

# Mixing of quasiparticle excitations and $\gamma$ vibrations in transitional nuclei

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Evidence of strong coupling of quasiparticle excitations with  $\gamma$  vibration is shown to occur in transitional nuclei. High-spin band structures in  $^{166,168,170,172}\text{Er}$  are studied by employing the recently developed multiquasiparticle triaxial projected shell model approach. It is demonstrated that a low-lying  $K = 3$  band observed in these nuclei, the nature of which has remained unresolved, originates from the angular-momentum projection of triaxially deformed two-quasiparticle (qp) configurations. Furthermore, it is predicted that the structure of this band depends critically on the shell filling: in  $^{166}\text{Er}$  the lowest  $K = 3$  2-qp band is formed from proton configuration, in  $^{168}\text{Er}$  the  $K = 3$  neutron and proton 2-qp bands are almost degenerate, and for  $^{170}\text{Er}$  and  $^{172}\text{Er}$  the neutron  $K = 3$  2-qp band becomes favored and can cross the  $\gamma$ -vibrational band at high rotational frequencies. We consider that these are a few examples in even-even nuclei, where the three basic modes of rotational, vibrational, and quasiparticle excitations coexist close to the yrast line.

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## I. INTRODUCTION

Major advances in experimental techniques have made it feasible to perform detailed measurements of atomic nuclei at the extremes of angular-momentum, isospin, and stability. Detailed spectroscopic studies have provided deep insights in our understanding of nuclear many-body problem. Band structures in some nuclei have been observed with many bands and up to extremely high angular momentum. With the near completion of the new advanced kind of  $\gamma$ -ray detector GRETTINA in USA, one would expect a vast amount of high-quality data covering the regions that have never been reached before.

The classification and the interpretation of the rich band structures is a challenge to nuclear theory. The three basic modes of excitations of rotational, vibrational, and quasiparticle constitute the primary origin of the observed bands in nuclei [1]. In spherical nuclei, the energy spectrum is primarily built on the quasiparticle excitations, while as in well-deformed nuclei rotational bands are observed and are classified using the Nilsson scheme. On the other hand, in transitional nuclei the excitation spectrum is quite rich and intricate and depict interplay of all the three modes of excitations [2–6].

Rotational bands built on vibrations in  $\beta$  and  $\gamma$  degree of deformation are observed in many transitional nuclei. In particular, well-developed  $\gamma$  bands are known to exist in most of the transitional regions of the nuclear chart and a considerable effort has been devoted to understand the detailed structure of these bands. These bands are traditionally interpreted in the phonon picture with the observed  $K = 2$  and 4 bands built on one- and two- $\gamma$ -phonon excitations [7–11].

Recently, these bands have been re-interpreted using the microscopic triaxial projected shell model (TPSM) [12–15]. It has been demonstrated that three-dimensional projection of angular momentum from the triaxially deformed vacuum state of an even-even system leads to  $K = 0, 2$ , and 4 bands that correspond to the ground,  $\gamma$ , and  $\gamma\gamma$  bands observed in nuclei. In a more recent development [16–19], the TPSM approach has been generalized to include quasiparticle (qp) excitations, and it was demonstrated that the projection from triaxially deformed qp states can result into various excited bands. These are the structures that couple  $\gamma$  vibration to qp excitations, based on which rotational bands are built. Thus, these bands have characteristics of all three excitation modes in nuclei and are, therefore, the best places to show up the interplay among them. These recent developments in TPSM approach have greatly enhanced the model predictability and may provide new insights into the observed bands with unknown structures. As a matter of fact, by using this approach, the interpretation of complicated band structures has reached a quantitative level [20,21].

In  $^{168}\text{Er}$  and  $^{170}\text{Er}$ , well-developed  $K = 3$  bands have been observed that are populated as intensively as  $\gamma$  bands [22–25]. The  $K = 3$  band is placed between the  $K = 2$  and  $K = 4$  bands, and, as a matter of fact, crosses the  $\gamma$  band in  $^{170}\text{Er}$  at about  $I = 12$  and becomes quite low in energy. The structure of these bands has remained unresolved and the purpose of the present work is to shed light on the origin of these low-lying  $K = 3$  bands. It is demonstrated, using the generalized TPSM approach [16,17], that these bands are examples of qp excitations that are admixed with  $\gamma$  vibration, and their correct description critically depends on the choice of the basis deformation. In the present work, we also evaluate the intra- and interband electromagnetic transition probabilities, and it is shown that the deformation used in the present work provides a

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better agreement for the transition calculations in comparison to our earlier work on the ground-state configuration only [26]. The TPSM approach has already been discussed in our earlier publications [12–17], and in the following we shall provide only a few details of the model that are relevant to the discussion of the results.

## II. OUTLINE OF THE THEORY

For even-even systems, the TPSM basis are composed of projected 0-qp state (or qp-vacuum  $|\Phi\rangle$ ), two-proton, two-neutron, and 4-qp configurations, that is,

$$\begin{aligned} &\hat{P}_{MK}^I |\Phi\rangle; \quad \hat{P}_{MK}^I a_{p_1}^\dagger a_{p_2}^\dagger |\Phi\rangle; \quad \hat{P}_{MK}^I a_{n_1}^\dagger a_{n_2}^\dagger |\Phi\rangle; \\ &\hat{P}_{MK}^I a_{p_1}^\dagger a_{p_2}^\dagger a_{n_1}^\dagger a_{n_2}^\dagger |\Phi\rangle, \end{aligned} \quad (1)$$

where the three-dimensional angular-momentum operator [27] is given by

$$\hat{P}_{MK}^I = \frac{2I+1}{8\pi^2} \int d\Omega D_{MK}^I(\Omega) \hat{R}(\Omega), \quad (2)$$

with  $\hat{R}(\Omega)$  being the rotation operator and  $D_{MK}^I(\Omega)$  the  $D$  function. The qp states are obtained by usual BCS calculations for the deformed single-particle states. Particle number is conserved on average through the introduction of the first-order Lagrange multipliers. The values of the corresponding neutron and proton chemical potentials are obtained by the constraint for given neutron and proton numbers of nuclei under consideration. This ensures the correct shell filling [27].

It is important to note that for the case of axial symmetry, the qp-vacuum state has  $K = 0$  [27], where as in the present case of triaxial deformation, the vacuum state  $|\Phi\rangle$ , as well as any configuration in (1), is a superposition of all possible  $K$  values. Rotational bands with the triaxial basis states in (1) are obtained by specifying different values for the  $K$ -quantum number in the angular-momentum projector in Eq. (2). The allowed values of the  $K$ -quantum number for a given intrinsic state are obtained through the following symmetry consideration. For  $\hat{S} = e^{-i\pi \hat{J}_z}$ , we have

$$\hat{P}_{MK}^I |\Phi\rangle = \hat{P}_{MK}^I \hat{S}^\dagger \hat{S} |\Phi\rangle = e^{i\pi(K-\kappa)} \hat{P}_{MK}^I |\Phi\rangle. \quad (3)$$

For the self-conjugate vacuum or 0-qp state,  $\kappa = 0$  and, therefore, it follows from the above equation that only  $K = \text{even}$  values are permitted for this state. For 2-qp states,  $a^\dagger a^\dagger |\Phi\rangle$ , the possible values for  $K$ -quantum number are both even and odd, depending on the structure of the qp state. For example, for a 2-qp state formed from the combination of the normal and the time-reversed states having  $\kappa = 0$ , only  $K = \text{even}$  values are permitted. For the combination of the two normal states,  $\kappa = 1$ , only  $K = \text{odd}$  states are permitted.

As in the earlier PSM calculations, we use the pairing plus quadrupole-quadrupole Hamiltonian [27]

$$\hat{H} = \hat{H}_0 - \frac{1}{2} \chi \sum_{\mu} \hat{Q}_{\mu}^\dagger \hat{Q}_{\mu} - G_M \hat{P}^\dagger \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^\dagger \hat{P}_{\mu}, \quad (4)$$

with the last term in (4) being the quadrupole-pairing force. The corresponding triaxial Nilsson mean-field Hamiltonian,

TABLE I. The deformation parameters used in the calculation for  $^{166,168,170,172}\text{Er}$ . The axial deformation  $\epsilon$  is taken from Ref. [29] (converted from  $\beta$  values given there to  $\epsilon$  by multiplying 0.95 factor). The triaxial deformation parameter is denoted by  $\epsilon'$ .

	$^{166}\text{Er}$	$^{168}\text{Er}$	$^{170}\text{Er}$	$^{172}\text{Er}$
$\epsilon$	0.325	0.321	0.319	0.314
$\epsilon'$	0.126	0.125	0.110	0.110

which is obtained by using the Hartree-Fock-Bogoliubov (HFB) approximation, is given by

$$\hat{H}_N = \hat{H}_0 - \frac{2}{3} \hbar \omega \left\{ \epsilon \hat{Q}_0 + \epsilon' \frac{\hat{Q}_{+2} + \hat{Q}_{-2}}{\sqrt{2}} \right\}. \quad (5)$$

Here  $\hat{H}_0$  is the spherical single-particle Hamiltonian, which contains a proper spin-orbit force [28]. The interaction strengths are taken as follows: The  $QQ$ -force strength  $\chi$  is adjusted such that the physical quadrupole deformation  $\epsilon$  is obtained as a result of the self-consistent mean-field HFB calculation [27]. The monopole pairing strength  $G_M$  is of the standard form

$$G_M = \left( G_1 \mp G_2 \frac{N-Z}{A} \right) \frac{1}{A} (\text{MeV}), \quad (6)$$

where  $- (+)$  is for neutron (proton). In the present calculation, we use  $G_1 = 20.12$  and  $G_2 = 13.13$ , which approximately reproduce the observed odd-even mass difference in this region. This choice of  $G_M$  is appropriate for the single-particle space employed in the model, where three major shells are used for each type of nucleons ( $N = 3, 4, 5$  for protons and  $N = 4, 5, 6$  for neutrons). The quadrupole pairing strength  $G_Q$  is assumed to be proportional to  $G_M$ , and the proportionality constant being fixed as 0.16. These interaction strengths are consistent with those used earlier for the same mass region [12,13,27]. Deformation parameters used to construct the qp basis are listed in Table I.

## III. RESULTS AND DISCUSSION

The angular-momentum projected energies from 0-qp, 2-qp, and 4-qp configurations, calculated with deformation parameters given above, are depicted in Figs. 1 and 2 for the four Er isotopes studied in the present work. These are the so-called band diagrams, defined in the projected shell model [27] approach. In these figures, the projected energies of only the lowest few 2- and 4-qp configurations are plotted for clarity. It has been already stated that the admissible  $K$  values for the triaxial vacuum state are  $K = 0, 2, 4, \dots$  and the projection from these possible values give rise to the ground-state band with  $K = 0$ ,  $\gamma$  band with  $K = 2$ ,  $\gamma\gamma$  band with  $K = 4$ , and etc. The calculated unperturbed band heads of  $\gamma$  and  $\gamma\gamma$  bands are roughly at energies of (relative to the ground state) 0.7033, 2.0643 for  $^{166}\text{Er}$ , 0.7009, 1.9651 for  $^{168}\text{Er}$ , 0.8659, 2.0730 for  $^{170}\text{Er}$ , and 0.8659, 2.073 for  $^{172}\text{Er}$  (all in MeV). The anharmonicity in  $\gamma$  vibration appears automatically from the calculations and correctly describes the experimental data [13,30,31].

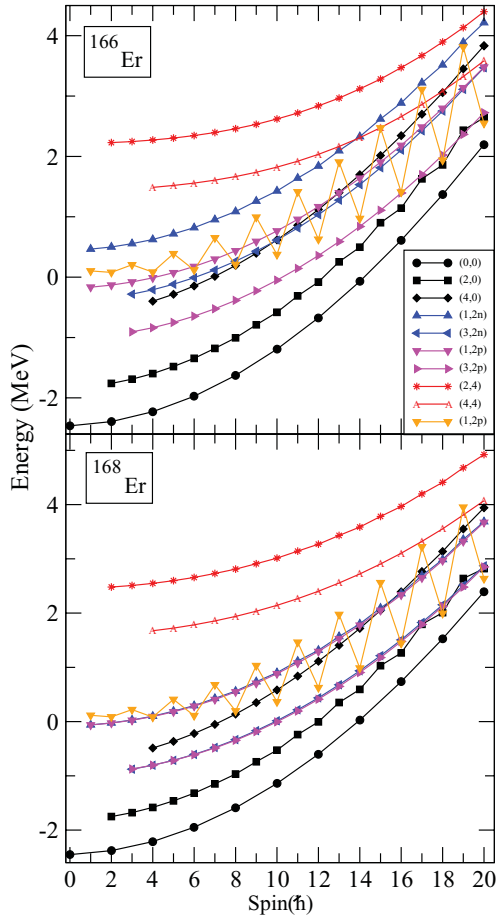


FIG. 1. (Color online) Band diagrams for  $^{166-168}\text{Er}$  isotopes. The labels  $(K, n\text{-qp})$  indicate the  $K$  value and the quasiparticle character of the configuration, for instance,  $(3, 2p)$  corresponds to the two-proton configuration with  $K = 3$ .

The projected bands from 2-qp states result into both even- and odd- $K$  values depending on the combination of the qp states. The bands with  $K = 1, 3, \dots$  are obtained by combining two normal states and are traditionally referred to as aligned bands. In many nuclei in the rare-earth region these aligned bands cross the ground-state band, giving rise to the phenomenon of backbending. Although for the four erbium isotopes studied in the present work, these aligned bands do not cross the ground-state band, they are noted to follow  $\gamma$  bands very closely and interact with them. It is quite interesting to observe from Figs. 1 and 2 that for  $^{166}\text{Er}$ , proton 2-qp band with  $K = 3$  is lower than the corresponding neutron band, for  $^{168}\text{Er}$  the two bands are nearly degenerate, and in cases of  $^{170,172}\text{Er}$  the neutron band is lower than the proton band. The relative change in the  $K = 3$  band structures is attributed to the shell filling of neutrons and protons. As neutron number increases, the neutron Fermi level changes, while the proton Fermi level remains almost unchanged for the isotopes. The proton and neutron character of the bands can be probed through measurement of  $g$  factors of the bands. Four-qp bands in Figs. 1 and 2 are observed to lie higher, and do not become yrast up to the highest angular-momentum state studied in the present work.

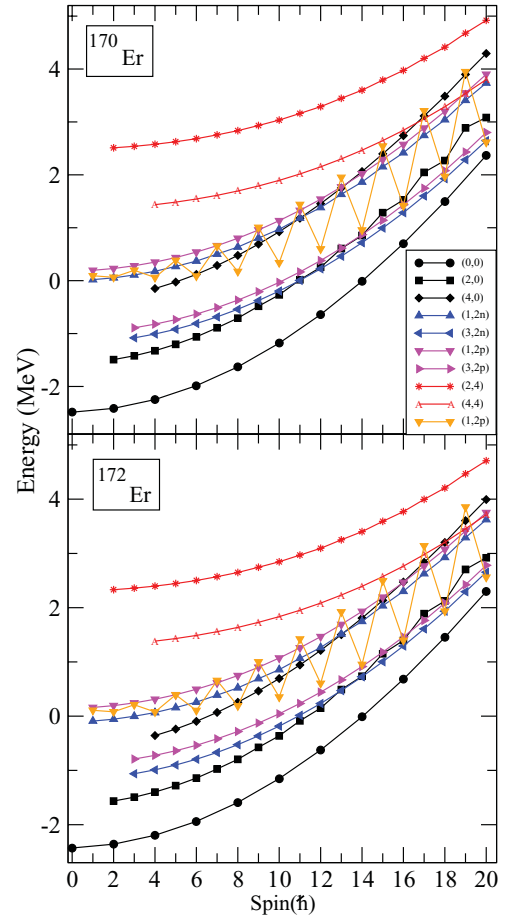


FIG. 2. (Color online) Same as in Fig. 1, but for  $^{170-172}\text{Er}$  isotopes.

In the second stage, the projected bands, obtained above, are then mixed through diagonalization of the shell model Hamiltonian in (4). In band diagrams of Figs. 1 and 2 only the lowest bands were shown, but in the diagonalization process the projected states employed is nearly 40 for all nuclei. Figure 3 depicts the calculated bands after diagonalization and also displays the corresponding available experimental data. It is important to point out that, although the bands in Fig. 3 are labeled as  $\gamma$ ,  $\gamma\gamma$ , and  $K = 3$  bands, these are only the dominant components in the wave function. The projected states after diagonalization are in general mixed. In particular, 2-qp  $K = 3$  band has a significant contribution from 0-qp  $K = 2$  configuration at higher angular momenta.

For  $^{166}\text{Er}$ , the agreement between the TPSM and the experimental energies for the yrast and the  $\gamma$  bands is exceedingly good. There is only  $I = 4$  bandhead state known for the  $\gamma\gamma$  band [30] and it is also reproduced quite well. It is noted from Fig. 3 that the  $K = 3$  band, which is a projected band from 2-qp proton configuration, is predicted above the known  $\gamma$  band but is lower than the  $\gamma\gamma$  band. We hope that future high-spin experimental studies shall be able to populate this band. In the lower panels of Fig. 3, the results for the other three studied isotopes also display a good agreement with the available experimental data. In  $^{168}\text{Er}$ , the known experimental data for the  $\gamma\gamma$  band [31] are also described correctly. The 2-qp proton and neutron  $K = 3$  bands are almost degenerate for

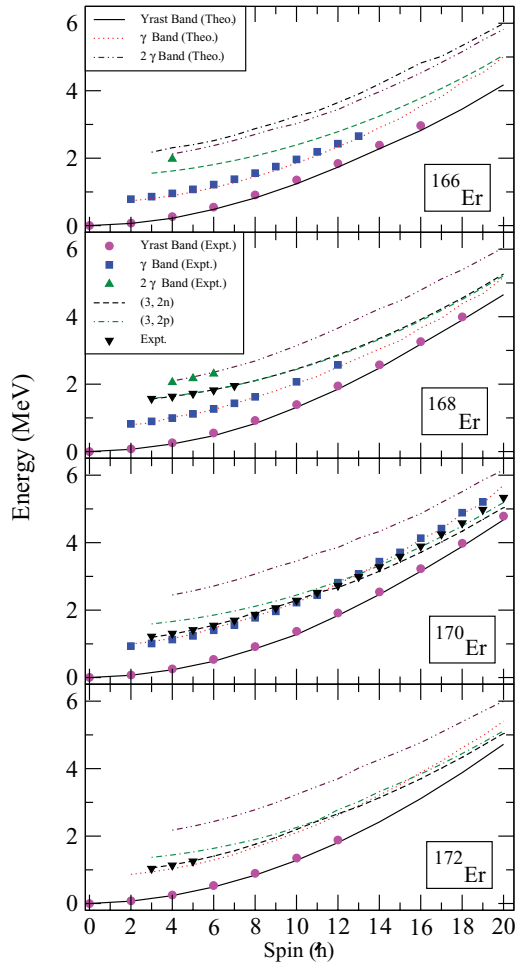


FIG. 3. (Color online) Comparison of the TPSM energies after configuration mixing with the available experimental data for  $^{166-172}\text{Er}$ . Data are taken from [22,30–32].

$^{168}\text{Er}$ , and the observed five states of this band are noted to be reproduced quite well. The interesting prediction is that there are two almost identical  $K = 3$  bands that have predominantly proton and neutron structures, respectively.

In  $^{170}\text{Er}$ , the  $K = 3$  band is populated as intensively as the  $\gamma$  band and is known up to  $I = 20$  [22]. Furthermore, this band crosses the  $\gamma$  band at  $I = 12$  and becomes the first excited band above this spin value. The present work reproduces these properties and what is more interesting is that the observed small staggering in the  $\gamma$  band at higher spins is also borne out by the TPSM results. The  $K = 3$  proton 2-qp band also crosses the  $\gamma$  band and becomes the second excited band above  $I = 14$ .

Figure 4 presents a more detailed comparison of the observed and the calculated  $K = 3$  bands for  $^{170}\text{Er}$ . The calculated  $K = 3$  2-qp neutron band agrees quite well with the experimental band, however, at the top of the band some discrepancies are quite evident. There could be several reasons for these discrepancies. The bulk of the discrepancy could be attributed to the fixed mean field assumed in the present study. The Nilsson potential is chosen for the mean field and is determined by the input deformation parameters  $\epsilon$  and  $\epsilon'$ .

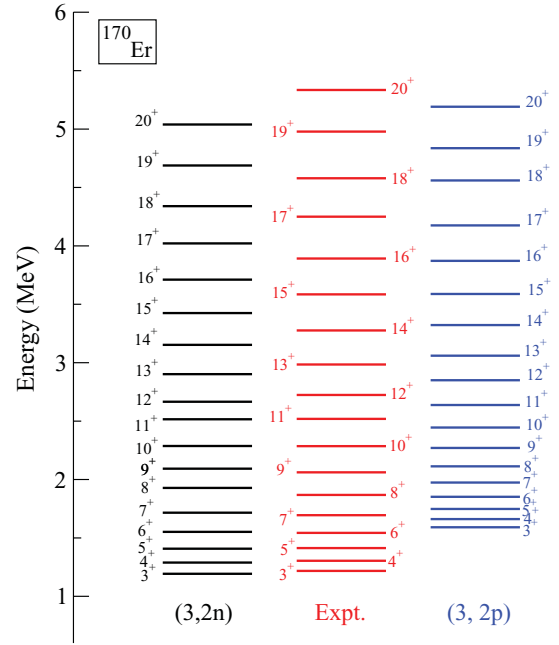


FIG. 4. (Color online) Detailed comparison of the calculated  $K = 3$  bands in  $^{170}\text{Er}$  with experimental data [22].

The pairing potential, on the other hand, is obtained from the monopole interaction using the BCS ansatz. In a more accurate self-consistent treatment, projection before variation, the mean field and the pairing potential are known to vary with qp excitation and angular momentum. Very similar to  $^{170}\text{Er}$ , the heavier isotope  $^{172}\text{Er}$  is also predicted to exhibit a band crossing picture between the  $K = 3$  band [32] and the  $\gamma$  band. The two bands follow very closely for the entire spin region, and interact with each other. Thus the  $K = 3$  band in both transitional  $^{170}\text{Er}$  and  $^{172}\text{Er}$  nuclei, although they have two-quasiparticle structure, interact strongly with the  $\gamma$  vibration.

To probe the mechanism behind the appearance of the  $K = 3$  band, close to the yrast line, in the Er isotopes we have studied the behavior of the projected energies as a function of deformation parameters  $\epsilon$  and  $\epsilon'$ . As an example, the variation of the projected energies are shown in Fig. 5 for  $^{170}\text{Er}$ . In the upper panel of the figure, the variation is depicted as a function of the axial deformation  $\epsilon$  with fixed triaxial deformation  $\epsilon' = 0.11$ . For low axial deformation values, the  $K = 3$  band is higher than the  $K = 1$  band. However, above  $\epsilon = 0.25$   $K = 3$  band depicts a large downward shift and becomes lower than the  $K = 1$  and  $\gamma\gamma$  band. The other bands are noted to be less sensitive to the axial deformation. The dependence of the projected energies on  $\epsilon'$ , shown in the lower panel of Fig. 5, is calculated for fixed  $\epsilon = 0.319$ . This dependence, first of all, clearly demonstrates that the ground-state minimum has  $\epsilon' = 0.11$  and this value has been considered in all our earlier calculations. This figure also shows that the  $K = 3$  band is less sensitive to  $\epsilon'$  as compared to the other bands. Therefore, it can be concluded from the present results that the appearance of the low-excitation  $K = 3$  band is primarily due to the axial deformation.

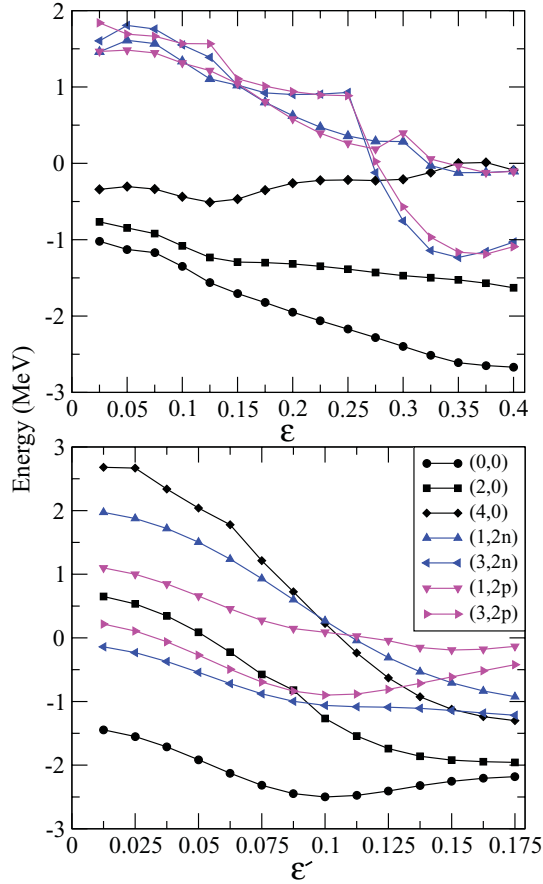


FIG. 5. (Color online) Behavior of the projected energies of various configurations as a function of axial and triaxial deformations for  $^{170}\text{Er}$ . In the upper panel, the projected energies have been evaluated for a fixed value of  $\epsilon' = 0.11$  and in the lower panel  $\epsilon = 0.319$  has been chosen.

The question obviously arises on the relevance of the triaxial deformation in the structure of  $K = 3$  bands. To investigate this question we have also performed axial projected shell model calculations using the original projected shell model code [33]. The parameters used in this study are exactly same as those used in the above triaxial study, except that now the

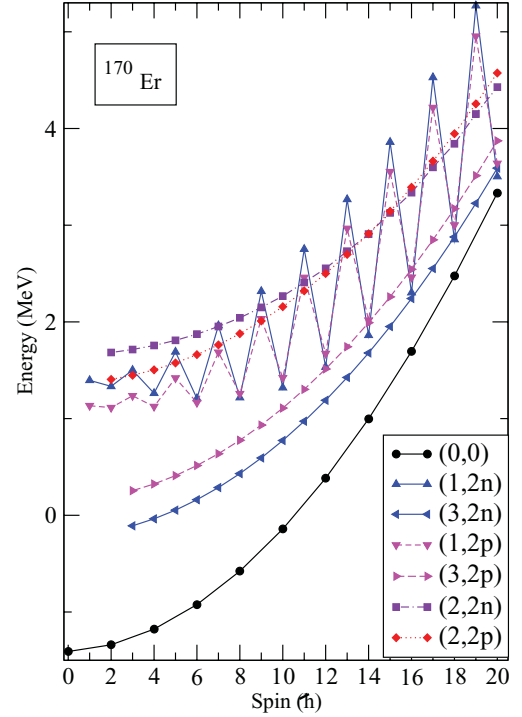


FIG. 6. (Color online) Band diagrams for  $^{170}\text{Er}$  by using axial projected shell model code [33].

triaxial deformation is absent. The advantage of the axial study is that it provides a direct information on the  $K$  structure of the quasiparticle states. The results of the axial study are plotted in Fig. 6. The ground-state band with  $K = 0$  is the projection from the qp-vacuum state with axial symmetry, and all other bands in Fig. 6 are the projected bands from the 2-qp states. The lowest 2-qp band is the neutron band with  $K = 3$  and is formed from the Nilsson states of  $[651]1/2$  and  $[633]7/2$ , and proton 2-qp band is formed from  $[523]7/2$  and  $[541]1/2$ . Although, the  $K = 3$  band is lower in this axial case as well, the obtained band structures are completely different from those in the triaxial study. First of all, as expected, there is no  $\gamma$  band as in the triaxial case and also in the observed data. Secondly, the bandhead of the  $K = 3$  band is higher in the axial case as compared to the corresponding experimental bandhead.

TABLE II. Comparison of known experimental yrast-band  $B(E2)$  values (in w.u., and associated errors in parentheses) and calculated ones for  $^{166,168,170,172}\text{Er}$  isotopes.

$(I, K)_i \rightarrow (I, K)_f$	$^{166}\text{Er}$ (expt.)	$^{166}\text{Er}$ (theor.)	$^{168}\text{Er}$ (expt.)	$^{168}\text{Er}$ (theor.)	$^{170}\text{Er}$ (expt.)	$^{170}\text{Er}$ (theor.)	$^{172}\text{Er}$ (theor.)
$(2, 0)_i \rightarrow (0, 0)_f$	214 (10)	245.02	207 (6)	242.51	208 (4)	244.96	241.07
$(4, 0)_i \rightarrow (2, 0)_f$	311 (20)	351.63	318 (12)	347.77		350.83	345.59
$(6, 0)_i \rightarrow (4, 0)_f$	347 (45)	390.38	440 (30)	385.67		388.15	383.01
$(8, 0)_i \rightarrow (6, 0)_f$	365 (50)	413.07	350 (20)	407.62	370 (30)	408.88	404.47
$(10, 0)_i \rightarrow (8, 0)_f$	371 (46)	429.60	302 (21)	423.73	320 (22)	423.20	419.99
$(12, 0)_i \rightarrow (10, 0)_f$		442.77	334 (22)	437.17	375 (20)	434.33	432.71
$(14, 0)_i \rightarrow (12, 0)_f$		453.01		449.09		443.35	443.66
$(16, 0)_i \rightarrow (14, 0)_f$		459.62		459.93		450.50	453.02
$(18, 0)_i \rightarrow (16, 0)_f$		461.29		469.96		445.86	460.55
$(20, 0)_i \rightarrow (18, 0)_f$		456.51		479.35		459.95	466.01



TABLE III. Calculated interband  $B(E2)$  values (in w.u.) from the  $K = 3$  to  $\gamma$  band for  $^{170,172}\text{Er}$  isotopes.

$(I, K)_i \rightarrow (I, K)_f$	$^{170}\text{Er}$ (theor.)	$^{172}\text{Er}$ (theor.)
$(4, 3)_i \rightarrow (2, 2)_f$	0.18	0.01
$(6, 3)_i \rightarrow (4, 2)_f$	0.68	0.03
$(8, 3)_i \rightarrow (6, 2)_f$	4.53	0.07
$(10, 3)_i \rightarrow (8, 2)_f$	91.27	0.64
$(12, 3)_i \rightarrow (10, 2)_f$	105.16	23.33
$(14, 3)_i \rightarrow (12, 2)_f$	40.04	110.24
$(16, 3)_i \rightarrow (14, 2)_f$	0.64	84.16
$(18, 3)_i \rightarrow (16, 2)_f$	0.29	0.23
$(20, 3)_i \rightarrow (18, 2)_f$	0.07	0.01

Therefore, although the axial deformation is important for the  $K = 3$  band to appear, the strong mixing with  $\gamma$  degree of freedom is crucial to explain its excitation and the rotational behavior.

In the present work we have also evaluated the  $B(E2)$  transition probabilities, which are presented in Table II along the yrast bands for the studied isotopes. Furthermore, we calculated the interband transitions between  $K = 3$  and  $\gamma$  bands for  $^{170,172}\text{Er}$  as these two bands cross for these isotopes, and the transitions are displayed in Table III. The  $B(E2)$  values have been calculated using the standard effective charges of  $e_p = 1.5e$  and  $e_n = 0.5e$ . It is evident from Table II that calculated  $B(E2)$  describe the known transitions well. For the interband transitions in  $^{170,172}\text{Er}$  between the  $K = 3$  and  $K = 2$  band shown in Table III, it is interesting to note that in the crossing region of the two bands (around spin 12), very enhanced  $B(E2)$  is predicted. The large inter-band  $B(E2)$  values indicate a considerable overlap between the wave functions, implying a strong mixing between the quasiparticles and the  $\gamma$  vibration.

#### IV. SUMMARY

In summary, we have presented evidences for strong coupling of quasiparticle excitations with  $\gamma$  vibration in transitional nuclei. High-spin band structures in a series of

erbium isotopes  $^{166,168,170,172}\text{Er}$  have been studied by using the recently developed multiquasiparticle triaxial projected shell model approach. The spotlight of the present investigation has been the  $K = 3$  band observed in some of these nuclei that is populated as strongly as the  $\gamma$  band. In the framework of triaxial angular-momentum projection, we have shown that this band has mainly a structure of triaxially deformed 2-qp state projected to the  $K = 3$  component. This is to compare with the traditional  $\gamma$  and  $\gamma\gamma$  bands, which are based on triaxially deformed 0-qp state projected to the  $K = 2$  and 4 components, respectively. It has been further shown that the detailed structure and position of the  $K = 3$  band depend sensitively on the shell filling. In  $^{166}\text{Er}$  the lowest  $K = 3$  2-qp band is formed from proton configuration, in  $^{168}\text{Er}$  the  $K = 3$  neutron and proton 2-qp bands are almost degenerate, and for  $^{170,172}\text{Er}$  the neutron  $K = 3$  2-qp band becomes favored. The prediction of systematic appearance of two  $K = 3$  bands with proton and neutron structures, close to the yrast line, awaits experimental confirmation.

The calculations presented in the present article have clearly demonstrated that a simple model based on schematic pairing plus quadrupole-quadrupole interaction with three-dimensional angular-momentum projection technique can describe the near yrast band structures in transitional nuclei in a quantitative manner. A drawback in the present analysis is the uncertainty in the strength parameters of the schematic interaction. In future studies we are planning to adopt a recently developed mapping procedure [34,35] to microscopically determine the strength parameters. In this new approach the energy surfaces obtained from the schematic effective interaction with free strength parameters are optimized to reproduce the energy surfaces retrieved from a realistic density functional approach.

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