Fluctuations of the number of participants and binary collisions in AA interactions at fixed centrality in the Glauber approach

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In the framework of the classical Glauber approach, the analytical expressions for the variance of the number of wounded nucleons and binary collisions in AA interactions at a given centrality are presented. Along with the optical approximation term, they contain additional contact terms arising only in the case of nucleus-nucleus collisions. The magnitude of the additional contributions, e.g., for PbPb collisions at Super Proton Synchrotron (SPS) energies, is larger than the contribution of the optical approximation at some values of the impact parameter. The sum of the additional contributions is in good agreement with the results of independent Monte Carlo simulations of this process. Due to these additional terms, the variance of the total number of participants for peripheral PbPb collisions and the variance of the number of collisions at all values of the impact parameter exceed several multiples of the Poisson variances. The correlator between the numbers of participants in colliding nuclei at fixed centrality is also analytically calculated.

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I. INTRODUCTION

At present, considerable attention is devoted to experimental and theoretical investigations of the multiplicity and transverse momentum fluctuations of charged particles in high-energy AA collisions (see [1–7] and references therein). One expects an increase of fluctuations in the case of freeze-out close to the QCD critical endpoint of the quark-gluon-plasma–hadronic-matter phase boundary line [8,9].

The aim of the present paper is to draw attention to another factor leading to the increase of fluctuations in the case of AA interactions, namely, the increase of fluctuations of the number of participants and binary collisions due to multiple-contact nucleon interactions in nucleus-nucleus collisions.

Clearly these fluctuations lead to fluctuations in the number of particle sources, and so they directly impact the multiplicity and transverse momentum fluctuations of produced charged particles and also the correlations between them (see, for example, [10–17]).

In this paper the analytical expressions for the variance of the number of wounded nucleons and binary collisions in AA interactions at a given centrality are obtained, taking into account multiple-contact NN interactions (so-called loop contributions). The calculations are fulfilled in the framework of the classical Glauber approach [18], having a simple probabilistic interpretation [19,20]. In contrast with purely Monte Carlo simulations, the analytical calculations enable us to understand the origin of the increased values of the fluctuations.

As a result, we demonstrate that multiple-contact NN interactions in AA scattering lead to the fact that, e.g., for PbPb collisions at SPS energies, the variance of the total number of participants for peripheral collisions and the variance of the number of collisions at all values of the impact parameter exceed a few multiples of the Poisson variances.

The paper is organized as follows. In Sec. II, in the framework of the classical Glauber approach, we present the analytical expression for the variance of the number of wounded nucleons in one of the colliding nuclei at a fixed value of the impact parameter. Along with the well-known optical contribution (which depends only on the total inelastic NN cross section), in the case of nucleus-nucleus collisions there is the additional contact term depending on the profile of the NN interaction probability in the impact parameter plane.

In Sec. III we calculate the correlator between the numbers of participants in colliding nuclei at fixed centrality, and as a consequence we find the variance of the total number of participants in both nuclei.

In Sec. IV, in the framework of the same approach, we present the analytical expression for the variance of the number of NN binary collisions in AA interactions at a given centrality. Along with the optical approximation term, this expression also contains other terms, which are the dominant ones. These terms correspond to multinucleon contact interactions and arise only in the case of nucleus-nucleus collisions.

The derivations of all formulas are presented in Appendixes A, B, and C.

Throughout the paper the results of numerical calculations are presented to illustrate the obtained analytical results. We also compare our analytical calculations with the results obtained purely from Monte Carlo simulations of nucleus-nucleus scattering.

Note that we restrict our consideration to the region of the impact parameter $\beta < R_A + R_B$, where the probability of inelastic interaction $\sigma_{AB}(\beta)$ of two nuclei with radii R_A and R_B is close to unity.

II. VARIANCE OF THE NUMBER OF PARTICIPANTS IN ONE NUCLEUS

At first we consider the variance $V[N_w^A(\beta)]$ of the number of participants $N_w^A(\beta)$ (wounded nucleons) at a fixed value

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of the impact parameter β in one of the colliding nuclei A. In the framework of a purely classical, probabilistic approach to nucleus-nucleus collisions, formulated in [18], we find for the mean value and for the variance of $N_w^A(\beta)$ the following expressions (see Appendix A):

$$\langle N_w^A(\beta)\rangle = AP(\beta), \tag{1}$$

$$V\big[N_w^A(\beta)\big] = AP(\beta)Q(\beta) + A(A-1)[Q^{(12)}(\beta) - Q^2(\beta)], \tag{2}$$

where $P(\beta) = 1 - Q(\beta)$. For $Q(\beta)$ and $Q^{(12)}(\beta)$ we have the following (where all integrations imply the integration over two-dimensional vectors in the impact parameter plane):

$$Q(\beta) = \int da\, T_A(a)[1 - f_B(a+\beta)]^B, \tag{3}$$

$$Q^{(12)}(\beta) = \int da_1 da_2 T_A(a_1)T_A(a_2)[1 - f_B(a_1+\beta) - f_B(a_2+\beta) + g_B(a_1+\beta, a_2+\beta)]^B, \tag{4}$$

with

$$f_B(a) \equiv \int db\, T_B(b)\sigma(a-b), \tag{5}$$

$$g_B(a_1, a_2) \equiv \int db\, T_B(b)\sigma(a_1-b)\sigma(a_2-b). \tag{6}$$

Here T_A and T_B are the profile functions of the colliding nuclei A and B. $\sigma(a)$ is the probability of inelastic interaction between two nucleons at impact parameter a. We assume that $\sigma(a)$, T_A, and T_B depend only on the magnitude of their two-dimensional vector argument. Hence $f_B(a) = f_B(|a|)$ and $Q(\beta) = Q(|\beta|)$.

Formula (1) and the first term in formula (2) correspond to the naive picture (the so-called optical approximation), implying that, in the case of AA collision at impact parameter β, one can use the binomial distribution for $N_w^A(\beta)$ (see, for example, [21,22]):

$$\wp_{\rm opt}\big(N_w^A\big) = C_A^{N_w^A}\, P(\beta)^{N_w^A}\, Q(\beta)^{A-N_w^A}, \quad P(\beta) = 1 - Q(\beta), \tag{7}$$

with an averaged probability $P(\beta)$ of inelastic interaction between a nucleon of nucleus A with nucleons of nucleus B. $P(\beta)$ is considered to be the same for all nucleons of nucleus A. In the optical approximation one has

$$\langle N_w^A(\beta)\rangle_{\rm opt} = AP(\beta), \quad V\big[N_w^A(\beta)\big]_{\rm opt} = AP(\beta)Q(\beta). \tag{8}$$

The whole expression (2) for the variance is the result of a more accurate calculation (see Appendix A) in which we first calculate the probabilities of all binary NN interactions, taking into account the impact-parameter plane positions of nucleons in nuclei A and B, and only then average over nucleon positions:

$$V\big[N_w^A(\beta)\big] = \langle N_w^A(\beta)^2\rangle - \langle N_w^A(\beta)\rangle^2, \tag{9}$$

where

$$\langle X\rangle \equiv \langle\langle \overline{X}\rangle_B\rangle_A \equiv \int \overline{X} \prod_{k=1}^{B} T_B(b_k)db_k \prod_{j=1}^{A} T_A(a_j)da_j. \tag{10}$$

Here $\overline{X}$ is the average value of some variate X at fixed positions of all nucleons in the nuclei A and B; $\langle\,\rangle_A$ and $\langle\,\rangle_B$ denote averaging over positions of these nucleons with corresponding nuclear profile functions.

In the limit $r_N \ll R_A, R_B$ formulas (5) and (6) reduce to

$$f_B(a) \approx \sigma_{NN}\, T_B(a),$$

$$g_B(a_1, a_2) \approx I(a_1 - a_2)\cdot T_B[(a_1+a_2)/2], \tag{11}$$

with

$$\sigma_{NN} \equiv \int db\, \sigma(b), \quad I(a) \equiv \int db\, \sigma(b)\sigma(b+a). \tag{12}$$

Note that in this limit $Q(\beta)$ and hence the mean value (1) and the first term of the variance (2) depend only on the integral inelastic NN cross section σ_{NN}, but $Q^{(12)}(\beta)$ in the second term of the variance (2) depends also on the shape of the function $\sigma(b)$ through the integral $I(a)$ (12).

Note also that using the simple approximation with the δ function, $\sigma(b) = \sigma_{NN}\delta(b)$, for NN interactions gives the same result (approaching the limit $r_N \ll R_A, R_B$) only for the optical part of the answer, which is expressed through $Q(\beta)$. If we use the approximation $\sigma(b) = \sigma_{NN}\delta(b)$ to calculate $Q^{(12)}(\beta)$ we get $I(a) = \sigma_{NN}^2\delta(a)$ and $g_B = \sigma_{NN}^2\delta(a_1 - a_2)T_B(a_1)$, which leads to infinite $Q^{(12)}(\beta)$ at $B \geqslant 2$. Meanwhile, for any correct approximation of $\sigma(b)$ with $\sigma(b) \leqslant 1$ (in correspondence with its probabilistic interpretation in the classical Glauber approach) we find a finite answer for $Q^{(12)}(\beta)$.

In the quantum case in the Glauber approximation, due to unitarity one has

$$\sigma(b) \equiv \sigma^{\rm in}(b) = \sigma^{\rm tot}(b) - \sigma^{\rm el}(b) = 2\,{\rm Im}\,\gamma(b) - |\gamma(b)|^2 \geqslant 0, \tag{13}$$

where $\gamma(b)$ is the amplitude of NN elastic scattering. This leads to the restrictions $0 \leqslant \sigma^{\rm tot}(b) \leqslant 4$, $0 \leqslant \sigma^{\rm el}(b) \leqslant 4$, and $0 \leqslant \sigma^{\rm in}(b) \leqslant 1$. So in the quantum case $\sigma(b)$ also admits a probabilistic interpretation [19,20].

In our numerical calculations we have used for $\sigma(b)$ the "black-disk" approximation

$$\sigma(b) = \theta(r_N - |b|) \tag{14}$$

and the Gauss approximation

$$\sigma(b) = \exp\big(-b^2/r_N^2\big). \tag{15}$$

In both cases $\sigma_{NN} = \pi r_N^2$. For the nuclear profile functions T_A and T_B we have used the standard Woods-Saxon approximation:

$$T_A(a) = \int dz\, \rho(r), \quad r^2 = a^2 + z^2,$$

$$\rho(r) = \rho_0\left(1 + \exp\frac{r - R_A}{\kappa}\right)^{-1}, \tag{16}$$

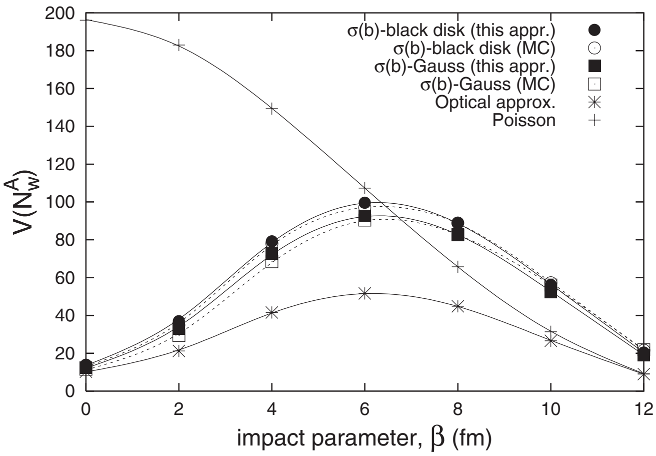

FIG. 1. Variance of the number of wounded nucleons in one nucleus for PbPb collisions at SPS energies ($\sigma_{NN} = 31$ mb) as a function of the impact parameter β (fm). The points • and ■ are results of numerical calculations from the analytical formulas (2)–(4), (11), and (12) using, respectively, the black-disk (14) and Gaussian (15) approximations for NN interactions; ○ and □ are results of independent Monte Carlo (MC) simulations using the black-disk (14) or Gaussian (15) approximation for NN interactions; ∗ is the optical approximation result (8) [the first term in formula (2)]; and + is the Poisson variance $V[N_w^A(\beta)] = \langle N_w^A(\beta)\rangle$. The curves are shown to guide the eyes.

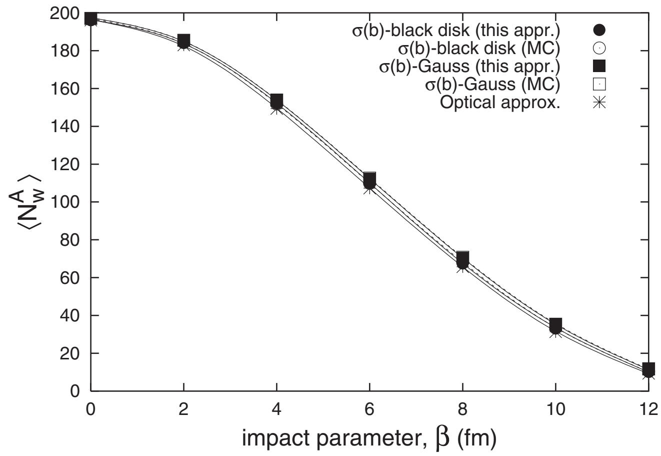

FIG. 2. The same as in Fig. 1 except for the mean number of wounded nucleons in one nucleus, calculated from formulas (1), (3), and (5) and from independent MC simulations; ∗ is the optical approximation result, calculated using formulas (1), (3), and (12).

with $R_A = R_0 A^{1/3}$, $R_0 = 1.07$ fm, $\kappa = 0.545$ fm, and ρ_0 fixed by the condition $\int da\, T_A(a) = 1$.

In Fig. 1 we present the the numerical evaluation of the contribution of the additional (contact) term in formula (2), using the example of PbPb collisions at SPS energies ($r_N = 1$ fm, $\sigma_{NN} = 31$ mb). For a control we have also carried out independent calculations of the mean values and the variances involved by using MC simulations of AA scattering, presenting the results in the same figure.

In Fig. 1 we see that the contact term in (2) is essential and that it gives approximately the same contribution to the variance of $N_w^A(\beta)$ in PbPb collisions at intermediate and large values of β as the first optical term. We see in Fig. 1 that the results of independent MC simulations of the $N_w^A(\beta)$ variance are in a good agreement with the results of the analytical calculations from formula (2) only if one takes into account the contact term.

We see also in Fig. 1 that, for peripheral AA collisions at large β, when $P(\beta)$ becomes small [$P(\beta) \ll 1$, $Q(\beta) \approx 1$] the optical approximation (7) reduces to the Poisson distribution with $V[N_w^A(\beta)]_{\text{opt}} \approx \langle N_w^A(\beta)\rangle$ (8).

The variance of $N_w^A(\beta)$ is larger than the Poisson one for peripheral PbPb collisions (at $\beta > 7$ fm) only because of the contact term, in correspondence with indications from experimental data on the dependence of multiplicity fluctuations on centrality at SPS and Relativistic Heavy Ion Collider (RHIC) energies [1,4].

The weak dependence of the results on the form of the NN interaction at nucleon distances is also seen. In the case of the black-disk (14) approximation for $\sigma(b)$ the results are systematically slightly higher than the results from the Gaussian (15) approximation with the same value of σ_{NN}.

In Fig. 2 we see that the mean value $\langle N_w^A(\beta)\rangle$ (1), in contrast to the variance, coincides with the optical approximation result (8) and depends only on σ_{NN} in the limit $r_N \ll R_A, R_B$. The MC simulations also confirm this result.

We would like to emphasize that the nontrivial term in expression (2) for the variance arises only in the case of nucleus-nucleus collisions. At $A = 1$ or $B = 1$ it vanishes. At $A = 1$ this is due to the explicit factor $A - 1$ in (2), and at $B = 1$ it is because in this case $Q^{(12)}(\beta) = Q^2(\beta)$. This result corresponds to the well-known fact that for nucleus-nucleus collisions the Glauber approach does not reduce to the optical approximation even in the limit $r_N \ll R_A, R_B$ (see, for example, [23]).

The additional term that arises in the expression for the variance (2) in the case of nucleus-nucleus collisions depends, as we have mentioned, not only on the integral value of the inelastic NN cross section $\sigma_{NN} = \int db\, \sigma(b)$ but also on the shape of the function $\sigma(b)$, i.e., on the details of the NN interaction at nucleon distances, which are much smaller than typical nuclear distances. In the quantum Glauber approach, this dependence corresponds to the fact that in the case of AA collisions, in contrast with pA collisions, loop diagrams of the type shown in Fig. 3 appear and one encounters the contact-term problem (see, for example, [23–25]).

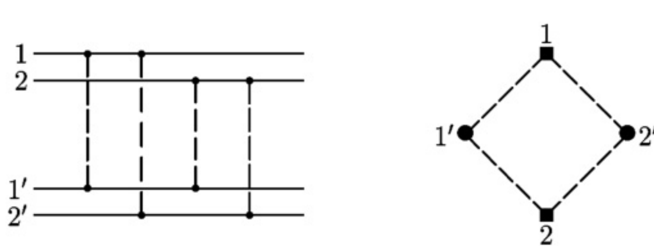

FIG. 3. An example of the loop diagram in AA collisions. 1 and 2 are nucleons of the nucleus A; 1′ and 2′ are nucleons of the nucleus B (see [23–25] for details).

The second term in formula (2) is the manifestation of this problem at the classical level. In the case of a tree diagram the "lengths" of the interaction links in the transverse plane are independent. As a consequence the result is expressed only through $P(\beta)$, the probability of the interaction of a nucleon of nucleus A with nucleons of nucleus B *averaged* over its position in nucleus A. $P(\beta)$ is the same for any nucleon of nucleus A. In the case of the loop diagram in Fig. 3 the "lengths" of the interaction links in the transverse plane are not independent, the result cannot be expressed only through the averaged probability $P(\beta)$, and correlation effects have to be taken into account.

III. VARIANCE OF THE TOTAL NUMBER OF PARTICIPANTS

Now we calculate the variance of the total number of participants $V[N_w^A(\beta) + N_w^B(\beta)]$ at a fixed value of the impact parameter β. Clearly, for the mean value we have simply

$$\langle N_w^A(\beta) + N_w^B(\beta)\rangle = \langle N_w^A(\beta)\rangle + \langle N_w^B(\beta)\rangle, \tag{17}$$

and by (9) for the variance we have

$$V\big[N_w^A(\beta) + N_w^B(\beta)\big] = V\big[N_w^A(\beta)\big] + V\big[N_w^B(\beta)\big] + 2\big\{\langle N_w^A(\beta)N_w^B(\beta)\rangle - \langle N_w^A(\beta)\rangle\langle N_w^B(\beta)\rangle\big\}. \tag{18}$$

In the naive optical approach there is no correlation between the numbers of participants in colliding nuclei at a fixed value of the impact parameter:

$$\langle N_w^A(\beta)N_w^B(\beta)\rangle_{\rm opt} = \langle N_w^A(\beta)\rangle_{\rm opt}\langle N_w^B(\beta)\rangle_{\rm opt} = \langle N_w^A(\beta)\rangle\langle N_w^B(\beta)\rangle.$$

More accurate calculations fulfilled in accordance with (9) and (10) (see Appendix B) lead to

$$\langle N_w^A(\beta)N_w^B(\beta)\rangle - \langle N_w^A(\beta)\rangle\langle N_w^B(\beta)\rangle = AB[Q^{(11)}(\beta) - Q(\beta)\widetilde{Q}(\beta)], \tag{19}$$

where

$$Q^{(11)}(\beta) = \int da\, db\, T_A(a)T_B(b)[1 - f_B(a+\beta)]^{B-1} \times [1 - f_A(b-\beta)]^{A-1}[1 - \sigma(a - b + \beta)], \tag{20}$$

$$\widetilde{Q}(\beta) = \int db\, T_B(b)[1 - f_A(b-\beta)]^A, \tag{21}$$

and

$$f_A(b) \equiv \int da\, T_A(a)\sigma(b-a) \approx \sigma_{NN}\, T_A(b). \tag{22}$$

$Q(\beta)$ and $f_B(a)$ are the same as in formulas (3), (5), and (11). Recall that in our approximation $f_A(b) = f_A(|b|)$ and $\widetilde{Q}(\beta) = \widetilde{Q}(|\beta|)$. Then $\widetilde{Q}(\beta)$ can be obtained from $Q(\beta)$ by a simple permutation of A and B. At $A = B$ we have $\widetilde{Q}(\beta) = Q(\beta)$.

The results of numerical calculations of the correlator (19) from formulas (20)–(22) for PbPb collisions at SPS energies together with the results obtained by independent MC simulations of these collisions are presented in Fig. 4.

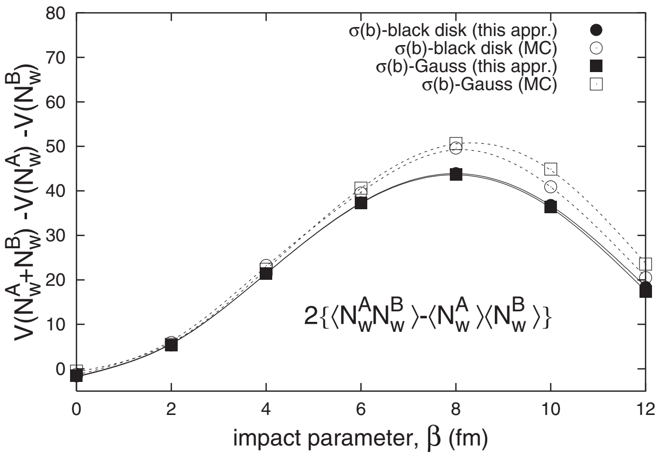

FIG. 4. The correlator between the numbers of wounded nucleons in colliding nuclei, calculated from analytical formulas (19)–(22) and from independent MC simulations. The symbols are the same as in Fig. 1.

Comparing Fig. 4 with Fig. 1 we see that the contribution of the correlator to the variance of the total number of participants at intermediate values of β is about half the variance for one nucleus, $V[N_w^A(\beta)]$, and is approximately equal to the contribution of the first optical term in (2). At large values of the impact parameter ($\beta \geqslant 10$ fm) the relative contribution of the correlator (19) to the total variance (18) is even greater. The results are again in a good agreement with the results obtained by MC simulations. [The small difference in the region 8–10 fm arises from use of the approximate formulas (11) and (22).]

In Figs. 5 and 6 we present the final results for the variance of the total number of participants in PbPb collisions at SPS energies, taking into account the contribution of the correlator. [Figure 6 is the same as Fig. 5 except for the scaled variance $V[N_w(\beta)]/\langle N_w(\beta)\rangle$, $N_w(\beta) \equiv N_w^A(\beta) + N_w^B(\beta)$.] We see in particular that the calculated variance of the total number of participants, $V[N_w(\beta)]$, is a few times larger than the Poisson variance in the impact parameter region of 8–12 fm.

IV. VARIANCE OF THE NUMBER OF BINARY COLLISIONS

In this section we present the results of the calculation of the variance of the number of NN collisions at a fixed value of the impact parameter β in the framework of the same classical Glauber approach [18] for nucleus-nucleus collisions. The details of the calculations are in Appendix C.

We found that the formula for the mean number of binary collisions again coincides with the well-known expression from the optical approximation [compare with formula (29) below]:

$$\langle N_{\rm coll}(\beta)\rangle = AB\chi(\beta), \tag{23}$$

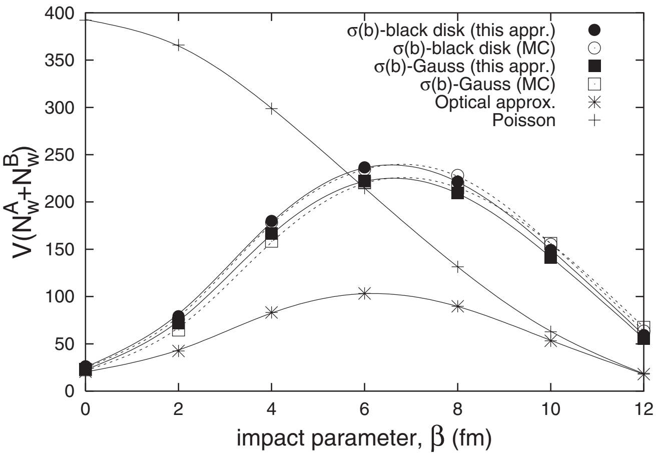

FIG. 5. The same as in Fig. 1 except for the variance of the total number of wounded nucleons $N_w(\beta) \equiv N_w^A(\beta) + N_w^B(\beta)$ in colliding nuclei. The variance $V[N_w(\beta)]$ is calculated from formulas (2)–(4), (11), and (12), taking into account the contribution of the correlator (18)–(22); + is the Poisson variance $V[N_w(\beta)] = \langle N_w(\beta)\rangle$.

where

$$\chi(\beta) \equiv \int da\, db\, T_A(a)T_B(b)\sigma(a - b + \beta) \approx \sigma_{NN} \int da\, T_A(a)T_B(a + \beta) \quad (24)$$

represents the averaged probability of NN interaction. The mean values of the number of collisions as a function of the impact parameter β are shown in Fig. 7.

In contrast to the mean value, the formula obtained for the variance of $N_{\rm coll}(\beta)$,

$$V[N_{\rm coll}(\beta)] = AB[\chi(\beta) + (B - 1)\chi_1(\beta) + (A - 1)\widetilde{\chi}_1(\beta) - (A + B - 1)\chi^2(\beta)], \quad (25)$$

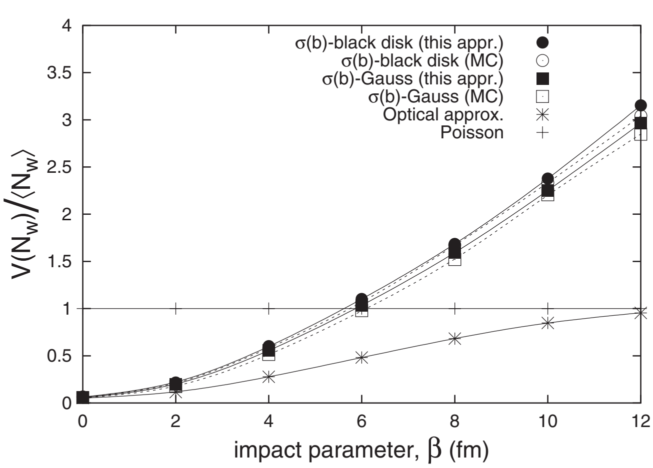

FIG. 6. The same as in Fig. 5 except for the scaled variance $V[N_w(\beta)]/\langle N_w(\beta)\rangle$ of the total number of wounded nucleons in colliding nuclei, $N_w(\beta) \equiv N_w^A(\beta) + N_w^B(\beta)$.

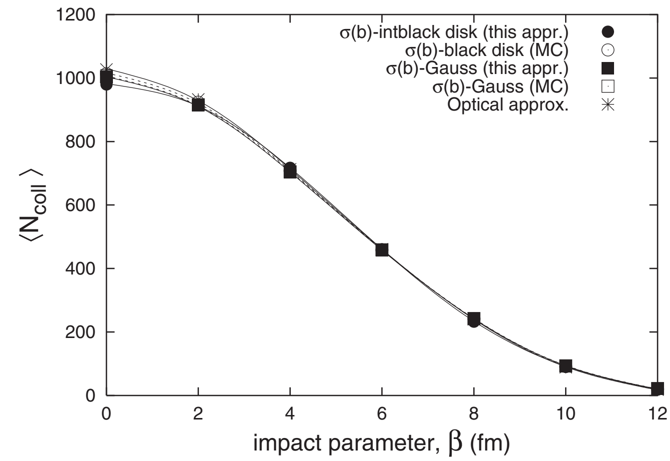

FIG. 7. The mean number of NN collisions in PbPb interactions at SPS energies calculated from formulas (23) and (24) and from independent MC simulations as a function of the impact parameter β (fm). The symbols are the same as in Fig. 1.

differs from the optical approximation result [see Eq. (30) below]. It depends not only on $\chi(\beta)$ (24) but also on

$$\chi_1(\beta) \equiv \int da\, T_A(a)\left(\int db\, T_B(b)\sigma(a - b + \beta)\right)^2 \approx \sigma_{NN}^2 \int da\, T_A(a)T_B^2(a + \beta) \quad (26)$$

and

$$\widetilde{\chi}_1(\beta) \equiv \int db\, T_B(b)\left(\int da\, T_A(a)\sigma(a - b + \beta)\right)^2 \approx \sigma_{NN}^2 \int da\, T_B(a)T_A^2(a + \beta). \quad (27)$$

$\widetilde{\chi}_1$ is obtained from χ_1 by permutation of A and B. (Recall that we consider T_A and T_B to depend only on the magnitudes of their two-dimensional vector arguments.) At $A = B$ we have $\widetilde{\chi}_1 = \chi_1$. Note also that in the limit $r_N \ll R_A, R_B$ the values χ, χ_1, and $\widetilde{\chi}_1$ and hence the variance (25) depend only on σ_{NN} and not on the form of the function $\sigma(b)$. [This was not the case for the variance of the number of wounded nucleons; see Sec. II after formula (12).]

For comparison we list below the optical approximation results, which assume a binomial distribution of $N_{\rm coll}(\beta)$ with averaged probability $\chi(\beta)$ of NN interaction (see, for example, [21,22]):

$$\wp_{\rm opt}(N_{\rm coll}) = C_{AB}^{N_{\rm coll}}\, \chi(\beta)^{N_{\rm coll}}\, [1 - \chi(\beta)]^{AB - N_{\rm coll}}. \quad (28)$$

In this case one has

$$\langle N_{\rm coll}(\beta)\rangle_{\rm opt} = AB\chi(\beta) \quad (29)$$

and

$$V[N_{\rm coll}(\beta)]_{\rm opt} = AB\chi(\beta)[1 - \chi(\beta)] = \langle N_{\rm coll}(\beta)\rangle[1 - \chi(\beta)]. \quad (30)$$

Note that for heavy nuclei $\chi(\beta)$ is small even for central collisions $[\chi(\beta) \sim r_N^2/R_A^2 \ll 1]$, so the distribution (28) and

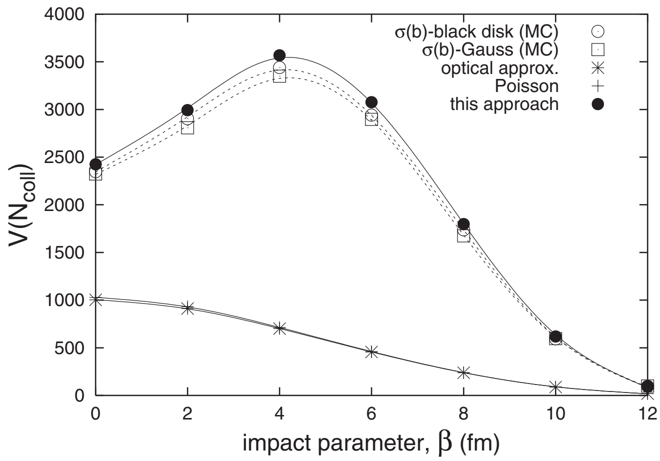

FIG. 8. The variance of the number of NN collisions in PbPb interactions at SPS energies as a function of the impact parameter β (fm). • is the result of calculations from analytical formulas (24)–(27); $*$ is the optical approximation result, calculated from formulas (24) and (30); $+$ is the Poisson variance $V[N_{\rm coll}(\beta)] = \langle N_{\rm coll}(\beta)\rangle$. The symbols are the same as in Fig. 1.

the variance in the optical approximation (30) practically coincide with the Poisson ones: $V[N_{\rm coll}(\beta)]_{\rm opt} \approx \langle N_{\rm coll}(\beta)\rangle$.

Note also that in the case of pA interactions ($A = 1$ or $B = 1$) our result (25) for the variance of the number of collisions coincides with formula (30) obtained from the optical approximation.

In Figs. 8 and 9 we present the results of our numerical calculations of the variance of the number of collisions from analytical formulas (24)–(27) in the case of PbPb scattering at SPS energies together with the results obtained from our independent Monte Carlo simulations of the scattering process. (Figure 9 is the same as Fig. 8 except for the scaled variance $V[N_{\rm coll}(\beta)]/\langle N_{\rm coll}(\beta)\rangle$.)

We see that the calculated variance of the number of collisions at all values of the impact parameter β is a few times larger than the Poisson one, whereas the variance given by the optical approximation practically coincides with the

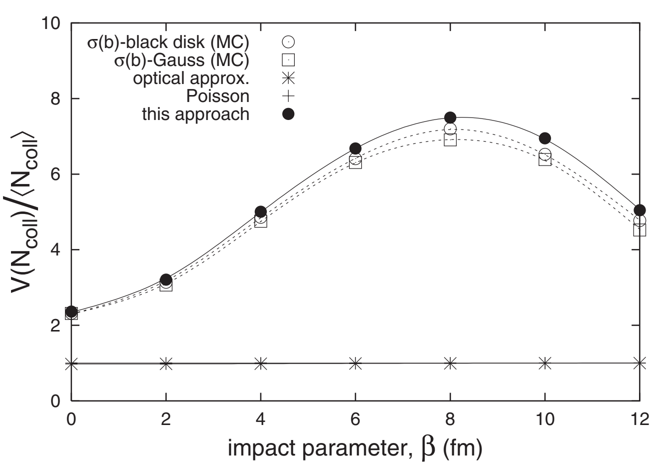

FIG. 9. The same as in Fig. 8 except for the scaled variance $V[N_{\rm coll}(\beta)]/\langle N_{\rm coll}(\beta)\rangle$ of the number of NN collisions.

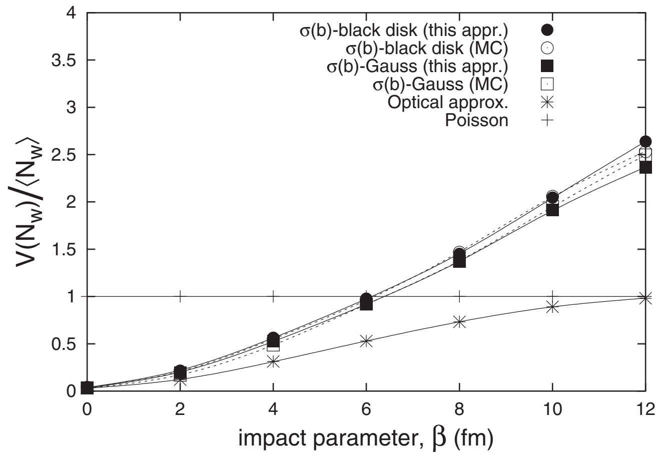

FIG. 10. The scaled variance of the total number of wounded nucleons. The same as in Fig. 6 except for the nucleon density distribution in nuclei (16), with a smaller value of the Woods-Saxon parameter $\kappa = 0.3$ fm.

Poisson one [see the remark after formula (30)]. The results obtained from independent Monte Carlo simulations confirm our analytical result. [The small difference again can be explained by the use of the approximate formulas (24), (26), and (27).]

We have also analyzed the dependence of fluctuations on the diffuseness of the nucleon density distribution in nuclei. To study this dependence, calculations with a smaller (0.3 fm) than standard (0.545 fm) value of the Woods-Saxon parameter κ (16) were performed, which corresponds to a model of the nucleus with a sharper edge (see Figs. 10 and 11).

The calculations confirm what one would expect from simple physical considerations: A more compact distribution of nucleons in nuclei does not change the mean number of wounded nucleons but reduces its fluctuation because the number of wounded nucleons is more strictly determined by the collision geometry. As a result, the scaled variance of the number of wounded nucleons decrease with κ (compare Figs. 6 and 10).

For the number of binary NN collisions, due to the more compact distribution of nucleons in nuclei the mean number of collisions increases along with its variance. Therefore the scaled variance of the number of binary collisions weakly depends on the variation of the parameter κ (compare Figs. 9 and 11). In both cases the contribution of the contact term plays a crucial role.

V. DISCUSSION AND CONCLUSIONS

We have shown that, although the so-called optical approximation gives correct results for the average number of wounded nucleons and binary collisions, the corresponding variances cannot be described within this approximation in the case of nucleus-nucleus interactions.

In the framework of the classical Glauber approach the analytical expression for the variance of the number of participants (wounded nucleons) in AA collisions at a fixed value of the impact parameter is presented. Along with the optical-approximation contribution, which depends only on the total inelastic NN cross section, in the case of nucleus-nucleus collisions there is the additional contact-term contribution, which depends on details of the NN interaction at nucleon distances.

In the classical Glauber approach this contact contribution arises from the interactions between two pairs of nucleons in colliding nuclei (a pair in one nucleus with a pair in another). Interactions of higher order than those between two pairs of nucleons do not contribute to the variance. The expression for the mean number of participants was proved to be exact in the optical approximation, based only on the averaged probability of interaction between single nucleons in projectile and target nuclei.

These results are obtained in the framework of a purely classical (probabilistic) Glauber approach [18]. However, it is possible that in the quantum case the one-loop expression for the variance and the "tree" expression for the mean number of participants and binary collisions will be exact.

Using the obtained analytical formulas, the numerical calculation of the variance of the number of participants in PbPb collisions at SPS energies was done as an example. We demonstrated that at intermediate and large impact parameter values the optical and contact-term contributions are of the same order and their sum is in a good agreement with the results of independent MC simulations of this process.

When calculating the variance of the total number of participants (in both nuclei) the correlation between the numbers of participants in colliding nuclei is taken into account. The analytical expression for the correlator at a fixed value of the impact parameter is obtained. The results of numerical calculations of the correlator for the same process of PbPb collisions show that, at intermediate and large values of the impact parameter, the correlator contribution to the variance of the total number of participants is about half of the variance in one nucleus, again in good agreement with independent MC simulations.

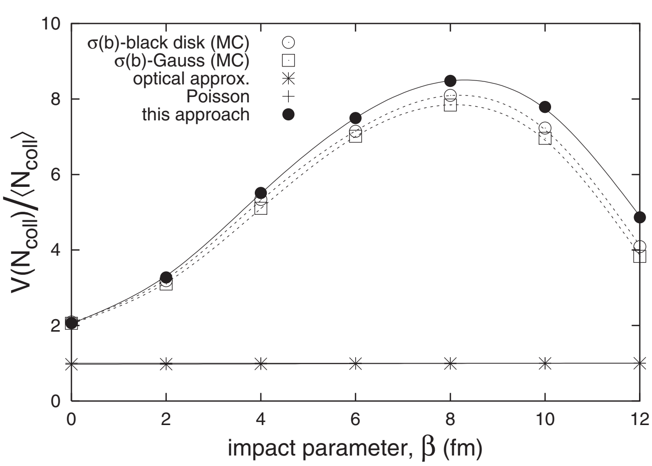

FIG. 11. The scaled variance of the number of binary NN collisions. The same as in Fig. 9 except for the nucleon density distribution in nuclei (16), with a smaller value of the Woods-Saxon parameter $\kappa = 0.3$ fm.

As a result, for peripheral PbPb collisions the variance of the total number of participants, calculated by taking into account the contributions of the correlator and the contact terms, is a few times larger than the Poisson variance.

In the framework of the same classical Glauber approach, the analytical expression for the variance of the number of NN binary collisions in AA interactions at a given centrality is also found. Along with the optical approximation term it also contains other terms, which are the dominant ones.

Due to these additional terms the variance of the number of collisions at all values of the impact parameter is several times higher than the Poisson one, whereas the variance given by the optical approximation practically coincides with the Poisson one. Again the results obtained by independent MC simulations confirm our analytical result.

Significantly, these additional contact terms in the expressions for the variances arise only in the case of nucleus-nucleus collisions. In the case of proton-nucleus collisions they are missing and the variances are well described by the optical approximation.

Note that we have used the simplest factorized approximation (A1) for the nucleon density distribution in nuclei and we do not take into account nucleon-nucleon correlations within one nucleus. Such correlations play a fundamental role in, for example, the description of particle production in nuclear collisions outside the domain kinematically available for production from NN scattering (the so-called cumulative phenomena) [26].

The additional contact contribution to the variance of the number of wounded nucleons, as we have found, arises from interactions between two pairs of nucleons in colliding nuclei, which must occur at the same position in the impact parameter plane. Taking into account nucleon-nucleon correlations within one nucleus must increase the probability of such configurations and hence the contribution of the contact term. However, numerical accounting of these effects is beyond the scope of the present paper.

Interestingly, the nontrivial contact terms in the variances (missing in the optical approximation) arise in our approach in the framework of the exploited factorized approximation for the nucleon density in nuclei, i.e., without taking into account nucleon-nucleon correlations within one nucleus.

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APPENDIX A: CALCULATION OF THE VARIANCE OF PARTICIPANTS IN ONE NUCLEUS

The geometry of an AB collision is depicted in Fig. 12. a_j and b_k are the two-dimensional vectors in the impact parameter plane. In the framework of the classical (probabilistic) approach [18] the dimensionless $\sigma(b)$ is the probability of inelastic interaction of two nucleons at the impact parameter value b [see also (13)]. T_A and T_B are the profile functions of

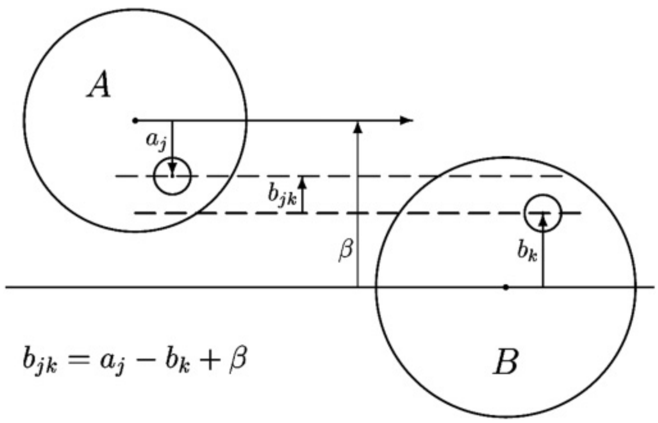

FIG. 12. Geometry of an AB collision.

the colliding nuclei A and B. For heavy nuclei the following factorization takes place:

$$T_A(a_1, \dots, a_A) = \prod_{j=1}^{A} T_A(a_j). \tag{A1}$$

For convenience we introduce the abbreviated notation

$$\int \hat{d}a = \int T_A(a)\, da = 1. \tag{A2}$$

All integrations imply integration over two-dimensional vectors in the impact parameter plane. In the new notation Eq. (10) takes the form

$$\langle X \rangle \equiv \langle \langle \overline{X} \rangle_B \rangle_A = \int \overline{X} \prod_{k=1}^{B} \hat{d}b_k \prod_{j=1}^{A} \hat{d}a_j. \tag{A3}$$

Recall that here $\overline{X}$ represents the average of some variate X at fixed positions of all nucleons in A and B, and $\langle\,\rangle_A$ and $\langle\,\rangle_B$ denote averaging over the positions of these nucleons.

We introduce the set of variates $X_1, \dots, X_A$ (each can be equal only to 0 or 1) as follows: $X_j = 1$ if the jth nucleon of nucleus A interacts with some nucleons of nucleus B, and $X_j = 0$ if the jth nucleon does not interact with any nucleons of nucleus B. The number of participants (wounded nucleons) in nucleus A in a given collision at the impact parameter β is equal to the sum of these variates:

$$N_w^A(\beta) = \sum_{j=1}^{A} X_j. \tag{A4}$$

Then we have for the mean value

$$\langle N_w^A(\beta) \rangle = \sum_{j=1}^{A} \langle X_j \rangle = \sum_{j=1}^{A} \langle \langle \overline{X_j} \rangle_B \rangle_A \tag{A5}$$

and for the variance of $N_w^A(\beta)$, we have

$$V[N_w^A(\beta)] \equiv \langle N_w^A(\beta)^2 \rangle - \langle N_w^A(\beta) \rangle^2,$$
$$\langle N_w^A(\beta)^2 \rangle = \left\langle \overline{\left(\sum_{j=1}^{A} X_j \right)^2} \right\rangle. \tag{A6}$$

First we calculate the mean value (A5). We denote by q_j and p_j the probabilities that the variate X_j will be equal to 0 or 1, respectively. Clearly, for given configurations of nucleons $\{a_j\}$ and $\{b_k\}$ in nuclei A and B,

$$q_j = \prod_{k=1}^{B} (1 - \sigma_{jk}), \qquad p_j = 1 - q_j, \tag{A7}$$

where

$$\sigma_{jk} \equiv \sigma(a_j - b_k + \beta) \tag{A8}$$

and

$$\overline{X_j} = 0 \cdot q_j + 1 \cdot p_j = p_j. \tag{A9}$$

Note that p_j and q_j are functions of $a_j, b_1, \dots, b_B$, and β:

$$q_j = q_j(a_j, \{b_k\}, \beta), \quad p_j = p_j(a_j, \{b_k\}, \beta). \tag{A10}$$

Recall that we restrict our consideration to the region of the impact parameter $\beta < R_A + R_B$, where the probability of inelastic nucleus-nucleus interaction $\sigma_{AB}(\beta)$ is close to unity. Otherwise one has to introduce in formula (A7) for q_j the factor $1/\sigma_{AB}(\beta)$, where

$$\sigma_{AB}(\beta) = 1 - \left\langle \left\langle \prod_{j=1}^{A} \prod_{k=1}^{B} (1 - \sigma_{jk}) \right\rangle_A \right\rangle_B \tag{A11}$$

and $\sigma_{AB} = \int d\beta\, \sigma_{AB}(\beta)$ is the so-called production cross section, which cannot be calculated in a closed form.

Substituting (A7)–(A9) into (A5) we have

$$\langle N_w^A(\beta) \rangle = A - \sum_{j=1}^{A} \langle \langle q_j \rangle_B \rangle_A. \tag{A12}$$

Averaging first over the positions of the nucleons in nucleus B we find

$$\langle q_j \rangle_B = (1 - \sigma_j)^B,$$

where we have introduced the short notation

$$\sigma_j \equiv \int \hat{d}b_1 \sigma_{j1} = \int db_1 T_B(b_1) \sigma(a_j - b_1 + \beta). \tag{A13}$$

Then averaging over the positions of the nucleons in nucleus A we have

$$\langle \langle q_j \rangle_B \rangle_A = \int \hat{d}a_j (1 - \sigma_j)^B, \tag{A14}$$

which is the same for any j, since a_j is the integration variable:

$$\langle \langle q_j \rangle_B \rangle_A = \int da_1 T_A(a_1)(1 - \sigma_1)^B \equiv Q(\beta). \tag{A15}$$

Then by (A12) we find

$$\langle N_w^A(\beta) \rangle = A[1 - Q(\beta)] = AP(\beta), \tag{A16}$$

which coincides with formula (1) if one takes into account the connection

$$\sigma_j = f_B(a_j + \beta) \tag{A17}$$

[see (5) and (A13)]. We see that the result for the mean number of participants (A16) is the same as in the optical approximation (8).

We calculate now by the same method the variance of $N_w^A(\beta)$. From (A6) we have

$$\langle N_w^A(\beta)^2\rangle = \sum_{j_1\neq j_2=1}^{A} \langle X_{j_1}X_{j_2}\rangle + \sum_{j=1}^{A}\langle X_j^2\rangle. \quad \text{(A18)}$$

Note that $\langle X_{j_1}X_{j_2}\rangle$ cannot be reduced to the product $\langle X_{j_1}\rangle\langle X_{j_2}\rangle$. In this case the optical approximation breaks for AB collisions.

Since by (A9)

$$\overline{X_j^2} = \overline{X_j} = p_j,$$

for the first sum in (A18) we find

$$\sum_{j=1}^{A}\langle X_j^2\rangle = \sum_{j=1}^{A}\langle X_j\rangle = \langle N_w^A(\beta)\rangle = AP(\beta). \quad \text{(A19)}$$

Because

$$\overline{X_{j_1}X_{j_2}} = \overline{X_{j_1}}\cdot\overline{X_{j_2}} = p_{j_1}p_{j_2} = 1 - q_{j_1} - q_{j_2} + q_{j_1}q_{j_2},$$

for the second sum in (A18) using (A15) we have

$$\sum_{j_1\neq j_2=1}^{A}\langle X_{j_1}X_{j_2}\rangle = A(A-1)[1-2Q(\beta)+Q^{(12)}(\beta)], \quad \text{(A20)}$$

where we have introduced

$$Q^{(12)}(\beta) \equiv \frac{1}{A(A-1)}\sum_{j_1\neq j_2=1}^{A}\langle\langle q_{j_1}q_{j_2}\rangle_B\rangle_A. \quad \text{(A21)}$$

We now calculate $Q^{(12)}(\beta)$. Averaging again over the positions of the nucleons in nucleus B, we have

$$\langle q_{j_1}q_{j_2}\rangle_B = (1-\sigma_{j_1}-\sigma_{j_2}+\sigma^{(j_1j_2)})^B,$$

where σ_{j_1} and σ_{j_2} are given by (A13) and

$$\sigma^{(j_1j_2)} \equiv \int \hat{d}b_1\sigma_{j_11}\sigma_{j_21} = \int db_1T_B(b_1)\sigma(a_{j_1}-b_1+\beta)\sigma(a_{j_2}-b_1+\beta). \quad \text{(A22)}$$

Then averaging over the positions of the nucleons in nucleus A we rewrite (A21) as

$$Q^{(12)}(\beta) = \int da_1da_2T_A(a_1)T_A(a_2)(1-\sigma_1-\sigma_2+\sigma^{(12)})^B, \quad \text{(A23)}$$

where from (A22)

$$\sigma^{(12)} = \int\hat{d}b_1\sigma_{11}\sigma_{21} = \int db_1T_B(b_1) \times\sigma(a_1-b_1+\beta)\sigma(a_2-b_1+\beta) \equiv g_B(a_1+\beta, a_2+\beta) \quad \text{(A24)}$$

[see Eq. (6)]. Substituting (A18), (A19), and (A20) into (A6) we find for the variance of $N_w^A(\beta)$

$$V[N_w^A(\beta)] = AQ(\beta)[1-Q(\beta)] + A(A-1)[Q^{(12)}(\beta)-Q^2(\beta)],$$

which coincides with formula (2) if we take into account (A17), (A23), and (A24).

APPENDIX B: CORRELATION BETWEEN THE NUMBERS OF PARTICIPANTS IN COLLIDING NUCLEI AT FIXED CENTRALITY

The calculations are similar to the ones in Appendix A (we use the same notation). Along with the set of variates $X_1,\ldots,X_A$ we introduce in a symmetric way the set of variates $\widetilde{X}_1,\ldots,\widetilde{X}_B$ (again each can be equal only to 0 or 1). $\widetilde{X}_k = 0\,(1)$ if the kth nucleon of the nucleus B does not interact (does interact) with nucleons of nucleus A. Then, similarly to (A4) for the number of participants (wounded nucleons) in a given event in the nucleus B we have

$$N_w^B(\beta) = \sum_{k=1}^{B}\widetilde{X}_k. \quad \text{(B1)}$$

Then

$$\langle N_w^A(\beta)N_w^B(\beta)\rangle = \sum_{j=1}^{A}\sum_{k=1}^{B}\langle\langle\overline{X_j\widetilde{X}_k}\rangle_B\rangle_A \quad \text{(B2)}$$

and similarly to (A9)

$$\overline{X_j\widetilde{X}_k} = P_{jk}(1,1), \quad \text{(B3)}$$

where $P_{jk}(1,1)$ is the probability that both variates X_j and $\widetilde{X}_k$ will be equal to 1. For the probability $P_{jk}(1,1)$ one finds

$$P_{jk}(1,1) = \sigma_{jk} + (1-\sigma_{jk})\rho_{jk}\widetilde{\rho}_{jk}, \quad \text{(B4)}$$

where σ_{jk} is the probability of the interaction of the jth nucleon of nucleus A with the kth nucleon of nucleus B [see formula (A8)], and ρ_{jk} is the probability of the interaction of the jth nucleon of nucleus A with at least one nucleon of nucleus B except for the kth nucleon (correspondingly, $\widetilde{\rho}_{jk}$ is the probability of the interaction of the kth nucleon of nucleus B with at least one nucleon of the nucleus A except for the jth nucleon):

$$\rho_{jk} = 1 - \prod_{k'=1(k'\neq k)}^{B}(1-\sigma_{jk'}),$$
$$\widetilde{\rho}_{jk} = 1 - \prod_{j'=1(j'\neq j)}^{A}(1-\sigma_{j'k}). \quad \text{(B5)}$$

Combining (B2)–(B5) and proceeding as in Appendix A, we obtain formulas (19)–(22).

APPENDIX C: FLUCTUATIONS OF THE NUMBER OF COLLISIONS

In this Appendix we calculate the variance of the number of NN collisions in AB interactions at a fixed value of centrality in the framework of the approach under consideration.

To calculate the number of collisions we define the set of the variates $Y_1,\ldots,Y_A$, which can take a value from 0 to B. If in a given event the jth nucleon of nucleus A interacts with

n nucleons of nucleus B, then $Y_j = n$. The number of NN collisions in the given event at impact parameter β can be expressed through these variates as

$$N_{\rm coll}(\beta) = \sum_{j=1}^{A} Y_j . \quad \text{(C1)}$$

Clearly again (see Appendix A)

$$P(Y_j = 0) = q_j = \prod_{k=1}^{B}(1-\sigma_{jk}). \quad \text{(C2)}$$

To calculate $P(Y_j = n)$ for $n = 1,\dots,B$ we introduce $\{k_1,\dots,k_n\}$, the sampling from the set $\{1,\dots,B\}$, and $\{k_{n+1},\dots,k_B\}$, the remainder after sampling. Then

$$P(Y_j = n) = \sum_{\{k_1,\dots,k_n\}} \sigma_{jk_1}\cdots\sigma_{jk_n}(1-\sigma_{jk_{n+1}})\cdots(1-\sigma_{jk_B}). \quad \text{(C3)}$$

Again we calculate the mean value of the number of collisions,

$$\langle N_{\rm coll}(\beta)\rangle = \sum_{j=1}^{A}\langle\langle\overline{Y_j}\rangle_B\rangle_A . \quad \text{(C4)}$$

For a given configuration $\{a_j\}$ and $\{b_k\}$ we have

$$\overline{Y_j} = \sum_{n=0}^{B} n\, P(Y_j = n). \quad \text{(C5)}$$

Using (C3) and averaging over the positions of the nucleons in nucleus B, we find

$$\langle\overline{Y_j}\rangle_B = \sum_{n=0}^{B} n\, C_B^n \sigma_j^n (1-\sigma_j)^{B-n} = B\,\sigma_j . \quad \text{(C6)}$$

We use the same notation as in Appendix A [see (A13)]. Then averaging over the positions of the nucleons in nucleus A we finally find

$$\langle N_{\rm coll}(\beta)\rangle = AB\chi(\beta), \quad \text{(C7)}$$

where

$$\chi(\beta) \equiv \int \hat{d}a_1\sigma_1 = \int \hat{d}a_1\hat{d}b_1\sigma_{11} = \int da_1 db_1 T_A(a_1)T_B(b_1)\sigma(a_1 - b_1 + \beta), \quad \text{(C8)}$$

and at $r_N \ll R_A, R_B$

$$\chi(\beta) \approx \sigma_{NN}\int da_1 T_A(a_1)T_B(a_1+\beta), \quad \text{(C9)}$$

which coincides with formulas (23) and (24). Comparing (C7) and (29) we see that the result for the mean number of collisions is the same as in the optical approximation.

In the remainder of this Appendix we calculate the variance of the number of collisions. To calculate the variance

$$V[N_{\rm coll}(\beta)] \equiv \langle N^2_{\rm coll}(\beta)\rangle - \langle N_{\rm coll}(\beta)\rangle^2 \quad \text{(C10)}$$

one must calculate

$$\langle N^2_{\rm coll}(\beta)\rangle = \left\langle\left(\sum_{j=1}^{A} Y_j\right)^2\right\rangle = \sum_{j_1\neq j_2=1}^{A}\langle Y_{j_1}Y_{j_2}\rangle + \sum_{j=1}^{A}\langle Y_j^2\rangle. \quad \text{(C11)}$$

Therefore we have to calculate the two sums

$$\sum_{j_1\neq j_2=1}^{A}\langle Y_{j_1}Y_{j_2}\rangle = \sum_{j_1\neq j_2=1}^{A}\langle\langle\overline{Y_{j_1}Y_{j_2}}\rangle_B\rangle_A \quad \text{(C12)}$$

and

$$\sum_{j=1}^{A}\langle Y_j^2\rangle = \sum_{j=1}^{A}\langle\langle\overline{Y_j^2}\rangle_B\rangle_A . \quad \text{(C13)}$$

To calculate the first sum we denote by $k'_1,\dots,k'_n$ the indices of the nucleons of nucleus B that interact only with nucleon j_1 of nucleus A. By $k''_1,\dots,k''_m$ we denote the indices of the nucleons that interact only with nucleon j_2 of nucleus A, and by $\bar{k}_1,\dots,\bar{k}_r$ we denote the indices of the nucleons that interact with both nucleons j_1 and j_2. By $k_1,\dots,k_{B-n-m-r}$ we denote the indices of the nucleons of nucleus B that do not interact with nucleons j_1 and j_2 of nucleus A. In this notation the probability $p_{j_1j_2}$ of such an event is

$$p_{j_1j_2} = p_{j_1}p_{j_2}, \quad \text{(C14)}$$

where

$$p_{j_1} = \prod_{i=1}^{r}\sigma_{j_1\bar{k}_i}\prod_{i=1}^{n}\sigma_{j_1k'_i}\prod_{i=1}^{m}(1-\sigma_{j_1k''_i})\prod_{i=1}^{B-r-m-n}(1-\sigma_{j_1k_i}), \quad \text{(C15)}$$

$$p_{j_2} = \prod_{i=1}^{r}\sigma_{j_2\bar{k}_i}\prod_{i=1}^{n}(1-\sigma_{j_2k'_i})\prod_{i=1}^{m}\sigma_{j_2k''_i}\prod_{i=1}^{B-r-m-n}(1-\sigma_{j_2k_i}). \quad \text{(C16)}$$

Using (C15) and (C16) we can rewrite $p_{j_1j_2}$ in the form

$$p_{j_1j_2} = \prod_{i=1}^{r}\sigma_{j_1\bar{k}_i}\sigma_{j_2\bar{k}_i}\prod_{i=1}^{n}\sigma_{j_1k'_i}(1-\sigma_{j_2k'_i})\prod_{i=1}^{m}(1-\sigma_{j_1k''_i})\sigma_{j_2k''_i} \times \prod_{i=1}^{B-r-m-n}(1-\sigma_{j_1k_i}-\sigma_{j_2k_i}+\sigma_{j_1k_i}\sigma_{j_2k_i}). \quad \text{(C17)}$$

The probability $P_{j_1j_2}(n,m,r)$ that nucleons j_1 and j_2 of nucleus A interact separately with n and m nucleons of nucleus B and interact simultaneously with r nucleons of nucleus B is equal to

$$P_{j_1j_2}(n,m,r) = \sum p_{j_1j_2}, \quad \text{(C18)}$$

where the sum is over all possible samplings $\{k'_1,\dots,k'_n\}$, $\{k''_1,\dots,k''_m\}$, and $\{\bar{k}_1,\dots,\bar{k}_r\}$ from the set $\{1,\dots,B\}$. After averaging (C18) over the positions of the nucleons in nucleus B we find

$$\langle P_{j_1 j_2}(n,m,r)\rangle_B = \frac{B!}{n!m!r!(B-r-m-n)!} z^r (y-z)^m (x-z)^n (1-x-y+z)^{B-r-m-n}, \tag{C19}$$

where we have used the short notation

$$x=\sigma_{j_1}, \quad y=\sigma_{j_2}, \quad z=\sigma^{(j_1 j_2)}. \tag{C20}$$

σ_{j_1} and σ_{j_2} are defined by (A13) and $\sigma^{(j_1 j_2)}$ is defined by (A22) in Appendix A. Then for the components of the first sum (C12) we have

$$\langle \overline{Y_{j_1} Y_{j_2}}\rangle_B = \sum_{r=0}^{B}\sum_{m=0}^{B-r}\sum_{n=0}^{B-r-m} (m+r)(n+r)\langle P_{j_1 j_2}(n,m,r)\rangle_B. \tag{C21}$$

After substitution of (C19) in (C21), the lengthy but straightforward calculation leads to the simple answer

$$\langle \overline{Y_{j_1} Y_{j_2}}\rangle_B = Bz + B(B-1)xy = B\sigma^{(j_1 j_2)} + B(B-1)\sigma_{j_1}\sigma_{j_2}. \tag{C22}$$

For the components of the second sum (C13) a similar but much simpler calculation yields

$$\langle \overline{Y_j^2}\rangle_B = B\sigma_j + B(B-1)\sigma_j^2. \tag{C23}$$

Averaging now over the positions of the nucleons in nucleus A, we can rewrite (C11) as

$$\langle N_{\rm coll}^2(\beta)\rangle = B\left(\sum_{j_1\neq j_2=1}^{A} (\langle\sigma^{(j_1 j_2)}\rangle_A + (B-1)\langle\sigma_{j_1}\sigma_{j_2}\rangle_A) + \sum_{j=1}^{A}\left(\langle\sigma_j\rangle_A + (B-1)\langle\sigma_j^2\rangle_A\right)\right)$$

$$= B\left(A(A-1)\int \hat{d}a_1 \hat{d}a_2 (\langle\sigma^{(12)}\rangle_A + (B-1)\langle\sigma_1\sigma_2\rangle_A) + A\int \hat{d}a_1 \left(\langle\sigma_1\rangle_A + (B-1)\langle\sigma_1^2\rangle_A\right)\right).$$

Recalling now that σ_1, σ_2, and $\sigma^{(12)}$ are given by formulas (A13) and (A24) of Appendix A, we obtain

$$\langle N_{\rm coll}^2(\beta)\rangle = AB[\chi(\beta) + (B-1)\chi_1(\beta) + (A-1)\widetilde{\chi}_1(\beta) + (A-1)(B-1)\chi^2(\beta)], \tag{C24}$$

with $\chi(\beta)$, $\chi_1(\beta)$, and $\widetilde{\chi}_1(\beta)$ defined by formulas (24), (26), and (27). Then using the definition (C10) and taking into account formula (C7) for $\langle N_{\rm coll}(\beta)\rangle$, we come to expression (25) for the variance of the number of collisions.

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