## Measurements of the pair production cross section close to the threshold energy

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We have measured the total cross section for production of  $e^+e^-$  pairs by photons in the Coulomb field of Ge nuclei. Up to  $10^{16} \gamma$  quanta per second were produced by thermal-neutron capture reactions of a natural gadolinium target, placed inside the high-flux reactor of the ILL Grenoble. Out of this stream of  $\gamma$  rays various strong discrete  $\gamma$  lines were extracted using the GAMS5 Bragg spectrometer of the ILL. A well-collimated beam of reflected  $\gamma$  rays irradiated a germanium detector placed inside an anti-Compton shield, consisting of eight bismuth germanate crystals, assembled as a pair spectrometer. Using this new technique we have measured, for the first time, the pair production cross section at only 18.2 keV above the 1022-keV threshold. Our measurements indicate that the pair production cross section, expressed in units of Bethe-Heitler, grows monotonously toward low energies and does not support the claim of other works of a maximum at ~65 keV.

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After the basic work of Bethe and Heitler [1] (hereafter referred to as BH) three quarters of a century ago, which provided the first theoretical description of the production of electron-positron  $(e^+e^-)$  pairs by photons in the Coulomb field of an atomic nucleus, numerous theoretical and experimental works followed, improving a step-by-step our understanding of this fundamental process. An early summary [2] and a collection of recent review articles [3] serve as a good reference to the subject.

In the past, the pair production process has been primarily studied at high photon energies  $E_{\gamma} > 10$  MeV, where it dominates over the other two interactions of  $\gamma$  rays with matter, Compton scattering, and photoelectric absorption. Those studies were driven by practical needs, such as the construction of protection shields against  $\gamma$  radiation. Strong bremsstrahlung beams were used, allowing measurements of the high-energy part of the total cross section for  $e^+e^-$  production  $\sigma_t$  with a precision as good as 1% [4], which was well reproduced by the calculations.

At the other end of the  $E_{\gamma}$  scale, close to the 1022-keV threshold for the  $e^+e^-$  production, the study of the pair production process is important from the point of view of the physics. Investigations of any deviation from the BH formula offer a way to probe the final-state interaction of electrons and positrons created in the field of an atomic nucleus.

The first systematic study of  $e^+e^-$  production in germanium at low photon energies [5] has shown an increase of the  $\sigma_t/\sigma_{\rm BH}$  prediction by a factor of 2 at a photon energy of 1077 keV. Several other studies published afterwards [6–10] have increased the range and accuracy of the  $\sigma_t$  values measured, as shown in Fig. 1.

In Fig. 1 a clear increase of the  $\sigma_t/\sigma_{BH}$  ratio is evident at low photon energies. An intriguing question is whether this

increase continues when approaching the 1022-keV threshold or if any fluctuations of the  $\sigma_t/\sigma_{\rm BH}$  ratio occur there. A possible resonancelike structure with a maximum at ~65 keV has been suggested in Ref. [11] (see Fig. 1 in Ref. [11]). Although one might question the result of Ref. [11], because of the nonmonochromatic radiation used [12], some other authors supported this finding [10,13]. The data points in Fig. 1 do not exclude a decrease of the  $\sigma_t/\sigma_{\rm BH}$  ratio below 60 keV, as the error bars do not allow any strict conclusions. Early calculations [14] predicted the  $\sigma_t/\sigma_{\rm BH}$  ratio to drop to a value of 1 when approaching the pair production threshold. However, later theoretical works indicated a monotonous increase of the ratio  $\sigma_t/\sigma_{\rm BH}$  [15,16].

One can also see that there are systematic discrepancies between the various experiments shown in Fig. 1, which are beyond statistical fluctuations. For instance, the  $\sigma_t/\sigma_{BH}$  values reported in the most recent study [9] are ~15% lower than the analogous values reported in Ref. [5]. Furthermore, the results of Ref. [9] are significantly more accurate than analogous data from another <sup>207</sup>Bi source measurement [7], even though experimental conditions would appear to be similar.

To clear these discrepancies, and to verify the eventual existence of a resonance structure in the  $\sigma_t/\sigma_{BH}$  function, more accurate measurements close to the threshold are needed. In Refs. [7] and [9] four weeks of measurement were needed to obtain a data point with 5–10% statistical error. It is hard to improve this result using similar techniques. Therefore, as commented by Pratt in a recent review [17], "we are not aware of any further work on low-energy pair production."

The lack of any progress over the past two decades was primarily due to the limited choice of suitable  $\gamma$  rays. To study the pair production just above threshold, where the cross section for the effect drops off quickly with decreasing photon energy, one needs  $\gamma$  rays with well-defined energies, ensuring that the cross section does not change significantly over the width of a  $\gamma$  line. Such  $\gamma$  rays can obtained from radioactive sources, such as <sup>60</sup>Co, <sup>152</sup>Eu, <sup>88</sup>Y, or <sup>207</sup>Bi. However, the

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FIG. 1. Previous measurements of the pair production cross section in germanium. The data are taken from Yamazaki (2005) [5], Sharma (1985) [6], Enyo (1980) [7], Coquette (1978) [8], De Braeckleer (1992) [9], and Avignone (1981) [10].

choice of lines from such sources with energies close to the 1022-keV threshold is limited. Moreover, there are limits on their intensities. For instance, the convenient 1086-keV  $\gamma$  ray from the <sup>152</sup>Eu source constitutes only ~7% of the total intensity of  $\gamma$  radiation emitted from this source (and less than 2% compared to the integral detector counts, including Compton-scattered photons) while all other  $\gamma$  rays contribute to background and increase the dead time on an acquisition system, thereby limiting the strength of the source used.

In the present Rapid Communication we propose a solution, overcoming the limitations discussed above, and report the first results obtained with this new technique. The principle is to produce a variety of strong  $\gamma$  rays, using thermal-neutron capture reactions, then individually select them, with a minimal energy width, using Bragg diffraction.

There are clear advantages in using single  $\gamma$ -ray lines. First, one significantly reduces the  $\gamma$ -ray intensity load on the Ge detector, while maintaining a high intensity for the  $\gamma$  ray of interest. Second, for a source such as <sup>207</sup>Bi, the Compton scattering of the higher-energy line may introduce extra background in the measurement of the lower-energy line, which was a serious problem in Ref. [7]. Third, the <sup>60</sup>Co source, used in previous measurements, emits two  $\gamma$ rays within a few picoseconds, a time interval which cannot be resolved by a Ge detector. This often complicates the pair-creation measurement.

Figure 2 shows a schematic drawing of our experimental system. We have used  $\gamma$  rays from the thermal-neutron capture reaction on a target T containing 3 g of natural Gd<sub>2</sub>O<sub>3</sub> powder, placed close to the core R of the high-flux reactor of the Institute Laue-Langevin in Grenoble. Approximately  $10^{16} \gamma$  quanta per second were produced by the target over a period of several weeks. A well-collimated incident beam of  $\gamma$  rays from the target  $\gamma_I$  with a profile of  $\sim 4 \times 40$  mm was scattered on Cu crystals of the GAMS spectrometer [18], located  $\sim 17.7$  m from the target and covering  $\sim 10^{-7}$  of the full solid angle. Bragg scattering allowed the selection of a small cut of the full  $\gamma$ -ray energy range, with an efficiency of up to 25 %. For example, a double reflection on two Cu111



FIG. 2. A schematic drawing of the experimental setup used in this work. Various elements are not drawn to the scale, but the dimensions relevant for determining the beam shape are given. In the figure the following notations are used: R, reactor core; T, target; W, walls of the reactor;  $C_{1,2}$ , diffracting copper crystals; Col, lead and heavy-metal collimators; Pb, lead shielding of the detector; BGO, segmented scintillator shield; Ge, HPGe detector. See the text for further details.

crystals produced a 15-keV-wide portion of the total spectrum at 1.1 MeV. The width of this cut refers to the full width at half maximum of a diffracted Gaussian profile. The intensity of the diffracted beam  $\gamma_S$  was a few kHz. Although the double-crystal arrangement acts as a monochromator, it does not completely prevent diffuse scattering of high-intensity gamma rays along the collimation paths. However, the probability of diffuse scattering is substantially lower than the coherent scattering by the monochromator.

The  $\gamma_S$  photons diffracted by the GAMS spectrometer were sent to a pair spectrometer, which consisted of a 30% relative efficiency coaxial Ge detector placed inside a conventional anti-Compton shield, consisting of eight bismuth germanate (BGO) crystals, encircling the Ge detector. The pair spectrometer was shielded by a 20-cm-thick layer of lead to protect it from the strong background radiation originating from the primary beam  $\gamma_U$ , which had an intensity of more than 10<sup>8</sup> photons per second (we note that for 1-MeV  $\gamma$  rays the diffraction angle  $\Theta$  is  ${\sim}2\,^\circ$  and the distance between the  $\gamma_S$  and  $\gamma_U$  beams at the front of the pair spectrometer was only a few centimeters). The  $\gamma_S$  beam was collimated over a total path of  $\sim 20$  m. It passed through a hole in the Pb shield surrounding the pair spectrometer and irradiated the central part of the Ge crystal, which was  $\sim$ 50 mm diam and  $\sim$ 50 mm thick, producing  $e^-e^+$  pairs. The Ge detector was used to measure the rate of  $\gamma$  quanta fully absorbed within the detector (full energy peak) as well as the number of  $e^-e^+$  pairs (corresponding to the so-called "double-escape" peak). The annihilating positron produced two  $\gamma$  quanta of 511 keV, which, after escaping the Ge detector, could be detected by a pair of diametrically opposed BGO crystals of the anti-Compton shield. Signals from BGO detectors were filtered through single-channel analyzers which selected only  $\gamma$  rays with energies of  $511 \pm 75$  keV. In addition, a fast coincidence condition, with a width of 50 ns, was imposed on the two 511-keV signals from the BGO detector. Under such conditions a 5- $\mu$ s window was opened in the acquisition system to register the corresponding "double-escape" signal from the Ge detector.



FIG. 3. (Color online) A comparison of a  $\gamma$ -ray singles spectrum (upper part) with the corresponding pair spectrum (lower part), which was shifted by 1022 keV. The crystal spectrometer was set such that diffraction of the 1065.1-, 1067.2-, and 1079.3-keV  $\gamma$  rays from the <sup>155</sup>Gd( $n_{\rm th}$ , $\gamma$ )<sup>156</sup>Gd reaction were directed toward the detector. The inset shows a logarithmic plot of a larger fragment of the complete singles spectrum.

A singles spectrum of  $\gamma_S$  photons, corresponding to scattering of the 1065.1-, 1067.2-, and 1079.3-keV  $\gamma$  rays from the <sup>155</sup>Gd( $n_{\rm th}$ , $\gamma$ )<sup>156</sup>Gd reaction is shown in Fig. 3 (the intensity-weighted average energy of the 1065.1- and 1067.2-keV doublet is 1066.0 keV). The intensity of the  $\gamma_S$  beam was 3.4 kHz and the Ge detector collected ~500 counts per second in the photo peaks of the 1066.0- and 1079.3-keV lines shown in Fig. 3. In the spectrum there are also weak lines at higher energies, for example, at 1186.0 keV, which corresponds to diffuse scattering of very intense  $\gamma$  lines from the <sup>155</sup>Gd( $n_{\rm th}$ , $\gamma$ )<sup>156</sup>Gd reaction, as there were ~10<sup>9</sup>  $\gamma$ rays cm<sup>-2</sup> s<sup>-1</sup> in the  $\gamma_I$  beam.

A spectrum of the  $e^+e^-$  energies as measured in this Rapid Communication for the 1066.0-keV line is shown in Fig. 3. In this spectrum, collected over 71 h, we observe 182(16) counts in the 44.0-keV peak, corresponding to the 1065.1-,

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1067.2-keV doublet from the <sup>155</sup>Gd( $n_{\rm th}$ , $\gamma$ )<sup>156</sup>Gd reaction. This result can be compared to the 118(21) counts in the 42-keV peak collected in ~680 h in Ref. [7] and ~180 ± 20 counts in the 42-keV peak seen in the top panel of Fig. 2 of Ref. [9]. In the latter case a few weeks of acquisition time was required to obtain a 5–10% statistical error.

In our measurement we observed ~7 background counts per day in a 1-keV energy bin near the 44.0-keV  $e^+e^-$  line, which is comparable to the best radioactive-source measurements [5,7,9]. This rather high level of background was due to the long 5- $\mu$ s Ge-BGO time correlation window. In the present experiment we did not select fast coincidences between Ge and BGO signals. Whereas the 511-keV signals from BGO have good timing, the "double-escape" Ge signals show significant time jitter at low energies. Consequently, a rather uncontrolled decrease in the efficiency of the detection and acquisition system appears after applying a constant-fraction



FIG. 4. (Color online) A single  $\gamma$ -ray spectrum (upper part) corresponding to diffraction of the 1040.4-keV line from the <sup>155</sup>Gd( $n_{\rm th}, \gamma$ )<sup>156</sup>Gd reaction is superposed to the according pair spectrum (lower part) shifted by 1022 keV. The inset shows a logarithmic plot of a larger fragment of the singles spectrum.



FIG. 5. (Color online) A comparison of a single (upper part) and  $e^+e^-$  pair (lower part) spectrum corresponding to the  $E_{\gamma} = 1186.0$  -keV line. The inset shows a logarithmic plot of a larger fragment of the singles spectrum.

discriminator (CFD) at very low energies. This is discussed in Ref. [7], where a somewhat arbitrary correction of 0.75(13) had to be applied to the data at  $E_{\gamma} = 1064$  keV, increasing the total error to 26% for this point. Avoiding fast coincidences allowed a measurement of the  $e^+e^-$  pair spectrum down to very low energies, as indicated by the constant level of background down to 12 keV in Fig. 3.

In this Rapid Communication we have performed the first measurement of the  $e^+e^-$  pair creation cross section at  $E_{\text{pair}} = 18.2 \text{ keV}$ , where the  $\sigma_{\text{BH}}$  estimate is an order of magnitude lower than at  $E_{\text{pair}} = 42 \text{ keV}$ , using the pair spectrometer described above and the 1040.4-keV  $\gamma$  line from the <sup>155</sup>Gd( $n_{\text{th}}, \gamma$ )<sup>156</sup>Gd reaction. The singles spectrum corresponding to the 1040.4-keV scattered photons is shown in Fig. 4. The count rate of the 1040.4-keV  $\gamma$  line was 270 counts per second. Apart from the 1040.4-keV line there are several weak lines due to diffuse scattering, among them, the 1154.1- and 1160.0-keV lines from the <sup>155</sup>Gd( $n_{\text{th}}, \gamma$ )<sup>156</sup>Gd reaction.

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The  $e^+e^-$  pair spectrum corresponding to the 1040.4-keV line, collected over 8 days, is shown in the lower part of Fig. 4. In the spectrum one observes a weak line at 18.2(2) keV, which is the double-escape peak corresponding to the  $E_{\gamma} =$ 1040.4-keV line. The spectrum is dominated by lines at 132.1 and 137.0 keV and a multiplet of lines in the energy region 161.9–165.2 keV. These lines originate from pair creation of the diffusely scattered transitions at 1154.1, 1159.0, and 1187.1 keV and the intense multiplet of transitions between 1183.9 and 1187.1 keV. Although the intensity of these lines is suppressed by more than a factor of 10 compared to the 1040.4-keV transition, their appearance in the pair spectrum is enhanced due the pair-creation cross section which is more than two orders of magnitude higher.

In the case of such weak effects, it is important to properly define the background level below the effect. We note that the shape of the background in Fig. 4 is similar to that seen in Fig. 3, being rather linear and slightly rising toward low energies with a cutoff at 12 keV. We therefore fitted the background with a linear function over the range 12–300 keV. A background level of 25 counts/channel, found near the 18.2-keV line, corresponds to the rate of 6 counts per day in a 1-keV wide energy bin, which is consistent with the background level seen in Fig. 3. Using this background fit we have obtained  $38 \pm 16$ ,  $295 \pm 17$ , and  $280 \pm 16$  counts in the 18.2-, 131.8-, and 137.0-keV peaks, respectively.

To compare our results to theoretical predictions a wellmeasured normalization point is needed. In the past highenergy points were preferred because of a higher cross section for pair production. In Ref. [5] the results were normalized at 2754 keV to the value  $\sigma_t/\sigma_{\rm BH} = 1.04$ , calculated at this energy in Refs. [19] and [20]. However, measurements at higher energies require corrections, which become essential for  $E_{\gamma} > 2$  MeV as discussed in Ref. [5], and introduce an

TABLE I. Results from the measurement of the pair production cross section in Ge, obtained in this work using the Ge-BGO pair spectrometer. The quoted errors of the single and coincidence rates are purely statistical. The error of the correction factor *C* in Eq. (1) contributes to the total error. The cross section data are in units of Bethe-Heitler normalized to 1.47 at  $E_{pair} = 164.0$  keV. See the text for further explanation.

$\frac{E_{\gamma}}{(\text{keV})}$	Single rate (s <sup>-1</sup> )	E <sub>pair</sub> (keV)	$N_{\rm pair}$ (counts)	С	$\sigma_t/\sigma_{ m BH}$
1040.4	285(5)	18.2(2)	38(16)	1.00(1)	3.7(16)
1066.0	445(9)	44.0(1)	182(16)	1.00(1)	2.22(20)
1154.1	17.2(5)	131.8(1)	295(17)	1.00(1)	1.57(10)
1159.0	13.9(5)	137.0(1)	280(15)	1.00(1)	1.66(11)
1180.3	150(3)	158.1(1)	300(19)	1.01(1)	1.55(10)
1186.0	1150(12)	164.0(1)	2450(50)	1.01(1)	1.47(4)
1230.8	290(9)	208.7(2)	135(13)	1.01(1)	1.32(13)
1242.5	250(8)	220.6(2)	115(12)	1.01(1)	1.14(13)
1263.5	340(12)	241.7(2)	232(17)	1.01(1)	1.31(11)
1277.5	370(14)	255.5(1)	301(18)	1.01(1)	1.34(10)
1323.4	170(4)	301.2(1)	530(24)	1.01(1)	1.33(7)
1517.4	71(3)	495.6(1)	1500(50)	1.01(1)	1.26(5)
1530.3	42(2)	508.5(1)	890(50)	1.01(1)	1.12(7)



FIG. 6. Results from the measurement of the pair production cross section in Ge as obtained in this work using Ge-BGO pair spectrometer. Data are in units of Bethe-Heitler normalized to calculations of [9] at ( $E_{pair} = 164.0$  keV). See the text for further explanation. The calculations are shown as a dotted line.

extra uncertainty. Therefore we decided to normalize our data at a rather low energy of  $E_{\gamma} = 1186.0$  keV, sufficiently close to other points to minimize corrections.

In Fig. 5 we show a pair spectrum corresponding to the  $E_{\gamma} = 1184.0$ -, 1186.0-, and 1187.1-keV lines from the<sup>155</sup>Gd( $n_{th},\gamma$ )<sup>156</sup>Gd reaction and the 1187.1-keV line from the <sup>157</sup>Gd( $n_{th},\gamma$ )<sup>158</sup>Gd reaction. The intensity-weighted mean energy of this quadruplet is 1186.0 keV and the corresponding pair line in Fig. 5 has been measured at 164.0 ± 0.1 keV. In the pair spectrum there is also a peak at 158.1 keV due to the  $E_{\gamma} = 1180.4$ - and 1180.3-keV lines from the same two reactions, respectively. This spectrum was collected over 13.5 h. The single rate in the 1186.0-keV quadruplet was 1133 counts per second.

We have also measured a few points above the 1186.0-keV line. All the data are summarized in Table I, where we show the results for  $\gamma$  lines used to measure the  $e^+e^-$  pair production

cross section in this Rapid Communication. The total cross section  $\sigma_t$  at an energy  $E_{\gamma}$  has been calculated from the formula

$$\sigma_t = \frac{N_{\text{pair}} \cdot t_{\text{single}} \cdot \epsilon_{\text{Ge}}}{N_{\text{single}} \cdot t_{\text{pair}} \cdot \epsilon_p} \times C, \qquad (1)$$

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where  $N_{\text{pair}}$  is the the number of counts in a double-escape peak at  $E_{\text{pair}} = E_{\gamma} - 1022 \text{ keV}$ ,  $t_{\text{pair}}$  is the measurement time,  $N_{\text{single}}$  is a number of counts in the photo peak at  $E_{\gamma}$ ,  $t_{\text{single}}$ is the acquisition time for the single measurement,  $\epsilon_{\text{Ge}}$  is the germanium detector efficiency at  $E_{\gamma}$  (in arbitrary units), and  $\epsilon_p$ is the germanium detector efficiency for collecting  $e^+e^-$  pair signals. We assumed after Ref. [5] that  $\epsilon_p = 1.0$ . This is well justified at low  $e^+e^-$  pair energies. However,  $\epsilon_p$  is lower than 1.0 at higher energies. These corrections were quantitatively investigated in Ref. [5] and are represented in formula (1) by factor *C*. This correction accounts for such effects as the escape of a positron or an electron from the Ge detector, multiple processes, and surface effects. As shown in Table I, the effect is negligible for  $\gamma$  energies below 1.5 MeV.

In the last column of Table I we show  $\sigma_t$  values in units of  $\sigma_{BH}$ , which are normalized to  $\sigma_t/\sigma_{BH} = 1.47$  at  $E_{pair} =$ 164.0 keV, a value calculated in Ref. [9]. These values are also displayed in Fig. 6 as a function of  $E_{pair}$ . In this figure we have sketched the calculations of Ref. [9] (dotted line), which is the standard calculation of Tseng and Prat [15,16], increased by a few-percent correction due to the  $e^+e^-$  final-state Coulomb interaction [9].

Our data points generally agree within experimental errors with the calculated trend of  $\sigma_t/\sigma_{\rm BH}$  for energies  $E_{\rm pair} > 130 \,\rm keV$ , where the pair production effect is quite well studied. We also observe an increase of the pair production cross section relative to BH predictions at low energies, in accordance with previous works [5,7–10]. The amplitude of this excess obtained in the present Rapid Communication agrees with previous measurements. There is no indication of any decrease of  $\sigma_t/\sigma_{\rm BH}$  below 60 keV.

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