High-spin yrast states in the *γ* **-soft nuclei 135Pr and 134Ce**

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High-spin states have been studied in $^{135}_{59}$ Pr, populated through the 116 Cd(²³Na,4*n*) reaction at 115 MeV, using the Gammasphere *γ* -ray spectrometer. The negative-parity yrast band has been significantly extended to spin ∼45¯*h* and excitation energy 21.5 MeV, showing evidence for several rotational alignments. The positive-parity yrast band of ¹³⁴₅₈Ce, populated through the *p4n* channel of this reaction, was also populated to spin ∼38*h* and excitation energy 18 MeV. Cranking calculations indicate that these nuclei are soft with respect to the triaxiality parameter γ and that several competing nuclear shapes occur at high spin.

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Nonaxial nuclear shapes, described by the triaxiality parameter γ in the polar representation of rotating quadrupole shapes [\[1\]](#page-3-0), are thought to become important for the 57La , 58Ce , $_{59}$ Pr, and $_{60}$ Nd isotopes beyond $A = 130$ that approach the $N = 82$ shell closure $[2-4]$. This nonaxial nuclear deformation is induced through core polarization by valence particles in anisotropic orbitals [\[5,6\]](#page-3-0). High-*j* particles from the bottom of a subshell prefer prolate nuclear shapes, while particles from the top of a subshell prefer an oblate shape [\[7\]](#page-3-0). The delicate interplay of such valence particles can therefore influence the overall shape of the nucleus, inducing triaxiality. Such is the case for these nuclei where valence protons occupy low-Ω orbitals from the bottom of the $πh_{11/2}$ subshell, while valence neutrons occupy high- Ω orbitals from the top of the *νh*_{11/2} subshell. Hence, this mass region provides an ideal environment for studying the shape-driving effects of specific valence particles. In addition, neutrons intruding from above the $N = 82$ shell gap can induce larger quadrupole deformation, *ε*₂.

The ideal triaxial shape is realized for $\gamma = \pm 30^{\circ}$ or -90° . In the Lund convention [\[1\]](#page-3-0), such shapes are equivalent with distinct "long," "short," and "intermediate" principal axes; it is only the axis about which the nucleus rotates that is different. For $\gamma = 30^{\circ}$, the nucleus rotates about the short axis; for $\gamma =$ −30◦, the nucleus rotates about the intermediate axis; and for

 $\gamma = -90^\circ$, it rotates about the long axis. The largest moment of inertia is achieved for $\gamma = 30^\circ$ and hence, classically, a triaxial body minimizes its energy for such motion. For the nucleus, however, which is a finite quantum system, strong coupling between collective and single-particle angular momenta can influence the way in which a triaxial nucleus rotates, or indeed precesses (wobbles) [\[8\]](#page-3-0).

This Brief Report presents new high-spin results for odd-*Z* ¹³⁵Pr, where the yrast band has been significantly extended to 45h; previous work on this nucleus can be found in Refs. $[9-11]$. In addition, the yrast band of 134 Ce $[12,13]$ has also been extended to spin approaching 40^h .

Experimental Details. High-spin states in 135Pr were populated with the $^{116}Cd(^{23}Na,4n\gamma)$ fusion-evaporation reaction. The experiment was performed at the Lawrence Berkeley National Laboratory, using a 115-MeV ²³Na beam supplied by the 88-inch cyclotron. This beam energy was chosen to primarily study the 134 Pr nucleus (5*n*) [\[14\]](#page-3-0). Two experiments were performed with different types of targets. A single, thin, self-supporting cadmium target of nominal thickness 1.2 mg*/*cm2 was used to study high-spin states, while a 1.0-mg*/*cm2 cadmium target on a thick lead backing of 15 mg*/*cm2 was used to allow lifetime measurements through the Doppler-shift attenuation method (DSAM) [\[15\]](#page-3-0). The Gammasphere *γ* -ray spectrometer [\[16,17\]](#page-3-0), containing 99 HPGe detectors, was used to record Compton-suppressed *γ* -ray coincidence events. Approximately 6×10^8 such events, of average *γ* -ray fold 4.5, were collected with the thin target, while 1.5×10^9 were collected with the backed target.

In order to provide channel selection, the bismuth germanate (BGO) anti-Compton shield elements of Gammasphere were used as a *γ* -ray fold and sum-energy selection device [\[18\]](#page-3-0). By removing the Hevimet collimators from the front of the HPGe detectors, the front of the BGO suppression shields were exposed, allowing *γ* rays to strike the shield elements directly. The number of BGO elements firing and

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their total summed energy were recorded for each event and added to the HPGe information off-line to provide fold *k* and sum-energy *H* information. High $k - H$ values, that retained [∼]60% (3*.*⁷ [×] 108 events) of the original data, were subsequently used in the off-line analysis to enhance the four-particle 135Pr channel relative to competing five-particle $(134\text{Pr}, 134\text{Ce})$ and six-particle (133Pr) channels. After this selection, the ratio of 135 Pr to 134 Pr was approximately doubled to 40% in the retained data.

Experimental Results. The thin-target data set was further unfolded into constituent triple (γ^3) coincidence events and re-played into a RADWARE-format [\[19\]](#page-3-0) cube containing 1.8×10^9 events. In order to establish the multipolarity of the transitions, angular-intensity ratios were measured. The negative-parity yrast band of 135Pr, obtained from this work, is shown in Fig. 1, while a double-gated *γ* -ray spectrum is presented in Fig. [2\(a\).](#page-2-0) The $I^{\pi} = 11/2^{-}$ state of ¹³⁵Pr is isomeric, with a half-life of 105 μ s [\[20\]](#page-3-0), and lies 317 keV above the $I^{\pi} = 3/2^{+}$ ground state $[21]$. The negative-parity yrast band of 135 Pr has been established firmly up to $I^{\pi} = 67/2^-$ and also extended to $I^{\pi} = (91/2^-)$. The work of Ref. [\[9\]](#page-3-0) is confirmed up to $I^{\pi} = 47/2^-$, with eleven new transitions placed above this state. In addition, positive-parity sidebands [\[9\]](#page-3-0) were confirmed up to $I^{\pi} = 39/2^{+}$ and $41/2^{+}$, but could not be significantly extended.

The yrast band of $134Ce$ has also been extended by two transitions, to a tentative spin and parity $I^{\pi} = (38^{+})$, from the present data; the level scheme is included in Fig. 1 and a double-gated spectrum is included in Fig. $2(b)$, where it can be seen that the topmost transitions have energies just above the previously known 1323 keV (34⁺ \rightarrow 32⁺) transition [\[12,13\]](#page-3-0). In addition, a weakly populated "superdeformed" band [\[22\]](#page-3-0) has been confirmed in ¹³⁴Ce.

Discussion. Experimental data for negative-parity yrast bands in odd-Pr isotopes are presented in Fig. [3](#page-2-0) in terms of total aligned angular momentum I_x and alignment i_x [\[23\]](#page-3-0), plotted as a function of rotational frequency, $\omega \approx E_y/2\hbar$. The yrast band of 134 Ce is also included in Fig. [3\(a\).](#page-2-0) In Fig. [3\(b\),](#page-2-0) a rotational reference, based on a configuration with a variable moment of inertia, $\mathcal{J}_{ref} = \mathcal{J}_0 + \omega^2 \mathcal{J}_1$, has been subtracted in each case, with Harris parameters [\[24\]](#page-3-0) $\mathcal{J}_0 = 17.0 \hbar^2 \text{ MeV}^{-1}$ and $\mathcal{J}_1 = 25.8 \hbar^4 \text{ MeV}^{-3}$ obtained from the S band of ¹³⁰Ce [\[25\]](#page-3-0). The data include ¹²⁵Pr $[26]$, ¹²⁷Pr $[27]$, ¹²⁹Pr $[28]$, ¹³¹Pr $[29]$, 133 Pr [\[30\]](#page-3-0), and 137 Pr [\[31\]](#page-3-0), in addition to the new results for $^{135}Pr.$

The yrast band of ¹³⁵Pr contains an odd- $h_{11/2}$ proton and hence the lowest-frequency alignment of *h*11*/*² quasiprotons is blocked. The large increase in i_x of 15h around $\omega =$ 0.45 MeV/ \hbar in ¹³⁵Pr, as shown in Fig. [3\(b\),](#page-2-0) then suggests the rotational alignment of both the second and third $h_{11/2}$ quasiprotons and the first pair of *h*11*/*² quasineutrons. The simultaneous alignment of $\pi h_{11/2}$ and $v h_{11/2}$ quasiparticles has also been observed in 139 Pm [\[32\]](#page-3-0). In 134 Ce, the first pair of $h_{11/2}$ quasiprotons (blocked in 135 Pr) align at the lower frequency of $\omega \sim 0.35$ MeV/ \hbar , with a sharp backbend, while the first pair of $h_{11/2}$ quasineutrons align at $\omega \sim 0.48 \text{ MeV}/\hbar$; see Fig. [3\(a\).](#page-2-0) An increase in i_x around $\omega = 0.65$ MeV/h suggests further structural changes in both 135 Pr and 134 Ce.

FIG. 1. (Color online) Partial level schemes deduced for 135Pr and ¹³⁴Ce from the present experiment. Transition energies are given in keV and are accurate to ± 0.3 keV, except those quoted as integers, which are accurate to ± 1 keV. The transitions above $I^{\pi} = 47/2^-$ in ¹³⁵Pr and 34⁺ in ¹³⁴Ce, labeled in red, are new.

Theoretical calculations for 135Pr have been performed in the framework of the configuration-dependent cranked Nilsson-Strutinsky (CNS) formalism without pairing [\[33,34\]](#page-3-0). Results have been previously discussed for the mass 130 region

FIG. 2. (Color online) Coincident *γ* -ray spectra for (a) the $(\pi, \alpha) = (-, -1/2)$ yrast band in ¹³⁵Pr, and (b) the $(+, 0)$ yrast band in 134Ce. Band members are labeled by their energies in keV, with new transitions denoted by diamonds. Contaminants from 135Pr are labeled by "C" in (b).

in Ref. [\[35\]](#page-3-0). In the present calculations, we have used the updated formalism presented in Ref. $[36]$ with $A = 150 \kappa$ and μ parameters [\[37\]](#page-3-0) defining the **l s** and **l**² strengths of the modified oscillator potential. Theoretical configurations are labeled, in relation to the 132 Sn core, using the shorthand notation $[p_1, n_1(n_2n_3)]$. Here p_1 represents the number of $\pi h_{11/2}$ particles, relative to $Z = 50$, and n_1 represents the number of $v h_{11/2}$ holes, relative to $N = 82$. The numbers in parentheses are only labeled if nonzero, with n_2 and n_3 being, respectively, the number of neutrons in the *νh*9*/*2*/f*7*/*² and *intruder orbitals from above the spherical* $N = 82$ *gap.* The generalized configurations for structures in 135 Pr may be written in full as

$$
\pi (d_{5/2}/g_{7/2})^{9-p_1} (h_{11/2})^{p_1}
$$

\n
$$
\otimes v (d_{3/2}/s_{1/2})^{-(6+n_2+n_3-n_1)} (h_{11/2})^{-n_1}
$$

\n
$$
\otimes v (h_{9/2}/f_{7/2})^{n_2} (i_{13/2})^{n_3}.
$$

Note also that with many neutron holes in the $N_{\text{osc}} = 4$ orbitals, some of them might be placed in orbitals of $d_{5/2}/g_{7/2}$ character.

Potential-energy surfaces for configurations with parity and signature $(\pi, \alpha) = (-, -1/2)$, fixed according to the observed high-spin band in 135 Pr, are presented for spin values $I = 75/2$ and $I = 91/2$ in Fig. 4. At the lower-spin value [see Fig. $4(a)$], the energy minimum is found at a small deformation, $\varepsilon_2 \approx 0.12$, $\gamma \approx -35^\circ$, corresponding to the valence-space configuration [3,4] with a maximum spin value, $I_{\text{max}} = 43.5$. Doppler-broadened line-shape analysis for the 1149-keV (63*/*2[−] → 59*/*2−) and 1227-keV (67*/*2[−] →

FIG. 3. (Color online) (a) Total aligned angular momentum *Ix* as a function of rotational frequency ω for the yrast bands in ¹³⁵Pr and ¹³⁴Ce. (b) Experimental alignment i_x as a function of rotational frequency *ω* for the negative-parity yrast bands in odd-*A* Pr isotopes.

63/2⁻) transitions in ¹³⁵Pr (see Fig. [1\)](#page-1-0) yields $Q_t \approx 3.3$ eb [\[11\]](#page-3-0), compatible with this predicted triaxial shape. Around 0.5-MeV higher in energy, however, is a secondary minimum at $\varepsilon_2 \approx 0.27$, $\gamma \approx 15^\circ$, where configurations of the type [3,4(21)] and [3,4(22)] are calculated low in energy.

FIG. 4. (Color online) Calculated potential-energy surfaces for negative-parity configurations at spin values $I = 75/2$ and $91/2$ (signature, $\alpha = -1/2$). The contour line separation is 0.25 MeV.

At $I = 91/2$ [see Fig. [4\(b\)\]](#page-2-0), corresponding to the highest spin value in the observed band, the larger-deformation minimum is calculated lowest in energy. Even though there is a close-to-spherical minimum, which comes only approximately 0.5-MeV higher in energy, it appears impossible that configurations corresponding to this shape could be assigned to the observed band, considering, for example, the *I*max value of the [3,4] configuration. Indeed, the high-spin range of the observed band is well described by the core-excited configurations specified above with three or four neutrons excited across the $N = 82$ gap. Similar configurations with one $i_{13/2}$ neutron have been assigned to triaxial bands in nearby 60Nd isotopes; see Refs. [38,39]. An axial prolate minimum, with $\varepsilon_2 \approx 0.4$,

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 $\gamma \approx 0^{\circ}$, also becomes competitive at the highest spins; see Fig. [4\(b\).](#page-2-0) This corresponds to the superdeformed shape (3:2 axes ratio) seen in 132 Ce and, indeed, 134 Ce [22].

In summary, the yrast band of 135Pr has been significantly extended to over $40\hbar$, and that of ¹³⁴Ce to 38 \hbar . Cranking calculations suggest that these nuclei are γ soft and that configurations involving high-*j* orbitals intruding from above $N = 82$ are favored at the highest spins.

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