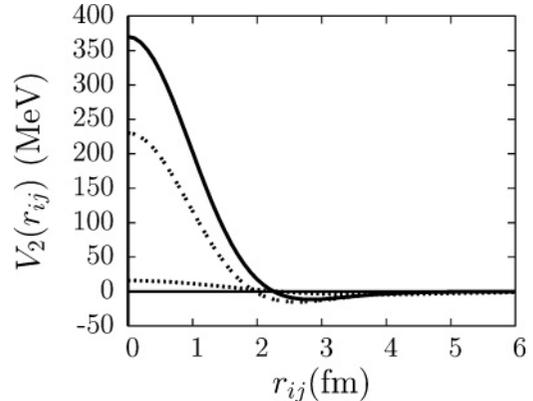


FIG. 1. Two body Ali-Bodmer type potentials $V_2(r_{ij})$ employed: straight line—AB1, [18], dashed line—AB2 [2], and dotted line—AB3 [19].

where the effective potential $V_{\text{eff}}(r)$ is given by

$$V_{\text{eff}}(r) = \frac{\mathcal{L}(\mathcal{L} + 1)}{r^2} - U_\lambda(r). \quad (19)$$

Our results are obtained using the shallowest adiabatic potential $U_\lambda(r)$.

III. RESULTS

Let us apply the formalism to $A\alpha$ systems where A is the number of α particles involved. The choice of the input potential is quite tricky as there is a wide range of $\alpha - \alpha$ interactions employed in the past in various investigations. These include deep local potentials sustaining a Pauli forbidden state and shallow potentials with a repulsive core, as well as nonlocal interactions. From the local potentials, the Ali-Bodmer one [18] is perhaps the most well known and used. It consists of two Gaussian terms the one describing the short-range repulsive behavior and the other the attractive exterior part,

$$V_{\alpha\alpha}(r_{ij}) = V_r e^{-\mu_r^2 r_{ij}^2} - V_a e^{-\mu_a^2 r_{ij}^2}. \quad (20)$$

Various choices of the constants lead to variant potentials. One such potential (designated as AB1) is with $V_r = 475$ MeV, $V_a = 130$ MeV, $\mu_r = 0.7 \text{ fm}^{-1}$, and $\mu_a = 0.475 \text{ fm}^{-1}$. Another version is the one employed by Consoni *et al.* [2] interaction (designated as AB2) in their variational calculations for the ground state of ^{12}C and subsequently used by other researchers as well. The parameter used are $V_r = 360$ MeV, $V_a = 130$ MeV, $\mu_r = 0.7 \text{ fm}^{-1}$, and $\mu_a = 0.475 \text{ fm}^{-1}$. The corresponding Coulomb potential used with the above strong

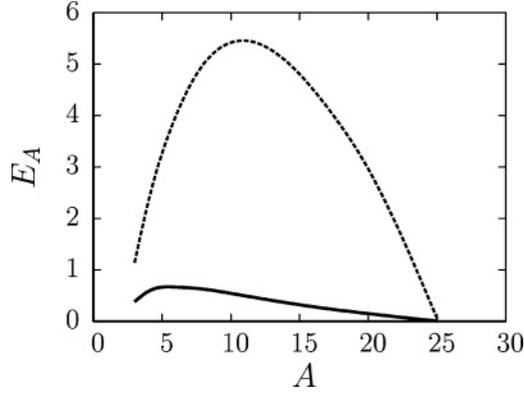


FIG. 2. The binding energy per particle, E_A/A as a function of the number of particles A for the Ali-Bodmer potentials AB1 (solid line) and AB2 (dotted line).

force is

$$V_C(r_{ij}) = \begin{cases} \frac{4e^2}{2r_C} \left[3 - \left(\frac{r_{ij}}{r_C} \right)^2 \right], & r_{ij} < r_C \\ \frac{4e^2}{r_{ij}}, & r_{ij} \geq r_C \end{cases} \quad (21)$$

with $r_C = 1.5$ fm. The Ali-Bodmer potential does not, however, give accurately the experimental resonant energy of ^8Be and in general is not fully adequate for describing the dilute 3α -boson system [19,20]. Yamada and Schuck [19] proposed a potential (designated as AB3) with with $V_r = 50$ MeV, $V_a = 34.101$ MeV, $\mu_r = 0.4 \text{ fm}^{-1}$, and $\mu_a = 0.3 \text{ fm}^{-1}$ with quite a softer short-range repulsion. It reproduces the resonant energy of ^8Be at $E_{2\alpha} = 92$ keV in agreement with experiment. The corresponding Coulomb potential used is the folded potential

$$V_{\text{Coul}}(r_{ij}) = \frac{4e^2}{r_{ij}} \text{erf}(ar_{ij}) \quad (22)$$

TABLE II. The ground state binding energies E_A (in MeV) and the average size r_{av} (in fm) obtained with the variant $\alpha - \alpha$ Ali-Bodmer AB2 potential of Ref. [2]. E_A^{exp} are the experimental binding energies for α -chain nuclei.

A	SIDE		SIDE-L		Other	
	E_A	r_{av}	E_A	r_{av}	E_A^{exp} [23]	HH [2]
3	5.71	2.23			7.27	6.60
4	10.27	2.40	5.15	2.60	14.43	
5	14.70	2.94	10.56	2.11	19.17	
6	19.03	3.17	15.11	2.52	28.48	
7	10.27	2.40	19.44	2.78	38.46	
10	34.28	3.00	31.17	2.82	59.09	
20	57.56	3.38	55.73	3.25		
50	69.70	3.62	69.88	3.49		
100			58.54	3.04		
200			44.83	2.76		
500			15.50	2.66		
600			5.15	2.60		

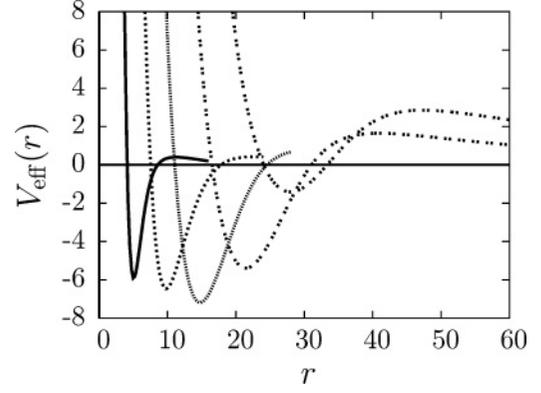


FIG. 3. The effective potentials $V_{\text{eff}}(r)A = 3, 6, 10, 17,$ and, $25,$ read from left to right, for the AB1 potential.

widely used in resonating group model calculations [21]. It is noted that this potential has been employed in Ref. [19] together with a three-body force. The three potentials are shown in Fig. 1 for comparison.

We see that while the Ali-Bodmer potentials AB1 and AB2 have a strong short-range repulsion which stimulates the Pauli state envisaged by the resonating group method (RGM) [21], the AB3 interaction in comparison has a very weak repulsion implying weak short-range correlations. It is noted here that the Consoni *et al.* potential AB2 sustains a two-body bound state at $E_2 = 1.75$ MeV.

Let us consider first the most repulsive AB1 potential. The results obtained with this potential with the SIDE equation, Eq. (4), (no need to use the SIDEL method), are given in Table I together with the average size r_{av} which is defined as the root-mean-square distance of the individual particle from the center of mass [22]

$$r_{\text{av}} = \left\langle \frac{1}{A} \sum_{i=1}^A (\vec{x}_i - \vec{X})^2 \right\rangle^{1/2} = \frac{\langle r^2 \rangle^{1/2}}{\sqrt{2A}}, \quad (23)$$

where \vec{X} is the center-of mass-coordinate. In Fig. 2 the binding energy per particle, E_A/A , as a function of the number of particles A is plotted. These results reveal that for the Ali-Bodmer potential AB1 the ^{20}Ne nucleus has a maximal stability while for the $A = 10$ system, i.e., the ^{40}Ca , the

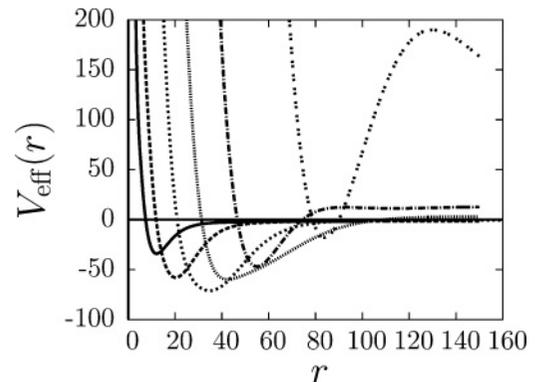


FIG. 4. Same as in Fig. 3 for the AB2 potential and for $A = 10, 20, 50, 100, 200,$ and $500.$

TABLE III. Same as for Table II for the AB3 potential.

A	SIDE		SIDE-L	
	E_A	r_{av}	E_A	r_{av}
3	0.11	3.01		
4	1.17	3.40		
10	10.98	3.64	8.51	3.48
20	38.06	3.78	34.59	3.69
50	186.10	3.94	177.58	3.90
100			647.54	4.00
200			2504.46	4.06
500			5602.25	4.08

binding energy has a maximum. By switching off the Coulomb force the maximal stability is shifted at $A \sim 13$ with $E_b = 40.61$ MeV and $E_b/A = 3.12$ MeV/particle while at $A = 80$ we obtain $E_b = 130.97$ MeV and $E_b/A = 1.64$ MeV/particle. For $A > 80$ the eigenpotential has a very long range and the numerics become cumbersome and difficult to control.

The importance of the short-range repulsion and of the existence of a bound state can be readily exposed by considering the AB2 potential [2]. The results obtained are given in Table II. It is seen that the binding energies are peaked around $A = 50$ and the formation of an α -particle systems ceased at $A \sim 600$. The binding energy per particle, E_A/A , as a function of the number of particles A is also plotted in Fig. 2.

It is interesting to see the behavior of the effective potential (19) for the two interactions, AB1 and AB2. They are plotted in Figs. 3 and 4. It is clear that they are strongly dependent on the characteristics of the pair potential. The minimum becomes deeper with increasing A until a critical value is reached. Thereafter the effective potential becomes shallower with increasing A , eventually generating a well in the positive energy region. Here the system does not sustain a bound state, and instead the eigenpotential generates the condition for a particle decay through the barrier. The two potentials, however, differ considerably in the possible number of particles needed to form a stable system, i.e., the critical value of A depends very much on the pair potential used.

Let us now turn our attention to the AB3 potential of Ref. [19] which is very shallow as compared with the AB1 and AB2 potentials. This potential has been used together with a three-body force. Although it generates the correct resonance for the $\alpha - \alpha$ system, it does not sustain any bound state (Pauli forbidden state) needed to interpret the existence of π in the $\ell = 0$ phase shifts at $k = 0$, i.e., $\delta_0(0) = \pi$, and therefore it is unrealistic. Nevertheless, it is of interest to see the results generated with this potential having such a weak short-range repulsion. The binding energy results obtained are given in Table III. The values obtained obviously are unrealistic and exemplify the importance of the short-range correlations.

IV. CONCLUSIONS

We employed a two-dimensional integrodifferential equation describing A -boson systems and taking the underlying two-body correlations exactly into account, to study the α -cluster model of nuclei. For large number of A , this equation is transformed to a simpler form which can be used in

studies for any arbitrary value of $A \geq 10$ without any major computational problem. Both equations, designated for pairs in an $\ell = 0$ state as SIDE and SIDEL respectively, have as input two-body forces only (including Coulomb forces).

In handling the problem, we faced the question of how to choose the $\alpha - \alpha$ interaction from the plethora of the available potentials. It is known that the $\alpha - \alpha$ potential should be strongly repulsive at short range. Within the the RGM theory the corresponding potential has a deep attractive well that reproduces the $\alpha - \alpha$ phase shifts but also sustains an unphysical Pauli-forbidden bound state. This unphysical state can be removed by two successive supersymmetric transformations [24] that results to a potential with a repulsive core having $1/r^2$ behavior at short distances. This behavior can be satisfactorily simulated using the Ali-Bodmer type potential which consists of two Gaussian terms generating a relatively strong repulsion at short distances and a shallow attractive well. The existence of the strong short-range repulsion, however, may give rise to numerical problems and thus various variant potentials with less repulsion were used in nuclear physics studies. To investigate the importance and the role played by these partly equivalent interactions we choose three potentials having quite different characteristics: The one with a strong repulsion at short distances [18], the second is that used in variational calculations by Consoni *et al.* [2] which sustains a bound state, and the third is the one employed by Yamada and Schuck in the Gross-Pitaevskii mean-field formalism together with a three body force [19] that has a very soft short range repulsion.

The use of a potential (AB1) that generates strong short-range correlations gave binding energies which are too weak and are peaked around $A = 10$, corresponding to the closed-shell nucleus ^{40}Ca while beyond $A \sim 26$ no formation of a nucleus consisting of α particles is possible.

The results obtained with the other two potentials (i.e., using a potential having a bound state or with soft repulsion at short distances to ease calculations) leads to unrealistic results. The existence of the bound state in the AB2 potential gave results which indicated that α clusters can be generated for very large number A (of the order of $A \sim 600$) while the lack of strong repulsive correlations in the AB3 potential gives rise to strongly bound systems with large numbers of particles.

It is interesting to note that the r_{av} does not change much with increasing A for all potentials used. It is not clear what the reason for this is, but it seems that this is related to a trade-off between the attraction of the strong force and the repulsion of the centrifugal and Coulomb terms that results in eigenpotentials which favor the existence of bound states.

We would like to mention here that such large α clusters could be formed in the center of red giants, where helium start burning. It could also formed, as was emphasized by Schramm *et al.*, [25], when quantum helium liquid is formed due to the crust evolution of the accretion on old neutron stars. Existing approximations of the description of such phenomena and structures are based mainly on Jastrow-type ansatze for two- and three-body correlations [26]. Although the present alternative method (SIDEL) includes two-body correlations only, the incorporation is exact and the numerics are tractable to any size of system. Therefore the use of this method in such studies is promising.

- [1] P. E. Hodgson, *Z. Phys. A* **349**, 197 (1994).
- [2] L. Consoni, T. K. Das, and H. T. Coelho, *Revista Brasileira de Fisica* **13**, 137 (1983).
- [3] S. I. Fedotov, O. I. Kartavtsev, V. I. Kochkin, and A. V. Malykh, *Phys. Rev. C* **70**, 014006 (2004).
- [4] E. O. Alt, P. Grassberger, and W. Sandhas, *Phys. Rev. C* **1**, 85 (1970).
- [5] H. Kamada and S. Oryu, *Nucl. Phys. A* **463**, 347 (1987).
- [6] S. Oryu, H. Kamada, H. Sekinea, T. Nishino, and H. Sekiguchi, *Nucl. Phys. A* **534**, 221 (1991).
- [7] Rizal Kurmiadi, Marsongkohadi, Zaki Su'ud, and Triyanta, *J. Phys. A* **16**, 57 (2005).
- [8] Y. Ronen, N. Barnea, and W. Leidemann, *Few-Body Syst.* **38**, 97 (2006).
- [9] Y. Funaki, M. Girod, H. Horiuchi, G. Röpke, P. Schuck, G. Röpke, A. Tohsaki, and T. Yamada, *J. Phys. G: Nucl. Part. Phys.* **37**, 064012 (2010).
- [10] M. Fabre de la Ripelle, *Ann. Phys. (NY)* **147**, 281 (1983).
- [11] M. Fabre de la Ripelle, H. Fiedeldey, and S. A. Sofianos, *Phys. Rev. C* **38**, 449 (1988).
- [12] M. Fabre de la Ripelle, S. A. Sofianos, and H. Fiedeldey, *Few-Body Syst.* **6**, 157 (1989).
- [13] M. Fabre de la Ripelle, S. A. Sofianos, and R. M. Adam, *Ann. Phys. (NY)* **316**, 107 (2005).
- [14] R. M. Adam and S. A. Sofianos, *Phys. Rev. A* **82**, 053635 (2010).
- [15] T. K. Das, S. Canuto, A. Kundu, and B. Chakrabarti, *Phys. Rev. A* **75**, 042705 (2007).
- [16] D. Blume and C. H. Greene, *Phys. Rev. A* **63**, 063601 (2001).
- [17] N. Barnea and M. Viviani, *Phys. Rev. C* **61**, 034003 (2000).
- [18] S. Ali and A. R. Bodmer, *Nucl. Phys.* **80**, 99 (1966).
- [19] T. Yamada and P. Schuck, *Phys. Rev. C* **69**, 024309 (2004).
- [20] Y. Suzuki and M. Takahashi, *Phys. Rev. C* **65**, 064318 (2002).
- [21] K. Wildermuth and Y. C. Tang, *A Unified Theory of the Nucleus* (Viweg, Braunschweig, Germany, 1977).
- [22] B. Chakrabarti, T. K. Das, and P. K. Debnath, *J. Low Temp. Phys.* **157**, 527 (2009).
- [23] A. C. Merchant and W. D. M. Rae, *Nucl. Phys. A* **549**, 431 (1992).
- [24] D. Baye, *Phys. Rev. Lett.* **58**, 2738 (1987).
- [25] S. Schramm and K. Langanke, and S. E. Koonin, *Astrophys. J.* **397**, 579 (1992).
- [26] H. M. Müller and K. Langanke, *Phys. Rev. C* **49**, 524 (1994).