Baryon kinetic energy loss in the color flux tube model

K. A. Lyakhov and H. J. Lee

Department of Nuclear and Energy Engineering, Jeju National University, Ara-dong 1, Jeju 690-756, South Korea

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One possible scenario of chromofield decay in its initial stage of evolution is Schwinger's mechanism in restricted volume. It is assumed that initial chromofield energy can be represented as a collection of color flux tubes (CFT) stretched between receding nuclei. CFT expands up to some length until its breakup followed by the production of soft partons. A new formula for initial chromofield energy density is derived from the MacLerran-Venugopalan model to calculate CFT tension. It considers two possible ansatzes for saturation momentum. Color charge screening by produced partons is taken into account as well. A new formula for evolution of produced parton multiplicities based on the Wigner representation of the phase-space density of probability is also derived.

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I. INTRODUCTION

After collision at ultrarelativistic energies, nuclei can be thought of as two massive sheets leaving a strong gluon field in their wake. In this case, leading partons (quarks and gluons) lose part of their energy due to soft-gluon radiation which forms a chromoelectromagnetic field [1]. It is convenient to split the whole dynamical process into three basic stages. In the first stage immediately after collision the nuclei, because of multiple soft gluons exchange, acquire a stochastic color charge. These color charges produce a multitude of color flux tubes (CFT) occupying space between receding slabs. Shortly afterwards these CFTs, or chromoelectromagnetic field energy, is converted into parton pairs. And finally rescatterings in a dense partonic system drive it quickly to local thermal equilibrium.

This paper deals with chromoelectromagnetic field decay with a subsequent parton plasma creation on the initial stage of the reaction. Parton plasma production is a nonperturbative mechanism. It can be described as the tunneling across the energetic gap of width $2m_\perp$ between the virtual energetic states inside the Dirac sea and continuum of real states or produced particles. A semiclassical or Wentzel-Kramers-Brillouin (WKB) approach due to its simplicity is the most commonly used method to tackle this problem [2,3]. However particle creation is, essentially, a quantum relativity effect. Therefore to study this process rigorously one has to solve the corresponding wave equations: Dirac equation for quark-antiquark pairs production, and Proca equation for gluon production. In infinite volume the problem for fermions was solved by Schwinger [4] by applying a Green's functions approach. However finite size effects should not be disregarded. For instance as pointed out in Ref. [5] they are quite essential to understanding baryon deceleration. As shown in Ref. [6] these effects can be studied within a Green's functions approach. Expansion of Green's functions on inverse field volume is used in this paper to get finite size corrections to Schwinger's results.

Finite size effects in a transverse plane are usually taken into account by applying MIT boundary conditions [7,9] for a radial wave function. In Refs. [8,9] the influence of finite CFT volume effects on parton production was investigated by solving Dirac and Klein-Gordon equations. We modified their results in order to take into account a CFT profile along its length corresponding to different choices of saturation momentum. It is also pointed out that averaging over all possible CFT volumes is necessary to take into account its expansion.

We applied results of the MacLerran-Venugopalan (MV) model [10] to calculate the dispersion of color charges over a CFT cross section and to calculate CFT tension. We found the relationship between the probability of CFT decay, baryon energy loss, and rapidity spectrum of produced partons.

As a further advance one can take our results for parton distribution functions as initial conditions for the parton cascade model (PCM) [11,12], or, assuming that immediate thermalization is justified, these distributions can be used as initial conditions for the thermal model as formulated in our previous work [13], where temperature field evolution is governed by a chromofield decay source term.

II. CFT TENSION AND INITIAL CHROMOFIELD ENERGY DENSITY

In this section we model the highly nonperturbative behavior of QCD in a confinement region in terms of CFT and MV models. Let us start from the QCD Lagrangian and corresponding field equations:

$$\mathcal{L}_{\mathrm{QCD}}(x) = \bar{\Psi}(x)[i\gamma^\mu \mathcal{D}_\mu - m]\Psi(x) - \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} \times (\text{sum runs over repeated indices}) \; a = 1 \ldots 8, \tag{1}$$

$$[i\gamma^\mu \mathcal{D}_\mu - m]\Psi(x) = 0; \tag{2}$$

$$\mathcal{D}_\mu F^a_{\mu\nu} = \bar{\Psi}(x)\gamma_\nu \frac{\lambda^a}{2}\Psi; \tag{3}$$

$$\mathcal{D}_\mu = \partial_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu. \tag{4}$$

It was observed quite long ago that confinement properties of this field can be modeled by the simple linear potential

$$g_s \frac{\lambda^a}{2} G^a_\mu \Rightarrow [A_0(z), 0]. \tag{5}$$

To calculate this potential we have to know the initial chromofield energy density. It can be derived by following the MV model and Color Glass Condensate (CGC) ideas formulated in Refs. [10,16,17]. According to the MV model random color charges are generated on the nuclear sheets as a result of the soft-gluon exchange on the nuclei interpenetration stage. These charges fluctuate from event to event, so that the mean areal charge vanishes. In each event these charges fluctuate from point to point in the transverse plane. Color charge density, $\rho(\eta, x_\perp)$, is introduced as a random variable which is a function of the coordinate in transverse plane and pseudorapidity $\eta = \ln(x^+/x^-)$. This random color charge has Gaussian distribution,

$$W_{y_0}[\rho] \sim \exp\left[-\frac{1}{2}\int d^2x_\perp \int_{-\infty}^{y_0} d\eta \frac{\rho^a(\eta, x_\perp)\rho^a(\eta, x_\perp)}{\lambda(\eta, x_\perp)}\right], \tag{6}$$

where y_0 is a leading parton rapidity corresponding to some moderate x_0. Following Ref. [17] it is convenient to introduce a new integrated-over-η random variable

$$\tilde{\rho}_{y_0}(x_\perp) = \int_{-\infty}^{y_0} d\eta \rho(\eta, x_\perp). \tag{7}$$

Then the distribution function in terms of a new variable is given by

$$\begin{aligned} P[\tilde{\rho}] &\sim \exp\left(-\frac{1}{2}\int d^2x_\perp \frac{\tilde{\rho}^a_{y_0}(x_\perp)\tilde{\rho}^a_{y_0}(x_\perp)}{\mu_{y_0}(x_\perp)}\right), \\ \mu_{y_0}(x_\perp) &= \int_{-\infty}^{y_0} d\eta \lambda(\eta, x_\perp). \end{aligned} \tag{8}$$

The value of parameter μ as shown in our previous paper [14] can be derived from the MV model correlation condition for color charge density averaged over the CFT cross section

$$\langle \tilde{\rho}^a(x_\perp)\tilde{\rho}^b(y_\perp)\rangle = \frac{\mu(x_\perp)}{\mathcal{A}}\delta^{ab}\delta(x_\perp - y_\perp), \tag{9}$$

where $\mathcal{A}$ is "charged spot"(CFT) cross section. Fluctuations of color charge are characterized by a certain scale in the transverse plane "charged spot", which is related to the saturation scale

$$r_a \approx \frac{1}{Q_{s,a}}, \quad a = p, t \tag{10}$$

introduced in high-density QCD [19,20]. In this paper we consider two variants to introduce saturation momentum. The first one corresponds, as shown below, to a CFT cross section which varies along its length between projectile and target slabs:

$$Q^2_{s,a}(x, x_\perp) = Q_0^2 N_a(x_\perp)x^{-\lambda}, \quad x = \frac{p_\perp}{\sqrt{s}}e^y, \tag{11}$$

where $N_a(x_\perp)$ is slab a baryon density per unit area, and Q_0 is some fitting parameter. The second variant for saturation momentum choice corresponds to the constant CFT radius along CFT length

$$\begin{aligned} Q^2_{s,p} &= Q^2_{s,t} = Q^2_s(x, x_\perp) \\ &= Q_0^2 \mathrm{Min}\{N_p(x_\perp), N_t(x_\perp)\}x^{-\lambda}, \quad x = \frac{p_\perp}{\sqrt{s}}e^y. \end{aligned} \tag{12}$$

In this variant, only interactions within a CFT cylinder are possible. Therefore in the noncentral collision case some fraction of partons remains intact because these partons do not have partners to produce strings. We shall consider only the first variant. Apparently results for the second variant can be derived trivially from the results for the first.

Since the transverse size of the baryonic slabs is much larger ($\sim \sigma_{NN}$, as assumed in previous work [13]), many string-like configurations (CFT) are stretched between receding slabs. Each such configuration connects the spots of opposite charges, $\mathcal{Q}_i$, like in the capacitor. Let us consider some CFTs in this configuration. A cross section $\mathcal{A}_a = \pi r_a^2$ corresponding to an intersection area with a slab surface at some point $x_\perp$ characterized by a radius vector and impact parameter of collision. Color charges on the spot in the projectile and target nucleus due to color neutrality of the CFT are of the same absolute value

$$\mathcal{Q}_p = \rho_p \mathcal{A}_p = \mathcal{Q}_t = \rho_t \mathcal{A}_t. \tag{13}$$

This can be also easily understood by assuming that CFT has a substructure represented as a batch of elementary quark-antiquark and *gluon* strings. The density of this strings is nothing else but a sum of distribution functions for valence quarks and gluons per unit area, and is obviously proportional to baryon number. However, in noncentral collisions the baryon number for colliding slabs is different. Therefore the smaller baryon density on projectile slab surface the larger color flux cross section is needed to encompass incoming strings from the target slab side and vice versa. On the other hand, smaller baryon density per unit area corresponds to smaller saturation momentum and therefore to a larger CFT cross section as pointed out in the previous sentence. In particular, for central collisions $\mathcal{A}_p = \mathcal{A}_t$. In the following discussion we consider a general case of noncentral collisions, and, to be certain, we assume that $\mathcal{A}_p > \mathcal{A}_t$. The chromoelectromagnetic field strength between capacitor plates is

$$\begin{aligned} \mathbf{V} &= \mathbf{V}_p + \mathbf{V}_t = \mathbf{E} + \mathbf{B}, \\ \mathbf{E} &= \mathbf{E}_p + \mathbf{E}_t, \\ \mathbf{B} &= \mathbf{B}_p + \mathbf{B}_t. \end{aligned} \tag{14}$$

Therefore the chromofield energy density within individual CFT is

$$\epsilon_i = \tfrac{1}{2}(\mathbf{E} + \mathbf{B})^2. \tag{15}$$

In the Abelian approximation the chromoelectric field strength in a flux tube produced by color charges on a nucleus surface can be obtained by the Gauss theorem [7]:

$$\int \mathbf{V}_a d^2\mathbf{s} = \mathcal{Q}_a. \tag{16}$$

According to a numerical solution of the classical Yang-Mills equation [1], only the longitudinal chromofield component is present initially. Therefore the last equation can be rewritten as

$$\left(E^i_z + B^i_z\right)_p = \frac{\mathcal{Q}^i_p}{\mathcal{A}_p} \equiv \rho^i_p. \tag{17}$$

By averaging over the ensemble of CFT stretched between slabs we obtain the mean field energy density

$$\epsilon_f(t_0)=\tfrac{1}{2}\langle \mathrm{Tr}\rho_p^2(x_\perp)\rangle+\tfrac{1}{2}\langle \mathrm{Tr}\rho_t^2(x_\perp)\rangle+\langle \mathrm{Tr}[\rho_p(x_\perp)\rho_t(x_\perp)]\rangle, \tag{18}$$

where Tr is taken over color indices. By using the color neutrality condition in a flux tube ((13)) the latter expression can be transformed to

$$\epsilon_f(t_0)=\frac{\langle \mathrm{Tr}\rho_p^2(x_\perp)\rangle}{2}\left(\frac{\mathcal{A}_p}{\mathcal{A}_t}+1\right)^2, \tag{19}$$

where averaging is assumed over the ensemble of CFT stretched between slabs. Assuming that quark and gluon charges are not correlated $\langle\rho_q\rho_g\rangle=0$, color charge density squared averaged over a CFT area for a slab in a projectile nucleus is given by

$$\begin{aligned}\langle \mathrm{Tr}\rho^2(x_\perp)\rangle &= R_p(x_\perp)\frac{\mu_q(x_\perp)+\mu_g(x_\perp)}{\mathcal{A}},\\ \mu_q(x_\perp)+\mu_g(x_\perp)&=g_s^2\left(N_c^2-1\right)\left(\frac{1}{2N_c}F_q(x_\perp)+N_cF_g(x_\perp)\right),\end{aligned} \tag{20}$$

where $F_q(x_\perp)$ and $F_g(x_\perp)$ are quark and gluon local densities of the slab. The projectile slab area fraction occupied by CFT is denoted by $R_p(x_\perp)$. The slab is assumed to be located in the vicinity of point $x_\perp$ in the transverse plane. This parameter takes into account a fraction of the slab area occupied by CFT, and its value depends on the impact parameter and initial beam energy. This factor is proportional to the fraction of nucleons in the slab which produce CFT. At higher energies as follows from Eq. (10) many CFT can be stretched between nucleons in colliding slabs. Indeed for the saturation momentum value $Q_s=1.2$ GeV, as proposed in Ref. [18], the CFT cross section is $\mathcal{A}=0.09$ fm^2. Then the maximum average number of CFT which can be produced by each nucleon in the central slab in a central gold-gold collision is

$$N_s=\frac{\sigma_{NN}}{\mathcal{A}N_B}\approx 5,$$

where $N_B=8.8$ is the baryon number for the central slab. We will call it a *nucleon activation degree* because the number of CFT produced by the nucleon corresponds to its "activation degree". Since each CFT has all produced CFT intersect only some fraction of overall slab surface. In our previous work [14] this fraction was equal to one since the whole slab area was assumed to be occupied by CFT.

As follows from Eq. (19) the initial chromofield energy density is given by

$$\begin{aligned}\epsilon_f(t_0) &= R_p(x_\perp)\left(N_c^2-1\right)\left(\frac{\mathcal{A}_p}{\mathcal{A}_t}+1\right)^2\frac{\mu_q(x_\perp)+\mu_g(x_\perp)}{2\mathcal{A}_p},\\ \mu_q(x_\perp) &= \frac{g_s^2}{2N_c}\int d^2k_\perp\int\frac{dx}{x}xf_v^S(x,k_\perp^2),\\ \mu_g(x_\perp) &= g_s^2N_c\int d^2k_\perp\int\frac{dx}{x}xG^S(x,k_\perp^2),\end{aligned} \tag{21}$$

where $xf_v^S(x,k_\perp)$ and $xG^S(x,k_\perp)$ are two-dimensional parton distribution functions within the slab area for valence quarks and gluons, respectively. Let us remind the reader that each slab is characterized by a baryon number per unit area and its area. In our previous work [14] this area was assumed equal to σ_{NN}. Valence quark distribution can be extracted from experimental data. The initial gluon distribution can be taken from Ref. [17]. It was obtained there by the interpolation between the color glass condensate and Balitskiy-Faddeev-Kuraev-Lipatov (BFKL) regimes

$$xG^S(x,k_\perp)=\frac{1}{\pi\gamma c\bar{\alpha}_s}\ln\left[1+\left(\frac{Q_s(x)^2}{k_\perp^2}\right)^\gamma\right]. \tag{22}$$

Since in the CFT model the energy density is uniform and time independent, the potential energy in the CFT gets larger linearly with increase of its length: $\Delta E(t)=\sigma z(t)$. For the sake of simplicity we assume that the CFT length increases with time as $z(t)=t$. However, in our previous work [14] it was demonstrated that there is a small deviation of CFT end trajectory from the light cone due to slab deceleration induced by strong chromofields and plasma back reactions.

The effect of dynamical color charge screening plays a crucial role in this problem. We take it into account only in a very simple way as was done by Matsui and Glendenning for a quark-antiquark string in their paper [15]. As shown in this work, the string tension is of only a half of its value without screening due to the interaction between the quark and antiquark of the produced pair. This result can be generalized quite straightforwardly for CFT by taking into account that the number of quark-antiquark and gluon strings inside CFT is proportional to the baryon number within a given CFT cross section. Hence according to Eq. (21) CFT tension is also only of a half of its unscreened value:

$$\Delta E=\tfrac{1}{2}\tilde{\epsilon}_f(t_0)V,\quad \tilde{\epsilon}_f(t_0)=\epsilon_f(t_0)+B, \tag{23}$$

where B is the bag constant. CFT volume V can be thought of for the axial symmetry case as a frustum because areas of the CFT intersecting projectile and target slab surface are not the same $Q_s^p\neq Q_s^t$ for a noncentral collision. By this way in the first approximation we can take into account the color charge screening effect to calculate transmission and reflection amplitudes for pair production.

CFT tension can be represented as

$$\sigma(x_\perp,z)=\tfrac{1}{2}\pi\wp^2(z)\tilde{\epsilon}_f(t_0)\quad\text{for}\quad r<\wp(z), \tag{24}$$

where the CFT profile is given by

$$\wp(z)=r_t+\frac{r_p-r_t}{L}z. \tag{25}$$

Since the chromofield outside CFT vanishes, the vector potential can be represented as varying in three characteristic

regions:

$$A_0(x_\perp, r, z) = \begin{cases} 0 & \text{for} \quad \mathrm{z < L}, \quad \mathrm{r > \wp(z)\,(region\ I)} \\ -\int_0^z \sigma(x_\perp, z')dz' & \text{for} \quad 0 \leqslant z < L, \quad r \leqslant \wp(z)\,(\text{region II}) \\ -\int_0^L \sigma(x_\perp, z)dz & \text{for} \quad z \geqslant L\,(\text{region III}) \end{cases}. \tag{26}$$

III. INITIAL PARTON RAPIDITY DISTRIBUTION

According to three possible interactions described by $\mathcal{L}_{\mathrm{QCD}}$, string can decay into quark-antiquark or gluon pairs or into tree gluons. It is believed that gluon self-interaction corresponding to the last decay mode is responsible for confinement. We assume that this mode is taken into account effectively by the linear CFT potential.

In Refs. [8,9] Shrödinger-like equations with linear potential in longitudinal direction were solved. Transmission and reflection amplitudes in the longitudinal direction are represented as combinations of parabolic cylinder functions. These results can be extended for expanding CFT by applying dynamic boundary conditions in the longitudinal direction. Indeed to show this let us consider a sequence of time instants $\{0 < t_1 < \cdots < t_n\}$. At each instant t_i CFT may acquire some length $L_i = L(t_i)$ (or decay at this length) so that $\{0 < L_1 < \cdots < L_n\}$ with corresponding reflection amplitude (transmission amplitude) squared. Therefore the *CFT survival probability* and *CFT decay probability* are given by

$$\mathfrak{p}_i = |R(\mathbf{p}, L_i, r)|^2 \tag{27}$$

and

$$\bar{\mathfrak{p}}_i = |T(\mathbf{p}, L_i, r)|^2, \tag{28}$$

respectively. The total density of the probability that the string will or will not decay over time period t which corresponds to CFT length $L(t)$ is given by summing up probabilities corresponding to all intermediate CFT lengths $0 < L_i = L(t_i) < L$:

$$p_{\mathrm{survive}} = \sum_{i=0}^{N} \mathfrak{p}_i, \tag{29}$$

$$p_{\mathrm{decay}} = \sum_{i=0}^{N} \bar{\mathfrak{p}}_i, \tag{30}$$

which are reduced to integration over boundary conditions. Time in this case plays the role of the continuously changing parameter.

As shown in the previous section, the CFT vector potential varies in three characteristic zones. In central collisions CFT has cylindrical shape, and this problem can be solved by separating the variables. The density of probability for parton pair production corresponds to values of wave function in zone III. The wave function in this region is represented as

$$\Psi_{B(F)}(\mathbf{p}, r, z) = T_{B(F)}(\mathbf{p}, L)\mathcal{R}(p_\perp, r, \phi) \times \exp(-iE_R t + ip_{z,R} z), \tag{31}$$

where $T_{B(F)}$ is transmission amplitude in longitudinal directions, $\mathcal{R}_{B(F)}$ is radial part of wave function, and ϕ is the azimuthal angle:

$$E_R = \omega(\mathbf{p}) + \tfrac{1}{2}\tilde{\epsilon}_f(t_0)V_f,$$
$$p_{z,R} = \sqrt{E_R^2 - m_\perp^2},$$

where $m_\perp^2 = p_\perp^2 + m^2$, and V_f is the CFT volume. Dependence on azimuthal angle is introduced as $e^{in\phi}$. The radial part of the wave function in the case of scalar or vector particles produced is expressed in terms of Bessel functions of n-th order:

$$\mathcal{R}(p_\perp, r, \phi) = e^{in\phi} J_n(p_\perp r). \tag{32}$$

Applying MIT boundary conditions along CFT surface $r = \frac{1}{Q_s}$, where Q_s leads to a discretization of transverse momentum.

To obtain the rapidity density of produced particles, as will be shown later, it is necessary to know the density of probability in phase space. Due to the uncertainty principle, the wave function squared due to its locality cannot be used for this purpose directly. However this problem can be solved by using Wigner's function. Wigner's function for gluons(quarks) is given by

$$W_{gg(q\bar{q})}(\mathbf{p}, \mathbf{r}) = \frac{\nu_{g,q}}{(2\pi)^3} \int d^3\mathbf{v} \Psi^\dagger_{B(F)}\left(z + \frac{v_z}{2}, r + \frac{v_r}{2}, \phi + \frac{\varphi}{2}\right) \times \Psi_{B(F)}\left(z - \frac{v_z}{2}, r - \frac{v_r}{2}, \phi - \frac{\varphi}{2}\right) e^{i\mathbf{pv}}, \quad \times \nu_g = N_c^2 - 1, \quad \nu_q = N_c N_f. \tag{33}$$

The wave function Ψ_B is a solution of the Proca equation for vector particles, and Ψ_F is a spinor solution of the Dirac equation. Integration over distance between two points along longitudinal coordinate leads to

$$\frac{1}{2\pi} \int dv_z e^{i(p_z - p_{z,R})v_z} = \delta(p_z - p_{z,R}) \tag{34}$$

which expresses a condition of energy conservation:

$$2\omega(\mathbf{p}) = -A_0. \tag{35}$$

Thus we get the following expressions for the Wigner's function corresponding to gluon (quark-antiquark) pair

production from the CFT of given length L_i:

$$W_{gg(q\bar{q})}(x_\perp, \mathbf{p}, r, L_i) = \hat{u}^\dagger \hat{u} \frac{v_{g(q)}}{2\pi} \sum_{n=0}^{\infty} \delta(p_z - p_{z,R}) |T_{B(F)}(p_\perp(n), p_z, L_i)|^2 \times \int dv_r v_r J_n\left[p_\perp(n)\left(r + \frac{v_r}{2}\right)\right] J_n\left[p_\perp(n)\left(r - \frac{v_r}{2}\right)\right] \times \int d\varphi e^{i(p_\perp v_r \cos(\varphi) - n\varphi)}, \quad (36)$$

where $\hat{u}$ is the spinor in the case of fermions. From conservation of energy it follows that

$$p_z(n) = \sqrt{\frac{A_0^2}{4} - [m_\perp(n)]^2} \quad (37)$$

which means that the number of transverse states to be considered obeys the inequality

$$m_\perp(n) \leqslant -\tfrac{1}{2} A_0. \quad (38)$$

Therefore an infinite sum over transverse states can be converted into a finite sum

$$\sum_{n=0}^{\infty} \delta(p_z - p_{z,R}) = \sum_{n=0}^{n_0} \delta[p_z - p_z(n)], \quad (39)$$

when n_0 can be found from $m_\perp(n_0) = -\frac{1}{2} A_0$.

According to Eq. (30) the total probability of CFT decay at time moment t is given by

$$P_{\text{decay}}(x_\perp, t) = 2\pi \int_0^t dt' v_t(t') \int_0^\infty r dr \times \int d^3\mathbf{p} W_{\text{decay}}[x_\perp, \mathbf{p}, r, L(t')], \quad (40)$$

where $W_{\text{decay}} = W_{q\bar{q}} + W_{gg}$, and

$$v_t = \frac{dL}{dt}$$

is CFT caps velocity. The transmission amplitude has to be changed to a reflection amplitude for Wigner's function which corresponds to the event that CFT of length L will not decay. We will call it the CFT *survival Wigner's function* $W_{\text{field}}(x_\perp, \mathbf{p}, r, L)$.

Now we can derive the relationship between energy stored in the chromofield and initial parton rapidity distribution. In the initial stage of the reaction all the kinetic energy lost by baryons after collision is converted into a multitude of CFT. The number of CFT stretched between slab i in a pairwise slab-slab collision is

$$n_i = \frac{R_i \sigma_0}{\mathcal{A}_i}, \quad (41)$$

where σ_0 is slab cross section, R_i is an average *nucleon activation degree*, and $\mathcal{A}_i$ is an average CFT cross section for slab i.

The average chromofield energy produced by N_0 pairwise slab collisions in nuclei overlap region is given by

$$E_f(\mathbf{b}) = \int_0^t dt' L(t') v_t(t') \int r dr \int d^3\mathbf{p} \sigma_0 \times \sum_{i=1}^{N_0} \mathcal{G}_i W_{\text{field}}(x_\perp^i, \mathbf{p}, r, t'), \quad \mathcal{G}_i = \frac{\sigma_i}{\mathcal{A}_i}, \quad (42)$$

where σ_i is an average tension of the CFT produced by the nucleon in the slab i, $x_\perp^i$ is slab i coordinate, and $\mathbf{b}$ is an impact parameter. *Nucleon activation degree* R_i is absorbed in a CFT cross section. In a continuous limit this formula is transformed to

$$E_f(\mathbf{b}) = \int d^2\mathbf{x}_\perp \epsilon_f(x_\perp, t),$$
$$\epsilon_f(x_\perp, t) = 2\pi \int_0^t dt' L(t') v_t(t') \int r dr \times \int d^3\mathbf{p} \mathcal{G} W_{\text{field}}(x_\perp, \mathbf{p}, r, t'),$$
$$\mathcal{G}(r, L, x_\perp) = \frac{\sigma(r_p, L)}{\mathcal{A}_p(x_\perp)}. \quad (43)$$

Therefore chromofield energy converted into parton plasma over time period t is given by

$$\epsilon_p(x_\perp, t) = 2\pi \int_0^t dt' L(t') v_t(t') \times \int r dr \int d^3\mathbf{p} \mathcal{G} W_{\text{decay}}(x_\perp, \mathbf{p}, r, t'). \quad (44)$$

From this expression the evolution of spectrum of produced partons can be readily obtained:

$$\epsilon_p(x_\perp, t) = \int d^3\mathbf{p} \Gamma(\mathbf{p}, t) = \int d^3\mathbf{p} \omega(\mathbf{p}) \frac{dN}{d^3\mathbf{p}}, \quad (45)$$

where $\Gamma = \Gamma_{q\bar{q}} + \Gamma_{gg}$. Thus the rapidity distribution of quarks and gluons generated in the slab-slab collision is calculated by the formula

$$\frac{dN_{q\bar{q}(gg)}(x_\perp, t)}{dy} = \int_{-\infty}^{\infty} d^2\mathbf{p}_\perp \Gamma_{q\bar{q}(gg)}(\mathbf{p}, t). \quad (46)$$

IV. CONCLUSIONS

In this paper we found new formulas for chromofield energy density and parton rapidity distribution evolution on the basis of the CFT model. To calculate a CFT tension, which is necessary to know in the CFT model, a new formula for initial chromofield energy density on the basis of the MV model is derived. The influence of centrality of collisions on parton multiplicity is also investigated. Obtained results can serve as a basis for numerical simulations targeted for the study of the initial stages of reaction. They may help to understand the relationship between nuclear parton distributions and spectra of produced particles.

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