High-spin yrast states in the γ -soft nuclei ¹³⁵Pr and ¹³⁴Ce

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(Received 15 June 2011; published 17 October 2011)

High-spin states have been studied in ${}^{135}_{59}$ Pr, populated through the 116 Cd(23 Na,4n) reaction at 115 MeV, using the Gammasphere γ -ray spectrometer. The negative-parity yrast band has been significantly extended to spin \sim 45 \hbar and excitation energy 21.5 MeV, showing evidence for several rotational alignments. The positive-parity yrast band of ${}^{134}_{58}$ Ce, populated through the p4n channel of this reaction, was also populated to spin \sim 38 \hbar and excitation energy 18 MeV. Cranking calculations indicate that these nuclei are soft with respect to the triaxiality parameter γ and that several competing nuclear shapes occur at high spin.

DOI: 10.1103/PhysRevC.84.047302

PACS number(s): 27.60.+j, 23.20.Lv, 21.10.Re

Nonaxial nuclear shapes, described by the triaxiality parameter γ in the polar representation of rotating quadrupole shapes [1], are thought to become important for the ${}_{57}$ La, ${}_{58}$ Ce, ₅₉Pr, and ₆₀Nd isotopes beyond A = 130 that approach the N = 82 shell closure [2–4]. This nonaxial nuclear deformation is induced through core polarization by valence particles in anisotropic orbitals [5,6]. High-*i* particles from the bottom of a subshell prefer prolate nuclear shapes, while particles from the top of a subshell prefer an oblate shape [7]. The delicate interplay of such valence particles can therefore influence the overall shape of the nucleus, inducing triaxiality. Such is the case for these nuclei where valence protons occupy low- Ω orbitals from the bottom of the $\pi h_{11/2}$ subshell, while valence neutrons occupy high- Ω orbitals from the top of the $vh_{11/2}$ subshell. Hence, this mass region provides an ideal environment for studying the shape-driving effects of specific valence particles. In addition, neutrons intruding from above the N = 82 shell gap can induce larger quadrupole deformation, ε_2 .

The ideal triaxial shape is realized for $\gamma = \pm 30^{\circ}$ or -90° . In the Lund convention [1], such shapes are equivalent with distinct "long," "short," and "intermediate" principal axes; it is only the axis about which the nucleus rotates that is different. For $\gamma = 30^{\circ}$, the nucleus rotates about the short axis; for $\gamma = -30^{\circ}$, the nucleus rotates about the intermediate axis; and for $\gamma = -90^{\circ}$, it rotates about the long axis. The largest moment of inertia is achieved for $\gamma = 30^{\circ}$ and hence, classically, a triaxial body minimizes its energy for such motion. For the nucleus, however, which is a finite quantum system, strong coupling between collective and single-particle angular momenta can influence the way in which a triaxial nucleus rotates, or indeed precesses (wobbles) [8].

This Brief Report presents new high-spin results for odd-Z 135 Pr, where the yrast band has been significantly extended to $45\hbar$; previous work on this nucleus can be found in Refs. [9–11]. In addition, the yrast band of 134 Ce [12,13] has also been extended to spin approaching $40\hbar$.

Experimental Details. High-spin states in ¹³⁵Pr were populated with the ¹¹⁶Cd(²³Na, $4n\gamma$) fusion-evaporation reaction. The experiment was performed at the Lawrence Berkeley National Laboratory, using a 115-MeV ²³Na beam supplied by the 88-inch cyclotron. This beam energy was chosen to primarily study the 134 Pr nucleus (5*n*) [14]. Two experiments were performed with different types of targets. A single, thin, self-supporting cadmium target of nominal thickness 1.2 mg/cm² was used to study high-spin states, while a 1.0-mg/cm² cadmium target on a thick lead backing of 15 mg/cm^2 was used to allow lifetime measurements through the Doppler-shift attenuation method (DSAM) [15]. The Gammasphere γ -ray spectrometer [16,17], containing 99 HPGe detectors, was used to record Compton-suppressed γ -ray coincidence events. Approximately 6×10^8 such events, of average γ -ray fold 4.5, were collected with the thin target, while 1.5×10^9 were collected with the backed target.

In order to provide channel selection, the bismuth germanate (BGO) anti-Compton shield elements of Gammasphere were used as a γ -ray fold and sum-energy selection device [18]. By removing the Hevimet collimators from the front of the HPGe detectors, the front of the BGO suppression shields were exposed, allowing γ rays to strike the shield elements directly. The number of BGO elements firing and

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their total summed energy were recorded for each event and added to the HPGe information off-line to provide fold k and sum-energy H information. High k - H values, that retained ~60% (3.7 × 10⁸ events) of the original data, were subsequently used in the off-line analysis to enhance the four-particle ¹³⁵Pr channel relative to competing five-particle (¹³⁴Pr, ¹³⁴Ce) and six-particle (¹³³Pr) channels. After this selection, the ratio of ¹³⁵Pr to ¹³⁴Pr was approximately doubled to 40% in the retained data.

Experimental Results. The thin-target data set was further unfolded into constituent triple (γ^3) coincidence events and replayed into a RADWARE-format [19] cube containing 1.8×10^9 events. In order to establish the multipolarity of the transitions, angular-intensity ratios were measured. The negative-parity yrast band of ¹³⁵Pr, obtained from this work, is shown in Fig. 1, while a double-gated γ -ray spectrum is presented in Fig. 2(a). The $I^{\pi} = 11/2^{-1}$ state of ¹³⁵Pr is isomeric, with a half-life of 105 μ s [20], and lies 317 keV above the $I^{\pi} = 3/2^+$ ground state [21]. The negative-parity yrast band of ¹³⁵Pr has been established firmly up to $I^{\pi} = 67/2^{-}$ and also extended to $I^{\pi} = (91/2^{-})$. The work of Ref. [9] is confirmed up to $I^{\pi} = 47/2^{-}$, with eleven new transitions placed above this state. In addition, positive-parity sidebands [9] were confirmed up to $I^{\pi} = 39/2^+$ and $41/2^+$, but could not be significantly extended.

The yrast band of ¹³⁴Ce has also been extended by two transitions, to a tentative spin and parity $I^{\pi} = (38^+)$, from the present data; the level scheme is included in Fig. 1 and a double-gated spectrum is included in Fig. 2(b), where it can be seen that the topmost transitions have energies just above the previously known 1323 keV ($34^+ \rightarrow 32^+$) transition [12,13]. In addition, a weakly populated "superdeformed" band [22] has been confirmed in ¹³⁴Ce.

Discussion. Experimental data for negative-parity yrast bands in odd-Pr isotopes are presented in Fig. 3 in terms of total aligned angular momentum I_x and alignment i_x [23], plotted as a function of rotational frequency, $\omega \approx E_{\gamma}/2\hbar$. The yrast band of ¹³⁴Ce is also included in Fig. 3(a). In Fig. 3(b), a rotational reference, based on a configuration with a variable moment of inertia, $\mathcal{J}_{ref} = \mathcal{J}_0 + \omega^2 \mathcal{J}_1$, has been subtracted in each case, with Harris parameters [24] $\mathcal{J}_0 = 17.0\hbar^2$ MeV⁻¹ and $\mathcal{J}_1 = 25.8\hbar^4$ MeV⁻³ obtained from the S band of ¹³⁰Ce [25]. The data include ¹²⁵Pr [26], ¹²⁷Pr [27], ¹²⁹Pr [28], ¹³¹Pr [29], ¹³³Pr [30], and ¹³⁷Pr [31], in addition to the new results for ¹³⁵Pr.

The yrast band of ¹³⁵Pr contains an odd- $h_{11/2}$ proton and hence the lowest-frequency alignment of $h_{11/2}$ quasiprotons is blocked. The large increase in i_x of 15 \hbar around $\omega =$ 0.45 MeV/ \hbar in ¹³⁵Pr, as shown in Fig. 3(b), then suggests the rotational alignment of both the second and third $h_{11/2}$ quasiprotons and the first pair of $h_{11/2}$ quasineutrons. The simultaneous alignment of $\pi h_{11/2}$ and $\nu h_{11/2}$ quasiparticles has also been observed in ¹³⁹Pm [32]. In ¹³⁴Ce, the first pair of $h_{11/2}$ quasiprotons (blocked in ¹³⁵Pr) align at the lower frequency of $\omega \sim 0.35$ MeV/ \hbar , with a sharp backbend, while the first pair of $h_{11/2}$ quasineutrons align at $\omega \sim 0.48$ MeV/ \hbar ; see Fig. 3(a). An increase in i_x around $\omega = 0.65$ MeV/ \hbar suggests further structural changes in both ¹³⁵Pr and ¹³⁴Ce.



FIG. 1. (Color online) Partial level schemes deduced for ¹³⁵Pr and ¹³⁴Ce from the present experiment. Transition energies are given in keV and are accurate to ± 0.3 keV, except those quoted as integers, which are accurate to ± 1 keV. The transitions above $I^{\pi} = 47/2^{-}$ in ¹³⁵Pr and 34^{+} in ¹³⁴Ce, labeled in red, are new.

Theoretical calculations for ¹³⁵Pr have been performed in the framework of the configuration-dependent cranked Nilsson-Strutinsky (CNS) formalism without pairing [33,34]. Results have been previously discussed for the mass 130 region



FIG. 2. (Color online) Coincident γ -ray spectra for (a) the $(\pi, \alpha) = (-, -1/2)$ yrast band in ¹³⁵Pr, and (b) the (+, 0) yrast band in ¹³⁴Ce. Band members are labeled by their energies in keV, with new transitions denoted by diamonds. Contaminants from ¹³⁵Pr are labeled by "C" in (b).

in Ref. [35]. In the present calculations, we have used the updated formalism presented in Ref. [36] with $A = 150 \kappa$ and μ parameters [37] defining the l·s and l² strengths of the modified oscillator potential. Theoretical configurations are labeled, in relation to the ¹³²Sn core, using the shorthand notation $[p_1, n_1(n_2n_3)]$. Here p_1 represents the number of $\pi h_{11/2}$ particles, relative to Z = 50, and n_1 represents the number of $\nu h_{11/2}$ holes, relative to N = 82. The numbers in parentheses are only labeled if nonzero, with n_2 and n_3 being, respectively, the number of neutrons in the $\nu h_{9/2}/f_{7/2}$ and $\nu i_{13/2}$ intruder orbitals from above the spherical N = 82 gap. The generalized configurations for structures in ¹³⁵Pr may be written in full as

$$\pi (d_{5/2}/g_{7/2})^{9-p_1} (h_{11/2})^{p_1} \\ \otimes \nu (d_{3/2}/s_{1/2})^{-(6+n_2+n_3-n_1)} (h_{11/2})^{-n} \\ \otimes \nu (h_{9/2}/f_{7/2})^{n_2} (i_{13/2})^{n_3}.$$

Note also that with many neutron holes in the $N_{\text{osc}} = 4$ orbitals, some of them might be placed in orbitals of $d_{5/2}/g_{7/2}$ character.

Potential-energy surfaces for configurations with parity and signature $(\pi, \alpha) = (-, -1/2)$, fixed according to the observed high-spin band in ¹³⁵Pr, are presented for spin values I = 75/2 and I = 91/2 in Fig. 4. At the lower-spin value [see Fig. 4(a)], the energy minimum is found at a small deformation, $\varepsilon_2 \approx 0.12$, $\gamma \approx -35^\circ$, corresponding to the valence-space configuration [3,4] with a maximum spin value, $I_{\text{max}} = 43.5$. Doppler-broadened line-shape analysis for the 1149-keV ($63/2^- \rightarrow 59/2^-$) and 1227-keV ($67/2^- \rightarrow$



FIG. 3. (Color online) (a) Total aligned angular momentum I_x as a function of rotational frequency ω for the yrast bands in ¹³⁵Pr and ¹³⁴Ce. (b) Experimental alignment i_x as a function of rotational frequency ω for the negative-parity yrast bands in odd-*A* Pr isotopes.

63/2⁻) transitions in ¹³⁵Pr (see Fig. 1) yields $Q_t \approx 3.3$ eb [11], compatible with this predicted triaxial shape. Around 0.5-MeV higher in energy, however, is a secondary minimum at $\varepsilon_2 \approx 0.27$, $\gamma \approx 15^\circ$, where configurations of the type [3,4(21)] and [3,4(22)] are calculated low in energy.



FIG. 4. (Color online) Calculated potential-energy surfaces for negative-parity configurations at spin values I = 75/2 and 91/2 (signature, $\alpha = -1/2$). The contour line separation is 0.25 MeV.

At I = 91/2 [see Fig. 4(b)], corresponding to the highest spin value in the observed band, the larger-deformation minimum is calculated lowest in energy. Even though there is a close-to-spherical minimum, which comes only approximately 0.5-MeV higher in energy, it appears impossible that configurations corresponding to this shape could be assigned to the observed band, considering, for example, the I_{max} value of the [3,4] configuration. Indeed, the high-spin range of the observed band is well described by the core-excited configurations specified above with three or four neutrons excited across the N = 82 gap. Similar configurations with one $i_{13/2}$ neutron have been assigned to triaxial bands in nearby ₆₀Nd isotopes; see Refs. [38,39]. An axial prolate minimum, with $\varepsilon_2 \approx 0.4$,

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- $\gamma \approx 0^{\circ}$, also becomes competitive at the highest spins; see Fig. 4(b). This corresponds to the superdeformed shape (3:2 axes ratio) seen in ¹³²Ce and, indeed, ¹³⁴Ce [22]. In summary, the yrast band of ¹³⁵Pr has been significantly

In summary, the yrast band of ¹³⁵Pr has been significantly extended to over 40 \hbar , and that of ¹³⁴Ce to 38 \hbar . Cranking calculations suggest that these nuclei are γ soft and that configurations involving high-*j* orbitals intruding from above N = 82 are favored at the highest spins.

This work was supported in part by the UK Engineering and Physical Sciences Research Council, the US National Science Foundation, the US Department of Energy, and the Swedish Science Research Council.

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