

Nuclear symmetry energy and the role of the tensor force

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Using the Hellmann-Feynman theorem we analyze the contribution of the different terms of the nucleon-nucleon interaction to the nuclear symmetry energy E_{sym} and the slope parameter L . The analysis is performed within the microscopic Brueckner-Hartree-Fock approach using the Argonne V18 potential plus the Urbana IX three-body force. We find that the main contribution to E_{sym} and L comes from the tensor component of the nuclear force.

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Nuclear symmetry energy, defined as the difference between the energies of neutron and symmetric matter, and in particular its density dependence, is a crucial ingredient needed to understand many important properties of isospin-rich nuclei and neutron stars [1–3]. Experimental information on the density dependence of the symmetry energy $E_{\text{sym}}(\rho)$ below, close to, and above saturation density ρ_0 can be obtained from the analysis of data of isospin diffusion measurements [4], giant [5] and pygmy resonances [6], isobaric analog states [7], isoscaling [8], or meson production in heavy ion collisions [9,10]. Accurate measurements of the neutron skin thickness $\delta R = \sqrt{\langle R_n^2 \rangle} - \sqrt{\langle R_p^2 \rangle}$ in heavy nuclei, via parity-violating electron scattering experiments [11,12] or by means of antiprotonic atom data [13,14], can also help to constrain $E_{\text{sym}}(\rho)$, since its derivative is strongly correlated with δR [15]. Additional information on $E_{\text{sym}}(\rho)$ can be extracted from the astrophysical observations of compact objects, which open a window into both the bulk and microscopic properties of nuclear matter at extreme isospin asymmetries [3]. In particular, the characterization of the core-crust transition in neutron stars [16–20] or the analysis of power-law correlations, such as the relation between the radius of a neutron star and the equation of state [21], can put stringent constraints on $E_{\text{sym}}(\rho)$. Theoretically, $E_{\text{sym}}(\rho)$ has been determined using both phenomenological and microscopic many-body approaches. Phenomenological approaches, either relativistic or nonrelativistic, are based on effective interactions that are frequently built to reproduce the properties of nuclei [22]. Since many of such interactions are built to describe systems close to the symmetric case, predictions at high asymmetries should be taken with care. Skyrme-Hartree-Fock [23] and relativistic mean field [24] calculations are the most popular ones among them. Microscopic approaches start from realistic nucleon-nucleon (NN) interactions that reproduce the scattering and bound state properties of the free two-nucleon system and include naturally the isospin dependence [25]. In-medium correlations are then built using many-body techniques that microscopically account for isospin asymmetric effects such as the difference in the Pauli blocking factors of neutrons and protons in asymmetric matter. Among the approaches of this type, the most popular ones are the Brueckner-Bethe-Goldstone (BBG) [26] and the Dirac-Brueckner-Hartree-Fock

(DBHF) [27] theories, the variational method [28], the correlated basis function (CBF) formalism [29], the self-consistent Green's function technique (SCGF) [30] or, recently, the $V_{\text{low } k}$ approach [31]. Nevertheless, in spite of the experimental [32] and theoretical [33] efforts carried out to study the properties of isospin-asymmetric nuclear systems, $E_{\text{sym}}(\rho)$ is still uncertain. Its value at saturation, E_{sym} , is more or less well established (~ 30 MeV), and its behavior below saturation is now much better known [34]. However, for densities above ρ_0 , $E_{\text{sym}}(\rho)$ is not well constrained yet, and the predictions from different approaches strongly diverge. Why $E_{\text{sym}}(\rho)$ is so uncertain is still an open question whose answer is related to our limited knowledge of the nuclear force, and in particular of its spin and isospin dependence [35–46].

In this work we analyze the contribution of the different terms of the NN interaction to E_{sym} and the slope parameter $L = 3 \rho_0 [\partial E_{\text{sym}}(\rho) / \partial \rho]_{\rho_0}$. The analysis is carried out with the help of the Hellmann-Feynman theorem [47] within the framework of the microscopic Brueckner-Hartree-Fock (BHF) approach [26]. We employ the Argonne V18 (Av18) potential [48] supplemented with the Urbana IX three-body force [49], which for use in the BHF approach is reduced to an effective two-body density-dependent force by averaging over the third nucleon [50]. We find that the tensor term of the nuclear force gives the largest contribution to both E_{sym} and L .

The BHF approach is the lowest order of the BBG many-body theory [26]. In this theory, the ground-state energy of nuclear matter is evaluated in terms of the so-called hole-line expansion, where the perturbative diagrams are grouped according to the number of independent hole-lines. The expansion is derived by means of the in-medium two-body scattering G matrix. The G matrix, which takes into account the effect of the Pauli principle on the scattered particles and the in-medium potential felt by each nucleon, has a regular behavior even for strong short-range repulsions, and it describes the effective interaction between two nucleons in the presence of a surrounding medium. In the BHF approach, the energy is given by the sum of only *two-hole-line* diagrams including the effect of two-body correlations through the G matrix. It has been shown by Song *et al.* [51] that the contribution to the energy from *three-hole-line* diagrams (which account for the effect of three-body correlations) is

TABLE I. Kinetic $\langle T \rangle$ and potential $\langle V \rangle$ energy contributions to E_{NM} , E_{SM} , E_{sym} , and L . Units are given in MeV.

	E_{NM}	E_{SM}	E_{sym}	L
$\langle T \rangle$	53.321	54.294	-0.973	14.896
$\langle V \rangle$	-34.251	-69.524	35.273	51.604
Total	19.070	-15.230	34.300	66.500

minimized when the so-called continuous prescription [52] is adopted for the in-medium potential, which is a strong indication of the convergence of the hole-line expansion. We adopt this prescription in our calculation.

The BHF approach does not give access to the separate contributions of the kinetic and potential energies in the correlated many-body state, because it does not provide the correlated many-body wave function $|\Psi\rangle$. However, it has been recently shown [53] that the Hellmann-Feynman theorem [47] can be used to calculate the ground-state expectation values of both contributions from the derivative of the total energy with respect to a properly introduced parameter. Writing the nuclear matter Hamiltonian as $H = T + V$, and defining a λ -dependent Hamiltonian $H(\lambda) = T + \lambda V$, the expectation value of the potential energy is given as

$$\langle V \rangle \equiv \frac{\langle \Psi | V | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \left(\frac{dE}{d\lambda} \right)_{\lambda=1}. \quad (1)$$

Then, the kinetic energy contribution $\langle T \rangle$ can be obtained simply by subtracting $\langle V \rangle$ from the total energy E .

In Table I we show the kinetic and potential energy contributions to the energy of neutron matter E_{NM} , symmetric matter E_{SM} , E_{sym} , and L at saturation ($\rho_0 = 0.187 \text{ fm}^{-3}$ in our calculation). We note that the kinetic contribution to E_{sym} is very small and negative. This is in contrast with the result for a free Fermi gas (FFG), whose contribution at ρ_0 is $\sim 14.4 \text{ MeV}$. A similar result has been recently found by Xu and Li [44]. According to these authors, this is due to the strong isospin dependence of the short-range NN correlations (SRC) induced by the tensor force. They have shown, in fact, that the increase of the kinetic energy of symmetric matter due to SRC is much larger than that of neutron matter, the kinetic part of the symmetry energy becoming then negative. We also note that the kinetic contribution to L is smaller than the corresponding one of the FFG ($L^{\text{FFG}} \sim 29.2 \text{ MeV}$). The major contribution to both E_{sym} and L is due to the potential part. Note that, in fact, this contribution is practically equal to the total value of E_{sym} , and it represents $\sim 78\%$ of L .

Tables II and III show the partial wave, and the spin (S) and isospin (T) channel decompositions of the potential part of E_{NM} , E_{SM} , E_{sym} , and L at ρ_0 . Contributions up to $J = 8$ have been considered. We observe that the spin-triplet ($S = 1$) and isospin-singlet ($T = 0$) channel, and in particular the 3S_1 wave, give the largest contribution to both E_{sym} and L . This is due, as we explicitly show in the following, to the effect of the tensor component of the nuclear force that dominates the potential contribution to the symmetry energy and L , mainly through the 3S_1 - 3D_1 channel. Note that this channel, which gives the major contribution to the energy of symmetric matter, does

TABLE II. Partial wave decomposition of the potential part of E_{NM} , E_{SM} , E_{sym} , and L . Units are given in MeV.

Partial wave	E_{NM}	E_{SM}	E_{sym}	L
1S_0	-23.070	-19.660	-3.410	-3.459
3S_1	0	-45.810	45.810	71.855
1P_1	0	4.904	-4.904	-18.601
3P_0	-5.321	-4.029	-1.292	-1.898
3P_1	16.110	10.720	5.390	21.949
3P_2	-16.000	-9.334	-6.666	-21.168
1D_2	-5.956	-3.201	-2.755	-11.033
3D_1	0	0.981	-0.981	-3.739
3D_2	0	-3.982	3.982	16.601
3D_3	0	-0.798	0.798	4.895
1F_3	0	0.694	-0.694	-3.348
3F_2	-0.695	-0.229	-0.466	-1.799
3F_3	2.000	0.821	1.179	4.883
3F_4	-0.796	-0.194	-0.602	-3.239
1G_4	-0.812	-0.247	-0.565	-3.036
3G_3	0	-0.001	0.001	0.441
3G_4	0	-0.213	0.213	0.449
3G_5	0	-0.057	0.057	0.650
1H_5	0	0.029	-0.029	0.107
3H_4	0.033	0.040	-0.007	0.232
3H_5	0.225	-0.033	0.258	0.968
3H_6	0.043	0.034	0.009	0.144
1I_6	-0.082	0.023	-0.105	-0.591
3I_5	0	-0.029	0.029	0.342
3I_6	0	0.067	-0.067	-0.819
3I_7	0	-0.021	0.021	0.239
1J_7	0	-0.027	0.027	0.385
3J_6	0.044	0.020	0.024	0.283
3J_7	-0.062	-0.060	-0.002	-0.313
3J_8	0.036	0.014	0.022	0.242
1K_8	0.031	0.021	0.010	0.169
3K_7	0	-0.011	0.011	0.138
3K_8	0	0.038	-0.038	-0.491
3L_8	0.021	0.006	0.015	0.166

not contribute to neutron matter. Note also that isospin-triplet ($T = 1$) channels give similar contributions to both E_{NM} and E_{SM} which almost cancel out in E_{sym} . Similar arguments have been pointed out by other authors [35–46].

Next, we analyze the role played by the different terms of the nuclear force, particularly the one of the tensor, in the determination of E_{sym} and L . To such end we apply the Hellmann-Feynman theorem to the separate components of the Av18 potential and the Urbana IX three-body force. The

TABLE III. Spin (S) and isospin (T) channel decomposition of the potential part of E_{NM} , E_{SM} , E_{sym} , and L . Units are given in MeV.

(S, T)	E_{NM}	E_{SM}	E_{sym}	L
(0, 0)	0	5.600	-5.600	-21.457
(0, 1)	-29.889	-23.064	-6.825	-17.950
(1, 0)	0	-49.836	49.836	90.561
(1, 1)	-4.362	-2.224	-2.138	0.450

Av18 potential has 18 components of the form $v_p(r_{ij})O_{ij}^p$ with $O_{ij}^{p=1,18}$

$$= 1, \vec{\tau}_i \cdot \vec{\tau}_j, \vec{\sigma}_i \cdot \vec{\sigma}_j, (\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j), S_{ij}, \\ S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j), \vec{L} \cdot \vec{S}, \vec{L} \cdot \vec{S}(\vec{\tau}_i \cdot \vec{\tau}_j), L^2, \\ L^2(\vec{\tau}_i \cdot \vec{\tau}_j), L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j), L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j), (\vec{L} \cdot \vec{S})^2, \\ (\vec{L} \cdot \vec{S})^2(\vec{\tau}_i \cdot \vec{\tau}_j), T_{ij}, (\vec{\sigma}_i \cdot \vec{\sigma}_j)T_{ij}, S_{ij}T_{ij}, (\tau_{zi} + \tau_{zj}),$$

with S_{ij} being the usual tensor operator, \vec{L} the relative orbital angular momentum, \vec{S} the total spin of the nucleon pair, and $T_{ij} = 3\tau_{zi}\tau_{zj} - \tau_i \cdot \tau_j$ the isotensor operator defined analogously to S_{ij} . Note that the last four operators break the charge independence of the nuclear interaction.

As we said above, the Urbana IX three-body force is reduced to an effective density-dependent two-body force when used in the BHF approach. For simplicity, in the following we refer to it as the reduced Urbana force. This force is made of three components of the type $u_p(r_{ij}, \rho)O_{ij}^p$, where $O_{ij}^{p=1,3} = 1, (\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j), S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j)$, introducing additional central, $\sigma\tau$, and tensor terms (see, e.g., Baldo and Ferreira in Ref. [50] for details).

The separate contributions to $E_{\text{NM}}, E_{\text{SM}}, E_{\text{sym}}$, and L from the various components of the Av18 potential and the reduced Urbana force are given in Table IV. The contribution from the tensor component to E_{sym} and L (contributions $\langle V_{S_{ij}} \rangle$ and $\langle V_{S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$ from the Av18 potential, and $\langle U_{S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$ from the reduced Urbana force) is 36.056 and 69.968 MeV, respectively. These results clearly confirm that the tensor force gives the largest contribution to both E_{sym} and L . The contributions from the other components are either negligible, as for instance the contributions from the charge symmetry breaking terms ($\langle V_{T_{ij}} \rangle, \langle V_{(\vec{\sigma}_i \cdot \vec{\sigma}_j)T_{ij}} \rangle, \langle V_{S_{ij}T_{ij}} \rangle$, and $\langle V_{(\tau_{zi} + \tau_{zj})} \rangle$), or they almost cancel out.

In summary, using the Hellmann-Feynman theorem we have evaluated the separate contributions of the different terms of the nuclear force to the nuclear symmetry energy E_{sym} and the slope parameter L . Our study has been done within the framework of the BHF approach using the Av18 potential plus an effective density-dependent two-body force deduced from the Urbana IX three-body one. Our results show that the potential part of the nuclear Hamiltonian gives the main contribution to both E_{sym} and L . The kinetic contribution to E_{sym} is very small and negative, in agreement with the recent results of Xu and Li [44]. We have performed a partial wave

TABLE IV. Separate contributions to $E_{\text{NM}}, E_{\text{SM}}, E_{\text{sym}}$, and L from the various components of the Av18 potential (denoted as $\langle V_i \rangle$) and the reduced Urbana force (denoted as $\langle U_i \rangle$). Units are given in MeV.

	E_{NM}	E_{SM}	E_{sym}	L
$\langle V_1 \rangle$	-31.212	-32.710	1.498	-5.580
$\langle V_{\vec{\tau}_i \cdot \vec{\tau}_j} \rangle$	-4.957	3.997	-8.954	-20.383
$\langle V_{\vec{\sigma}_i \cdot \vec{\sigma}_j} \rangle$	-0.319	-0.382	0.063	2.392
$\langle V_{(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	-5.724	-11.388	5.664	2.521
$\langle V_{S_{ij}} \rangle$	-0.792	1.912	-2.704	-4.998
$\langle V_{S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	-4.989	-37.592	32.603	47.095
$\langle V_{\vec{L} \cdot \vec{S}} \rangle$	-7.538	-1.754	-5.784	-12.251
$\langle V_{\vec{L} \cdot \vec{S}(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	-2.671	-6.539	3.868	3.969
$\langle V_{L^2} \rangle$	11.850	13.610	-1.760	1.521
$\langle V_{L^2(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	-2.788	0.270	-3.058	-14.262
$\langle V_{L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j)} \rangle$	1.265	1.383	-0.118	1.405
$\langle V_{L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	0.051	0.008	0.043	-0.341
$\langle V_{(\vec{L} \cdot \vec{S})^2} \rangle$	4.194	5.682	-1.488	-0.327
$\langle V_{(\vec{L} \cdot \vec{S})^2(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	5.169	-6.190	11.359	31.368
$\langle V_{T_{ij}} \rangle$	0.003	0.039	-0.036	-0.022
$\langle V_{(\vec{\sigma}_i \cdot \vec{\sigma}_j)T_{ij}} \rangle$	-0.017	-0.106	0.089	0.042
$\langle V_{S_{ij}T_{ij}} \rangle$	0.004	0.079	-0.075	-0.124
$\langle V_{(\tau_{zi} + \tau_{zj})} \rangle$	-0.084	-0.001	-0.083	-0.331
$\langle U_1 \rangle$	2.985	3.251	-0.266	-0.630
$\langle U_{(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	2.254	3.999	-1.745	-7.228
$\langle U_{S_{ij}(\vec{\tau}_i \cdot \vec{\tau}_j)} \rangle$	-0.935	-7.092	6.157	27.768

and a spin-isospin channel decomposition of the potential part of E_{sym} and L , showing that the major contribution to them is given by the spin-triplet ($S = 1$) and isospin-singlet ($T = 0$) channel. As we have explicitly shown, this is due to the dominant effect of the tensor force, which gives the largest contribution to both E_{sym} and L . In conclusion, our results confirm the critical role of the tensor force in the determination of the symmetry energy and its density dependence.

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