

Landau parameters for asymmetric nuclear matter with a strong magnetic field

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The Landau Fermi-liquid parameters are calculated for charge-neutral asymmetric nuclear matter in beta equilibrium at zero temperature in the presence of a very strong magnetic field with relativistic mean-field models. Due to the isospin structure of the system, with different populations of protons and neutrons and spin alignment to the field, we find nonvanishing Landau mixing parameters. The existence of quantized Landau levels for the charged sector has some impact on the Landau parameters with the presence of discretized features in those involving the proton sector. Using the Fermi-liquid formalism singlet and triplet excited quasiparticle states are analyzed, and we find that in-medium effects and magnetic fields are competing; however, the former are more important in the interaction energy range considered. It is found that for magnetic field strengths $\log_{10} B(G) \leq 17$ the relatively low polarization of the system produces mild changes in the generalized Landau parameters with respect to the unmagnetized case, while for larger strengths there is a resolution of the degeneracy of the interaction energies of quasiparticles in the system. As an application we calculate the incompressibility and first sound velocities to illustrate how this formalism can be used to obtain physical information from the system.

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I. INTRODUCTION

Asymmetric nuclear matter is presently an important topic proposed for study in experiments at radioactive beam facilities such as FAIR [1] at GSI, SPIRAL2 [2] at GANIL, ISAC-III at TRIUMF [3], or FRIB [4] at MSU, among others [5]. These will allow the investigation of nuclei in regions of the nuclear chart far from the stability line. These nuclear regions, where the isospin asymmetry ratio, given by the ratio of the proton vector number density, n_p^v , to baryonic vector number density, n_B , defined as $Y_p = n_p^v/n_B$, largely departs from the 0.5 value, are of interest in the study of stability of exotic nuclei as in the neutron rich nuclei. Recently studied examples of those are the isotopes of Ca [6] with $Z = 20$ or the isotones $N = 30$ of Ca and Sc with CLARA experiment [7] in Legnaro. The study of observables related to nuclei far from the isospin stability line allows for an improved description of neutron-rich environments as those of interest in astrophysical scenarios of neutron star matter. They are relevant to interiors of compact stellar objects such as neutron stars (NSs) arising in the aftermath of a supernova event. In this context, the rapid deleptonization of the NS following the gravitational collapse of the inner regions with the proton-electron capture process makes matter more neutron rich since (anti) neutrinos diffuse out of the star [8,9]. Also supernova matter can be considered to be constituted by a set of nuclei, in the nuclear statistical equilibrium (NSE) approximation, and presents a distribution of nuclei shifted to the $N > Z$ region [10,11].

On Earth, the high temperature (T) and small baryonic chemical potential (μ_B) region of matter phase space has

been somewhat tested [12] at RHIC, and it will be possible to further test with other heavy-ion experiments such as ALICE [13] at CERN. Although this improvement of our knowledge of the phases of matter is certainly valuable, the phase diagram of nuclear matter relevant for the equation of state (EOS) of NSs is that of *cold* asymmetric nuclear matter in the low-temperature range.

Historically, most of the existing literature on the nuclear matter EOS characterization [14] has neglected the input from external fields; in particular, in the case of magnetic fields, this is claimed due to the tiny value of the nuclear magnetic moment [15]. On terrestrial experiments recent estimations of the magnetic field strength that could be produced dynamically at CERN or BNL energies [16] are on the order of $B \approx 10^{17}$ – 10^{19} G. In nature, we have another indication of sources of intense magnetic fields of astrophysical origin such as magnetars [17]. Magnetars are neutron stars which may have *surface* magnetic fields $B \approx 10^{15}$ G [18–20] discovered in the x-ray and γ -ray electromagnetic spectrum (for a review see [21]). They are identified with the anomalous x-ray pulsars (AXPs) and soft γ -ray repeaters [22].

Taking as reference the critical field B_c^e at which the electron cyclotron energy is equal to the electron mass, $B_c^e = 4.414 \times 10^{13}$ G, we define $B^* = B/B_c^e$. It has been shown by several authors [23–26] that magnetic fields larger than $B^* = 10^5$ will affect the EOS of compact stars. In particular, field-theoretical descriptions based on several parametrizations of the nonlinear Walecka model (NLWM) [27] show an overall similar behavior. According to the scalar virial theorem [28] the *interior* magnetic field strength could be as large as $B = 10^{18}$ G so, in principle, this is the maximum field strength that is meaningful to consider.

In 1959 the formal theory for treatment of low-temperature (non-superfluid) fermion systems, known as normal *Fermi liquids*, was developed by Landau [29,30] to describe the

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behavior of ${}^3\text{He}$ below 100 mK. With this Fermi-liquid theory (FLT) (see a recent reference [31]) the excited states in the system could be described as quasiparticles (qp's) as long as these states have sufficiently long lifetimes. At low temperature, the small excitation energy (compared to the chemical potential) will assure this fact. In the context of the FLT, the so-called *Landau parameters* can parametrize the interaction energy between a pair of qp's in the medium. Previous works have attempted to partially study the behavior of Landau parameters for nonmagnetized symmetric nuclear matter or neutron matter [32–34] or in magnetized matter under the presence of a magnetic field either in a nonrelativistic formalism [35–37] or in magnetized matter without considering the B field including exchange terms in a relativistic way [38].

In this work we will be interested in calculating the Landau Fermi-liquid parameters for an isospin asymmetric nuclear system in beta equilibrium and in charge neutrality under the effect of an intense magnetic field. The FLT used in this case must describe relativistically the more general condition of a magnetized nonpure isospin system to account for the fact that the intense magnetic field can modify isospin populations and partially align nucleon magnetic moments with respect to the case of vanishing magnetic field. In addition, the different dynamics of proton and neutron sectors under the presence of a magnetic field (including the existence of anomalous nucleon magnetic moments) will have effects in the Landau parameter computation showing discretized or continuous features for protons and neutrons, respectively. In Sec. II we introduce the relativistic Lagrangian model used in this work and the generalized formalism of the FLT for charge-neutral isospin asymmetric hadronic systems under the presence of a magnetic field. In Sec. III we discuss the explicit form of the matricial structure of the coefficients describing the interaction of qp's in the magnetized system through the Landau parameters. In Sec. IV we analyze the obtained coefficients for either individual spin quantum numbers or total spin (singlet or triplet) for the qp excitations for the electrically neutral system configurations calculated under beta equilibrium and the Landau parameter behavior in presence of a strong magnetic field. We apply those coefficients to the derivation of the incompressibility and first sound velocity as an example of this useful formalism to extract information on the dense and magnetized medium. Finally, in Sec. V, we summarize and draw some conclusions.

II. THE FORMALISM

For the description of the EOS of neutron star matter, we employ a relativistic field-theoretical approach in which the baryons, neutrons (n), and protons (p) interact via the exchange of $\sigma - \omega - \rho$ mesons in the presence of a uniform magnetic field B along the z axis. The Lagrangian density for the TM1 parametrization [39] of the nonlinear Walecka model (NLWM) reads [27]

$$\mathcal{L} = \sum_{b=n,p} \mathcal{L}_b + \sum_{m=\sigma,\omega,\rho} \mathcal{L}_m + \sum_{l=e} \mathcal{L}_l. \quad (1)$$

The baryon ($b = n, p$), meson ($m = \sigma, \omega$ and ρ), and lepton ($l = e$) Lagrangians are given by ($c = \hbar = 1$)

$$\mathcal{L}_b = \bar{\Psi}_b \left(i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - m_b + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \tau_{3b} \gamma_\mu \rho^\mu - \frac{1}{2} \mu_{N\kappa b} \sigma_{\mu\nu} F^{\mu\nu} \right) \Psi_b, \quad (2)$$

$$\mathcal{L}_m = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3!} \kappa \sigma^3 - \frac{1}{4!} \lambda \sigma^4 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4!} \xi g_\omega^4 (\omega_\mu \omega^\mu)^2 - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} P^{\mu\nu} P_{\mu\nu}, \quad (3)$$

$$\mathcal{L}_l = \bar{\Psi}_l (i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l) \Psi_l, \quad (4)$$

where Ψ_b, Ψ_l are the baryon and lepton Dirac fields, respectively.

The nucleon isospin z projection for the proton (neutron) is denoted by $\tau_{3p} = 1$ ($\tau_{3n} = -1$). The nucleon mass is m_b ($m_b = m_n = m_p = 938$ MeV), its charge is q_b , and the baryonic anomalous magnetic moments (AMMs) are introduced via the coupling to the electromagnetic field tensor with $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ and strength κ_b . In particular, $\kappa_n = -1.91315$ for the neutron and $\kappa_p = 1.79285$ for the proton. m_l and q_l are the mass and charge of the lepton. We will consider the simplest model where the leptonic sector is formed just by electrons ($l = e$), with no anomalous magnetic moment, providing charge neutrality in this astrophysical scenario. Despite there being a nonzero electron AMM its value [15] is tiny when compared to that in the hadronic sector and it was shown that this contribution is negligible for the magnetic fields of interest in astrophysics if properly introduced [40]. The mesonic and electromagnetic field strength tensors are given by their usual expressions: $\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $P_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The electromagnetic field is assumed to be externally generated (and thus has no associated field equation), and only frozen-field configurations will be considered in this work. To calculate the thermodynamic conditions in this charge-neutral asymmetric nuclear system in beta equilibrium with an intense magnetic field, isoscalar and isovector current conservation must be imposed. Explicitly, the following conditions are fulfilled: (i) electrical charge neutrality, (ii) conservation of baryonic charge, and (iii) mesonic field equation self-consistency. In addition, we will assume throughout this work that neutrinos scape freely and therefore there is no neutrino trapping.

The field equations of motion are determined from the Euler-Lagrange equations arising from the Lagrangian density in Eq. (1). Under the conditions of the present calculation, a relativistic mean field (RMF) approximation will be used so that the space-time varying fields are replaced by a homogeneous value, $\phi(x_\mu) \rightarrow \phi$. In this way, the Dirac equation for a nucleon is given by

$$\left[i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - (m_b - g_\sigma \sigma) - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \tau_{3b} \gamma_\mu \rho^\mu - \frac{1}{2} \mu_{N\kappa b} \sigma_{\mu\nu} F^{\mu\nu} \right] \Psi_b = 0, \quad (5)$$

where the effective baryon mass is $m^* = m_b - g_\sigma \sigma$. For leptons,

$$(i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l)\Psi_l = 0. \quad (6)$$

For meson fields we obtain

$$m_\sigma^2 \sigma + \frac{1}{2} \kappa \sigma^2 + \frac{1}{3!} \lambda \sigma^3 = g_\sigma (n_p^s + n_n^s) = g_\sigma n^s, \quad (7)$$

$$m_\omega^2 \omega^0 + \frac{1}{3!} \xi g_\omega^4 (\omega^{02} - \omega^2) \omega^0 = g_\omega (n_p^v + n_n^v) = g_\omega n_B, \quad (8)$$

$$m_\omega^2 \boldsymbol{\omega} + \frac{1}{3!} \xi g_\omega^4 (\omega^{02} - \omega^2) \boldsymbol{\omega} = g_\omega \mathbf{j}_B, \quad (9)$$

$$m_\rho^2 \rho^0 = \frac{1}{2} g_\rho (n_p^v - n_n^v) = \frac{1}{2} g_\rho n_3, \quad (10)$$

$$m_\rho^2 \boldsymbol{\rho} = \frac{1}{2} g_\rho \mathbf{j}_3, \quad (11)$$

where we use the notation as in the work of Matsui [34] and n_B is the baryonic (vector) particle number density constructed as the sum of (vector) particle number density of protons (n_p^v) and neutrons (n_n^v), $n_B = n_p^v + n_n^v$.

The baryon current $\mathbf{j}_B = \mathbf{j}_p + \mathbf{j}_n$ is also the sum of the proton (\mathbf{j}_p) and neutron (\mathbf{j}_n) currents. $n^s = n_p^s + n_n^s$ is the scalar density constructed from that of protons (n_p^s) and neutrons (n_n^s). $n_3 = n_p^v - n_n^v$ and $\mathbf{j}_3 = \mathbf{j}_p - \mathbf{j}_n$ are the isoscalar particle number density and isovector baryon current, respectively. When solving for equilibrium conditions in the nuclear system governed by Eqs. (5)–(11) we impose $\mathbf{j}_B = 0$ and $\mathbf{j}_3 = 0$.

In the present work we will discuss neutral β -equilibrium matter, and will solve self-consistently the following set of equations:

(i) Electrical charge neutrality

$$n_p^v = n_e^v. \quad (12)$$

(ii) Baryonic number conservation with

$$n_B = n_p^v + n_n^v. \quad (13)$$

(iii) β -equilibrium

$$\mu_n = \mu_p + \mu_e. \quad (14)$$

(iv) The equations for the mean mesonic fields, which read as follows: For the σ field,

$$g_\sigma \sigma = \frac{g_\sigma^2}{m_\sigma'^2} n^s, \quad (15)$$

with $m_\sigma'^2 = m_\sigma^2 + \frac{1}{2} \kappa \sigma + \frac{1}{3!} \lambda \sigma^2$. For the ω^0 field we get

$$g_\omega \omega^0 = \frac{g_\omega^2}{m_\omega'^2} n_B, \quad (16)$$

with $m_\omega'^2 = m_\omega^2 + \frac{1}{3!} \xi g_\omega^4 \omega^{02}$. For the ρ^0 field we get

$$m_\rho^2 \rho^0 = \frac{1}{2} g_\rho (n_p^v - n_n^v). \quad (17)$$

The explicit expressions for the chemical potentials in Eq. (14) are given by

$$\mu_p = E_F^p + g_\omega \omega^0 + \frac{1}{2} g_\rho \rho^0, \quad (18)$$

$$\mu_n = E_F^n + g_\omega \omega^0 - \frac{1}{2} g_\rho \rho^0, \quad (19)$$

where E_F^p and E_F^n are, respectively, the proton and neutron effective Fermi energies. For the electron we have $\mu_e = E_F^e$.

When the Dirac equation for nucleons Eq. (5) is solved, a magnetic field B in the z direction given by $\mathbf{B} = B \hat{\mathbf{k}}$ is used. The energies for the quasiprotons and quasineutrons in the medium with spin z projection, s , are given by the following expressions [23]:

$$\epsilon_{v,s}^p = \sqrt{k_z^2 + (\sqrt{m_p^{*2} + 2\nu q_p B - s\mu_N \kappa_p B})^2} + g_\omega \omega^0 + \frac{1}{2} g_\rho \rho^0, \quad (20)$$

$$\epsilon_s^n = \sqrt{k_z^2 + (\sqrt{m_n^{*2} + k_\perp^2 - s\mu_N \kappa_n B})^2} + g_\omega \omega^0 - \frac{1}{2} g_\rho \rho^0, \quad (21)$$

where $\nu = n + \frac{1}{2} - \text{sgn}(q_b) \frac{s}{2} = 0, 1, 2, \dots$ enumerates the quantized Landau levels for protons with electric charge q_p . The quantum number s is $+1$ for spin-up and -1 for spin-down quasiparticles. At the Fermi level $\epsilon_{v,s}^p = \mu_p$ and $\epsilon_s^n = \mu_n$.

Due to the fact that the magnetic field is taken in the z direction, it is useful to define three-momentum (\mathbf{k}) components along parallel (k_z) and perpendicular (k_\perp) directions. For neutrons, the surface of constant energy is an ellipsoid while, for protons, constant energy surfaces are formed by circumferences on nested cylinders with the radius labeled by the Landau level; see next section. In this work we are mainly interested in hadronic properties, and electron dynamics will be such that at high B field strengths, they will mostly be in the low Landau levels. For completeness we write the expressions of the scalar and vector densities for protons and neutrons for both spin polarizations as follows [23]:

$$\begin{aligned} n_p^s &= \frac{q_p B m_p^*}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} \sum_s \frac{\sqrt{m_p^{*2} + 2\nu q_p B - s\mu_N \kappa_p B}}{\sqrt{m_p^{*2} + 2\nu q_p B}} \\ &\quad \times \ln \left| \frac{k_{F,v,s}^p + E_F^p}{\sqrt{m_p^{*2} + 2\nu q_p B - s\mu_N \kappa_p B}} \right|, \\ n_n^s &= \frac{m_n^*}{4\pi^2} \sum_s \left[E_F^n k_{F,s}^n - \bar{m}_n^2 \ln \left| \frac{k_{F,s}^n + E_F^n}{\bar{m}_n} \right| \right], \\ n_p^v &= \frac{q_p B}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} \sum_s k_{F,v,s}^p = \sum_{\nu,s} n_{p\nu s}, \\ n_n^v &= \frac{1}{2\pi^2} \sum_s \left[\frac{1}{3} (k_{F,s}^n)^3 - \frac{1}{2} s \mu_N \kappa_n B \right. \\ &\quad \left. \times \left(\bar{m}_n k_{F,s}^n + E_F^n \left(\arcsin \left(\frac{\bar{m}_n}{E_F^n} \right) - \frac{\pi}{2} \right) \right) \right] = \sum_s n_{ns}, \end{aligned} \quad (22)$$

where $k_{F,v,s}^p$, $k_{F,s}^n$ are the Fermi momenta of protons and neutrons related to the proton and neutron effective Fermi

energies, E_F^p and E_F^n , respectively, by

$$k_{F,\nu,s}^{p2} = E_F^{p2} - [\sqrt{m_p^{*2} + 2\nu q_p B} - s\mu_N \kappa_p B]^2, \quad (23)$$

$$k_{F,\nu,s}^{n2} = E_F^{n2} - \bar{m}_n^2, \quad (24)$$

$\bar{m}_n = m_n^* - s\mu_N \kappa_n B \equiv \bar{m}_n^s$, and $n_{p\nu s}$ and $n_{n s}$ are the partial proton and neutron vector densities associated, respectively, with the ν proton Landau level with spin s and the neutron spin s level. The summation in ν in the above expressions terminates at ν_{\max} , the largest value of ν for which the square of Fermi momentum of the charged particle is still positive and corresponds to the closest integer from below defined by the ratio

$$\nu_{\max} = \left\lfloor \frac{(E_F^p + s\mu_N \kappa_p B)^2 - m_p^{*2}}{2|q_p| B} \right\rfloor. \quad (25)$$

For electrons $k_{F,\nu,s}^e = \sqrt{E_F^{e2} - (m_e^2 + 2\nu q_e B)}$ and ν_{\max} does not have the same value as that for protons.

III. LANDAU FERMI-LIQUID PARAMETERS

We now calculate the Landau parameters using a generalized formulation of the Landau theory of Fermi liquids [31] from the variations of the energy density of the system, \mathcal{E} , given by [32]

$$\begin{aligned} \mathcal{E} = & \frac{1}{2} \frac{g_\omega^2 m_\omega^2}{m_\omega^4} n_B^2 - \frac{1}{2} \frac{g_\omega^2 m_\omega^2}{m_\omega^4} j_B^2 + \frac{1}{4!} \frac{\xi g_\omega^8}{m_\omega^8} n_B^4 + \frac{1}{4!} \frac{\xi g_\omega^8}{m_\omega^8} j_B^4 \\ & - \frac{2}{4!} \frac{\xi g_\omega^8}{m_\omega^8} j_B^2 n_B^2 + \frac{1}{8} \frac{g_\rho^2}{m_\rho^2} n_3^2 - \frac{1}{8} \frac{g_\rho^2}{m_\rho^2} j_3^2 + \sum_{i,\nu,s} n_i^p E_{i\nu,s}^p \\ & + \sum_{i,s} n_i^n E_{is}^n + \sum_{i,\nu,s} n_i^l E_{i\nu,s}^l + \frac{1}{2} \frac{m^2}{g_\sigma^2} (m_b - m^*)^2 \\ & + \frac{1}{3!} \frac{\kappa}{g_\sigma^2} (m_b - m^*)^3 + \frac{1}{4!} \frac{\lambda}{g_\sigma^2} (m_b - m^*)^4 + \frac{B^2}{2}, \quad (26) \end{aligned}$$

where we have defined $n_i^p = n^p(k_i, \nu, s_i)$ as the occupation number of the quasiprotons and $n_i^n = n^n(k_i, s_i)$ for the quasineutrons. For leptons the occupation number is $n_i^l = n^l(k_i, \nu, s_i)$. Also we have defined the energies

$$E_{i\nu,s}^p = \sqrt{\mathcal{K}_{zi}^{p2} + (\sqrt{m_p^{*2} + 2\nu q_p B} - s\mu_N \kappa_p B)^2}, \quad (27)$$

$$E_{is}^n = \sqrt{\mathcal{K}_{zi}^{n2} + (\sqrt{m_n^{*2} + \mathcal{K}_{\perp i}^2} - s\mu_N \kappa_n B)^2}, \quad (28)$$

and analogously for electrons, $E_{i\nu,s}^l$. We use generalized three-momenta depending on isospin

$$\mathcal{K}_i^j = \mathbf{k}_i - \mathcal{V}_i^j, \quad j = p, n, \quad (29)$$

with $\mathcal{V}^p = g_\omega \boldsymbol{\omega} + \frac{1}{2} g_\rho \boldsymbol{\rho}$ and $\mathcal{V}^n = g_\omega \boldsymbol{\omega} - \frac{1}{2} g_\rho \boldsymbol{\rho}$. The equation for the nucleon effective mass can be written as

$$m^* = m_b - g_\sigma \sigma = m_b - \frac{g_\sigma^2}{m_\sigma'^2} (n_p^s + n_n^s), \quad (30)$$

where

$$n_n^s = \sum_{i,s} \frac{n_{is}^n m^*}{E_{is}^n} \left(1 - \frac{s\mu_N \kappa_n B}{\sqrt{m_n^{*2} + \mathcal{K}_{\perp i}^2}} \right), \quad (31)$$

$$n_p^s = \sum_{i,\nu,s} \frac{n_{i,\nu,s}^p m^*}{E_{i\nu,s}^p} \frac{\bar{m}_{\nu s}^p}{\bar{m}_{\nu s}^p + s\mu_N \kappa_p B}, \quad (32)$$

where the following definition has been used:

$$\bar{m}_{\nu s}^p = \sqrt{m^{*2} + 2q_p \nu B} - s\kappa_p \mu_N B = \tilde{\epsilon}_{\nu s}^p - s\kappa_p \mu_N B. \quad (33)$$

The vector current for quasiprotons is written as

$$\mathbf{j}_p = \sum_{i,\nu,s} \frac{\mathcal{K}_i^p n_i^p}{[\mathcal{K}_i^{2p} + (\bar{m}_{\nu s}^p)^2]^{1/2}}, \quad (34)$$

and for quasineutrons,

$$\mathbf{j}_n = \sum_{i,s} n_{is}^n \left[\frac{\mathcal{K}_{\perp i}^n}{E_{is}^n} \left(1 - \frac{s\mu_N \kappa_n B}{\sqrt{m_n^{*2} + \mathcal{K}_{\perp i}^2}} \right) + \frac{\mathcal{K}_{zi}^n}{E_{is}^n} \right], \quad (35)$$

so that \mathbf{j}_B and \mathbf{j}_3 can be constructed.

According to the FLT [31] the first variation of the energy density of the system, \mathcal{E} , with respect to the occupation number for qp's with isospin of j th type, n_i^j , defines the qp energy, ϵ_i^j . Let us notice that in reduced notation the index i means (i, ν, s) for quasiprotons and (i, s) for quasineutrons:

$$\delta \mathcal{E} = \sum_{i,j} \epsilon_i^j \delta n_i^j, \quad (36)$$

and

$$\epsilon_i^p = E_{i,\nu,s}^p + \frac{g_\omega^2}{m_\omega'^2} n_B + \frac{g_\rho^2}{4m_\rho^2} n_3, \quad (37)$$

$$\epsilon_i^n = E_{is}^n + \frac{g_\omega^2}{m_\omega'^2} n_B - \frac{g_\rho^2}{4m_\rho^2} n_3. \quad (38)$$

The single-quasiparticle energy has, each, two explicit contributions, one due to the motion under the influence of a strong quantizing magnetic field and another due to the motion in a medium with mesonic self-interacting fields. To calculate the Landau parameters we use the standard approach [31], but generalizing to the case when there are external fields. These will allow us to extract information on the interaction energies of quasiparticles of spin z projection (s, s') and isospin (i, j) and, if they are protons, different Landau levels (ν, ν') in the system. In a generalized system with a qp state the Landau parameters are calculated as the second derivative of the energy density of the system, \mathcal{E} , with respect to the qp state with occupation number $n_{i,s'}^j$, that is, j isospin, spin s' , and momentum \mathcal{K}_i^j (if they are quasiprotons they include the additional quantum number ν , in the way $n_{i\nu s'}^j$). Using Eq. (36) it is defined

$$f_{rlls's'}^{ij} = \frac{\partial \epsilon_{r,s}^i}{\partial n_{i,s'}^j}. \quad (39)$$

From the original formulation by Landau of the pure neutron system without considering spin degrees calculated in [29,30] the above expression generalizes to a $(2\nu_{\max} + 2) \times (2\nu_{\max} + 2)$ matrix, f_L , in isospin and spin space. In this way we have a characterization of the nuclear system when an external quantizing magnetic field is considered,

$$f_L = \begin{pmatrix} f_{ilss'vv'}^{pp} & f_{ilss'v}^{pn} \\ f_{ilss'v'}^{np} & f_{ilss'}^{nn} \end{pmatrix}. \quad (40)$$

The detailed calculation of the matrix elements is given in the Appendix 1.

In order to obtain the relativistic Landau parameters we must consider in the context of the FLT that the interactions of the effective quasiparticles in the system will take place close to the Fermi surfaces, since the lifetime of these excitations varies inversely with the departure of its energy E from the Fermi energy $\tau \approx \frac{1}{(E-E_F)^2}$.

Since the matrix elements in Eq. (40) have dimensions of energy divided by number density it seems convenient to define new dimensionless coefficients by multiplying them by the density of states at each Fermi level for quasiprotons, $N_p^{s,\nu}$, and quasineutrons, N_n^s , at the Fermi surface with a given spin projection. In the case of protons the quantized level filling must be carefully considered and the definition of density of states at the Fermi level and in the Landau level ν with polarization s is

$$\begin{aligned} N_p^{s,\nu} &= \frac{1}{V} \sum_{k_z} \delta(\epsilon_{k_z\nu,s}^p - E_F^p) = \frac{|q_p|B}{2\pi L} \sum_{k_z} \delta(\epsilon_{k_z\nu,s}^p - E_F^p) \\ &\rightarrow \frac{|q_p|B}{2\pi^2} \int_0^\infty dk_z \delta(\epsilon_{k_z\nu,s}^p - E_F^p) = \frac{|q_p|B}{2\pi^2} \\ &\times \int_0^\infty d\epsilon_{k_z\nu,s}^p \frac{\epsilon_{k_z\nu,s}^p}{k_z} \delta(\epsilon_{k_z\nu,s}^p - E_F^p) = \frac{|q_p|B}{2\pi^2} \frac{E_F^p}{k_{F,\nu,s}^p}, \end{aligned} \quad (41)$$

where $\epsilon_{k_z\nu,s}^p = E_{k_z\nu,s}^p (V_z^p = 0)$ with $E_{k_z\nu,s}^p$ defined by Eq. (27), and $k_{F,\nu,s}^p$ is the proton Fermi momentum Eq. (23).

Summing over all possible levels we have

$$N_p^s = \left(\frac{\partial n_s^p}{\partial \epsilon_s^p} \right)_{E_F^p} = \sum_\nu N_p^{s,\nu} = \frac{q_p B}{2\pi^2} \sum_\nu \frac{E_F^p}{k_{F,\nu,s}^p}, \quad (42)$$

where $k_{F,\nu,s}^p = \sqrt{E_F^{p2} - (\bar{m}_{\nu s}^p)^2}$. For neutrons,

$$\begin{aligned} N_n^s &= \left(\frac{\partial n_s^n}{\partial \epsilon_s^n} \right)_{E_F^n} \\ &= \frac{E_F^n}{2\pi^2} \left\{ k_{F,s}^n - s \mu_N \kappa_n B \left[\arcsin \left(\frac{\bar{m}_n^s}{E_F^n} \right) - \frac{\pi}{2} \right] \right\}. \end{aligned} \quad (43)$$

For the mixed states and due to the fact that the $f_{ss'}^{ij}$ arise from two-body operators we can define the density,

$$N_{ij}^{ss'} = \sqrt{N_i^s N_j^{s'}}. \quad (44)$$

Notice that this is not the only possible definition for the mixed density of states; however, the correct limit for isospin pure systems and the $B = 0$ limit can be recovered when used.

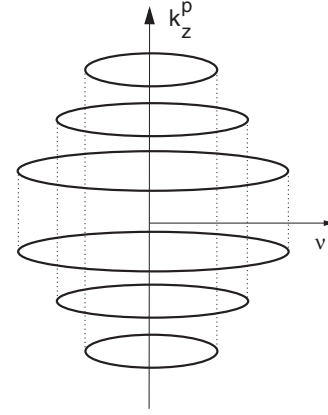


FIG. 1. Cylindrical shape of constant energy for quasiprotons. For spin projection s the Fermi surfaces are concentric circles with values $k_z^p = k_{F,\nu,s}^p$ and radius $\bar{m}_{\nu s}^p$. Notice that as ν grows k_z^p is smaller.

Using this prescription, dimensionless coefficients $F_{ss'}^{ij}$ can be defined [29–31] so that

$$F_{ij}^{ss'} = N_{ij}^{ss'} f_{ss'}^{ij}. \quad (45)$$

The Fermi surfaces are selected between the energy surfaces given by Eqs. (27) and (28) in the system at equilibrium. For protons these are cylindrical holes with k_z^p taking values $\pm k_{F,\nu,s}^p$ at the Landau level defined by ν for a given spin projection s as shown in Fig. 1.) For each spin projection, $s = \pm 1$, and a given ν , the proton Fermi surface are two circles with $k_z^p = \pm k_{F,\nu,s}^p$. For neutrons, instead, the Fermi surface for given s is shown in Fig. 2) as an ellipsoid of maximum perpendicular momentum

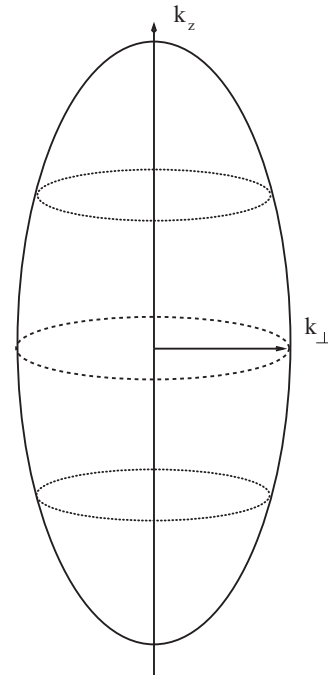


FIG. 2. Ellipsoidal shape of constant energy for quasineutrons. For spin projection s the Fermi surface has maximum values $|k_{z,s}^{n,\max}| = k_{F,s}^n$ and $|k_{\perp,s}^{n,\max}| = k_{F,\perp,s}^n$.

$|\mathbf{k}|_{\perp,s}^{n,\max} = k_{F\perp,s}^n$. This maximum Fermi momentum in the transverse direction is

$$|\mathbf{k}|_{\perp,s}^{n,\max} = k_{F\perp,s}^n = \sqrt{(E_F^n + s\kappa_n\mu_N B)^2 - m^{*2}}, \quad (46)$$

while in the z direction the k_z^n value attains a maximum value,

$$|k|_{z,s}^{n,\max} = k_{Fz}^n = \sqrt{E_F^n^2 - (\bar{m}_s^n)^2}. \quad (47)$$

In regular FLT the relevant interaction takes place on the Fermi surfaces and then the Landau parameters depend on the density n_B and the angle θ between qp three momenta being possible to perform their expansion in Legendre polynomials, $P_l(\cos(\theta))$. Taking averages over angular dependence of the Landau parameters shows that the only terms remaining are the $l=0$ terms. Then, for a pure isospin system the relations $f_l = \frac{f_{s,s} + f_{s,-s}}{2}$ and $g_l = \frac{f_{s,s} - f_{s,-s}}{2}$ hold and in this way values for $F_0 = \tilde{N}_0 f_0$ and $G_0 = \tilde{N}_0 g_0$ were the first to be historically obtained by Landau [29,30] and can be obtained from indirect experimentally measured observables [34]. On general grounds we can define auxiliary combinations of Landau parameters,

$$F_{ij}^+ = \frac{F_{ij}^{++} + F_{ij}^{+-}}{2}, \quad F_{ij}^- = \frac{F_{ij}^{--} + F_{ij}^{-+}}{2}, \quad (48)$$

$$G_{ij}^+ = \frac{F_{ij}^{++} - F_{ij}^{+-}}{2}, \quad G_{ij}^- = \frac{F_{ij}^{--} - F_{ij}^{-+}}{2}, \quad (49)$$

and, from them, we define combinations with singlet ($S=0$) or triplet ($S=1$) qp states as

$$F_{ij}^{(S=0)} = F_{ij}^+ + F_{ij}^-, \quad F_{ij}^{(S=1)} = G_{ij}^+ + G_{ij}^-. \quad (50)$$

If the $B \rightarrow 0$ limit is taken, we recover from the previous expression the usual F_0 and G_0 in the normal FLT.

In our work, in a similar way, and in order to evaluate the angular averages of coefficients over qp Fermi surfaces, we must take into account the different dynamical behavior due to the qp electrical charge. For protons there are several Landau levels that can be populated over cylindrical holes with $k_z^p = \pm k_{Fv,s}^p$. The average of a function $\tilde{f}(k_z^p, s, s', v, v')$ is performed as

$$\langle \tilde{f} \rangle_{ss'} = \frac{\sum_{v,v'} \tilde{f}(k_z^p, s, s', v, v') N_p^{vs} N_p^{v's'}}{\sqrt{N_p^s N_p^{s'}}}. \quad (51)$$

This definition gives the correct $B \rightarrow 0$ limit as the one obtained with a regular FLT $B=0$ calculation. For neutrons the Fermi surface is an ellipsoid defined by Eq. (28) and the average of a given function $\tilde{g}(k_z^n, s)$ should be performed using the fact that it presents axial symmetry. We will perform an integration over k_{\perp}^n resulting from the projection of the Fermi volume over the plane, that is, a disk \mathcal{S}_{\perp} of radius $|\mathbf{k}|_{\perp,s}^{n,\max}$ as given by Eq. (46). Then, in order to average we must replace the k_z^n by its value on the Fermi surface,

$$k_z^n = \pm \sqrt{E_F^n^2 - [\bar{m}_s^n(k_{\perp})]^2}, \quad (52)$$

with $\bar{m}_s^n(k_{\perp}) = \sqrt{k_{\perp}^2 + m^{*2}} - s\mu_N\kappa_n B$, up to the maximum value, and integrate $\tilde{g}(k_z^n(k_{\perp}), s)$ over the disk \mathcal{S}_{\perp} in the XY

plane. The average of the function \tilde{g} finally reads as

$$\begin{aligned} \langle \tilde{g} \rangle_s &= \frac{\int_{\mathcal{S}_{\perp}} \tilde{g} dS}{\int_{\mathcal{S}_{\perp}} dS} \\ &= \frac{\int_0^{2\pi} \int_0^{\sqrt{(E_F^n + s\kappa_n\mu_N B)^2 - m^{*2}}} \tilde{g}(k_z^n(k_{\perp}), s) k_{\perp} dk_{\perp} d\phi}{\int_0^{2\pi} \int_0^{\sqrt{(E_F^n + s\kappa_n\mu_N B)^2 - m^{*2}}} k_{\perp} dk_{\perp} d\phi}. \end{aligned} \quad (53)$$

IV. RESULTS

We have solved self-consistently Eqs. (12)–(21) for a beta equilibrium charge-neutral nuclear homogeneous system, constituted by protons, neutrons, and electrons, under the influence of a strong quantizing magnetic field.

In Fig. 3 we show proton population fractions for particles with magnetic moments polarized parallel (Y_p^+) (solid line) and antiparallel (Y_p^-) (dashed line) to the magnetic field B for different baryonic densities $n_B/n_0 = 0.1, 0.5, 2, 4$ as a function of the logarithm (base 10) of the magnetic field strength. Due to the tiny value of the baryon magnetic moment, magnetic field strengths larger than $\log B(G) > 16$ are needed so that there is a rapid increase (decrease) of the up (down) proton fraction. For a given magnetic field strength, the differences between fractions of protons polarized parallel and antiparallel to the magnetic field direction are larger for the smaller densities. This behavior is a consequence of the energy balance of the interaction of the proton orbit magnetic momentum and AMM with the magnetic field. Due to the positive charge and the fact that $\kappa_p > 0$, a quasiparticle energy is lowered by aligning spins with B , and the opposite for spin down. Let us remind that for densities below saturation density, $n_B = n_0 = 0.145 \text{ fm}^{-3}$, the spatially nonhomogeneous systems are energetically preferred over the uniform ones with the onset of *pasta phases* [11]; however this density can be of interest to the study of low-density neutron-rich gas among the expected pasta structures.

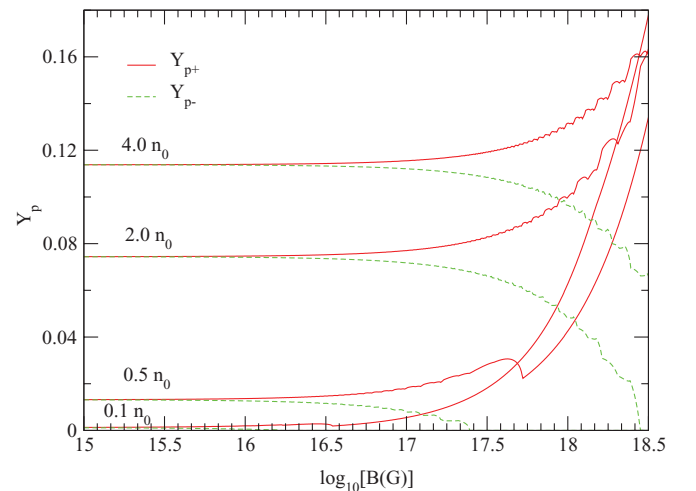


FIG. 3. (Color online) Proton fraction for the up (red solid line) and down (green dashed line) spin as a function of the logarithm (base 10) of the magnetic field strength at baryonic densities $n_B/n_0 = 0.1, 0.5, 2, 4$.

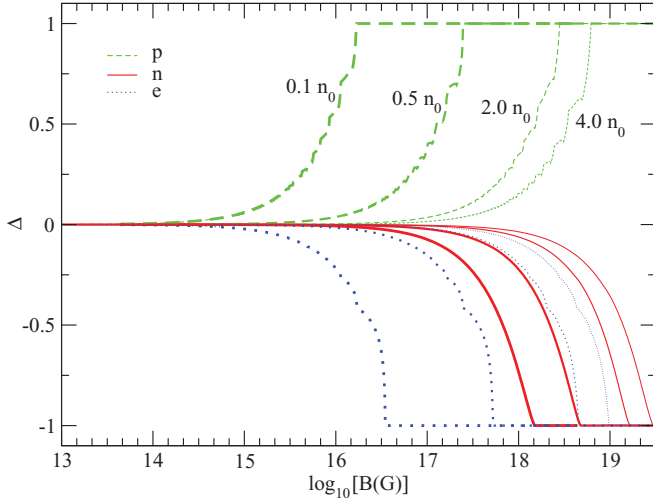


FIG. 4. (Color online) Proton (dashed green), neutron (solid red), and electron (dotted blue) relative polarization as a function of logarithm (base 10) of the magnetic field strength (in gauss) for $n_B/n_0 = 0.1, 0.5, 2, 4$.

In Fig. 4 we show relative polarization, $\Delta_i = (n_i^+ - n_i^-)/(n_i^+ + n_i^-)$, where n_i^\pm are the vector number densities of i -particle population, for neutrons (solid line), protons (dashed line), and electrons (dotted line) as a function of the logarithm (base 10) of the magnetic field strength (in gauss) for $n_B/n_0 = 0.1, 0.5, 2, 4$. Increasing densities are depicted with decreasing line width. Hadrons and leptons show different behavior according to the associated sign and value of their magnetic moment.

At $n_B/n_0 = 0.1$ and for fields $\log[B(G)] > 16$ there is a complete alignment of the proton sector where they will all be in the $n = 0$ Landau level, while the opposite charge of electrons force them to be in the antialigned state with $n = 0$ for a slightly smaller field. Instead, the antipolarization of neutrons occurs due to the different interaction ($\kappa_n < 0$) of the neutron AMM with the magnetic field. In this sense, this low-density case could somewhat illustrate the properties of the homogeneous neutron gas and it remains to be seen whether the interaction of the B field and other effects [42] could prevent the formation of proposed exotic superfluid states in the interior of NSs.

From Fig. 4 it is also seen that as density grows the magnetic field strength needed to polarize a given population fraction is bigger, due to competing effects in Fermi energies. The different behavior between protons and neutrons is due to the orbital magnetic moment contributing only for protons. In general, it is quite difficult to polarize neutrons, and Δ_n remains always below Δ_p (in absolute value).

A. Landau parameters

In Figs. 5 and 6 we show the normalized Landau parameters $F_{ij}^{ss'}$, averaged according to Eq. (51) for the protons and calculated with the maximum transverse Fermi momentum for the neutrons, for the different combinations of isospin $\{i, j\} = \{n, p\}$ as functions of the logarithm (base 10) of the

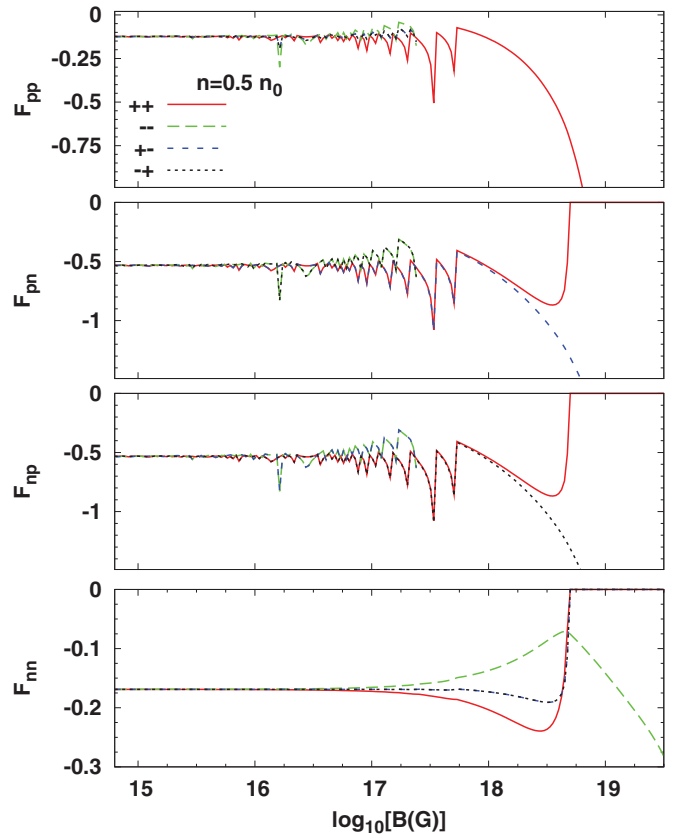


FIG. 5. (Color online) Normalized Landau parameters $F_{ij}^{ss'}$ as functions of the logarithm of the magnetic field strength for $s, s' = \pm 1$ at densities $n_B/n_0 = 0.5$. For the neutrons we took the maximum transverse Fermi momentum and for the protons we have averaged with respect to the Landau levels.

magnetic field strength (in gauss) for $n_B/n_0 = 0.5, n_B/n_0 = 2$, respectively. F_{pp}, F_{pn}, F_{np} , and F_{nn} are depicted from top to bottom in each. We can see the different polarization components, $(++)$, $(+-)$, $(-+)$, $(--)$, in solid, long-dashed, short-dashed, and dotted lines respectively. The peaks appearing in the dimensionless coefficients involving protons, that is, (F_{pp}, F_{pn}, F_{np}) , are due to the fact that the level density depends on the Landau level n and spin projection s . Instead, for neutrons the behavior is smooth since Fermi surfaces are ellipsoids. We can see that the parameters $F_{ij}^{ss'}$ for the low-density case in Fig. 5 are negative, signaling attractive interaction, while for the high-density case they are always positive, or repulsive, for the magnetic field strengths considered in this work. Results for B field strengths with $\log_{10}[B(G)] > 18$ should be taken with care and considered as an extrapolation. At low B the hopping behavior is smooth due to the large number of Landau levels populated and as B grows the number of levels decreases and the separation between them increases (see Fig. 1). For sufficiently large B , $k_{F,v,s=-1}^p = 0$, and beyond that strength the density of states, N_p^- , and all components in the F_{pp} except for the $(++)$ are not defined. The same happens for F_{pn}, F_{np} , since they involve the proton spin-down component. For these components there is a symmetry by replacing simultaneously spin and isospin

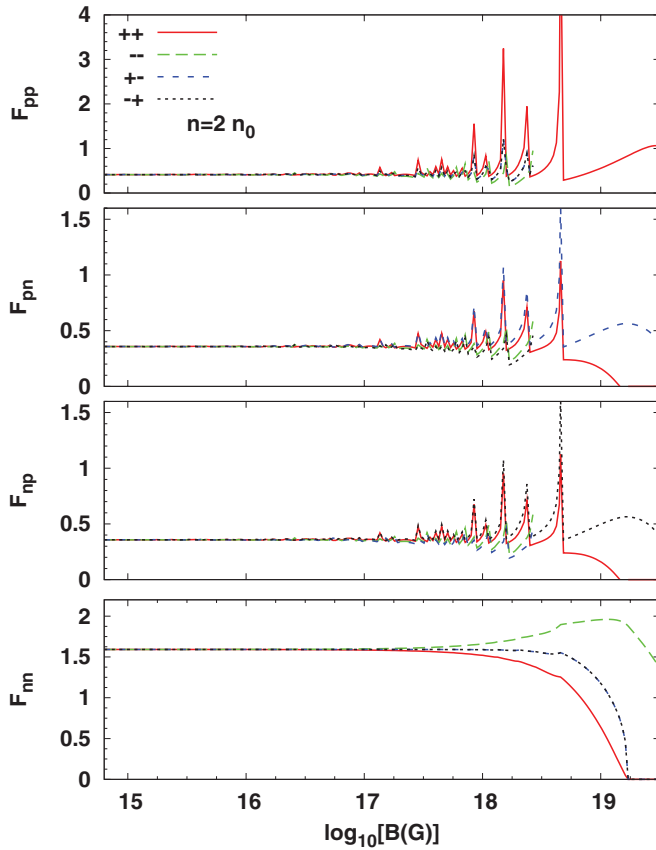


FIG. 6. (Color online) Normalized Landau parameters $F_{ij}^{ss'}$ as functions of the logarithm of the magnetic field strength for $s, s' = \pm 1$ at densities $n_B/n_0 = 2$.

indexes; in this way, for instance, the F_{pn}^{+-} and F_{np}^{-+} are equal. For strengths of $B \approx 4 \times 10^{18}$ G all the neutrons are polarized down and the only nonvanishing components are those involving this fraction.

At low densities and for components with mixed spin the quasiparticle states are more bound than the same spin components. Protons tend to polarize up, as B grows, filling the low Landau levels, and the qp interaction becomes more attractive; however for neutrons this behavior reverses since, the larger B is, the smaller the neutron fraction populates the system and the more repulsive the medium reacts to the formation of a qp state. Then, this causes quasineutron excitations to be less bound on an absolute scale. In the high-density case (6), the spikes signal the hopping of the Landau levels, and the clearly more repulsive interaction than shown in the low-density case (5) where the magnetization is stronger. The mixed states involving protons are less repulsive than the F_{nn} in the mostly neutron populated system.

In Figs. 7 and 8 we show the F_{ij}^S normalized Landau coefficients according to the definitions in Eq. (50) with a definite value of total spin, S , for all combinations of isospin $\{i, j\} = \{n, p\}$ as a function of the logarithm (base 10) of the magnetic field strength (in gauss) for $n_B/n_0 = 0.5$ and $n_B/n_0 = 2$, respectively. F_{pp} , F_{pn} , F_{np} , and F_{nn} are depicted from top to bottom in each figure. The singlet (solid line) and triplet (dashed line) state parameters present a different

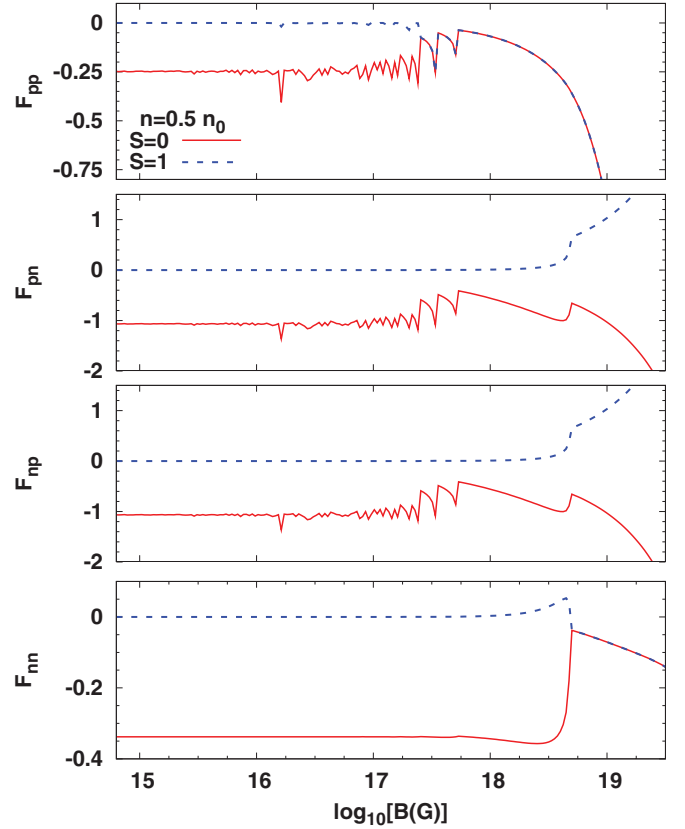


FIG. 7. (Color online) Normalized Landau parameters F_{ij}^S as functions of the logarithm of the magnetic field strength for $S = 0, 1$ for densities $n_B/n_0 = 0.5$.

behavior due to their construction. As seen in Fig. 7 for the low-density case, due to cancellations of negative components ($++$, $+-$, $-+$, $--$), the triplet states have small binding. Notice that for the F_{pp} the well-defined values are those where $k_{F,v,s}^p$ is real ($v \leq v_{\max}$). For singlet states the energy is lowered with respect to triplet states, for different isospin components (pn , np). At higher densities, $n_B/n_0 = 2$, as in Fig. 5, the different components $F_{ij}^{ss'}$ are all positive, and, therefore, the singlet and triplet binding behave in a similar way to the low-density case; however, in absolute value the singlet case will have repulsive character. This analysis of Landau parameters could be of potential interest to obtain transport coefficients from the components of their expansion in cylindrical harmonics, due to the axial symmetry introduced by the existence of a strong quantizing magnetic field. From this expansion information on low-temperature nuclear system observables [41] such as effective mass, spin diffusivity, and viscosity could be accessed. Additional contributions from exchange terms to the calculated parameters should also be considered.

B. Incompressibility and first sound velocity

We next apply the above expressions of the Landau parameters and calculate the incompressibility and the first sound velocity of β -equilibrium matter.

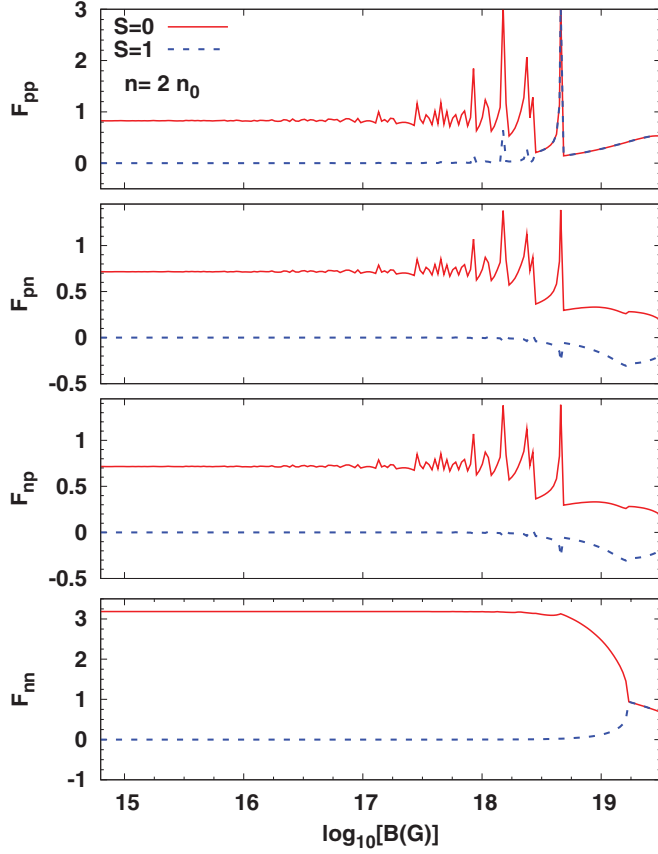


FIG. 8. (Color online) Normalized Landau parameters F_{ij}^S as functions of the logarithm of the magnetic field strength for $S = 0, 1$ for densities $n_B/n_0 = 2$.

The incompressibility is defined by [34]

$$K = 9n_B \frac{\partial^2 E}{\partial n_B^2}. \quad (54)$$

For asymmetric nuclear matter we obtain

$$K = 9n_B \frac{\partial}{\partial n_B} \left(\mu_p \frac{1+t}{2} + \mu_n \frac{1-t}{2} \right). \quad (55)$$

In the presence of a strong magnetic field, and in terms of the zero-order Landau parameters, the incompressibility is given by

$$\begin{aligned} K = 9n_B & \left[\left(\frac{1+t}{2} \right)^2 \sum_{\nu\nu's's'} \left(f_0^{pp\nu\nu's's'} + \frac{\delta_{\nu\nu'}\delta_{ss'}}{N_{p\nu's's'}} \right) \xi_{p\nu s} \xi_{p\nu's'} \right. \\ & + \left(\frac{1-t^2}{4} \right) \sum_{\nu's's'} f_0^{np\nu's's'} \xi_{ns} \xi_{p\nu's'} + \left(\frac{1-t}{2} \right)^2 \\ & \times \sum_{ss'} \left(f_0^{nnss'} + \frac{\delta_{ss'}}{N_{ns'}} \right) \xi_{ns} \xi_{ns'} + \left(\frac{1-t^2}{4} \right) \\ & \left. \times \sum_{\nu s s'} f_0^{pn\nu s s'} \xi_{ns'} \xi_{p\nu s} \right], \quad (56) \end{aligned}$$

where we have introduced the partial proton $\xi_{p\nu s}$ and neutron ξ_{ns} fractions defined as

$$\xi_{p\nu s} = \frac{n_{p\nu s}}{n_p}, \quad \xi_{ns} = \frac{n_{ns}}{n_n}, \quad (57)$$

with $n_{p\nu s}$ and n_n defined in (22), and we have used the equalities

$$\frac{\partial P}{\partial n_p} = \sum_{\nu s} \frac{\partial P}{\partial n_{p\nu s}} \xi_{p\nu s}, \quad \frac{\partial P}{\partial n_n} = \sum_s \frac{\partial P}{\partial n_{ns}} \xi_{ns}. \quad (58)$$

The sound velocity is directly obtained from the pressure

$$\begin{aligned} c_1^2 &= \frac{\partial P}{\partial E} = \sum_{i=p,n} \frac{\partial P}{\partial n_i} \frac{\partial n_i}{\partial n_B} \frac{\partial n_B}{\partial E} \\ &= \frac{1}{\bar{\mu}} \left[\frac{1+t}{2} \sum_{\nu s} \frac{\partial P}{\partial n_{p\nu s}} \xi_{p\nu s} + \frac{1-t}{2} \sum_s \frac{\partial P}{\partial n_{ns}} \xi_{ns} \right], \quad (59) \end{aligned}$$

where we have used the relation

$$\bar{\mu} = \frac{\partial E}{\partial n_B} = \sum_{i=p,n} \frac{\partial E}{\partial n_i} \frac{\partial n_i}{\partial n_B} = \mu_p \frac{1+t}{2} + \mu_n \frac{1-t}{2}. \quad (60)$$

Substituting in the above equation the pressure

$$P = \mu_p n_p + \mu_n n_n - E \quad (61)$$

and making use of the fraction $\xi_{p\nu s}$ and ξ_{ns} introduced in Eq. (57), we obtain for the sound velocity

$$c_1^2 = \frac{K}{9\bar{\mu}} \quad (62)$$

with K defined in Eq. (56).

In Fig. 9 we show the averaged nuclear incompressibility in beta equilibrium asymmetric nuclear matter as a function of the magnetic field strength for densities $n_B/n_0 = 0.5$ (solid line) and $n_B/n_0 = 1$ (dashed line). We chose these two densities to show typical behavior at low density and intermediate

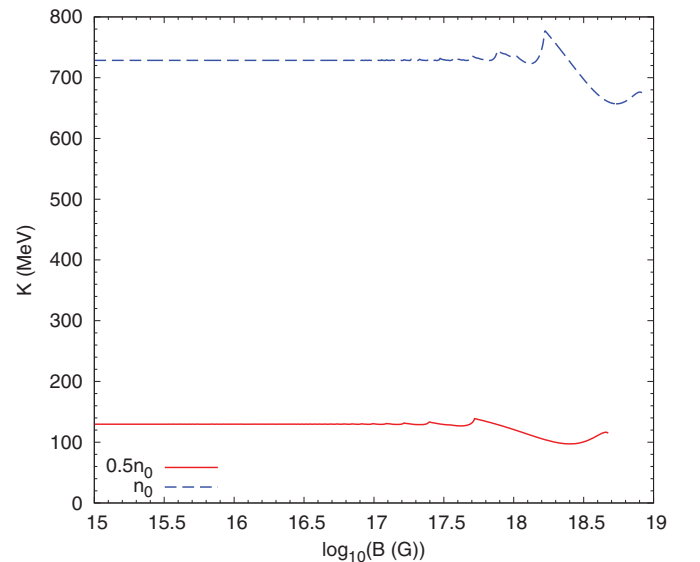


FIG. 9. (Color online) Averaged nuclear incompressibility as a function of the logarithm of the magnetic field strength for densities $n_B/n_0 = 0.5$ (solid line) and $n_B/n_0 = 1$ (dashed line).

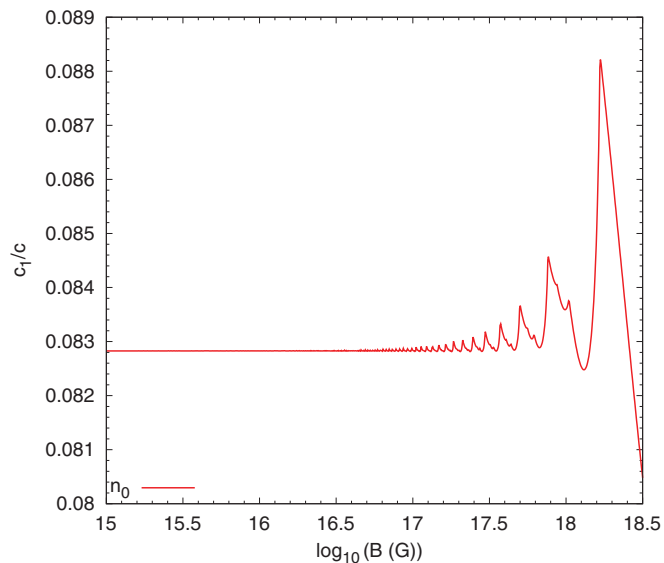


FIG. 10. (Color online) Averaged first sound velocity (in units of c) as a function of the logarithm of the magnetic field strength for a density $n_B/n_0 = 1$.

density, and as we can see there is a sizable difference in the subsaturation density case from the saturation density. This observable is very important in the behavior of pressure waves in the core-collapse supernovae. Let us recall that if we were to consider symmetric nuclear matter without B fields this observable achieves negative values signaling the presence of instabilities in the homogeneous nuclear system for these densities. It remains for future work to see what is the actual shape in the chemical phase space of first-order transition in beta equilibrium nuclear matter when passing from the crust phases to homogeneous phases. At higher values of the magnetic field strength the oscillation features arise clearly due to the prominent effect of the Landau-level filling. Note, however, that these values have been calculated averaging the Landau parameters by integrating in momentum space for neutrons and summing over available Landau levels. This is done in order to have an indication of the average trend since they show some directional dependence as a magnetic field in z direction is introduced in the system.

In Fig. 10 we show the averaged first sound in beta equilibrium asymmetric nuclear matter as computed from the nuclear incompressibility as a function of the magnetic field strength for a density $n_B/n_0 = 1$. We can see that as the magnetic field strength is larger, the oscillations in the first sound velocity are more prominent showing the effect of reducing the available phase space for neutrons and protons by gradually lowering the number of Landau levels filled by protons and integrating over the ellipsoidal shape of Fermi surface for neutrons.

V. SUMMARY

In the present paper we have derived for the first time the Landau parameters for charge neutral homogeneous asymmetric nuclear matter under very strong magnetic fields. In

particular, we have considered cold stellar matter in β equilibrium in the presence of fields with strengths $\log_{10}[B(G)] < 18$ as allowed by the scalar virial theorem. We have used a RMF model to describe the nuclear system so that our calculation may be extended to high densities. However, no exotic matter such as hyperons or kaon condensates has been considered. In our work, we have only considered matter densities from subsaturation density, $n_B/n_0 = 0.1$, to densities before a possible quark transition. The present results at subsaturation densities should be taken with care, as a first approach to a realistic description of the effect of strong magnetic fields on the cold stellar matter. Also we have not considered the existence of superfluid neutrons at subsaturation densities.

We find that the nuclear system needs at least magnetic field strengths larger than $B \approx 10^{16}$ G to show some noticeable magnetization in excitation spectrum. Protons tend to polarize aligning spins with B field direction, while neutrons and electrons do the opposite. Landau coefficients $F_{ij}^{ss'}$ show the interaction energy of qp excitations with isospins i and j and spins s and s' , respectively, and are calculated from the energy density \mathcal{E} of the system in charge neutrality as a two-body operator obtained from its second derivative with respect to the occupation numbers of the qp excitation. In the excitations involving quasiprotons the discrete Landau levels play an important role quantizing the momentum states in the direction perpendicular to the field. In this way there is a discrete feature associated in the Landau coefficients. For small B the number of allowed levels is large and is hardly visible on the plots; however for higher B the number of levels decreases and they are more separated. For neutrons instead, the lack of quantized Landau levels implies a smooth behavior of the parameters as functions of magnetic field strength.

At low densities the effect of the magnetic field is small on the individual polarization since the polarization induced is weak (for fields with $\log_{10}[B(G)] < 17$) and gives an attractive overall effect with respect to unmagnetized systems. The dominant component drives the tendency of the parameter; in this way the proton sector is more bound and so are the pp , pn , np components, while for neutrons they are less bound since the number of neutrons decreases when B increases. In this case, for situations of complete polarization the Landau parameters become largely negative. In the high-density cases the interaction is increased in the way to unbind the qp excitations. The medium effects are such that the interaction is repulsive and the magnetic field tends to decrease this effect in the less populated fractions while increasing in the more populated ones.

For the singlet qp configuration the medium effects play a role of binding at low densities while at high densities the opposite occurs. The B field tends to reverse this tendency (if not too strong) and can be considered a competing effect. As for the triplet, for the qp spin configurations the in-medium effects play no role, while the B field brings extra binding in the pp or nn channel at low densities, while the opposite occurs at high densities. The effect of the B field is reversed in the pn or np channels; binding (unbinding) is present at high (low) densities.

The calculation we have performed could have potential interest to size the relevance of the inclusion of a magnetic

field in astrophysical scenarios with realistic description of nuclear systems in beta equilibrium by relativistic mean-field theoretical models. We have obtained singlet and triplet interaction energies of qp states and further work could relate these, in a consistent fashion, to relativistic and (possibly) superfluid hadronic components in this type of system.

As an application we have calculated the incompressibility and the first sound velocity of cold β -equilibrium matter. It was shown that the incompressibility and first sound velocity show a distinctive oscillatory behavior due to the progressive lowering of the Landau levels for protons and ellipsoidal shape for the neutron Fermi surface. These quantities were calculated using the averaging procedure as explained, since they show some directional dependence in the momentum space as a magnetic field direction is introduced in the problem.

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APPENDIX

In this appendix we provide the explicit form of the quasiparticle interaction matrix elements depending on spin and isospin and the mass and current derivatives appearing in the calculation of the Landau parameters in Sec. III.

1. Interaction matrix elements

The matrix elements appearing in f_L in Eq. (40) in Sec. III read as follows:

(i) For the quasiproton-quasiproton interaction,

$$\begin{aligned} f_{ilss'v'v'}^{pp} &= \frac{\partial \epsilon_{iv,s}^p}{\partial n_{ls'v'}^p} = \frac{\bar{m}_{vs}^p}{\epsilon_{ivs}^p} \frac{\partial \bar{m}_{vs}^p}{\partial n_{ls'v'}^p} - \frac{k_{zi}}{\epsilon_{ivs}^p} \frac{\partial \mathcal{V}_z^p}{\partial n_{ls'v'}^p} + A_+ = \frac{\bar{m}_{vs}^p}{\epsilon_{ivs}^p} \frac{m^*}{\bar{m}_{vs}^p + s\kappa_p \mu_N B} \frac{\partial m^*}{\partial n_{ls'v'}^p} - \frac{k_{zi}^p}{\epsilon_{ivs}^p} \left[A_+ \frac{\partial j_z^p}{\partial n_{ls'v'}^p} + A_- \frac{\partial j_z^n}{\partial n_{ls'v'}^p} \right] + A_+ \\ &= -\frac{g_\sigma^2}{m_\sigma'^2} \frac{\bar{m}_{vs}^p m^* \bar{m}_{v's'}^p m^*}{\epsilon_{ivs}^p \tilde{\epsilon}_{v's'}^p \epsilon_{lv's'}^p \tilde{\epsilon}_{v's'}^p} \frac{1}{1 + \frac{g_\sigma^2}{m_\sigma'^2} (I_1 + I_2)} - \frac{k_{zi}^p k_{zl}^p}{\epsilon_{ivs}^p \epsilon_{lv's'}^p} \frac{A_+ + (A_+^2 - A_-^2) b_n}{D} + A_+. \end{aligned} \quad (A1)$$

(ii) For quasineutron-quasineutron interaction,

$$\begin{aligned} f_{ilss'}^{nn} &= \frac{\partial \epsilon_{is}^n}{\partial n_{ls'}^n} = A_+ - \frac{k_{zi}^n}{\epsilon_{is}^n} \frac{\partial \mathcal{V}_z^n}{\partial n_{ls'}^n} + \frac{\epsilon_{\perp i}^n - s\kappa_n \mu_N B}{\epsilon_{is}^n \epsilon_{\perp i}^n} \left[m^* \frac{\partial m^*}{\partial n_{ls'}^n} - \mathbf{k}_{i\perp}^n \frac{\partial \mathcal{V}_\perp^n}{\partial n_{ls'}^n} \right] \\ &= A_+ - \frac{k_{zi}^n}{\epsilon_{is}^n} \left[A_- \frac{\partial j_z^p}{\partial n_{ls'}^p} + A_+ \frac{\partial j_z^n}{\partial n_{ls'}^n} \right] + \frac{\epsilon_{\perp i}^n - s\kappa_n \mu_N B}{\epsilon_{is}^n \epsilon_{\perp i}^n} \left[m^* \frac{\partial m^*}{\partial n_{ls'}^n} - \mathbf{k}_{i\perp}^n \left(A_+ \frac{\partial j_\perp^n}{\partial n_{ls'}^n} \right) \right] \\ &= A_+ - \frac{k_{zi}^n k_{zl}^n}{\epsilon_{is}^n \epsilon_{ls'}^n} \frac{A_+ + (A_+^2 - A_-^2) b_p}{D} - \frac{g_\sigma^2}{m_\sigma'^2} \frac{(\epsilon_{\perp i}^n - s\kappa_n \mu_N B) m^* (\epsilon_{\perp i}^n - s'\kappa_n \mu_N B) m^*}{\epsilon_{is}^n \epsilon_{\perp i}^n \epsilon_{ls'}^n \epsilon_{\perp l}^n} \frac{1}{1 + \frac{g_\sigma^2}{m_\sigma'^2} (I_1 + I_2)} \\ &\quad - \frac{(\epsilon_{\perp i}^n - s\kappa_n \mu_N B) (\epsilon_{\perp l}^n - s'\kappa_n \mu_N B)}{\epsilon_{is}^n \epsilon_{\perp i}^n \epsilon_{ls'}^n \epsilon_{\perp l}^n} \mathbf{k}_{i\perp}^n \cdot \mathbf{k}_{l\perp}^n \frac{A_+}{1 + A_+ I_3}. \end{aligned} \quad (A2)$$

(iii) For quasiproton-quasineutron interaction,

$$\begin{aligned} f_{ilss'v'}^{pn} &= \frac{\partial \epsilon_{isv}^p}{\partial n_{ls'v'}^n} = A_- + \frac{m^*}{\bar{m}_{vs}^p + s\kappa_p \mu_N B} \frac{\bar{m}_{vs}^p}{\epsilon_{ivs}^p} \frac{\partial \bar{m}^*}{\partial n_{ls'v'}^n} - \frac{k_{zi}}{\epsilon_{ivs}^p} \left[A_+ \frac{\partial j_z^p}{\partial n_{ls'v'}^p} + A_- \frac{\partial j_z^n}{\partial n_{ls'v'}^n} \right] \\ &= A_- - \frac{g_\sigma^2}{m_\sigma'^2} \frac{m^* \bar{m}_{vs}^p (\epsilon_{\perp l}^n - s'\kappa_n \mu_N B) m^*}{\epsilon_{ivs}^p \tilde{\epsilon}_{vs}^p \epsilon_{ls'v'}^n \epsilon_{\perp l}^n} \frac{1}{1 + \frac{g_\sigma^2}{m_\sigma'^2} (I_1 + I_2)} + \frac{k_{zi}}{\epsilon_{ivs}^p} \frac{k_z}{\epsilon_{ls'v'}^n} \frac{A_-}{D}. \end{aligned} \quad (A3)$$

(iv) For quasineutron-quasiproton interaction,

$$\begin{aligned} f_{ilss'v'}^{np} &= \frac{\partial \epsilon_{is}^n}{\partial n_{ls'v'}^p} = A_- - \frac{k_{zi}}{\epsilon_{is}^n} \left[A_- \frac{\partial j_z^p}{\partial n_{ls'v'}^p} + A_+ \frac{\partial j_z^n}{\partial n_{ls'v'}^n} \right] + \frac{\epsilon_{\perp i}^n - s\kappa_n \mu_N B}{\epsilon_{is}^n \epsilon_{\perp i}^n} \left[m^* \frac{\partial m^*}{\partial n_{ls'v'}^p} - \mathbf{k}_{i\perp}^n \left(A_+ \frac{\partial j_\perp^n}{\partial n_{ls'v'}^n} \right) \right] \\ &= A_- - \frac{g_\sigma^2}{m_\sigma'^2} \frac{\epsilon_{\perp i}^n - s\kappa_n \mu_N B}{\epsilon_{is}^n \epsilon_{\perp i}^n} \frac{m^* \bar{m}_{v's'}^p}{\epsilon_{lv's'}^p \tilde{\epsilon}_{v's'}^p} \frac{1}{1 + \frac{g_\sigma^2}{m_\sigma'^2} (I_1 + I_2)} + \frac{k_{zi}}{\epsilon_{is}^n} \frac{k_{zl}}{\epsilon_{is'v'}^p} \frac{A_-}{D}. \end{aligned} \quad (A4)$$

The indexes in labels in expressions Eqs. (A1)–(A4) have been accordingly written with six numbers (i, ν, s), (l, ν', s') for quasiproton interaction, and four numbers (i, s), (l, s') for quasineutron interaction. For isospin mixing terms [quasiproton (quasineutron)–quasineutron (quasiproton)] there are 5 numbers (i, ν, s), (l, s') in the label. We have used auxiliary definitions in the combination of coupling constants,

$$A_{\pm} = \frac{g_{\omega}^2}{m_{\omega}^2} \pm \frac{g_{\rho}^2}{4m_{\rho}^2}, \quad (\text{A5})$$

and the proton effective energy

$$\epsilon_{i\nu s}^p = \sqrt{k_{zi}^2 + (\bar{m}_{\nu s}^p)^2}, \quad (\text{A6})$$

with an auxiliary mass $\bar{m}_{\nu s}^p = \sqrt{m^{*2} + 2q_p \nu B} - s\kappa_p \mu_N B = \bar{\epsilon}_{\nu s}^p - s\kappa_p \mu_N B$.

For neutrons we define an auxiliary energy (which can be interpreted as a neutron momentum dependent mass),

$$\epsilon_{\perp i}^n = \sqrt{m_n^{*2} + k_{\perp i}^2}, \quad (\text{A7})$$

and the effective energy for neutrons,

$$\epsilon_{is}^n = \sqrt{k_{zi}^2 + (\epsilon_{\perp i}^n - s\kappa_n \mu_N B)^2}. \quad (\text{A8})$$

We also define

$$D = (1 + A_+ b_n)(1 + A_+ b_p) - A_-^2 b_p b_n,$$

where the coefficients b_p and b_n are

$$b_p = \sum_j \frac{n_{js''\nu''}^p}{\epsilon_{js''\nu''}^p} \left[1 - \left(\frac{k_{jz}}{\epsilon_{js''\nu''}^p} \right)^2 \right] = \frac{q_p B}{2\pi^2} \sum_{s,\nu} \frac{k_{F\nu s}^p}{E_F^p}, \quad (\text{A9})$$

$$b_n = \sum_j \frac{n_{js''}^n}{\epsilon_{js''}^n} \left[1 - \left(\frac{k_{jz}}{\epsilon_{js''}^n} \right)^2 \right] = \frac{1}{2\pi^2} \sum_s \left\{ \frac{(k_{F_s}^n)^3}{3E_F^n} - \frac{s\kappa_n \mu_N B}{2E_F^n} \left[\bar{m}_{F_s}^n - E_F^n \left[\frac{\pi}{2} - \arcsin \left(\frac{\bar{m}_n}{E_F^n} \right) \right] \right] \right\}, \quad (\text{A10})$$

and $k_{F_s}^n = \sqrt{E_F^n^2 - \bar{m}_n^2}$. The explicit form for the integrals I_1 , I_2 , and I_3 is given by

$$I_1 = \sum_{j\nu''s''} n_{j\nu''s''}^p \frac{\partial^2 \epsilon_{j\nu''s''}^p}{\partial m^{*2}} = \frac{q_p B}{2\pi^2} \sum_{\nu,s} \int_0^{k_{F_s}^p} dx \frac{1}{\sqrt{k_z^2 + \bar{m}_p^2}} \left(\frac{m^{*2}(s\kappa_p \mu_N B)}{(\bar{\epsilon}_{\nu s}^p)^3} + \frac{\bar{m}_p}{\bar{\epsilon}_{\nu s}^p} - \frac{\bar{m}_p m^{*2}}{(\bar{\epsilon}_{\nu s}^p)^2} \frac{1}{(\sqrt{k_z^2 + \bar{m}_p^2})^2} \right) \\ = \frac{q_p B}{2\pi^2} \sum_{\nu,s} \left[\frac{\bar{m}_{\nu s}^p 2q_p \nu B + m^{*2} \bar{\epsilon}_{\nu s}^p}{(\bar{\epsilon}_{\nu s}^p)^3} \ln \left(\frac{E_F^p + k_{F\nu s}^p}{\bar{m}_{\nu s}^p} \right) - \frac{m^{*2} k_{F\nu s}^p}{E_F^p (\bar{\epsilon}_{\nu s}^p)^2} \right], \quad (\text{A11})$$

$$I_2 = \sum_{js''} n_{js''}^n \frac{\partial^2 \epsilon_{js''}^n}{\partial m^{*2}} = \sum_s \int_{\bar{m}_n}^{E_F^n} dx (x + s\kappa_n \mu_N B) \left\{ \frac{m^{*2} s\kappa_n \mu_N B + x(x + s\kappa_n \mu_N B)^2}{(x + s\kappa_n \mu_N B)^3} \ln \left[\frac{\sqrt{(E_F^n)^2 - x^2} + E_F^n}{x} \right] - \frac{m^{*2} \sqrt{(E_F^n)^2 - x^2}}{(x + s\kappa_n \mu_N B)^2 E_F^n} \right\} \\ = \frac{1}{2\pi^2} \sum_s \int_{\bar{m}_n}^{E_F^n} dx \left\{ \left[\frac{m^{*2} s\kappa_n \mu_N B}{(x + s\kappa_n \mu_N B)^2} + x \right] \ln \left(\frac{E_F^n + \sqrt{(E_F^n)^2 - x^2}}{x} \right) - \frac{m^{*2} \sqrt{(E_F^n)^2 - x^2}}{E_F^n x + s\kappa_n \mu_N B} \right\}. \quad (\text{A12})$$

The integral I_2 can be written out into several pieces $I_2 = I_2^a + I_2^b + I_2^c$:

$$I_2^a = \frac{1}{2\pi^2} \sum_s \int_{\bar{m}_n}^{E_F^n} dx \frac{m^{*2} s\kappa_n \mu_N B}{(x + s\kappa_n \mu_N B)^2} \ln \left(\frac{E_F^n + \sqrt{(E_F^n)^2 - x^2}}{x} \right), \quad (\text{A13})$$

$$I_2^a = \sum_s - \frac{(s\kappa_n \mu_N B) m^{*2}}{2\pi^2} \left\{ \frac{E_{F_s}^n}{(s\kappa_n \mu_N B) \alpha} \left[\ln \left(\frac{(E_{F_s}^n)^2 + s\kappa_n \mu_N B E_{F_s}^n}{\alpha k_{F_s} + (E_{F_s}^n)^2 + s\kappa_n \mu_N B \bar{m}_n} \right) - \ln \left(\frac{E_F^n + s\kappa_n \mu_N B}{\bar{m} + s\kappa_n \mu_N B} \right) \right] - \frac{1}{(s\kappa_n \mu_N B)} \left[\ln \left(\frac{\bar{E}_F^n}{E_{F_s}^n + k_{F_s}^n} \right) - \ln \left(\frac{E_{F_s}^n}{\bar{m}} \right) \right] - \frac{1}{(s\kappa_n \mu_N B + \bar{m})} \ln \left(\frac{k_{F_s}^n + E_{F_s}^n}{\bar{m}} \right) \right\}, \quad (\text{A14})$$

$$I_2^b = \frac{1}{2\pi^2} \sum_s \int_{\bar{m}_n}^{E_F^n} dx x \ln \left(\frac{E_F^n + \sqrt{(E_F^n)^2 - x^2}}{x} \right), \quad (\text{A15})$$

$$I_2^c = \frac{-1}{4\pi^2} \sum_s \left[\bar{m}^2 \ln \left(\frac{k_{F_s}^n + E_{F_s}^n}{\bar{m}} \right) - k_{F_s}^n E_{F_s}^n \right], \quad (\text{A16})$$

$$I_2^c = \frac{-m^{*2}}{2\pi^2} \sum_s \int_{\bar{m}_n}^{E_F^n} dx \frac{\sqrt{(E_F^n)^2 - x^2}}{E_F^n(x + s\kappa_n\mu_N B)}, \quad (\text{A17})$$

$$I_2^c = \frac{-m^{*2}}{2\pi^2 E_F^n} \sum_s \left\{ -\alpha \ln \left(\frac{(E_F^n)^2 + s\kappa_n\mu_N B E_F^n}{\alpha k_{Fs}^n + (E_F^n)^2 + s\kappa_n\mu_N B \bar{m}} \right) + \alpha \ln \left(\frac{E_F^n + s\kappa_n\mu_N B}{\bar{m} + s\kappa_n\mu_N B} \right) + s\kappa_n\mu_N B \left[\frac{\pi}{2} - \arctan \left(\frac{\bar{m}}{k_{Fs}^n} \right) \right] - k_{Fs}^n \right\}, \quad (\text{A18})$$

with $\alpha = \sqrt{(E_F^n)^2 - (s\kappa_n\mu_N B)^2}$. The integral I_3 can be written as

$$I_3 = \sum_{js''} n_{js''}^n \left[\frac{\epsilon_{j\perp}^n - s''\kappa_n\mu_N B}{\epsilon_{js''}^n \epsilon_{j\perp}^n} \left(1 - \frac{\epsilon_{j\perp}^n - s''\kappa_n\mu_N B}{2\epsilon_{js''}^n \epsilon_{j\perp}^n} k_{j\perp}^2 \right) + \frac{s''\kappa_n\mu_N B}{2\epsilon_{js''}^n \epsilon_{j\perp}^n} k_{j\perp}^2 \right]. \quad (\text{A19})$$

We write the integral as several pieces $I_3 = I_3^a + I_3^b + I_3^c$. For I_3^a we have $I_3^a = I_2^b$. For I_3^b ,

$$I_3^b = \frac{-1}{4\pi^2 E_F^n} \sum_s \int_{\bar{m}_n}^{E_F^n} dx \left\{ \frac{[(x + s\kappa_n\mu_N B)^2 - m^{*2}] \sqrt{(E_F^n)^2 - x^2}}{(x + s\kappa_n\mu_N B)} \right\}, \quad (\text{A20})$$

$$I_3^b = \frac{-1}{4\pi^2 E_F^n} \sum_s \left\{ \frac{1}{3} (k_{Fs}^n)^3 - \frac{s\kappa_n\mu_N B}{2} \bar{m} k_{Fs}^n + (E_F^n)^2 \frac{s\kappa_n\mu_N B}{2} \left[\frac{\pi}{2} - \arctan \left(\frac{\bar{m}_n}{k_{Fs}^n} \right) \right] \right\} \\ + \frac{m^{*2}}{4\pi^2 E_F^n} \sum_s \left\{ -\alpha \left[\ln \left(\frac{(E_F^n)^2 + s\kappa_n\mu_N B E_F^n}{\alpha k_{Fs}^n + (E_F^n)^2 + s\kappa_n\mu_N B \bar{m}_n} \right) - \ln \left(\frac{E_F^n + s\kappa_n\mu_N B}{\bar{m}_n + s\kappa_n\mu_N B} \right) \right] + s\kappa_n\mu_N B \left[\frac{\pi}{2} - \arctan \left(\frac{\bar{m}}{k_{Fs}^n} \right) \right] - k_{Fs}^n \right\}. \quad (\text{A21})$$

For I_3^c ,

$$I_3^c = \frac{s\kappa_n\mu_N B}{4\pi^2} \sum_s \int_{\bar{m}_n}^{E_F^n} dx \left\{ \frac{[(x + s\kappa_n\mu_N B)^2 - m^{*2}] \ln \left(\frac{\sqrt{(E_F^n)^2 - x^2} + E_F^n}{x} \right)}{(x + s\kappa_n\mu_N B)^2} \right\}, \quad (\text{A22})$$

$$I_3^c = \frac{s\kappa_n\mu_N B}{4\pi^2} \sum_s \left\{ -\bar{m} \ln \left(\frac{E_F^n + k_{Fs}^n}{\bar{m}} \right) + E_F^n \left(\frac{\pi}{2} - \arctan \left(\frac{\bar{m}}{k_{Fs}^n} \right) \right) \right\} \\ - \frac{s\kappa_n\mu_N B m^{*2}}{4\pi^2} \sum_s \left\{ \frac{-E_F^n}{s\kappa_n\mu_N B \alpha} \left[\ln \left(\frac{(E_F^n)^2 + s\kappa_n\mu_N B E_{Fs}^n}{\alpha k_{Fs}^n + (E_F^n)^2 + s\kappa_n\mu_N B \bar{m}} \right) - \ln \left(\frac{E_F^n + s\kappa_n\mu_N B}{\bar{m} + s\kappa_n\mu_N B} \right) \right] \right\} \\ + \frac{1}{s\kappa_n\mu_N B} \left[\ln \left(\frac{E_F^n}{k_{Fs}^n + E_F^n} \right) - \ln \left(\frac{E_F^n}{\bar{m}} \right) \right] + \frac{1}{s\kappa_n\mu_N B + \bar{m}} \ln \left(\frac{k_{Fs}^n + E_F^n}{\bar{m}} \right). \quad (\text{A23})$$

2. Effective mass derivatives

The mass derivative appearing in Eqs. (A1)–(A4) $\frac{\partial m^*}{\partial n_{ls'v'}^p} = \frac{\partial(m_b - g_\sigma \sigma)}{\partial n_{ls'v'}^p}$ can be obtained using the equation of motion for the scalar σ field in Eq. (7). Then we can write

$$\frac{\partial m^*}{\partial n_{ls'v'}^p} = -\frac{g_\sigma^2}{m^{\prime 2}} \frac{\bar{m}_{v's'}^p m^*}{\epsilon_{lv's'}^p \tilde{\epsilon}_{v's'}^p} \frac{1}{1 + \frac{g_\sigma^2}{m^{\prime 2}} \left(\sum_{jv''s''} n_{jv''s''}^p \frac{\partial^2 \epsilon_{jv''s''}^p}{\partial m^{*2}} + \sum_{js''} n_{js''}^n \frac{\partial^2 \epsilon_{js''}^n}{\partial m^{*2}} \right)}, \quad (\text{A24})$$

$$\frac{\partial m^*}{\partial n_{ls'}^n} = -\frac{g_\sigma^2}{m^{\prime 2}} \frac{(\epsilon_{l\perp}^n - s'\kappa_n\mu_N B) m^*}{\epsilon_{ls'}^n \epsilon_{l\perp}^n} \frac{1}{1 + \frac{g_\sigma^2}{m^{\prime 2}} \left(\sum_{jv''s''} n_{jv''s''}^p \frac{\partial^2 \epsilon_{jv''s''}^p}{\partial m^{*2}} + \sum_{js''} n_{js''}^n \frac{\partial^2 \epsilon_{js''}^n}{\partial m^{*2}} \right)}. \quad (\text{A25})$$

In the limit where sums are converted into integrals it is found that

$$\frac{\partial m^*}{\partial n_{ls'v}^p} = -\frac{g_\sigma^2}{m_\sigma'^2} \frac{\bar{m}_{v's'}^p m^*}{\epsilon_{lv's'}^p \bar{\epsilon}_{v's'}^p} \frac{1}{1 + \frac{g_\sigma^2}{m_\sigma'^2} (I_1 + I_2)}, \quad (\text{A26})$$

$$\frac{\partial m^*}{\partial n_{ls'}^n} = -\frac{g_\sigma^2}{m_\sigma'^2} \frac{(\epsilon_{l\perp}^n - s' \kappa_n \mu_N B) m^*}{\epsilon_{lv's'}^n \epsilon_{l\perp}^n} \frac{1}{1 + \frac{g_\sigma^2}{m_\sigma'^2} (I_1 + I_2)}, \quad (\text{A27})$$

with the integrals I_1 and I_2 having the form given in Eqs. (A11) and (A13).

3. Current derivatives

The baryonic currents Eqs. (34)–(35) in the asymmetric nuclear system can be written in terms of the components in directions parallel $\hat{\mathbf{k}}$ and perpendicular $\hat{\mathbf{n}}_\perp$ to the external magnetic field. Using explicitly the expression for the effective momentum $\mathcal{K}_i^j = \mathbf{k}_i - \mathcal{V}_i^j$, $j = p, n$ we have, for protons,

$$\mathbf{j}_p = j_{pz} \hat{\mathbf{k}}, \quad (\text{A28})$$

with

$$j_{pz} = \sum_{js''v''} n_{js''v''}^p \frac{k_{zj} - \mathcal{V}_{pz}}{\epsilon_{js''v''}^p}, \quad (\text{A29})$$

and for neutrons,

$$\mathbf{j}_n = \sum_{js''} n_{js''}^n \left[\frac{k_{zj} - \mathcal{V}_{nz}}{\epsilon_{js''}^n} \hat{\mathbf{k}} + \frac{(\epsilon_{j\perp}^n - s'' \kappa_n \mu_N B)(k_{j\perp} - \mathcal{V}_{n\perp})}{\epsilon_{js''}^n \epsilon_{j\perp}^n} \hat{\mathbf{n}}_\perp \right]. \quad (\text{A30})$$

Using the conventions as defined in Matsui's work [34] $\mathbf{j}_B = \mathbf{j}_p + \mathbf{j}_n$ and $\mathbf{j}_3 = \mathbf{j}_p - \mathbf{j}_n$ the derivatives of the baryonic isovector current components are obtained assuming that the macroscopic currents in equilibrium will vanish; i.e., $\mathbf{j}_B = \mathbf{0}$ and $\mathbf{j}_3 = \mathbf{0}$. In this way the derivative of the baryonic vector current (z and \perp components) with respect to the proton density n_{ls}^p are

$$\frac{\partial \mathbf{j}_B|_z}{\partial n_{ls}^p} = \left[\frac{k_{lz}^p}{\epsilon_{ls}^p} - \frac{g_\rho^2}{4m_\rho^2} J_1 \frac{\partial \mathbf{j}_3|_z}{\partial n_{ls}^p} \right] \left(1 + \frac{g_\omega^2}{m_\omega^2} J_2 \right)^{-1}, \quad (\text{A31})$$

with integrals J_1 and J_2 that can be decomposed as $J_1 = b_p - b_n$ and $J_2 = b_p + b_n$. We also have

$$\frac{\partial \mathbf{j}_B|_\perp}{\partial n_{ls}^p} = 0. \quad (\text{A32})$$

For the derivatives of \mathbf{j}_B with respect to the neutron density n_{ls}^n we have

$$\frac{\partial \mathbf{j}_B|_z}{\partial n_{ls}^n} = \left[\frac{k_{lz}^n}{\epsilon_{ls}^n} - \frac{g_\rho^2}{4m_\rho^2} J_1 \frac{\partial \mathbf{j}_3|_z}{\partial n_{ls}^n} \right] \left(1 + \frac{g_\omega^2}{m_\omega^2} J_2 \right)^{-1}, \quad (\text{A33})$$

$$\frac{\partial \mathbf{j}_B|_\perp}{\partial n_{ls}^n} = \left[\frac{(\epsilon_{\perp l}^n - s \kappa_n \mu_N B) \mathbf{k}^n|_\perp}{\epsilon_{\perp l}^n \epsilon_{is}^n} + \frac{g_\rho^2}{4m_\rho^2} J_3 \frac{\partial \mathbf{j}_3|_\perp}{\partial n_{ls}^n} \right] \times \left(1 + \frac{g_\omega^2}{m_\omega^2} J_3 \right)^{-1}, \quad (\text{A34})$$

with the integral J_3 which can be written out as $J_3 = I_3$. If we now calculate the derivatives of the \mathbf{j}_3 current with respect to n_{ls}^p ,

$$\frac{\partial \mathbf{j}_3|_z}{\partial n_{ls}^p} = \left[\frac{k_z^p}{\epsilon_s^p} - \frac{g_\omega^2}{m_\omega^2} J_1 \frac{\partial \mathbf{j}_B|_z}{\partial n_{ls}^p} \right] \left[1 + \frac{g_\rho^2}{4m_\rho^2} J_2 \right]^{-1}, \quad (\text{A35})$$

$$\frac{\partial \mathbf{j}_3|_\perp}{\partial n_{ls}^p} = 0, \quad (\text{A36})$$

and with respect to neutron density n_{ls}^n ,

$$\frac{\partial \mathbf{j}_3|_z}{\partial n_{ls}^n} = \left[\frac{-k_{lz}^n}{\epsilon_{ls}^n} - \frac{g_\omega^2}{m_\omega^2} J_1 \frac{\partial \mathbf{j}_B|_z}{\partial n_{ls}^n} \right] \left[1 + \frac{g_\rho^2}{4m_\rho^2} J_2 \right]^{-1}, \quad (\text{A37})$$

$$\frac{\partial \mathbf{j}_3|_\perp}{\partial n_{ls}^n} = \left[\frac{-(\epsilon_{\perp l}^n - s \mu_N \kappa_n B) \mathbf{k}^n|_\perp}{\epsilon_{\perp l}^n \epsilon_{is}^n} + \frac{g_\omega^2}{m_\omega^2} J_3 \frac{\partial \mathbf{j}_B|_\perp}{\partial n_{ls}^n} \right] \times \left[1 + \frac{g_\rho^2}{4m_\rho^2} J_3 \right]^{-1}. \quad (\text{A38})$$

To solve the coupled set of Eqs. (A31)–(A38) for the derivatives of \mathbf{j}_B and \mathbf{j}_3 the following formula can be used in a straightforward way. For the system

$$y_1 = \frac{a + by_2}{1 + d}, \quad y_2 = \frac{c + ey_1}{1 + f}, \quad (\text{A39})$$

the solutions are

$$y_1 = \frac{a(1 + f) + bc}{(1 + d)(1 + f) - be}, \quad y_2 = \frac{c(1 + d) + ea}{(1 + d)(1 + f) - be}. \quad (\text{A40})$$

- [1] T. Nilsson, *Eur. Phys. J.* **1**, 1 (2008).
[2] P. Jardin *et al.*, *Rev. Sci. Instrum.* **81**, 02A909 (2010).
[3] P. Schmor *et al.*, Development and Future Plans at ISAC, LINAC2004, Lübeck, Germany, August 2004.
[4] FRIB website, [<http://www.frib.msu.edu/about>].
[5] Y. Blumenfeld, *Nucl. Instrum. Methods B* **266**, 4074 (2008).
[6] B. Bastin *et al.*, *Phys. Rev. Lett.* **99**, 022503 (2007).
[7] J. J. Valiente-Dobón *et al.*, *Phys. Rev. Lett.* **102**, 242502 (2009).
[8] M. A. Perez-Garcia, *Eur. Phys. J. A* **44**, 77 (2010), C. J. Horowitz and M. A. Perez-Garcia, *Phys. Rev. C* **68**, 025803 (2003).

- [9] A. Rabhi and C. Providência, *J. Phys. G: Nucl. Part. Phys.* **37**, 075102 (2010); D. P. Menezes and C. Providência, *Phys. Rev. C* **69**, 045801 (2004); P. K. Panda, C. Providência, and D. P. Menezes, *ibid.* **82**, 045801 (2010).
[10] H. Tsuruta and A. G. W. Cameron, *Can. J. Phys.* **43**, 2056 (1965).
[11] C. J. Horowitz, M. A. Perez-Garcia, and J. Piekarewicz, *Phys. Rev. C* **69**, 045804 (2004); C. J. Horowitz, M. A. Perez-Garcia, J. Carriere, D. K. Berry, and J. Piekarewicz, *Phys. Rev. C* **70**, 065806 (2004); C. J. Horowitz, M. A. Perez-Garcia, D. K. Berry, and J. Piekarewicz, *ibid.* **72**, 035801 (2005); S. S. Avancini, L. Brito, J. R. Marinelli, D. P. Menezes, M. M. W. de

- Moraes, C. Providência, and A. M. Santos, *ibid.* **79**, 035804 (2009); S. S. Avancini, S. Chiacchiera, D. P. Menezes, and C. Providência, *ibid.* **82**, 055807 (2010).
- [12] H. Caines, [arXiv:0911.3211v1](https://arxiv.org/abs/0911.3211v1) [nucl-ex].
- [13] K. Aamodt *et al.* (ALICE Collaboration), *J. Instrum.* **3**, S08002 (2008).
- [14] H. Muther, *Prog. Part. Nucl. Phys.* **30**, 1 (1993).
- [15] W.-M. Yao *et al.* (Particle Data Group), *J. Phys. G* **33**, 1 (2006).
- [16] V. V. Skokov *et al.*, *Int. J. Mod. Phys. A* **24**, 5925 (2009).
- [17] N. K. Glendenning, *Compact Stars* (Springer-Verlag, New-York, 2000).
- [18] V. V. Usov, *Nature (London)* **357**, 472 (1992).
- [19] B. Paczyński, *Acta Astron.* **42**, 145 (1992).
- [20] R. C. Duncan and C. Thompson, *Astrophys. J.* **392**, L9 (1992); C. Thompson and R. C. Duncan, *MNRAS* **275**, 255 (1995).
- [21] A. K. Harding and D. Lai, *Rep. Prog. Phys.* **69**, 2631 (2006).
- [22] SGR/APX online catalogue, [<http://www.physics.mcgill.ca/~pulsar/magnetar/main.html>].
- [23] A. Broderick, M. Prakash, and J. M. Lattimer, *Astrophys. J.* **537**, 351 (2000).
- [24] A. Rabhi, C. Providência, and J. da Providência, *J. Phys. G: Nucl. Part. Phys.* **35**, 125201 (2008).
- [25] A. Rabhi, H. Pais, P. K. Panda, and C. Providência, *J. Phys. G: Nucl. Part. Phys.* **36**, 115204 (2009).
- [26] S. Chakrabarty, *Phys. Rev. D* **54**, 1306 (1996); S. Chakrabarty, D. Bandyopadhyay, and S. Pal, *Phys. Rev. Lett.* **78**, 2898 (1997).
- [27] B. D. Serot and J. D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986); J. Boguta and A. R. Bodmer, *Nucl. Phys. A* **292**, 413 (1977).
- [28] D. Lai and S. Shapiro, *Astrophys. J.* **383**, 745 (1991).
- [29] L. D. Landau, *Zh. Eksp. Teor. Fiz.* **30**, 1058 (1956) [*Sov. Phys. JETP* **3**, 920 (1957)].
- [30] L. D. Landau, *Zh. Eksp. Teor. Fiz.* **32**, 59 (1957) [*Sov. Phys. JETP* **5**, 101 (1957)].
- [31] G. Baym and C. Pethick, *Landau Fermi-Liquid Theory* (John Wiley & Sons, New York, 1991).
- [32] J. C. Caillon, P. Gabinski, and J. Labarsouque, *Nuc. Phys. A* **696**, 623 (2001).
- [33] J. C. Caillon, P. Gabinski, and J. Labarsouque, *J. Phys. G* **28**, 189 (2002).
- [34] T. Matsui, *Nucl. Phys. A* **370**, 365 (1981).
- [35] M. A. Perez-Garcia, *Phys. Rev. C* **77**, 065806 (2008).
- [36] M. A. Perez-Garcia, J. Navarro, and A. Polls, *Phys. Rev. C* **80**, 025802 (2009).
- [37] M. A. Perez-Garcia, *Phys. Rev. C* **80**, 045804 (2009).
- [38] P. Bernardos, S. Marcos, R. Niembro, and M. L. Quelle, *Phys. Lett. B* **356**, 175 (1995).
- [39] Y. Sugahara and H. Toki, *Prog. Theor. Phys.* **92**, 803 (1994).
- [40] Robert C. Duncan, [arXiv:astro-ph/0002442v1](https://arxiv.org/abs/astro-ph/0002442v1).
- [41] Nicolas Chamel and Pawel Haensel, *Living Rev. Relativity* **11**, 10 (2008).
- [42] M. A. Perez-Garcia, J. Silk, and J. R. Stone, *Phys. Rev. Lett.* **105**, 141101 (2010).