PHYSICAL REVIEW C **84**, 061901(R) (2011)

Universal hydrodynamics and charged hadron multiplicity at energies available at the CERN Large Hadron Collider

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(Received 31 August 2011; published 7 December 2011)

Time evolution of a "little bang" created in heavy-ion collisions can be divided into two phases, the pre-equilibrium and hydrodynamic. At what moment does the evolution become hydrodynamic and is there any universality in the hydrodynamic flow? To answer these questions we briefly discuss various versions of hydrodynamics and their applicability conditions. In particular, we elaborate on the idea of "universal" (all-order resumed) hydrodynamics and propose a simple new model for it. The model is motivated by results obtained recently via the Anti–de Sitter and conformal field correspondence. Finally, charged hadron multiplicities in heavy-ion collisions at the Relativistic Heavy Ion Collider and Large Hadron Collider are discussed. At the freeze-out, the multiplicities can be related to total entropy produced in the collision. Assuming the universal hydrodynamics to hold, we calculate the entropy production in the hydro stage of the collision. We end up speculating about a connection between the multiplicity growth and the temperature dependence of the quark-gluon plasma viscosity.

DOI: 10.1103/PhysRevC.84.061901 PACS number(s): 12.38.Aw, 12.38.Mh, 24.10.Nz, 47.75.+f

Introduction. This Rapid Communication contains some further developments of the ideas put forward in our paper [1]. There we argued that entropy production in the strongly coupled quark-gluon plasma (sQGP) should be computed using an all-order resummed hydrodynamics and that the resummation makes it possible to provide reliable estimates, even starting from very short thermalization times. The main goal of this Rapid Communication is to connect this proposal to some recent theory developments based on the Anti–de Sitter and conformal field theory (AdS-CFT) setting [2], which support our ideas, as well as to address the phenomenological question of charged particle multiplicity production in heavy-ion collisions at the LHC, to be detailed below. Let us stress here that the entropy production is only one of several applications for which an all-order resummation might be important. There are additional interesting phenomena in which matter gradients are large and applicability limits of standard hydrodynamics are in question. Let us give here two examples of those.

As recent studies have shown, fluctuations of initial-state density in heavy-ion collisions are the origin of sound waves. By freeze-out, these waves reach large distances, comparable to the fireball radius itself, and are observed as fluctuations of angular harmonics in the particle distributions. It is remarkable that amplitudes of up to ninth harmonics have been measured, displaying good agreement with hydrodynamics [3–5]. Yet the questions as to how to treat these fluctuations in nonequilibrium and from what initial times they can be evolved hydrodynamically remain unanswered.

"Mach cones" induced in the matter by quenching jets [6,7] present another application of the sound waves in heavy-ion physics. Unlike sounds from the previous example, the jet-induced waves were studied in detail within the AdS-CFT context [8]. At late stages, the results were shown to agree with hydrodynamics. However, when exactly the hydrodynamics becomes applicable is still an open question, despite the availability of exact AdS-CFT solutions. The issue becomes even more important with the first Large Hadron Collider (LHC) data on jets, revealing events with huge amounts of energy, ∼100 GeV, deposited by a jet. This calls for studies of the full nonlinear settings, beyond the linearized sound-wave approximation.

Below we discuss initial conditions for hydrodynamics from the perspective of the AdS-CFT results. We also propose in a new, all-order resumed, hydrodynamics model for Bjorken explosion. Later, we use this model in order to compute the entropy production in the hydro phase. Phenomenological relevance to the data on charged particle multiplicities is also discussed. We summarize and provide additional discussions.

Multiplicities in pp and AA collisions. One of the first discoveries made by the LHC is a rapid rise with energy of multiplicities of charged hadrons produced both in proton-proton (*pp*) and heavy-ion collisions. The discovery is especially dramatic in heavy-ion collisions, where most of the existing models have failed to predict the data.

The first ALICE data on charged particle multiplicity in lead-lead collisions are [9]

$$\left.\frac{dN}{d\eta}\right|_{\text{PbPb}} (2.76\ \text{TeV}) = 1584 \pm 76, \tag{1}$$

combined with the earlier data from the Relativistic Heavy Ion Collider (RHIC); these imply the multiplicity in nucleus-nucleus (*AA*) collisions growing with the (center-of-mass) energy per nucleon as

$$\left.\frac{dN}{d\eta}\right|_{\text{PbPb}} (E_{NN}) \sim E_{NN}^{0.30}. \tag{2}$$

The corresponding power in the pp collisions is 0.22, and thus the ratio of the two also grows with the energy

$$\left.\frac{dN}{d\eta}\right|_{\mathrm{PbPb}} \Big/ \left.\frac{dN}{d\eta}\right|_{pp} \sim E_{NN}^{0.08}. \tag{3}$$

From the RHIC energy ($E = 0.2$ TeV) to the LHC, the double ratio is

$$\frac{\left.\frac{dN}{d\eta}\right|_{\mathrm{PbPb,LHC}} \Big/ \left.\frac{dN}{d\eta}\right|_{pp,\mathrm{LHC}}}{\left.\frac{dN}{d\eta}\right|_{\mathrm{AuAu,RHIC}} \Big/ \left.\frac{dN}{d\eta}\right|_{pp,\mathrm{RHIC}}} = 1.23. \tag{4}$$

This noticeable change with energy calls for a theoretical explanation. (An increase in the atomic number, 197 for Au and 208 for Pb, explains only 0.055 of it.)

Particle production in heavy-ion collisions proceeds via two basic phases: (i) a prethermalization phase and (ii) a hydrodynamical stage. Theoretical frameworks used for their descriptions are very different.

The first one is based on a perturbative quantum chromodynamics (pQCD) cascade of gluons, described by high-energy evolution equations including gluon saturation effects, or a color glass condensate (CGC). CGC relies on the emergence of a semihard scale, the saturation momentum

$$Q_s^2 \sim A^{1/3} x^{-\lambda}, \quad \lambda = 0.25\text{–}0.30, \tag{5}$$

related to the density of gluons with a longitudinal momentum fraction x. Within the CGC approach, many quantities become universal and simply scale with the saturation scale, the property known as geometrical scaling. As an example of this, the particle's p_t spectra in pp collisions are found to have a dependence of the type $f(p_t/Q_s)$ [10].

If the hypothesis of geometrical scaling is true, then a CGC-based estimate for the *AA*-*pp* multiplicity ratio should be energy independent [see, however, Ref. [11], which discusses the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) effect on the saturation scale]. Yet, experiments observe a prominent growth with energy. Another observation is that the CGC-based multiplicity estimates tend to underestimate it at the LHC. In particular, Ref. [12] underestimates the observed multiplicity by ~35%: $dN/d\eta|_{\mathrm{PbPb}}(2.76\ \mathrm{TeV}) \simeq 1175$.[1]

The second phase of heavy-ion collision process is hydrodynamic, and it produces particles (entropy) due to finite viscosity. While the viscosity itself grows, from a strongly coupled regime at the beginning of the evolution to hadronic matter at its end, and even gets very large near freeze-out, the main entropy production still happens at the very beginning. This is so because the viscosity coefficient gets multiplied by flow gradients, which rapidly decrease with the evolution time. Below, we will discuss the effect of viscosity on the multiplicity growth.

When does the hydro stage start? This question is not well posed unless we specify what exactly is meant by "hydro" and what is meant by its "start." To define a starting moment is relatively easy: For any theory and an approximation to it, the approximation is considered as valid as long as the two deviate from each other within a preset accuracy (say, 1%).

[1]In the latter [13] the results of Ref. [12] were revised toward a much better agreement with the data.

The question of defining "hydro" has different meaning and depends on what approximation is used. We will mention three cases here: (i) "Ideal hydrodynamics" is a collective description that includes local quantities only, such as pressure and energy density. The accuracy and validity of the "ideal hydro" approximation depends on viscous corrections to this local approximation, which contain first gradients of the flow of matter. (ii) Navier-Stokes hydrodynamics (NS) includes these viscous terms, and its accuracy is estimated by *next* terms involving *two* gradients. (iii) "Resummed hydrodynamics" (RH) includes in some approximate form all higher-order gradients. Accuracy of this approximation is given by deviations from first-principles nonequilibrium calculations.

Obviously, as the accuracy of approximation increases from (i) to (iii), its applicability regions widen. In connection with heavy-ion collision processes, it means that "the beginning of the hydro stage" moves toward increasingly earlier times.

Conformal "resummed hydrodynamics." When talking about all-order resummed hydro, it is convenient to introduce viscosity as a momenta-dependent function. In Ref. [1] we extracted it from an AdS-CFT computed sound dispersion curve. In Ref. [14] we took a more formal approach, which lead us to propose the following model:

$$\eta(\omega, k^2) = \frac{\eta_0}{1 - 1/2k^2 - i\omega\tau_R}. \tag{6}$$

Here $\eta_0 = 1/2$ in dimensionless units in which $2\pi T = 1$ and that corresponds to the celebrated ratio of viscosity to entropy density equal to $1/4\pi$ [15]. In these units, $\tau_R = 2 - \ln 2$ and is the relaxation time of the Israel-Stewart (IS) model [16]. The model (6) reproduces well the small ω and k expansion up to fifth order.

We consider Bjorken flow [17] as a model for the explosion. It has the simplest geometry: There is no dependence on two transverse coordinates, as well as on space-time rapidity $y = (1/2)\ln[(t-x)/(t+x)]$. What is left is a dependence on the proper time $\tau = \sqrt{t^2 - x^2}$ only. In these coordinates, the metric is $ds^2 = -d\tau^2 + \tau^2 dy^2 + d\vec{x}_\perp^2$, and we will not write any further details, as those are well known. In the Bjorken flow, there are no spatial variations ($\vec{k} = 0$) and our model (6) reduces back to IS. It is well known that additional nonlinear terms contribute to the entropy production that is not governed by the viscosity term only. However, the entropy is produced mostly at the beginning of the expansion, when viscous terms are dominant. It is especially true for the case of very early thermalization. This is why a more or less reliable estimate of entropy production can emerge only if we know the dissipation tensor at very large ω.

Let us introduce the dimensionless variable $w = \tau T$. Then, within the all-order hydrodynamic approximation, the entropy production equation can be written with some "universal function" of this variable

$$\frac{dw}{d\ln\tau} = F(w), \tag{7}$$

Solving (7) one finds a time dependence of the temperature, from the initial time τ_i to the final (freeze-out) time τ_f

$$\tau(w_f) = \tau(w_i)\exp\left[\int_{w_i}^{w_f}\frac{dw'}{F(w')}\right], \quad T(w) = w/\tau(w). \quad (8)$$

The *final* values T_f, τ_f should be read off the experimental data [there is evidence that T_f is about the same at the RHIC and LHC while τ_f grows with E_{NN}, and hence the total entropy (multiplicity) also grows].

From these experimental data, one may use the solution and trace back to the *initial* values for the thermalization time and temperature. However, Eq. (8) provides only one relation between the two. In the plane (τ_i, T_i) it defines a curve. [This is similar to field theory renormalization group (RG) flows of couplings.] An additional condition, to be detailed below, is needed in order to fix the absolute values of the initial conditions.

The function $F(w)$ can be expanded in powers of $1/w$, with the coefficients of the expansion being higher-order viscosities. Thanks to the AdS-CFT correspondence, for conformal $\mathcal{N}=4$ plasma the expansion terms are known up to third order [18,19]

$$F(w)/w = \frac{2}{3} + \frac{1}{3w}\bar\eta - \frac{1}{3w^2}\frac{\bar\eta(\ln 2 - 1)}{3\pi} + \frac{15 - 2\pi^2 - 45\ln(2) + 24[\ln(2)]^2}{972\pi^3 w^3} + O(1/w^4). \quad (9)$$

The first term corresponds to the ideal hydro. The second one is NS, with $\bar\eta = 1/3\pi$, while the third one is second order including nonlinear terms, beyond IS. At large w the series is convergent. We will argue below that hydro is a reasonably good approximation for $w \geqslant w_0 \simeq 0.4$. For illustration purposes we give here values of these terms at w_0, normalized to the first term:

$$(3/2)F(w_0)/w_0 = 1 + 0.1326 + 0.0107 - 0.0189. \quad (10)$$

It is clear that the NS term is still very important. The next terms are an order of magnitude smaller. Moreover, we would like to stress the sign alternating feature of these higher-order terms. As a result, being resummed, these terms contribute less than each of them separately.

To get such qualitative behavior we proposed a new and very simple "resummation model" with a new (positive) parameter α

$$F(w)/w = \frac{2}{3} + \frac{\bar\eta}{3(w+\alpha)}. \quad (11)$$

This model obviously expands into a sign-alternating geometric series. The important feature is in the small w behavior, which gets regularized. One might want to relate α either to the relaxation time τ_R of IS or to the expansion terms in Eq. (9). However, we argue that the most natural choice is simply $\alpha = \bar\eta$: To eliminate any self-heating at the early times, α shall be bigger than $\bar\eta$, $\alpha \geqslant \bar\eta$. $\alpha = \bar\eta$ appears to be the optimal model choice: It leads to $T(\tau) \sim \tau^0$ at small τ, which is consistent with Ref. [20] and CGC-based estimates. This choice maximizes the amount of entropy that can be produced within the model (11). Larger α will drive the hydro to appear

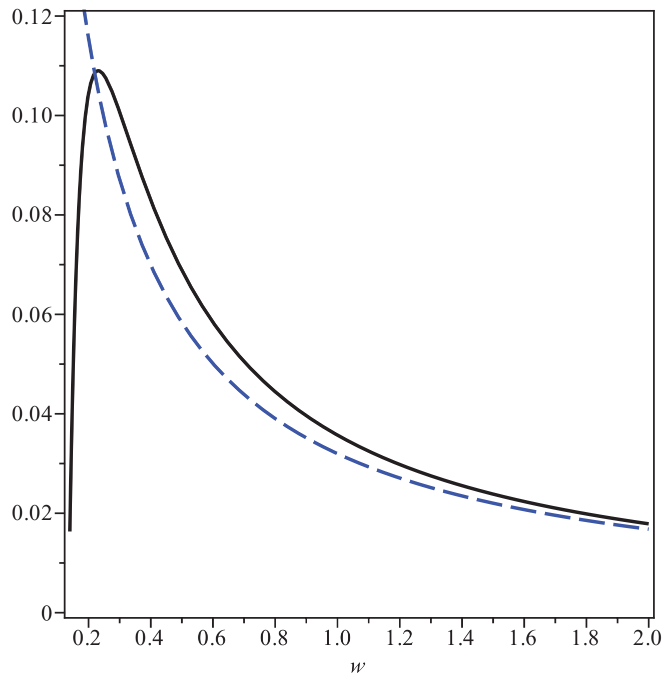

FIG. 1. (Color online) The solid line is our model for $F(w)/w - 2/3$; the dashed line is its known large w asymptotics (9).

to be more ideal. Figure 1 compares this model function with the known asymptotics at large w given by (9).

AdS-CFT-based studies of equilibration. AdS-CFT correspondence provides a possibility to study strongly coupled plasmas. We will not elaborate here on any details but will only refer to some relevant results. Following the first applications of the AdS-CFT to equilibrium properties (such as the equation of state) and near-equilibrium kinetic coefficients (such as viscosity), it was further realized that the duality provides a unique opportunity to study nonequilibrium problems from first principles based on well-developed gravitational tools. From the fifth-dimensional perspective, a fall of an object under gravitational force is equivalent to a relaxation process, proceeding from UV to IR. This was clearly demonstrated in Ref. [21] for an elastic membrane falling under its own weight.

Without citing the full list of the AdS-CFT-based studies of nonequilibrium phenomena, we refer to two recent works [19,22], relevant for this Rapid Communication. Both papers address the question as to what extent a sQGP explosion deduced from exact numerical solutions of the Einstein equations in AdS_5 agrees with a hydro evolution. Relying on these studies one can answer the questions posed above, namely, "what is hydro?" and "when does it start?," at least for the conformal plasma in study. As seen from Fig. 3 of Ref. [22], full numerical solutions of the Einstein equations agree with the NS hydro at quite early times. A similar analysis was performed in Ref. [19]. Starting from a number of artificial initial conditions (which, to some extent, are equivalent to introducing nuclei with arbitrary structure functions) the authors of Ref. [19] traced the exact time evolution from the gravity side. It was found that, starting from some initial w_i, the evolution of *all* trajectories converged to a universal behavior of the form (7). Figure 4 of this work displays this convergence

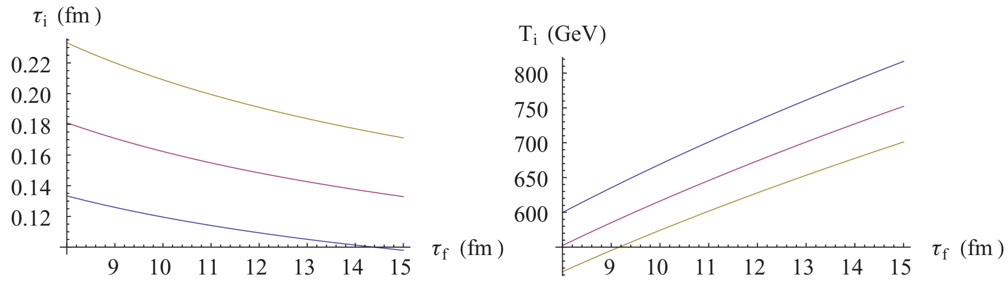

FIG. 2. (Color online) The initial time and temperature as a function of the freezout time τ_f. Left panel: $w_i = 0.4$ (bottom curve), $w_i = 0.5$ (middle curve), $w_i = 0.6$ (upper curve). Right panel: function of the freezout time τ_f. Left panel: $w_i = 0.6$ (bottom curve), $w_i = 0.5$ (middle curve), $w_i = 0.4$ (upper curve).

and can be used to define w_i, which is the "beginning of hydro." We conclude that, depending on the accuracy requested,

$$w_i(\text{few percents}) \approx 0.40, \quad w_i(\text{half percent}) \approx 0.65. \tag{12}$$

One of these values provides the second relation between T_i and τ_i, which, together with (8), fixes the initial conditions uniquely. Obviously, our model function should be used for $w > w_i$ only. Its accuracy can be estimated from comparison with the asymptotics (9) (Fig. 1). As seen from the figure, the accuracy is ~1% or even better. It is also important to note that in both studies mentioned above, the convergence between the exact and hydro results happens when the viscosity-induced asymmetries are still very large, $O(1)$. Emergence of the ideal hydrodynamics (small asymmetries) can be also seen in those results: It happens at noticeably later times.

The entropy production. The model (11) makes it possible to consider a small w limit with the function F being well regularized. Within this model, the proper time as a function of w can be found analytically:

$$\frac{\tau}{\tau_i} = \left(\frac{w}{w_i}\right)^{\frac{3\alpha}{2\alpha+\bar\eta}} \left(\frac{2w + 2\alpha + \bar\eta}{2w_i + 2\alpha + \bar\eta}\right)^{\frac{3}{2} - \frac{3\alpha}{2\alpha+\bar\eta}}. \tag{13}$$

The entropy density $s = 4k_B T^3$. Assuming R, the ratio between the experimentally measured multiplicity and the prethermalization one, to coincide with the ratio between the finite and initial entropies, we have

$$R = \frac{s\tau}{s_i\tau_i} = \left(\frac{w}{w_i}\,\frac{2w_i + 2\alpha + \bar\eta}{2w + 2\alpha + \bar\eta}\right)^{3 - \frac{6\alpha}{2\alpha+\bar\eta}}. \tag{14}$$

At the end of the evolution

$$\tau_f \sim w_f^{3/2} \to \infty.$$

R goes to its limiting value

$$R = \left(\frac{2w_i + 2\alpha + \bar\eta}{2w_i}\right)^{3 - \frac{6\alpha}{2\alpha+\bar\eta}} \simeq 1 + \frac{2\alpha + \bar\eta}{2w_i}\left(3 - \frac{6\alpha}{2\alpha + \bar\eta}\right).$$

For our choice $\alpha = \bar\eta$ [2]

$$R = \left(\frac{2w_i + 3\bar\eta}{2w_i}\right) \approx 1.39, \ldots, 1.24, \tag{15}$$

where the numerical values 0.4, ..., 0.65 were used for w_i. Thus, our model can nicely recover the missing 35% in the total multiplicity production at the LHC at $E_{NN} = 2.76$ TeV. This also supports $w_i \simeq 0.5$ as the correct choice for the initial condition.

More on hydro initial conditions. As we argued above, hydro evolution (8), supplemented by a universal value of w_i provides a means to estimate both the initial temperature T_i and initial time τ_i from the finite data. It makes sense to take as a final temperature T_f the value of 170 GeV, which is the QCD critical temperature. The freeze-out time τ_f is not well known, and neither we can be certain about our estimate of w_i. Varying these parameters we can still provide a reasonable estimate for the initial data. We do it in Fig. 2, which displays τ_i and T_i as a function of τ_f for three values of $w_i = 0, 4$, 0.5, 0.6.

Summary and discussion. Inspired by the results derived via the AdS-CFT, we argued that for a rapidity-independent geometry, a nonequilibrium explosion rapidly converges to a universal hydrodynamical function $F(w)$, for which we proposed a new simple model. Using this model we estimated the amount of entropy produced in the hydrodynamic stage and found it to constitute ~30% of the total. This compliments the perturbative studies such as those of Ref. [12], and recovers the "missing entropy" in heavy-ion collisions at the LHC. In addition, we were able to provide estimates both for the initial temperature $T_i \sim 600$–800 GeV and for the thermalization time $\tau_i \sim 0.1$–0.2 fm.

The key question, of course, is to what extent our arguments and estimates are indeed applicable to QCD in real heavy-ion collisions. The QCD plasma is believed to be strongly coupled. From the lattice studies, thermodynamics of QCD is also known to display a near-conformal behavior ($p/T^4, \epsilon/T^4 \sim$ const for $T > 2T_c$). However, QCD is presumably not a theory with a gravity dual, and there is little information about its transport coefficients.

In particular, there is no reason for the viscosity-to-entropy ratio to have the same universal value $1/4\pi$ as in theories with gravity duals. Indeed, presently available phenomenological estimates of the viscosity favor a larger value for the ratio: Ref. [23] obtained $\eta/s \simeq 2\ (1/4\pi)$ at the RHIC and $\eta/s \simeq 2.5\ (1/4\pi)$ at the LHC. First studies of the higher flow harmonics support these findings [3,4]. Furthermore, this ratio is expected to increase with the temperature, because the coupling becomes weaker. Perturbative studies, such as

[2] Choosing α even several times the value of our choice would not affect much the results, because R is a relatively flat function of α.

of Ref. [24], relate η/s to an interplay between $gg \to gg$ and $gg \to ggg$ cross sections, which are $O[\alpha_s(T)^2]$ and $O[\alpha_s(T)^3]$. respectively (processes with more gluons in the final state can be also considered [25]).

Nonperturbative studies relate viscosity to an interplay between gluon-gluon and gluon-monopole scattering processes. In the latter case, there is no coupling constant in the cross section (the electric charge times magnetic is the integer); the monopole density, computed on the lattice, decreases with temperature. Reference [26] predicted a rise in η/s as a function of temperature (Fig. 14 of this reference), from $\eta/s \simeq 2\,(1/4\pi)$ at $T = 2T_c$ to $\eta/s \simeq 2.6\,(1/4\pi)$ at $T = 4T_c$, roughly corresponding to the initial conditions at the RHIC and LHC.

If the QCD plasma were conformal, R would not depend on the collision energy E_{NN}, and we would obtain the same prediction for the RHIC and LHC. However, as we have noted in the beginning, experimentally it is not true. We are to speculate that this extra multiplicity observed at the LHC (relative to the RHIC normalization) may originate from the viscosity growth as a function of temperature. We further conjecture that our "universal resummed hydrodynamics" should, in some form, be valid in any theory and perhaps the same value of w_i parameter will be true in QCD. Then, the extra entropy produced, between the RHIC and LHC, can be ascribed to viscosity growth. Relying on our "resummed hydrodynamics" result we get

$$\frac{R(\mathrm{LHC})}{R(\mathrm{RHIC})} \approx 1 + \frac{3[\bar\eta(\mathrm{LHC}) - \bar\eta(\mathrm{RHIC})]}{2w_i + 3\bar\eta(\mathrm{RHIC})}. \tag{16}$$

Substituting $\bar\eta(\mathrm{RHIC}) \approx 2(1/3\pi)$, $\omega_i = 0.4$ we find that in order to get the $23 - 5.5 = 16.5\%$ (4) of the unaccounted extra multiplicity (double-ratio) growth at LHC, one would need the relative viscosity growth $[\bar\eta(\mathrm{LHC}) - \bar\eta(\mathrm{RHIC})]/\bar\eta(\mathrm{RHIC}) \simeq 0.4$, which is in the expected range.

The ultimate knowledge about QCD transport properties will come from a systematic study of various hydrodynamical phenomena, beyond entropy production discussed in this Rapid Communication. The most promising ones are sound waves, already discussed in the Introduction.

Acknowledgments. M.L. is very grateful to the BNL and Stony Brook Nuclear Theory Groups for hospitality during the period when this work was completed. The work of M.L. is partially supported by the Marie Curie Grant No. PIRG-GA-2009-256313.

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