

**Strong decay of low-lying  $S_{11}$  and  $D_{13}$  nucleon resonances to pseudoscalar mesons and octet baryons**

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Partial decay widths of lowest-lying nucleon resonances  $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $D_{13}(1520)$ , and  $D_{13}(1700)$  to the pseudoscalar mesons and octet baryons are studied within a chiral constituent quark model. Effects of the configurations mixing between the states  $|N_8^2 P_M\rangle$  and  $|N_8^4 P_M\rangle$  are considered, taking into account  $SU(6) \otimes O(3)$  breaking effects. In addition, possible contributions of the strangeness components in the  $S_{11}$  resonances are investigated. Experimental data for the partial decay widths of the  $S_{11}$  and  $D_{13}$  resonances are well reproduced. Contributions from five-quark components in the  $S_{11}$  resonances are found crucial in reproducing the partial widths. Predictions for coupling constants of the four nucleon resonances to pseudoscalar mesons and octet baryons, crucial issues in the photo- and hadron-induced meson production reactions, are reported.

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**I. INTRODUCTION**

Production of mesons with hidden or open strangeness via electromagnetic or hadronic probes, in the baryon resonance energy range, is subject to extensive experimental and theoretical investigations. In this realm, partial decay widths of resonances to meson-baryon final states, as well as the relevant coupling constants, are crucial, but not well enough known [1], ingredients in our understanding of the reaction mechanisms and also of the nature of those resonances. Phenomenological approaches, dealing with the above ingredients, arise mainly from two families of formalisms: effective Lagrangians based on meson-baryon degrees of freedom [2–30] and QCD based/inspired models [31–46].

Among the low-lying nucleon excitations, the  $S_{11}(1535)$  resonance plays a special role due to its large  $\eta N$  decay width [1], though its mass is very close to the threshold of the decay. Moreover, in the  $KY$  production reactions the importance of the  $S_{11}(1650)$  is well established. For the two other first orbitally excited (quark model prediction) nucleon resonances,  $D_{13}(1520)$  and  $D_{13}(1700)$ , the couplings to the pseudoscalar meson and octet baryons seem to be rather weak, but the first one is known to intervene significantly in polarization asymmetries.

The observables of interest in this paper are partial decay widths. Experimental values are available [1] for all four resonances' decay to  $\pi N$  and  $\eta N$  final states, as well as for the  $S_{11}(1650)$  and  $D_{13}(1700)$  resonances to  $K\Lambda$ , although with rather large uncertainties. However, in spite of extensive studies mentioned above, to our knowledge no single formalism has reproduced *simultaneously* those partial widths. The only exception here is a very recent comprehensive study [46] based on the  $1/N_C$  expansion approach. In addition to the fact that a large number of investigations concentrate on the  $S_{11}$  resonances, recent copious photoproduction data have not yet been fully exploited by sophisticated coupled-channels

phenomenological approaches. The main motivation of the present work is then to study those partial decay widths within a QCD inspired formalism and shed light on the structure of those baryons.

The theoretical frame of the present work is based on a chiral constituent quark model ( $\chi$ CQM), complemented with the  $SU(6) \otimes O(3)$  symmetry-breaking effects. The outcomes of those formalisms are compared to the known [1] partial decay widths of the above-mentioned resonances. This approach gives satisfactory results for the  $D_{13}$  resonances but misses partly the data for  $S_{11}$ .

Attempting to alleviate the observed theoretical and experimental discrepancies, the  $\chi$ CQM is subsequently complemented with including contributions from higher Fock components, namely five-quark configurations. Actually, several authors [47–54] have shown that contributions from the five-quark components are quite significant in describing the properties of baryons and their electromagnetic and strong decays, especially contributions from the  $qqqq\bar{q} \rightarrow M(\gamma) + qqq$  transitions. For recent reviews on five-quark components in baryons; see Refs. [55–57].

The extended  $\chi$ CQM allows us to reproduce the known partial decay widths for both  $S_{11}$  resonances. Following the successful results obtained for low-lying baryon resonances, we put forward predictions for the coupling constants of those resonances to seven meson-baryon final states, i.e.,  $\pi^0 p$ ,  $\pi^+ n$ ,  $\eta p$ ,  $K^+ \Lambda$ ,  $K^0 \Sigma^+$ ,  $K^+ \Sigma^0$ ,  $\eta' p$ .

The present manuscript is organized in the following way: In Sec. II, we present the theoretical formalism which includes the wave functions, strong decays, and the resulting transition amplitudes for the nucleon resonances  $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $D_{13}(1520)$ , and  $D_{13}(1700)$  to the pseudoscalar mesons and octet baryons. Numerical results are given in Sec. III and, finally, Sec. IV contains a summary and conclusions.

**II. THEORETICAL FORMALISM**

In Sec. II A, we present the wave functions of the nucleon resonances  $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $D_{13}(1520)$ , and  $D_{13}(1700)$ . Section II B embodies a brief review of the formalism for the strong decay of the baryon resonances to meson-baryon in a  $\chi$ CQM, where we derive transition coupling amplitudes for

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the above four nucleon resonances to the  $\pi N$ ,  $\eta N$ ,  $K\Lambda$ ,  $K\Sigma$ , and  $\eta' N$  channels.

### A. Wave functions

In the  $\chi$ CQM, complemented with five-quark components, a baryon is a superposition of a three- and five-quark mixture and the wave function can be written as

$$|B\rangle = A_3|qqq\rangle + A_5|qqqq\bar{q}\rangle, \quad (1)$$

with  $A_3$  and  $A_5$  the probability amplitudes for the corresponding  $qqq$  and  $qqqq\bar{q}$  states, respectively.

For the three-quark components, we employ the wave functions in traditional three-quark  $\chi$ CQM. In the  $SU(6) \otimes O(3)$  conserved case, the general form for the wave functions of the octet baryons, the  $N(\frac{2}{8}P_M)_{S^-}$  and  $N(\frac{4}{8}P_M)_{S^-}$  states, can be expressed as

$$|B(\frac{2}{8}S_S)_{\frac{1}{2}^+, S_z}\rangle = \frac{1}{\sqrt{2}} \left( |B\rangle_{\lambda} \left| \frac{1}{2}, s_z \right\rangle_{\lambda} + |B\rangle_{\rho} \left| \frac{1}{2}, s_z \right\rangle_{\rho} \right) \varphi_{000}^s(\vec{\lambda}, \vec{\rho}), \quad (2)$$

$$|N(\frac{2}{8}P_M)_{S^-, S_z}\rangle = \frac{1}{2} \sum_{m, s_z} C_{1m, \frac{1}{2} s_z}^{SS_z} \left[ \left( |N\rangle_{\rho} \left| \frac{1}{2}, s_z \right\rangle_{\lambda} + |N\rangle_{\lambda} \left| \frac{1}{2}, s_z \right\rangle_{\rho} \right) \varphi_{11m}^{\rho} \times (\vec{\lambda}, \vec{\rho}) + \left( |N\rangle_{\rho} \left| \frac{1}{2}, s_z \right\rangle_{\rho} - |N\rangle_{\lambda} \left| \frac{1}{2}, s_z \right\rangle_{\lambda} \right) \varphi_{11m}^{\lambda}(\vec{\lambda}, \vec{\rho}) \right], \quad (3)$$

$$|N(\frac{4}{8}P_M)_{S^-, S_z}\rangle = \frac{1}{\sqrt{2}} \sum_{m, s_z} C_{1m, \frac{3}{2} s_z}^{SS_z} \left[ |N\rangle_{\rho} \left| \frac{3}{2}, s_z \right\rangle_{\rho} \varphi_{11m}^{\rho}(\vec{\lambda}, \vec{\rho}) + |N\rangle_{\lambda} \left| \frac{3}{2}, s_z \right\rangle_{\lambda} \varphi_{11m}^{\lambda}(\vec{\lambda}, \vec{\rho}) \right], \quad (4)$$

where  $|B\rangle_{\rho(\lambda)}$  denotes the mixed symmetric flavor wave function of the three-quark system for the corresponding baryon and  $|\frac{1}{2}, s_z\rangle_{\rho(\lambda)}$  and  $|\frac{3}{2}, s_z\rangle_{\rho(\lambda)}$  are the mixed symmetric and symmetric spin wave functions of the three-quark system, respectively.  $\varphi_{Nlm}(\vec{\lambda}, \vec{\rho})$  is the harmonic oscillator basis orbital wave function for the three quarks with the subscripts  $Nlm$  being the corresponding quantum numbers. Finally,  $C_{1m, s_z}^{SS_z}$  are the Clebsch-Gordan coefficients for the coupling of the orbital and spin of the three-quark system to form a baryon state with spin  $S$  and  $z$ -component  $S_z$ . The explicit forms for all of the above flavor, spin, and orbital wave functions can be found in Ref. [54].

Taking into account the breakdown of  $SU(6) \otimes O(3)$  symmetry due to either the color-magnetic [58] or flavor-magnetic [59] hyperfine interactions between the quarks, one can express the wave functions of the  $S_{11}$  and  $D_{13}$  resonances in terms of the given  $N(\frac{2}{8}P_M)_{S^-}$  and  $N(\frac{4}{8}P_M)_{S^-}$  wave functions, Eqs. (3) and (4), by introducing the configuration mixing angles  $\theta_S$  and  $\theta_D$

$$\begin{pmatrix} |S_{11}(1535)\rangle \\ |S_{11}(1650)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_S & -\sin\theta_S \\ \sin\theta_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} |N(\frac{2}{8}P_M)_{\frac{1}{2}^-}\rangle \\ |N(\frac{4}{8}P_M)_{\frac{1}{2}^-}\rangle \end{pmatrix}, \quad (5)$$

$$\begin{pmatrix} |D_{13}(1520)\rangle \\ |D_{13}(1700)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_D & -\sin\theta_D \\ \sin\theta_D & \cos\theta_D \end{pmatrix} \begin{pmatrix} |N(\frac{2}{8}P_M)_{\frac{3}{2}^-}\rangle \\ |N(\frac{4}{8}P_M)_{\frac{3}{2}^-}\rangle \end{pmatrix}. \quad (6)$$

For the octet baryons, other than the lowest-lying  $S_{11}$  and  $D_{13}$ , the configuration mixing effects are not so significant. So, for those baryons we take the wave functions within the exact  $SU(6) \otimes O(3)$  symmetry.

For the five-quark components of  $S_{11}(1535)$ , we use the wave functions given in Ref. [53],

$$\begin{aligned} \psi_{r,s} = & \sum_{a,b,c} \sum_{Y,y,T_z,t_z} \sum_{S_z,s_z} C_{[31]_a[211]_a}^{[1^4]} C_{[F]_b[S]_c}^{[31]_a} [F]_{b,Y,T_z} [S]_{c,S_z} \\ & \times [211; C]_a(Y, T, T_z, y, \bar{t}, t_z | 1, 1/2, t) \\ & \times (S, S_z, 1/2, s_z | 1/2, s) \bar{\chi}_{y,t_z} \bar{\xi}_{s_z} \varphi_{[5]}. \end{aligned} \quad (7)$$

In fact, this general wave function is appropriate for the five-quark components in all the low-lying nucleon resonances with  $S^P = \frac{1}{2}^-$ , albeit with different probabilities for five-quark components.

As reported in Ref. [53], there are five different flavor-spin configurations which may form five-quark components in the resonances with negative parity. If the hyperfine interaction between the quarks is assumed to depend on flavor and spin, the energy of the second and third configurations should be about 80 and 200 MeV higher than the first configuration, respectively. Since  $S_{11}(1535)$  and  $S_{11}(1650)$  are the first two orbital excitations of the nucleon with spin 1/2, the configurations with low energies, namely the first two five-quark configurations should be the most appropriate ones to form higher Fock components in those two resonances. Moreover, the contribution of the second five-quark configuration is very similar to that of the first one, because of the same flavor structure, which rules out the five-quark components with light quark and antiquark pairs in the  $S_{11}$  resonances. Actually, the transition elements between all five five-quark configurations and the octet baryons differ just by constant factors. Therefore, the contributions from all five configurations are similar, albeit with appropriate probability amplitudes. Consequently, the first configuration is enough for us to study the strong decays of  $S_{11}(1535)$  and  $S_{11}(1650)$ . The wave functions for the five-quark components in  $S_{11}(1535)$  and  $S_{11}(1650)$  then reduce to the following form:

$$\begin{aligned} \psi_{5q} = & \sum_{abc} C_{[31]_a[211]_a}^{[1^4]} C_{[211]_b[22]_c}^{[31]_a} [4]_X [211]_F(b) [22]_S(c) \\ & \times [211]_C(a) \bar{\chi}_{s_z} \varphi(\{\xi_i\}), \end{aligned} \quad (8)$$

the explicit form of which is given in Ref. [52].

Following Eq. (5), the introduction of five-quark wave functions leads to

$$|S_{11}(1535)\rangle = A_3 [\cos\theta_S |N(\frac{2}{8}P_M)_{\frac{1}{2}^-}\rangle - \sin\theta_S |N(\frac{4}{8}P_M)_{\frac{1}{2}^-}\rangle] + A_5 \psi_{5q}, \quad (9)$$

$$|S_{11}(1650)\rangle = A'_3 [\sin\theta_S |N(\frac{2}{8}P_M)_{\frac{1}{2}^-}\rangle + \cos\theta_S |N(\frac{4}{8}P_M)_{\frac{1}{2}^-}\rangle] + A'_5 \psi_{5q}. \quad (10)$$

The probability amplitude for the five-quark component in a baryon can be related to the coupling  $s_q \langle \hat{V}_{\text{cou}} \rangle_{3q}$  between the

$qqq$  and  $qqqq\bar{q}$  configurations in the corresponding baryon

$$A_{5q} = \frac{5q \langle \hat{V}_{\text{cou}} \rangle_{3q}}{M - E_5}, \quad (11)$$

with  $E_5$  the energy of the five-quark component. Given that the resonances considered here have negative parity, all of the quarks and antiquark in the five-quark system should be in their ground states. Hence, we can take  $\hat{V}_{\text{cou}}$  to be of the following form:

$$\hat{V}_{\text{cou}} = 3V(r_{34}) \frac{\hat{\sigma}_3 \cdot \vec{p}_3}{2m_3} \chi_{00}^{45} C_{00}^{45} F_{00}^{45} \varphi_{00}(\vec{p}_4 - \vec{p}_5) b_4^\dagger(\vec{p}_4) d_5^\dagger(\vec{p}_5), \quad (12)$$

where  $\chi_{00}^{45}$ ,  $C_{00}^{45}$ ,  $F_{00}^{45}$ , and  $\varphi_{00}(\vec{p}_4 - \vec{p}_5)$  denote the spin, flavor, color, and orbital singlets of the quark and antiquark pair, respectively, and  $b_4^\dagger(\vec{p}_4)$  and  $d_5^\dagger(\vec{p}_5)$  are the creation operators for a quark and antiquark pair with momentum  $\vec{p}_4$  and  $\vec{p}_5$ , respectively.  $V(r_{34})$  is the coupling potential which depends on the relative coordinate  $|\vec{r}_3 - \vec{r}_4|$ . We then obtain

$$\frac{\langle \psi_{5q} | \hat{V}_{\text{cou}} | N(\frac{2}{8} P_M)_{\frac{1}{2}^-} \rangle}{\langle \psi_{5q} | \hat{V}_{\text{cou}} | N(\frac{4}{8} P_M)_{\frac{1}{2}^-} \rangle} = -2 \quad (13)$$

and

$$\frac{A'_{5q}}{A_{5q}} = \frac{\sin\theta_S - \frac{1}{2}\cos\theta_S}{\cos\theta_S + \frac{1}{2}\sin\theta_S} \frac{M_{S_{11}(1535)} - E_5}{M_{S_{11}(1650)} - E_5}. \quad (14)$$

Here we emphasize that the considered  $D_{13}$  resonances are not relevant for five-quark components issues. Actually, all of the quarks and antiquark should be in their ground states (lowest energy) to form the negative parity. The spin configuration of the four-quark subsystem then is limited to be  $[31]_S$ , for which the total spin of the four-quark subsystems is  $S = 1$ , in order to combine with the antiquark to form the required total spin  $3/2$ . For the configurations with spin  $[31]_S$ , the flavor-spin overlap factors between such five-quark configurations and the  $D_{13}$  states vanish. Therefore,

the probabilities for these five-quark components in the  $D_{13}$  resonances are 0. Some additional five-quark configurations, other than those given in Ref. [53], could also be considered, for instance, the configurations with the antiquark orbitally excited ( $l_{\bar{q}} = 2, 4, \dots$ ), the ones in which the four-quark subsystem with spin symmetry  $[4]_S$  ( $S_4 = 2$ ), or the ones given in Ref. [49] with the four quark subsystem orbital symmetry  $[31]_X$  and orbital momentum  $L_4 = 2, 4, \dots$ . However, all those configurations have very high energies, far away from the lowest-lying  $D_{13}$  resonances masses.

Finally, we do not consider the five-quark components in the ground states of octet baryons in this manuscript, because, on the one hand, their probabilities in the baryons are very small [48,60] and, on the other hand, their contributions to electromagnetic and strong decays of nucleon resonances are negligible [52]. Actually, the five-quark configurations in the ground states of octet baryons cannot transit to three-quark components of the first orbitally excited baryon resonances due to the vanishing flavor-spin overlap factors.

## B. Formalism for strong decay

It is well known that the pseudoscalar meson-quark coupling, in the tree level approximation, takes the form

$$H_M = \sum_j \frac{g_A^q}{2f_M} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \partial^\mu \phi_M, \quad (15)$$

where  $\psi_j$  and  $\phi_M$  are the quark and pseudoscalar fields, respectively, and  $g_A^q$  is the axial coupling constant for the constituent quarks, the value of which is in the range 0.7–1.26 [3,61,62].  $f_M$  denotes the decay constant of the corresponding meson; the empirical values for the decay constants of  $\pi$ ,  $K$ ,  $\eta$ , and  $\eta'$  are  $f_\pi = 93$  MeV,  $f_K = 113$  MeV,  $f_\eta = 1.2f_\pi$ ,  $f_{\eta'} = -0.58f_\pi$ .

In the framework of the nonrelativistic  $qqq$  quark model, the coupling, Eq. (15), takes the following form:

$$H_M^{\text{NR}(3)} = \sum_j \frac{g_A^q}{2f_M} \left( \frac{\omega_M}{E_f + M_f} \sigma \cdot \vec{P}_f + \frac{\omega_M}{E_i + M_i} \sigma \cdot \vec{P}_i - \sigma \cdot \vec{k}_M + \frac{\omega_M}{2\mu} \sigma \cdot \vec{p}_j \right) X_M^j \exp\{-i\vec{k}_M \cdot \vec{r}_j\}, \quad (16)$$

where  $\vec{k}_M$  and  $\omega_M$  are the three-momentum and energy of the final meson,  $\vec{P}_{i(f)}$  and  $M_{i(f)}$  denote the mass and three-momentum of the initial (final) baryon,  $\vec{p}_j$  and  $\vec{r}_j$  the three-momentum and coordinate of the  $j^{\text{th}}$  quark, and  $\mu$  is the reduced mass of the initial and final  $j^{\text{th}}$  quark which emits the meson. Finally,  $X_M^j$  is the flavor operator for emission of the meson from the corresponding  $j^{\text{th}}$  quark, given by following expressions:

$$\begin{aligned} X_{\pi^0}^j &= \lambda_3^j, & X_{\pi^\pm}^j &= \mp \frac{1}{\sqrt{2}} (\lambda_1^j \mp \lambda_2^j), \\ X_{K^\pm}^j &= \mp \frac{1}{\sqrt{2}} (\lambda_4^j \mp \lambda_5^j), & X_{K^0}^j &= \mp \frac{1}{\sqrt{2}} (\lambda_6^j \mp \lambda_7^j), \\ X_\eta^j &= \cos\theta \lambda_8^j - \sin\theta \sqrt{\frac{2}{3}} \mathcal{I}, & X_{\eta'}^j &= \sin\theta \lambda_8^j + \cos\theta \sqrt{\frac{2}{3}} \mathcal{I}, \end{aligned} \quad (17)$$

where  $\lambda_i^j$  are the SU(3) Gell-Mann matrices, and  $\mathcal{I}$  is the unit operator in the SU(3) flavor space, and  $\theta$  denotes the mixing angle between  $\eta_1$  and  $\eta_8$ , leading to the physical  $\eta$  and  $\eta'$

$$\eta = \eta_8 \cos\theta - \eta_1 \sin\theta, \quad (18)$$

$$\eta' = \eta_8 \sin\theta + \eta_1 \cos\theta, \quad (19)$$

where  $\theta = -23^\circ$  [63].

Taking into account the five-quark components in the resonances, we have to calculate the transition coupling amplitudes for  $qqqq\bar{q} \rightarrow qqq + M$ . The reduced form of the coupling in Eq. (15) reads

$$\begin{aligned} H_M^{\text{NR}(5)} &= \sum_j \frac{g_A^q}{2f_M} C_{\text{XFSC}}^j(m_i + m_f) \bar{\chi}_z^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \chi_z^j X_M^j \\ &\times \exp\{-i\vec{k}_M \cdot \vec{r}_j\}, \end{aligned} \quad (20)$$

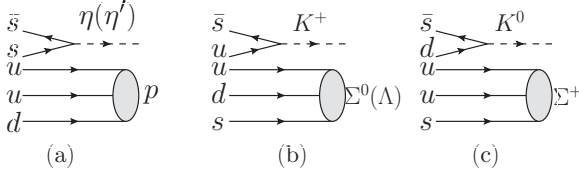


FIG. 1. Strangeness component transit in the  $S_{11}$  resonances to  $\eta p$  or  $\eta' p$  (a),  $K^+ \Lambda$  or  $K^+ \Sigma^0$  (b), and  $K^0 \Sigma^+$  (c).

where  $m_i$  and  $m_f$  denote the constituent masses of the quark and antiquark which combine to form a pseudoscalar meson and  $C_{\text{XFSC}}^j$  denotes the overlap between the three-quark configuration of the final baryon and the residual orbital-flavor-spin-color configuration of the three-quark system that is left in the initial  $qqqq\bar{q}$  after the combination of the  $j^{\text{th}}$  quark with the antiquark into a final meson. The transitions  $qqqq\bar{s} \rightarrow B + M$  scheme is shown in Fig. 1. where three quarks of the five-quark system go as spectators to form the final three-quark baryon, and the fourth quark gets combined with the strange antiquark to form a meson:  $K$ ,  $\eta$ , or  $\eta'$ .

The transition coupling amplitude for a resonance to a pseudoscalar meson and a octet baryon then is obtained by calculating the following matrix element:

$$T^{MB} = \langle B(\frac{2}{8}S_S)_{\frac{1}{2}^+} | (H_M^{\text{NR}(3)} + H_M^{(5)}) | N^* \rangle \equiv T_3^{MB} + T_5^{MB}, \quad (21)$$

TABLE I. Transition coupling amplitudes  $T_3^{MB}$  for the low-lying  $S_{11}$  and  $D_{13}$  resonances to meson-baryon final states. Note that the full amplitudes are obtained by multiplying each term by the following expressions:  $\frac{g_A^q}{2f_M} \omega_3 [(a_M - \frac{b_M}{3}) \frac{k_M^2}{\omega_3^2} - 3b_M] \exp\{-\frac{k_M^2}{6\omega_3^2}\}$  for  $S_{11}$  and  $\frac{g_A^q}{2f_M} (a_M - \frac{b_M}{3}) \frac{k_M^2}{\omega_3^2} \exp\{-\frac{k_M^2}{6\omega_3^2}\}$  for  $D_{13}$  resonances. Here,  $\omega_3$  is the harmonic oscillator parameter for the three-quark components  $a_M = 1 + \frac{\omega_M}{E_f + M_f}$  and  $b_M = \frac{\omega_M}{2\mu}$ .

	$S_{11}(1535)$	$S_{11}(1650)$
$\pi^0 p$	$\frac{\sqrt{2}}{9}(2\cos\theta_S - \sin\theta_S)$	$\frac{\sqrt{2}}{9}(2\sin\theta_S + \cos\theta_S)$
$\pi^+ n$	$-\frac{2}{9}(2\cos\theta_S - \sin\theta_S)$	$-\frac{2}{9}(2\sin\theta_S + \cos\theta_S)$
$\eta p$	$\frac{\sqrt{2}}{3}(\cos\theta_S + \sin\theta_S)(\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)$	$\frac{\sqrt{2}}{3}(\sin\theta_S - \cos\theta_S)(\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)$
$K^0 \Lambda$	$-\frac{1}{\sqrt{6}}\cos\theta_S$	$-\frac{1}{\sqrt{6}}\sin\theta_S$
$K^0 \Sigma^+$	$-\frac{1}{9}(\cos\theta_S + 4\sin\theta_S)$	$-\frac{1}{9}(\sin\theta_S - 4\cos\theta_S)$
$K^+ \Sigma^0$	$-\frac{1}{9\sqrt{2}}(\cos\theta_S + 4\sin\theta_S)$	$-\frac{1}{9\sqrt{2}}(\sin\theta_S - 4\cos\theta_S)$
$\eta' p$	$\frac{\sqrt{2}}{3}(\cos\theta_S + \sin\theta_S)(\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)$	$\frac{\sqrt{2}}{3}(\sin\theta_S - \cos\theta_S)(\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)$
	$D_{13}(1520)$	$D_{13}(1700)$
$\pi^0 p$	$-\frac{2}{9}(2\cos\theta_D - \frac{1}{\sqrt{10}}\sin\theta_D)$	$-\frac{2}{9}(2\sin\theta_D + \frac{1}{\sqrt{10}}\cos\theta_D)$
$\pi^+ n$	$\frac{2\sqrt{2}}{9}(2\cos\theta_D - \frac{1}{\sqrt{10}}\sin\theta_D)$	$\frac{2\sqrt{2}}{9}(2\sin\theta_D + \frac{1}{\sqrt{10}}\cos\theta_D)$
$\eta p$	$-\frac{2}{3}(\cos\theta_D + \frac{1}{\sqrt{10}}\sin\theta_D)(\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)$	$-\frac{2}{3}(\sin\theta_D - \frac{1}{\sqrt{10}}\cos\theta_D)(\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)$
$K^0 \Lambda$	$\frac{1}{\sqrt{3}}\cos\theta_D$	$\frac{1}{\sqrt{3}}\sin\theta_D$
$K^0 \Sigma^+$	$\frac{1}{9}(\sqrt{2}\cos\theta_D + \frac{4}{\sqrt{5}}\sin\theta_D)$	$\frac{1}{9}(\sqrt{2}\sin\theta_D - \frac{4}{\sqrt{5}}\cos\theta_D)$
$K^+ \Sigma^0$	$-\frac{1}{9\sqrt{2}}(\sqrt{2}\cos\theta_D + \frac{4}{\sqrt{5}}\sin\theta_D)$	$-\frac{1}{9\sqrt{2}}(\sqrt{2}\sin\theta_D - \frac{4}{\sqrt{5}}\cos\theta_D)$
$\eta' p$	$-\frac{2}{3}(\cos\theta_D + \frac{1}{\sqrt{10}}\sin\theta_D)(\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)$	$-\frac{2}{3}(\sin\theta_D - \frac{1}{\sqrt{10}}\cos\theta_D)(\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)$

the resulting transition coupling amplitudes  $T_3^{MB}$  and  $T_5^{MB}$  for the  $S_{11}$  and  $D_{13}$  resonances to  $\pi^0 p$ ,  $\pi^+ n$ ,  $\eta p$ ,  $K^+ \Lambda$ ,  $K^0 \Sigma^+$ ,  $K^+ \Sigma^0$ , and  $\eta' p$  channels are shown in Tables I and II, respectively. Notice that (Table I), within the exact  $SU(6) \otimes O(3)$  symmetry, the matrix elements for transition  $N(\frac{4}{8}P_M)_S \rightarrow K \Lambda$  vanish, and, hence, the decay widths of  $S_{11}(1650)$  and  $D_{13}(1700)$  to  $K \Lambda$  are null. Moreover, the transition elements for  $5q \rightarrow MB$  do not vanish when  $k_M = 0$ , and it may enhance or depress the transitions  $S_{11} \rightarrow MB$  significantly near the meson-baryon threshold (Table II). Finally, the strangeness component does not transit to  $\pi^0 p$ , since the matrix element of the flavor operator  $X_{\pi^0}^j$  between the  $s\bar{s}$  pair is 0.

To obtain the relevant expressions for partial decay widths, we take the Lagrangian for  $N^*MB$  coupling in the hadronic level to be of the following form:

$$\mathcal{L}_{S_{11}BM} = -i g_{S_{11}BM} \bar{\psi}_B \phi_M \psi_{S_{11}} + \text{H.c.}, \quad (22)$$

$$\mathcal{L}_{D_{13}BM} = \frac{1}{m_M} g_{D_{13}BM} \bar{\psi}_B \partial_\mu \phi_M \psi_{D_{13}}^\mu + \text{H.c.}, \quad (23)$$

where  $\bar{\psi}_B$  and  $\psi_{S_{11}}$  denote the Dirac spinor fields for the final baryon and the  $S_{11}$  resonances, respectively, and  $\phi_M$  is the scalar field for the final meson.

For the  $D_{13}$  resonances, with spin 3/2, we employ the Rarita-Schwinger vector-spinor fields  $\psi_{D_{13}}^\mu$  [64,65], which are

TABLE II. Transition coupling amplitudes  $T_5^{MB}$ . Note that the full amplitudes are obtained by multiplying each term by the following expression,  $\frac{g_A}{2f_M} C_{35} \exp\{-\frac{3k_M^2}{20\omega_3^2}\}$ , with  $C_{35}$  related to the harmonic oscillator parameter for the three- and five-quark components as  $C_{35} = (\frac{2\omega_3\omega_5}{\omega_3^2+\omega_5^2})^3$ .

$\pi^0 p$	$\pi^+ n$	$\eta p$	$K^+ \Lambda$	$K^0 \Sigma^+$	$K^+ \Sigma^0$	$\eta' p$
0	0	$\frac{2}{\sqrt{3}} m_s (2\cos\theta + \sqrt{2}\sin\theta)$	$\frac{1}{\sqrt{3}} (m + m_s)$	$\sqrt{2} (m + m_s)$	$-(m + m_s)$	$\frac{2}{\sqrt{3}} m_s (2\sin\theta - \sqrt{2}\cos\theta)$

defined as

$$\psi_{D_{13}}^\mu(S_z) = \sum_{ms} C_{1m, \frac{1}{2}^3}^{\frac{3}{2} S_z} \epsilon_m^\mu u_s. \quad (24)$$

One can directly obtain the transition coupling amplitudes for  $N^* \rightarrow MB$  in the hadronic level using the Lagrangian, Eq. (23). Then, the coupling constants  $g_{N^*MB}$  are extracted by comparing the transition coupling amplitudes  $T^{MB}$  in the quark model to those in the hadronic model.

With the resulting coupling constants, the strong decay widths for the  $S_{11}$  and  $D_{13}$  resonances to the pseudoscalar meson and octet baryon read

$$\Gamma_{S_{11} \rightarrow MB} = \frac{1}{4\pi} g_{S_{11}MB}^2 \frac{E_f + M_f}{M_i} |\vec{k}_M|, \quad (25)$$

$$\Gamma_{D_{13} \rightarrow MB} = \frac{1}{12\pi} \frac{1}{m_M^2} g_{D_{13}MB}^2 \frac{E_f - M_f}{M_i} |\vec{k}_M|^3. \quad (26)$$

Note that in the center-of-mass frame of the initial resonance,  $\vec{P}_i = 0$ ,  $\vec{k}_M$ , and  $E_f$  can be related to the masses of the initial and final hadrons as

$$|\vec{k}_M| = |\vec{P}_f| = \frac{\sqrt{[M_i^2 - (M_f + m_M)^2][M_i^2 - (M_f - m_M)^2]}}{2M_i}, \quad (27)$$

$$E_f = \sqrt{|\vec{k}_M|^2 + M_f^2} = \frac{M_i^2 - m_M^2 + M_f^2}{2M_i}. \quad (28)$$

For decay channels with thresholds above the mass of the initial resonance, off-shell effects are taken into account by putting  $|\vec{k}_M| = 0$  and introducing the form factor [4]

$$F = \frac{\Lambda^4}{\Lambda^4 + (q_{N^*}^2 - M_{N^*}^2)^2}, \quad (29)$$

with the cutoff parameter  $\Lambda = 1$  GeV and where  $q_{N^*}$  denotes the threshold of the corresponding channel. In fact, this form factor affects mainly the  $N^* \rightarrow \eta' N$  process, since thresholds for all other channels are below or slightly above the masses of the four resonances.

### III. NUMERICAL RESULTS

In this section our results for partial decay widths  $\Gamma_{N^* \rightarrow MB}$  and coupling constants  $g_{N^*MB}$  are reported for the four investigated resonances, with  $MB \equiv \pi^0 p, \pi^+ n, \eta p, K^+ \Lambda, K^0 \Sigma^+, K^+ \Sigma^0$ , and  $\eta' p$ . The starting point, Sec. III A, is the standard  $\chi$ CQM. In Sec. III B we introduce  $SU(6) \otimes O(3)$  breaking and, finally, in Sec. III C, the five-quark components are

embodied for the  $S_{11}$  resonances. For the partial decay widths, we compare our results to the experimental values reported in PDG [1] and produce predictions for yet unmeasured channels.

#### A. Pure $qqq$ configuration and exact $SU(6) \otimes O(3)$ symmetry

Within this simplest configuration, there are three input parameters: quarks' masses and harmonic oscillator parameter. For the constituent quarks' masses, we use the traditional  $qqq$  quark model values [31,35,59], namely  $m \equiv m_u = m_d = 340$  MeV and  $m_s = 460$  MeV.

The scale of the oscillator parameter,  $\omega_3$ , can be inferred from the empirical radius of the proton via  $\omega_3 = 1/\sqrt{\langle r^2 \rangle}$ , which leads to  $\omega_3 \simeq 250$  MeV, for  $\sqrt{\langle r^2 \rangle} \simeq 1$  fm. However, since the photon couples to  $u$  and  $d$  quarks through  $\rho$  and  $\omega$  mesons, the measured proton charge radius may reflect partly the vector meson propagator [66]. Moreover, the pion cloud has some influence on the measured proton charge radius. Consequently, the intrinsic size of the proton still has some model dependence, and, hence, the oscillator parameter  $\omega_3$  might deviate from 250 MeV, within the range 100–400 MeV [31,35,48,50].

Figure 2 shows the decay widths for  $S_{11}(1535) \rightarrow \pi N, \eta N$  (left panel) and  $S_{11}(1650) \rightarrow \pi N, \eta N$  (right panel) as a function of  $\omega_3$ . The solid and dashed curves are our results and the horizontal lines give the bands reported in PDG [1].

The width for  $S_{11}(1535) \rightarrow \pi N$  (solid curve) falls in the experimental range (dash-dotted lines) for  $300 \lesssim \omega_3 \lesssim 340$  MeV, while for  $S_{11}(1535) \rightarrow \eta N$  the dashed curve and dash-dot-dotted lines lead to  $300 \lesssim \omega_3 \lesssim 380$  MeV. Accord-

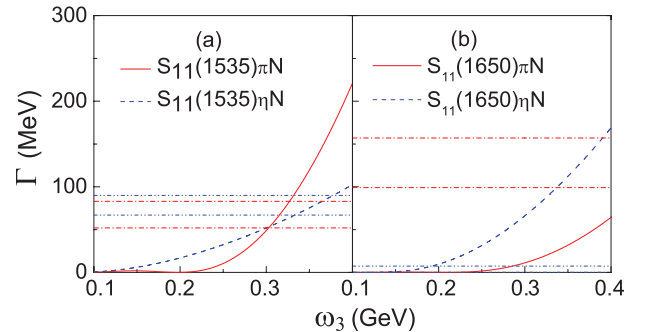


FIG. 2. (Color online) Partial decay widths of  $S_{11}(1535)$  and  $S_{11}(1650)$  to  $\pi N$  and  $\eta N$  channels as a function of the harmonic oscillator parameter  $\omega_3$ . Results of the present work are depicted in solid and dashed curves for  $S_{11} \rightarrow \pi N$  and  $S_{11} \rightarrow \eta N$ , respectively, without the  $SU(6) \otimes O(3)$  breakdown effects. The horizontal lines are the bands given in PDG for  $S_{11} \rightarrow \pi N$  (dash-dotted line) and  $S_{11} \rightarrow \eta N$  (dash-dot-dotted line).

ingly, in the former range for  $\omega_3$ , the simple  $qqq$  configuration allows reproducing the decay widths of  $S_{11}(1535)$  in both  $\pi N$  and  $\eta N$  channels.

The situation with respect to the second  $S_{11}$  resonance differs dramatically. In the whole  $\omega_3$  range, the calculated  $S_{11}(1650) \rightarrow \pi N$  width (solid curve) underestimates the experimental band (dash-dotted lines). For the  $\eta N$  decay channel, predicted values (dashed curve) agree with experimental band (dash-dot-dotted lines) below  $\omega_3 \approx 200$  MeV, where  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  turns out to be vanishing. It is also worthwhile mentioning that, within exact  $SU(6) \otimes O(3)$  symmetry,  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda} = 0$  and, hence, disagrees with the experimental value [1]:  $4.8 \pm 0.7$  MeV.

In summary, the pure  $qqq$  configuration, within exact  $SU(6) \otimes O(3)$ , is not appropriate to describe the  $S_{11}(1650)$  resonance properties. Consequently, one has to consider the  $SU(6) \otimes O(3)$  breakdown effects.

### B. Pure $qqq$ configuration and broken $SU(6) \otimes O(3)$ symmetry

As discussed in Sec. II A,  $SU(6) \otimes O(3)$  symmetry-breaking effects can be related to the mixing angles  $\theta_S$  and  $\theta_D$ . Several predictions on those angles are available (for a recent review, see, e.g., Ref. [67]). Here we will extract ranges for both angles and discuss them with respect to the two most common approaches leading to  $SU(6) \otimes O(3)$  symmetry breaking, namely one-gluon-exchange (OGE) [31,68–72] and one-boson-exchange (OBE) models [59]. Those approaches have raised some controversy [73,74]. Given that both the sign and the magnitude of the mixing angles in those approaches differ (see, e.g., Refs. [67,75]) and that, even within a given approach, the sign depends on the convention used [31,67] or on the exchanged mesons included [76], in Appendix we give values obtained within each approach in line with the de Swart [77] convention for  $SU(3)$ . In order to investigate the sign and range for  $\theta_S$ , in this section we report our numerical results for partial decay widths of  $S_{11}(1535)$  and  $S_{11}(1650)$  to  $\pi N$  and  $\eta N$  as a function of  $\omega_3$  for six values of  $\theta_S$ , namely  $\pm 15^\circ$ ,  $\pm 30^\circ$ ,  $\pm 45^\circ$ , and compare them to the data ranges.

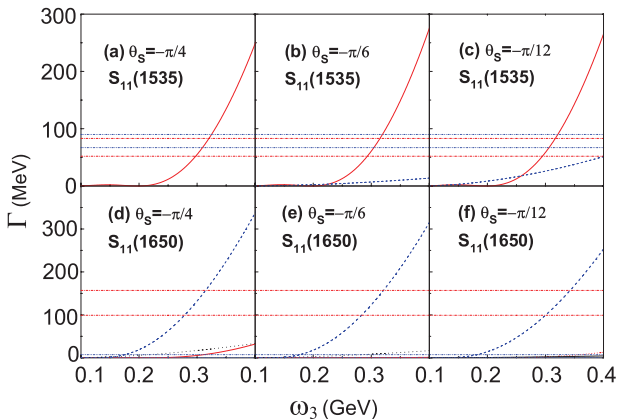


FIG. 3. (Color online) Decay widths of  $S_{11}(1535)$  (upper panels) and  $S_{11}(1650)$  (lower panels) as a function of  $\omega_3$ , with  $\theta_S$  taken to be  $-\frac{\pi}{4}$ ,  $-\frac{\pi}{6}$ , and  $-\frac{\pi}{12}$ , respectively. The dotted curves are our results for  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$ , and the other ones are as in Fig. 2.

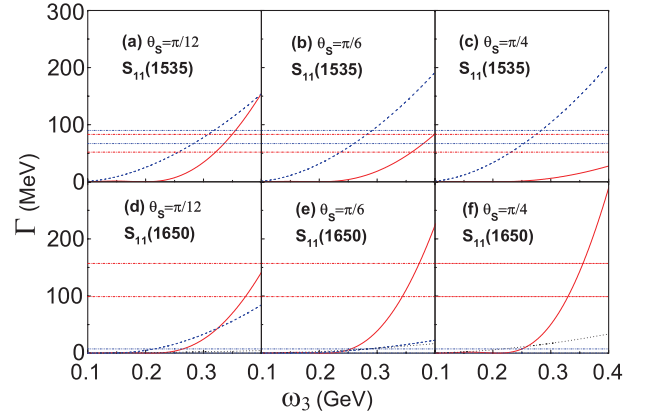


FIG. 4. (Color online) Same as shown in Fig. 3 but with  $\theta_S$  taken to be  $\frac{\pi}{12}$ ,  $\frac{\pi}{6}$ , and  $\frac{\pi}{4}$ .

In Fig. 3 the strong decay partial widths  $\Gamma_{S_{11} \rightarrow \pi N}$  and  $\Gamma_{S_{11} \rightarrow \eta N}$  for  $S_{11}(1535)$  (upper panel) and  $S_{11}(1650)$  (lower panel) are shown as a function of  $\omega_3$ , with negative values for  $\theta_S$ . Conventions for the curves are the same as in Fig. 2, and, due to  $SU(6) \otimes O(3)$  symmetry breaking,  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$  gets nonvanishing values, depicted with dotted curves. The experimental bands for this latter width are not shown, because they are almost identical to those for  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$ .

At all three mixing angles, our predictions for  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$  and  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$  fall in the experimental bands for  $\omega_3 \approx 300$  MeV, while the model underestimates  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$  and  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  very badly. Accordingly, within our approach, negative values for  $\theta_S$  lead to unacceptable results compared to the data.

In Fig. 4 the strong decay partial widths  $\Gamma_{S_{11} \rightarrow \pi N}$  and  $\Gamma_{S_{11} \rightarrow \eta N}$  for both  $S_{11}$  resonances, as well as  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$ , are depicted as a function of  $\omega_3$  with positive values for  $\theta_S$ . For the  $S_{11}(1535)$  resonance, we obtain good agreement with data for  $\theta_S = 15^\circ$  and  $\omega_3 \approx 300$  MeV for both  $\pi N$  and  $\eta N$  decay widths. This is also the case at all angles for  $S_{11}(1650) \rightarrow \pi N$ , but for  $\omega_3 \approx 350$  MeV.

To go further in our investigation, we fix the harmonic parameter at  $\omega_3 = 340$  MeV and calculate partial widths and coupling constants for two extreme positive values of the mixing angle,  $\theta_S = 15^\circ$  and  $35^\circ$ . Moreover, we extend our study to the  $D_{13}(1520)$  and  $D_{13}(1700)$  resonances, with the relevant mixing angle, also at two extreme values,  $\theta_D = 0^\circ$  and  $17.5^\circ$ . Results obtained within this procedure are hereafter referred to as model A. In Table III, we present our results for the strong decay partial widths  $\Gamma_{\pi N}$ ,  $\Gamma_{\eta N}$ , and  $\Gamma_{K\Lambda}$  for the low-lying  $S_{11}$  and  $D_{13}$  resonances studied.

Within model A, the reduced  $\chi^2$  per data point is 10.3. However, this large value is due to  $S_{11} \rightarrow \eta N$ ,  $K\Lambda$  decay channels. It is worthwhile mentioning that for the five  $D_{13}$  partial decay widths, we get  $\chi^2_{d.p.} = 0.7$ . Here,  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$  is well reproduced, while  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$  is overestimated at the level of  $3\sigma$  and  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  underestimated by roughly  $2\sigma$ . For the remaining two other channels, large uncertainties on  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$  (from both experiment and the model) and on  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$  (mainly the model) do not lead to reliable conclusions. Because of those undesirable features,

TABLE III. Strong decay partial widths (in MeV) for the  $S_{11}$  and  $D_{13}$  resonances in the three-quark model with broken  $SU(6) \otimes O(3)$  symmetry.

$N^*$	$\Gamma_{\text{tot}}$	$\pi N$	$\eta N$	$K\Lambda$	Ref.
$S_{11}(1535)$	$150 \pm 25$	$68 \pm 15$	$79 \pm 11$		PDG [1]
		$51 \pm 21$	$121 \pm 15$		Model A
$S_{11}(1650)$	$165 \pm 20$	$128 \pm 29$	$3.8 \pm 3.6$	$4.8 \pm 0.7$	PDG [1]
		$81 \pm 22$	$28 \pm 22$	$9 \pm 6$	Model A
$D_{13}(1520)$	$115 \pm 15$	$69 \pm 6$	$0.26 \pm 0.05$		PDG [1]
		$66 \pm 7$	$0.19 \pm 0.01$		Model A
		$72 \pm 11$	$0.26 \pm 0.07$		Jayalath <i>et al.</i> [46]
$D_{13}(1700)$	$100 \pm 50$	$10 \pm 5$	$0.5 \pm 0.5$	$1.5 \pm 1.5$	PDG [1]
		$13 \pm 10$	$0.5 \pm 0.5$	$0.1 \pm 0.1$	Model A
		$12 \pm 13$	$\leq 0.15$	$\leq 0.03$	Jayalath <i>et al.</i> [46]

we postpone to the next section the discussion on results from other sources, as well as the extraction of coupling constants.

For both  $D_{13}$  resonances, model A allows us to reproduce satisfactorily enough (Table III) the known partial widths and agrees with values obtained within the  $1/N_C$  expansion framework [46]. Model A is, hence, appropriate to put forward predictions for  $D_{13}$ -meson-baryon coupling constants. In Table IV, our predictions for  $\Gamma_{D_{13}MB}$  for seven meson-baryon sets are reported. To end this section, we summarize our main findings within a traditional  $qqq\chi$ CQM, complemented with  $SU(6) \otimes O(3)$  breakdown effects, and use the following input values for adjustable parameters:  $\omega_3 = 340$  MeV,  $15^\circ \leq \theta_S \leq 35^\circ$ , and  $0^\circ \leq \theta_D \leq 17.5^\circ$ .

Model A is found to be appropriate for the  $D_{13}$  resonances, given that the partial decay widths show agreements with the PDG values that range from reasonable to good. Therefore, we do not push our studies further with respect to the  $D_{13}(1520)$  and  $D_{13}(1700)$  resonances.

The main shortcomings of model A concern  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$  and the fact that, for the  $S_{11}(1650)$  resonance, central values for all three channels show significant discrepancies with those reported in PDG. This latter point remains problematic because of large uncertainties. In an attempt to alleviate these disagreements with respect to the  $S_{11}$  resonances, in the next section we consider possible contributions from higher Fock components.

### C. Mixed $qqq$ and $qqqq\bar{q}$ configuration and broken $SU(6) \otimes O(3)$ symmetry

To produce numerical results, seven input parameters are needed, the values of which are discussed below.

- (i) *Constituent quarks' masses.* Due to the introduction of five-quark components, the masses to be used are smaller than those we adopted in Sec. III A, while

dealing with pure three-quark states. In line with Ref. [52], we take  $m = 290$  MeV and  $m_s = 430$  MeV.

- (ii) *Oscillator parameters.* Following the results presented in Sec. III A, we fix the oscillator parameter at  $\omega_3 = 340$  MeV. For the five-quark components, a commonly used value for the oscillator parameter,  $\omega_5 = 600$  MeV, is adopted.
- (iii) *Mixing angle.* In Sec. III B, we showed that to fit the decay widths, the mixing angle should be in the range  $15^\circ \leq \theta_S \leq 35^\circ$ . In the following, this angle is treated as an adjustable parameter.
- (iv) *Probabilities of five-quark components.* The probabilities of the five-quark components in  $S_{11}(1535)$  ( $P_{5q} = A_{5q}^2$ ) and  $S_{11}(1650)$  ( $P'_{5q} = A'^2_{5q}$ ) are also adjustable parameters in our model search.

The latter three adjustable parameters have been extracted by mapping out the whole phase space defined by  $15^\circ \leq \theta_S \leq 35^\circ$  and from 0 to 100% for five-quark probabilities in both  $S_{11}(1535)$  and  $S_{11}(1650)$ . The calculated observables include the partial decay widths of both  $S_{11}$  resonances to  $\pi N$  and  $\eta N$ , as well as  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$ . Sets  $[\theta_S, P_{5q}, P'_{5q}]$  leading [78] to decay widths within ranges reported in PDG have been singled out. For each partial width, extreme values for those parameters then are retained as model ranges, namely

$$26.8^\circ \leq \theta_S \leq 29.8^\circ; \quad 21\% \leq P_{5q} \leq 30\%; \\ 11\% \leq P'_{5q} \leq 18\%. \quad (30)$$

The obtained model is hereafter called model B.

As an example, Fig. 5 illustrates how the known ranges for the partial decay widths allow us to determine ranges for the five-quark components' probabilities. For each decay width, intersections of the model curve with the horizontal bands taken from PDG determine the extreme values for the relevant five-quark probability.

TABLE IV. Coupling constants for  $D_{13}$  resonances to pseudoscalar meson and octet baryon within model A.

$N^*$	$\pi^0 p$	$\pi^+ n$	$\eta p$	$K^+ \Lambda$	$K^0 \Sigma^+$	$K^+ \Sigma^0$	$\eta' p$
$D_{13}(1520)$	$-1.51 \pm 0.07$	$2.13 \pm 0.10$	$-8.33 \pm 0.20$	$3.44 \pm 0.08$	$0.99 \pm 0.14$	$-0.69 \pm 0.09$	$2.11 \pm 0.05$
$D_{13}(1700)$	$-0.35 \pm 0.17$	$0.50 \pm 0.25$	$0.93 \pm 0.91$	$1.43 \pm 1.43$	$-2.80 \pm 0.05$	$1.98 \pm 0.04$	$1.67 \pm 0.52$

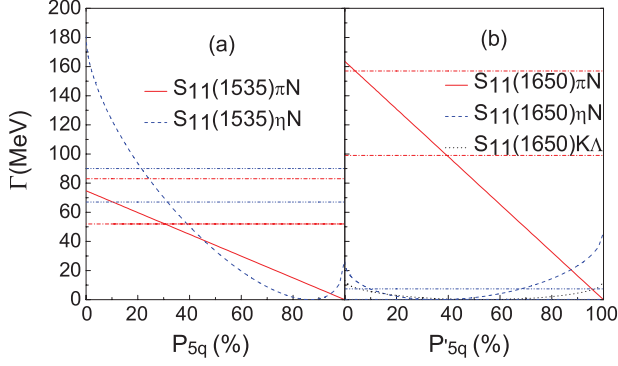


FIG. 5. (Color online) Partial decay widths (in MeV) for  $S_{11}$  resonances as a function of five-quark components,  $\theta_S = 28^\circ$ . The curves are the same as those shown in Fig. 4.

Notice that the probability range for five-quark component in  $S_{11}(1535)$  given above is compatible with previous results [52,53] obtained within  $\chi$ CQM approaches. The latter one [53] puts an upper limit of  $P_{5q} \leq 45\%$ , based on the axial charge study of the resonance, whereas the former one [52], dedicated to the electromagnetic transition  $\gamma^* N \rightarrow S_{11}(1535)$ , reports  $25\% \leq P_{5q} \leq 65\%$ .

### 1. Partial decay widths $\Gamma_{S_{11} \rightarrow MB}$

The resulting numerical partial decay widths, within both models A and B, are reported in Table V and compared with the PDG data [1] as well as with results from other authors, based on various approaches [4,6,7,14,20,22,26,38,46]. Comparing

results of models A and B with the data for all five channels shows clearly the superiority of model B. The  $\chi^2_{d.p.}$  is 0.15 instead of 19.9 in the case of model A.

The most striking feature here is that  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$  is nicely reproduced, which was not the case with previous configurations, namely pure  $qqq$  without or with  $SU(6) \otimes O(3)$  symmetry breaking. Moreover,  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$  agrees with PDG values within better than  $1\sigma$ . The range for  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  is significantly reduced within model B with respect to the model A result and is compatible with the PDG value within less than  $1\sigma$ . Narrow experimental widths for  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$  and  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$  are well reproduced by the model B, with uncertainties comparable to those of the data. In the following, we proceed with comparisons with results from other sources.

The most complete set of results comes from a very recent comprehensive study [46] of all known partial decay widths for 16 baryon resonances within the framework of the  $1/N_C$  expansion in the next-to-leading-order (NLO) approximation. Results for the  $S_{11}(1535)$  decay channels from that work and model B are in excellent agreement. For the  $S_{11}(1650)$ , given that the authors of Ref. [46] use branching fractions data in PDG for  $\eta N$  and  $K\Lambda$  channels, rather than the branching ratios, we postpone the comparisons to Sec. III C3.

The Pitt-ANL [14] multichannel analysis of  $\pi N \rightarrow \pi N, \eta N$ , produces rather small total widths for  $S_{11}(1535)$  and large one for  $S_{11}(1650)$ . Those features underestimate  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$  and  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$  and overestimate  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$ . However,  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  comes out in agreement with PDG and model B results.

An extensive coupled-channels analysis [4,5] studied all available data in 2002 by use of an isobar approach for the

TABLE V. Strong decay widths (in MeV) for  $S_{11}(1535)$  and  $S_{11}(1650)$ .

$N^*$	$\Gamma_{\text{tot}}$	$\pi N$	$\eta N$	$K\Lambda$	Approach	Ref.	
$S_{11}(1535)$	$150 \pm 25$	$68 \pm 15$	$79 \pm 11$			PDG [1]	
		$51 \pm 21$	$121 \pm 15$		Model A	Present work	
	$112 \pm 19$	$58 \pm 5$	$79 \pm 11$		Model B	Present work	
		$57 \pm 19$	$73 \pm 44$		$1/N_C$ -NLO	Jayalath <i>et al.</i> [46]	
		$39 \pm 5$	$57 \pm 6$		Coupled-channels	Vrana <i>et al.</i> [14]	
		$129 \pm 8$	$46 \pm 1$	$68 \pm 1$		Coupled-channels	Penner-Mosel [4]
		136	34.4	56.2		Coupled-channels	Shyam [20]
			$42 \pm 6$	$70 \pm 10$		PWA	Arndt <i>et al.</i> [22]
			21.3	65.7		Chiral unitary	Inoue <i>et al.</i> [6]
		95	42	51		Chiral quark model	Golli <i>et al.</i> [38]
		165	64	89		K-Matrix	Ceci <i>et al.</i> [26]
		142		71		Disp. Rel.	Aznauryan [7]
	195		97		Isobar	Aznauryan [7]	
	$S_{11}(1650)$	$165 \pm 20$	$128 \pm 29$	$3.8 \pm 3.6$	$4.8 \pm 0.7$		PDG [1]
$81 \pm 22$			$28 \pm 22$	$9 \pm 6$	Model A	Present work	
$202 \pm 40$		$143 \pm 5$	$4.5 \pm 3.0$	$4.8 \pm 0.7$	Model B	Present work	
		$149 \pm 4$	$12 \pm 2$		Coupled-channels	Vrana <i>et al.</i> [14]	
		$138 \pm 7$	$90 \pm 6$	$1.4 \pm 0.8$	Coupled-channels	Penner-Mosel [4]	
		133	71.9	2.5	Coupled-channels	Shyam [20]	
		144	86	1.4	13	Chiral quark model	Golli <i>et al.</i> [38]
		233	149	37		K matrix	Ceci <i>et al.</i> [26]
		85		3.2		Disp. Rel.	Aznauryan [7]
		125		6.9		Isobar	Aznauryan [7]



following processes:  $\gamma N \rightarrow \gamma N, \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma, \omega N$  and  $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma, \omega N$ . That work describes successfully four of the five decay channels, albeit with a few tens of free parameters, with the main shortcoming being the underestimate of  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$ . Interpreting the  $pN \rightarrow pN\eta$  data, within an effective Lagrangian approach [20], underestimates all partial decay widths, except  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$ .

The latest available results from SAID [22], in 2005, analyzing  $\pi N$  elastic scattering and  $\eta N$  production data, give a smaller  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$  with respect to PDG, which is compatible with the PDG value for  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$ .

A chiral unitary approach [6] dedicated to the  $S$ -wave meson-baryon interactions reproduces well  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$  but underestimates  $\Gamma_{S_{11}(1535) \rightarrow \pi N}$  by more than a factor of 2.

A recent chiral quark model [38], concentrating on the meson scattering and  $\pi$  and  $\eta$  electroproduction amplitudes, leads to a rather small total width for both resonances, underestimating all  $\pi N$  and  $\eta N$  partial decay widths by roughly  $2\sigma$  and overestimating  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$  by more than  $10\sigma$ . The authors conclude, however, that the  $S_{11}(1535)$  resonance is dominated by a genuine three-quark state.

Results of a  $K$ -matrix approach [26] for  $\pi N$  and  $\eta N$  final states provide realistic values for all considered partial widths, except for  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$ . Finally, in Ref. [7], studying the  $\eta N$  final states, dispersion relations lead to values in agreement with data, while the isobar model tends to overestimate  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$ .

The ambitious EBAC [79] program offers a powerful frame to study the properties of baryons, including partial decay widths [80], extraction of which requires nonambiguous determination of the poles positions [81]; a topic under extensive investigations [81–88].

## 2. Coupling constants $g_{S_{11}MB}$

In Table VI, predictions for the relevant resonance-meson-baryon coupling constants,  $g_{S_{11}MB}$ , from models A and B are given in a particle basis.

In order to emphasize the most sensitive decay channels to the five-quark components in  $S_{11}(1535)$ , we compare results from models A and B. For  $K^+\Sigma^0$  and  $K^0\Sigma^+$ , we observe variations by a factor of 2 between the two models, with central values differing from each other by more than  $4\sigma$ . Next are  $K^+\Lambda$  and  $\eta p$ , with about 30% differences and  $2\sigma$ . The other three channels ( $\pi^0 p, \pi^+ n, \eta' p$ ) show no significant sensitivities to the five-quark components.

In the case of  $S_{11}(1650)$ , similar sensitivities are observed. However, the rather small branching ratios to those final

states require substantial experimental effort and sophisticated phenomenological approaches, e.g., for  $\gamma p \rightarrow K^0\Sigma^+, K^+\Sigma^0$ . In Table VI, results from a chiral unitary approach [6] are also reported, showing compatible values with those of model B for  $K^+\Sigma^0, K^0\Sigma^+$ , and  $\eta p$ . For the other three channels the two sets differ by roughly 60%.

In Table VII, predictions in an isospin basis are reported for model B and other sources. Additional results reported in the literature and limited to fewer channels are also discussed below.

Within an isobar approach [23], a combined analysis [24] of the data available in 2005 on pseudoscalar meson photoproduction has extracted coupling constants in an isospin basis, with uncertainties of around  $\pm 60\%$ . The reported couplings  $g_{S_{11}(1535)\pi N}$  and  $g_{S_{11}(1535)\eta N}$  are compatible with the model B predictions within  $2\sigma$ , while discrepancies between the two approaches for  $g_{S_{11}(1535)K\Lambda}$  and  $g_{S_{11}(1535)K\Sigma}$  reach factors 3 to 4 and  $4\sigma$ . For the second resonance, results from the two calculations agree within  $1\sigma$  for  $g_{S_{11}(1650)\pi N}, g_{S_{11}(1650)\eta N}$ , and  $g_{S_{11}(1650)K\Lambda}$ , with only significant disagreement observed for  $g_{S_{11}(1650)K\Sigma}$ . Copious data released since then, if interpreted within the same approach, might bring in new insights into the coupling constants. Results from a recent SU(6) extended chiral Lagrangian [30], embodying 11 meson-baryon final states, are also reported in Table VII and show consistent values between that approach and model B for  $g_{S_{11}(1535)\eta N}, g_{S_{11}(1535)K\Lambda}, g_{S_{11}(1535)K\Sigma}$ , and  $g_{S_{11}(1650)K\Lambda}$ .

An effective Lagrangian focused on interpreting [20]  $\eta$  production data in  $NN$  and  $\pi N$  collisions leads to  $g_{S_{11}(1535)\eta N} = 2.2$  and  $g_{S_{11}(1650)\eta N} = 0.55$ , compatible with our values. Another effective Lagrangian approach [18] studying  $\eta$  and  $\eta'$  production data in the same reactions gives  $g_{S_{11}(1535)\eta' p} = 3.7$ , about only 10% higher than the value given by model B.

Here, we wish to make a few comments with respect to the relative values of some of the coupling constants.

- (i) While the  $\eta NN$  coupling constant is known to be smaller than that of  $\pi NN$ , the ratio  $|g_{S_{11}(1535)\eta N}/g_{S_{11}(1535)\pi N}|$  comes out to be significantly larger than 1. This result is in line with the finding [43] that, in the soft pion limit,  $\pi NN^*$  coupling vanishes due to chiral symmetry, while that of  $\eta NN^*$  remains finite.
- (ii) The ratio  $|g_{S_{11}(1535)K\Lambda}/g_{S_{11}(1535)\eta N}|$  takes the value  $1.3 \pm 0.3$ , within an isobar model [8] interpreting  $J/\psi \rightarrow \bar{p}p\eta$  and  $\psi \rightarrow \bar{p}K^+\Lambda$  data, larger than the results reported in Table VII. Dressed versus bare mass

TABLE VI.  $S_{11}$ -meson-baryon coupling constants ( $g_{S_{11}MB}$ ) in a particle basis.

$N^*$	$\pi^0 p$	$\pi^+ n$	$\eta p$	$K^+\Lambda$	$K^0\Sigma^+$	$K^+\Sigma^0$	$\eta' p$	Ref.
$S_{11}(1535)$	$-0.58 \pm 0.13$	$0.82 \pm 0.18$	$-2.57 \pm 0.17$	$1.42 \pm 0.11$	$0.95 \pm 0.20$	$-0.62 \pm 0.09$	$3.09 \pm 0.20$	Model A
	$-0.63 \pm 0.03$ $\pm 0.39$	$0.89 \pm 0.04$ $\pm 0.56$	$-2.07 \pm 0.15$ $\pm 1.84$	$1.76 \pm 0.02$ $\pm 0.92$	$1.81 \pm 0.06$ $\pm 2.12$	$-1.28 \pm 0.04$ $\pm 1.50$	$3.33 \pm 0.10$	Model B [6]
$S_{11}(1650)$	$-0.70 \pm 0.10$	$0.94 \pm 0.19$	$0.84 \pm 0.40$	$0.67 \pm 0.25$	$-1.42 \pm 0.21$	$0.95 \pm 0.10$	$-1.61 \pm 0.79$	Model A
	$-0.94 \pm 0.02$	$1.33 \pm 0.03$	$0.35 \pm 0.12$	$0.51 \pm 0.03$	$-2.17 \pm 0.05$	$1.53 \pm 0.04$	$-1.62 \pm 0.14$	Model B

TABLE VII.  $S_{11}$ -meson-baryon coupling constants ( $g_{S_{11}MB}$ ) in an isospin basis.

$N^*$	$\pi N$	$\eta N$	$K\Lambda$	$K\Sigma$	$\eta'N$	Approach	Ref.
$S_{11}(1535)$	$-1.09 \pm 0.05$	$-2.07 \pm 0.15$	$1.76 \pm 0.02$	$2.21 \pm 0.07$	$3.3 \pm 0.1$	Model B	Present work
	$\pm(0.62 \pm 0.32)$	$\pm(0.97 \pm 0.45)$	$\pm(0.55 \pm 0.32)$	$\pm(0.55 \pm 0.32)$		PWA	Sarantsev <i>et al.</i> [24]
	$\pm 0.6$	$\pm 2.1$	$\pm 1.7$	$\pm 2.4$		Chiral Lagrangian	Gamermann <i>et al.</i> [30]
$S_{11}(1650)$	$-1.64 \pm 0.03$	$0.35 \pm 0.14$	$0.53 \pm 0.04$	$-2.66 \pm 0.06$	$-1.62 \pm 0.14$	Model B	Present work
	$\pm(1.05 \pm 0.45)$	$\pm(0.63 \pm 0.32)$	$\pm(0.32 \pm 0.32)$	$\pm(0.71 \pm 0.39)$		PWA	Sarantsev <i>et al.</i> [24]
	$\pm 1.2$	$\pm 0.8$	$\pm 0.6$	$\pm 1.7$		Chiral Lagrangian	Gamermann <i>et al.</i> [30]

considerations [89], might affect the reported ratio in Ref. [8]. Investigation of the same reaction within a unitary chiral approach [6,27] puts that ratio around 0.5 to 0.7, smaller than our result.

- (iii) The ratio  $|g_{S_{11}(1650)K\Sigma}/g_{S_{11}(1650)K\Lambda}|$  turns out to be around 5. Actually,  $S_{11}(1650)$  is dominant by the state  $N(\frac{4}{8}P_M)_{\frac{1}{2}^-}$ , which cannot transit to  $K\Lambda$  channel. Moreover, there is a cancellation between the contributions from  $qqq \rightarrow K\Lambda$  and  $qqqq\bar{q} \rightarrow K\Lambda$ , which leads also to a very small decay width  $\Gamma_{S_{11}(1650) \rightarrow K\Lambda}$ . In addition, the threshold for  $S_{11}(1650) \rightarrow K\Sigma$  decay channel being very close to the mass of  $S_{11}(1650)$ , contributions from the five-quark component enhance significantly the coupling constant  $g_{S_{11}(1650)K\Sigma}$ .
- (iv) It is worth noting that the coupling constants  $g_{S_{11}\eta N}$ ,  $g_{S_{11}K\Sigma}$ , and  $g_{S_{11}\eta'N}$  for  $S_{11}(1535)$  and  $S_{11}(1650)$  have opposite signs. Moreover, the ratio  $|g_{S_{11}(1535)K\Sigma}/g_{S_{11}(1650)K\Sigma}|$  is close to unity. These features might lead to significant cancellations in the interference terms in  $KY$  photo- and/or hadron-induced productions.
- (v) In Tables VI and VII, one finds the following orderings for magnitudes of the coupling constants, predicted by model B, and in Refs. [29,30], noted below as (a), (b), and (c), respectively.

For  $S_{11} \equiv S_{11}(1535)$

*In the particle basis:*

$$(a) : |g_{S_{11}\pi^0 p}| < |g_{S_{11}\pi^+ n}| < |g_{S_{11}K^+\Sigma^0}| < |g_{S_{11}K^+\Lambda}| \\ \approx |g_{S_{11}K^0\Sigma^+}| < |g_{S_{11}\eta p}| < |g_{S_{11}\eta' p}|, \quad (31)$$

$$(b) : |g_{S_{11}\pi^0 p}| \approx |g_{S_{11}K^+\Sigma^0}| < |g_{S_{11}\pi^+ n}| \\ \approx |g_{S_{11}K^0\Sigma^+}| < |g_{S_{11}\eta p}| < |g_{S_{11}K^+\Lambda}|. \quad (32)$$

The main feature of our result (a) is that the strongest couplings are found the hidden strangeness sector, while those for open strangeness channels come out in between  $\pi N$  and  $\eta N$  final states.

Inequalities in (b) come from a recent unitarized chiral effective Lagrangian [29], in which both  $S_{11}(1535)$  and  $S_{11}(1650)$  are dynamically generated. Within that model, the coupling to  $K^+\Sigma^0$  is highly suppressed and that to  $K^+\Lambda$  turns out to be larger than the coupling to  $\eta p$ .

*In the isospin basis:*

$$(a') : |g_{S_{11}\pi N}| < |g_{S_{11}K\Lambda}| < |g_{S_{11}\eta N}| \approx |g_{S_{11}K\Sigma}| < |g_{S_{11}\eta' N}|, \quad (33)$$

$$(c') : |g_{S_{11}\pi N}| < |g_{S_{11}K\Lambda}| < |g_{S_{11}\eta N}| \approx |g_{S_{11}K\Sigma}|. \quad (34)$$

Results from a chiral Lagrangian study [30], (c'), give the same ordering for couplings as in model B. It is also the case for results from a chiral unitary approach [6], while another chiral unitary approach [44], distinguishing dynamically generated resonances from genuine quark states, leads to

$$|g_{S_{11}\pi N}| < |g_{S_{11}K\Lambda}| < |g_{S_{11}\eta N}| < |g_{S_{11}K\Sigma}|. \quad (35)$$

For  $S_{11} \equiv S_{11}(1650)$

*In the particle basis:*

$$(a) : |g_{S_{11}\eta p}| < |g_{S_{11}K^+\Lambda}| < |g_{S_{11}\pi^0 p}| < |g_{S_{11}\pi^+ n}| \\ < |g_{S_{11}K^+\Sigma^0}| < |g_{S_{11}\eta' p}| < |g_{S_{11}K^0\Sigma^+}|, \quad (36)$$

$$(b) : |g_{S_{11}K^+\Lambda}| < |g_{S_{11}\pi^0 p}| < |g_{S_{11}\pi^+ n}| \approx |g_{S_{11}K^+\Sigma^0}| \\ < |g_{S_{11}\eta p}| < |g_{S_{11}K^0\Sigma^+}|. \quad (37)$$

In our model, the ordering in strangeness sector is separated by  $\pi N$ , according to the fact that the relevant disintegration channel is above or below the resonance mass.

The main differences between the results from model B and those in Ref. [29] concern couplings to  $K^+\Lambda$  and  $\eta p$ .

*In the isospin basis:*

$$(a') : |g_{S_{11}K\Lambda}| < |g_{S_{11}\eta N}| < |g_{S_{11}\pi N}| \approx |g_{S_{11}\eta' N}| < |g_{S_{11}K\Sigma}|, \quad (38)$$

$$(c') : |g_{S_{11}\eta N}| \lesssim |g_{S_{11}K\Lambda}| < |g_{S_{11}\pi N}| < |g_{S_{11}K\Sigma}|. \quad (39)$$

Here, again, model B and Ref. [30] lead basically to identical orderings.

To end this section, we would like to emphasize the following point, with respect to the importance of five-quark components. Our model leads to the probability for the strangeness component in  $S_{11}(1650)$  being smaller than that for the five-quark component in  $S_{11}(1535)$ . Moreover, the probability amplitude turns out to be positive for  $S_{11}(1535)$  but negative for  $S_{11}(1650)$ .

Taking the ranges determined for probabilities [Eq. (30)], one gets  $-77.4 \leq A_{5q}/A'_{5q} \leq -72.5$ . This latter range and that for  $\theta_S$ , embodied in Eq. (14), allow extracting values for the energy of the strangeness component,  $1641.60 \leq E_5 \leq 1649.99$  MeV. The coupling between  $qqq$  and  $qqqq\bar{q}$  in the corresponding baryon  ${}_{5q}\langle \bar{V}_{\text{cou}} \rangle_{3q}$ , Eq. (14), turns out to be negative for both  $S_{11}$  resonances.

TABLE VIII. Strong decay widths (in MeV) for  $S_{11}(1650)$ .

$\Gamma_{\text{tot}}$	$\pi N$	$\eta N$	$K \Lambda$	Approach	Ref.
$165 \pm 20$	$128 \pm 29$	$3.8 \pm 3.6$	$4.8 \pm 0.7$	BR	PDG [1]
	$143 \pm 5$	$4.5 \pm 3.0$	$4.8 \pm 0.7$	Model B	Present work
	$128 \pm 29$	$10.7 \pm 5.8$	$11.5 \pm 6.6$	BF	PDG [1]
	$148 \pm 8$	$9.7 \pm 6.7$	$7.9 \pm 0.3$	Model C	Present work
	$133 \pm 33$	$12.5 \pm 11.0$	$11.5 \pm 6.4$	$1/N_C$ -NLO	Jayalath <i>et al.</i> [46]

### 3. Branching fraction versus branching ratio considerations

As mentioned earlier, in PDG [1] estimates for both branching fractions (BF) to meson-baryon states and branching ratios (BR), ( $\Gamma_{MB}/\Gamma_{\text{total}}$ ), are reported. In the case of the  $S_{11}$  resonances considered here, those estimates are not identical for  $S_{11}(1650) \rightarrow \eta N, K \Lambda$ . In the present work we have used BR. However, a very recent work [46] has adopted BF. In order to compare the results of this latter work with those of model B, we have investigated the drawback of using BF instead of BR in our approach. Accordingly, a third model, hereafter called model C, was obtained.

Though we extract simultaneously the partial decay widths for both  $S_{11}$  resonances, the above changes in the data do not affect results for the  $S_{11}(1535)$ . In Table VIII, results from PDG, Ref. [46] and our models B and C are given for  $S_{11}(1650)$ . The  $\chi_{\text{d.p.}}^2$  for the three models are comparable, namely 0.15 (model B), 0.25 (model C), and 0.19 (Ref. [46]).

Model C leads to results in agreement with the two other sets within the uncertainties therein. Comparing models B and C, we observe that the most sensitive width is  $\Gamma_{S_{11}(1650) \rightarrow K \Lambda}$  and, to a lesser extent,  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$ , while  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  increases very slightly.

In Table IX, results for coupling constant from models B and C are reported. We find, of course, the same features as for partial decay widths. In addition, given the associated uncertainties, it turns out that  $\Gamma_{S_{11}(1650) \rightarrow \eta' N}$  and  $\Gamma_{S_{11}(1650) \rightarrow K \Sigma}$  change very slightly within the two models.

Those trends are also present in the coupling constants given in particle basis (Table X).

Taking into account the associated uncertainties to the coupling constants, model C does not significantly modify the coupling constants ordering obtained in Sec. III C2 for model B.

To end this section, we give the phase space defined by model C:

$$\begin{aligned} 24.7^\circ \leq \theta_S \leq 30.0^\circ; \quad 19.8\% \leq P_{5q} \leq 31\%; \\ 3.0\% \leq P'_{5q} \leq 12.6\%. \end{aligned} \quad (40)$$

Compared to model B, Eq. (30), the ranges for  $\theta_S$  and  $P_{5q}$  increase slightly. The most significant change concerns  $P'_{5q}$ ,

which goes from  $11\% \leq P'_{5q} \leq 18$  to  $3\% \leq P'_{5q} \leq 13$ . This feature shows the sensitivity of  $\Gamma_{S_{11}(1650) \rightarrow K \Lambda}$  and, to a lesser extent, that of  $\Gamma_{S_{11}(1650) \rightarrow \eta N}$  to the five-quark components in  $S_{11}(1650)$ .

## IV. SUMMARY AND CONCLUSIONS

Within a constituent quark approach, we studied the properties of four low-lying baryon resonances with respect to their partial decay widths to seven meson-baryon channels and associated resonance-meson-baryon coupling constants. The starting point was the simplest chiral constituent quark model ( $\chi$ CQM). The second step consisted in introducing  $SU(6) \otimes O(3)$  breaking effects. Finally, five-quark components in the  $S_{11}$  resonances were implemented and investigated.

The outcome of the present work is reported below, focusing on the considered nucleon resonances [ $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $D_{13}(1520)$ , and  $D_{13}(1700)$ ] and their strong decays to  $\pi N$ ,  $\eta N$ ,  $\eta' N$ ,  $K \Lambda$ , and  $K \Sigma$  final states. Within the  $\chi$ CQM, the only adjustable parameter ( $\omega_3$ ) did not allow us to reproduce the partial widths of resonances. Introducing  $SU(6) \otimes O(3)$  breaking, via configuration mixing angles  $\theta_S$  and  $\theta_D$ , brought significant improvements with respect to the decay widths of the  $D_{13}$  resonances but missed the data for the  $S_{11}$  resonances partial decay widths. Nevertheless, this second step allowed us to fix the value of  $\omega_3$  and extract ranges for the mixing angles, treated as free parameters. In an attempt to alleviate this unsatisfactory situation, possible roles for the five-quark component in the baryons' wave functions were investigated. Given that the latter issue is irrelevant with respect to the  $D_{13}$  resonances, the properties of which were well described in the second step, the final phase of our study was devoted to the  $S_{11}$  resonances.

We calculated the partial decay widths  $S_{11}(1535) \rightarrow \pi N$ ,  $\eta N$  and  $S_{11}(1650) \rightarrow \pi, \eta N, K \Lambda$  in the whole phase space defined by the mixing angle  $\theta_S$  and the probability of five-quark components in each of the two resonances. Regions of the phase space allowing us to reproduce the data for those widths were selected. Accordingly, that procedure allowed us to extract ranges for partial widths, with a decay threshold below the relevant resonance mass and resonance-meson-

TABLE IX.  $S_{11}(1650)$ -meson-baryon coupling constants ( $g_{S_{11}MB}$ ) in isospin basis.

$\pi N$	$\eta N$	$K \Lambda$	$K \Sigma$	$\eta' N$	Approach	Ref.
$-1.64 \pm 0.03$	$0.35 \pm 0.14$	$0.53 \pm 0.04$	$-2.66 \pm 0.06$	$-1.62 \pm 0.14$	Model B	Present work
$-1.66 \pm 0.05$	$0.55 \pm 0.16$	$0.62 \pm 0.09$	$-2.49 \pm 0.16$	$-1.74 \pm 0.24$	Model C	Present work

TABLE X.  $S_{11}(1650)$ -meson-baryon coupling constants ( $g_{S_{11}MB}$ ) in particle basis.

$\pi^0 p$	$\pi^+ n$	$\eta p$	$K^+ \Lambda$	$K^0 \Sigma^+$	$K^+ \Sigma^0$	$\eta' p$	Ref.
$-0.94 \pm 0.02$	$1.33 \pm 0.03$	$0.35 \pm 0.14$	$0.51 \pm 0.03$	$-2.17 \pm 0.05$	$1.53 \pm 0.04$	$-1.62 \pm 0.14$	Model B
$-0.96 \pm 0.03$	$1.36 \pm 0.04$	$0.55 \pm 0.16$	$0.62 \pm 0.09$	$-2.03 \pm 0.13$	$1.44 \pm 0.09$	$-1.74 \pm 0.24$	Model C

baryon coupling constants for the following meson-baryon combinations:  $\pi^0 p$ ,  $\pi^+ n$ ,  $\eta p$ ,  $K^+ \Lambda$ ,  $K^0 \Sigma^+$ ,  $K^+ \Sigma^0$ , and  $\eta' p$ .

The main findings of the present work are summarized below with respect to the approaches studied in describing the properties of the four low-lying nucleon resonances.

- (i) The chiral constituent quark approach in three-quark configuration and exact  $SU(6) \otimes O(3)$  symmetry is not appropriate to reproduce the known partial decay widths.
- (ii) Introducing symmetry-breaking effects due to one-gluon-exchange mechanism allows us to account for the partial decay width of the  $D_{13}(1520)$  and  $D_{13}(1700)$  resonances but not for those of the  $S_{11}$  resonances.
- (iii) Complementing the formalism with five-quark components in the  $S_{11}$  resonances leads to satisfactory results with respect to all known partial decay widths investigated here.
- (iv) The complete formalism puts ranges on the three adjustable parameters, namely the mixing angle between configurations  $|N_8^2 P_M\rangle$  and  $|N_8^4 P_M\rangle$  and the five-quark component probabilities in  $S_{11}(1535)$  and  $S_{11}(1650)$  resonances.
- (v) For  $S_{11}(1535)$ , the most sensitive entities to the five-quark component turn out to be  $\Gamma_{S_{11}(1535) \rightarrow \eta N}$ ,  $g_{S_{11}K^+ \Sigma^0}$ ,  $g_{S_{11}K^0 \Sigma^+}$ , and  $g_{S_{11}\eta p}$ , all with sizable magnitudes.

- (vi) For  $S_{11}(1650)$ , the same trends as for  $S_{11}(1535)$  are observed. In addition,  $\Gamma_{S_{11}(1650) \rightarrow \pi N}$  undergoes significant change due to the five-quark mixture. Here, the  $\eta N$  channel has a smaller width and coupling constant compared to the  $S_{11}(1535)$  case.

Further, interpretation of recent data, obtained using electromagnetic and/or hadronic probes, within approaches with a reasonable number of free parameters, is very desirable. Within the present extended  $\chi$ CQM approach, analysis of the  $\gamma p \rightarrow \eta p$  data is underway [90].

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#### APPENDIX: $S_{11}(1535)$ AND $S_{11}(1650)$ RESONANCES MIXING ANGLE IN ONE-GLUON-EXCHANGE AND ONE-BOSON-EXCHANGE MODELS

The mixing angle  $\theta_S$  can be obtained by diagonalizing the following matrix:

$$\begin{pmatrix} \langle N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}} | N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle, \langle N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}} | N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle \\ \langle N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}} | N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle, \langle N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}} | N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle \end{pmatrix}, \quad (\text{A1})$$

where  $H_{\text{hyp}}$  is the hyperfine interaction between the quarks. In the OGE [58] and OBE models [59], the explicit forms of  $H_{\text{hyp}}$  are

$$H_{\text{hyp}}^{\text{OGE}} = \sum_{i < j} \frac{2\alpha_s}{3m_i m_j} \left[ \frac{8\pi}{3} \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} \left( \frac{3\vec{S}_i \cdot \vec{r}_{ij} \vec{S}_j \cdot \vec{r}_{ij}}{r_{ij}^2} - \vec{S}_i \cdot \vec{S}_j \right) \right], \quad (\text{A2})$$

$$H_{\text{hyp}}^{\text{OBE}} = \sum_{i < j} \sum_F \frac{g^2}{4\pi} \frac{1}{12m_i m_j} \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \left( \left\{ \vec{\sigma}_i \cdot \vec{\sigma}_j \left[ \frac{\mu^2 e^{-\mu r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right] \right\} + \left( \frac{3\vec{\sigma}_i \cdot \vec{r}_{ij} \vec{\sigma}_j \cdot \vec{r}_{ij}}{r_{ij}^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \frac{\mu^2 e^{-\mu r_{ij}}}{r_{ij}} \left( 1 + \frac{3}{\mu r_{ij}} + \frac{3}{\mu^2 r_{ij}^2} \right) \right). \quad (\text{A3})$$

#### 1. OGE model

The OGE hyperfine interaction leads to the following matrix elements:

$$\langle N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OGE}} | N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle = -C, \quad (\text{A4})$$

$$\langle N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OGE}} | N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle = C, \quad (\text{A5})$$

$$\langle N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OGE}} | N(\frac{2}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle = C, \quad (\text{A6})$$

$$\langle N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OGE}} | N(\frac{4}{8}P_M)_{\frac{1}{2}^-}, S_z \rangle = 0, \quad (\text{A7})$$

with the constant  $C = \frac{2\alpha_s}{m^2} \omega_3^3 \pi^{-\frac{1}{2}}$ , where  $m$  and  $\omega_3$  are the light quark mass and the harmonic oscillator parameter, respectively. We then obtain  $\theta_S^{\text{OGE}} \simeq 32^\circ$ .

Here a comment is in order with respect to the sign of  $\theta_S$ . As, reported in Ref. [67], a non ambiguous entity with respect to that sign is the following ratio:

$$\mathcal{R} = \frac{\langle N | H_m | N(^4P_M)_{\frac{1}{2}^-} \rangle}{\langle N | H_m | N(^2P_M)_{\frac{1}{2}^-} \rangle}, \quad (\text{A8})$$

with  $H_m$  the pseudovector couplings at the tree level. The ratio  $\mathcal{R}$  is a constant determined by  $SU(6) \otimes O(3)$  symmetry.

Notice that, in the present work, we have adopted the convention introduced by Koniuk and Isgur [31], where wave functions are in line with the  $SU(3)$  conventions of de Swart [77]. In this frame, the constant  $\mathcal{R}$  gets a negative value, and the relevant mixing angle for the  $S$  wave,  $\theta_S$ , turns out positive. However, in line with the Hey, Litchfield, and Cashmore [91] analysis, Isgur and Karl, in their early works [68,70–72], used another convention, for which  $\mathcal{R} = +1$  and  $\theta_S < 0$ . In the literature, both conventions are being used, often without explicit mention of the utilized convention.

## 2. OBE model

The OBE hyperfine interaction results in

$$\langle N(^2P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OBE}} | N(^2P_M)_{\frac{1}{2}^-}, S_z \rangle = 5V_{11} - 7V_{00}, \quad (\text{A9})$$

$$\langle N(^2P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OBE}} | N(^2P_M)_{\frac{1}{2}^-}, S_z \rangle = -8T_{11}, \quad (\text{A10})$$

$$\langle N(^4P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OBE}} | N(^2P_M)_{\frac{1}{2}^-}, S_z \rangle = -8T_{11}, \quad (\text{A11})$$

$$\langle N(^4P_M)_{\frac{1}{2}^-}, S_z | H_{\text{hyp}}^{\text{OBE}} | N(^4P_M)_{\frac{1}{2}^-}, S_z \rangle = 4V_{11} - 2V_{00} + 8T_{11}, \quad (\text{A12})$$

where  $V_{00}$ ,  $V_{11}$ , and  $T_{11}$  are constants from the orbital integral

$$V_{00} = \langle \varphi_{00} | \frac{g^2}{4\pi} \frac{1}{12m_i m_j} \left[ \frac{\mu^2 e^{-\mu r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right] | \varphi_{00} \rangle, \quad (\text{A13})$$

$$V_{11} = \langle \varphi_{1m} | \frac{g^2}{4\pi} \frac{1}{12m_i m_j} \left[ \frac{\mu^2 e^{-\mu r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right] | \varphi_{1m} \rangle, \quad (\text{A14})$$

$$T_{11} = \langle \varphi_{1m} | \frac{g^2}{4\pi} \frac{1}{12m_i m_j} \frac{\mu^2 e^{-\mu r_{ij}}}{r_{ij}} \left( 1 + \frac{3}{\mu r_{ij}} + \frac{3}{\mu^2 r_{ij}^2} \right) | \varphi_{1m} \rangle. \quad (\text{A15})$$

Taking the same values for the parameters as in Ref. [59], we obtain  $\theta_S = -13^\circ$ . However, if one considers contributions from the vector-meson exchanges, the absolute value of  $\theta_S$  might be decreased or the sign might change [74,76].

Relevance of the OGE versus the OBE has been studied by several authors, see, e.g., Refs. [36,40,92,93], favoring the OGE mechanism, and endorsed by the present work, as the origin of the  $SU(6) \otimes O(3)$  symmetry breakdown.

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