Chiral QCD sum rules for open-charm mesons

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QCD sum rules for chiral partners in the open-charm meson sector are presented at nonzero baryon net density or temperature. We focus on the differences between pseudo-scalar and scalar as well as vector and axial-vector D mesons and derive the corresponding Weinberg-type sum rules. This allows for the identification of such QCD condensates which drive the nondegeneracy of chiral partners in the lowest order of the strong coupling α_s and which, therefore, may serve as "order parameters" for chiral restoration (or elements thereof).

I. INTRODUCTION

Chiral symmetry and its breaking pattern represent important features of strong interaction physics. The non-degeneracy of chiral partners of hadrons is considered to be a direct hint to the spontaneous chiral symmetry breaking in nature which characterizes the QCD vacuum. In fact, the distinct difference of isovector-vector and isovector–axial-vector spectral functions deduced from τ decays [1,2] gives empirical and precise evidence for the breaking of chiral symmetry. The low-energy strengths of the mentioned spectral functions, concentrated in the resonances $\rho(770)$ and $a_1(1260)$, deviate strongly from each other and from perturbative QCD predictions. (For a dynamical interpretation of the two spectra see Refs. [3,4].) This exposes clearly the strong nonperturbative effects governing the low-energy part of the hadron spectrum. The spontaneous symmetry breaking is quantified by the chiral condensate $\langle \bar{q}q \rangle$, which plays an important role in the Gell-Mann-Oakes-Renner relation connecting hadronic quantities and quark degrees of freedom (cf. [5,6]).

Weinberg-type chiral sum rules for differences of moments between light vector and axial-vector spectral functions have been developed for vacuum [7,8] and for a strongly interacting medium at finite temperature [9]. It is the aim of the present paper to present analog sum rules for the scalar and pseudoscalar as well as vector- and axial-vector mesons composed essentially of a heavy and a light quark, e.g., $u\bar{c}$ and $d\bar{c}$ realized in vacuum as $D^\pm(1867)$, $J^P = 0^-$, $D^0(1865)$, $J^P = 0^-$, $D_0^*(2400)^0$, $J^P = 0^+$, $D^*(2007)^0$, $J^P = 1^-$, $D^*(2010)^\pm$, $J^P = 1^-$, $D_1(2420)^0$, $J^P = 1^+$ (there is no confirmed charged scalar or axial-vector state in the open-charm sector) [10]. Such open-charm degrees of freedom will be addressed in the near future by the CBM [11] and PANDA [12] collaborations at the Facility for Antiproton and Ion Research (FAIR) in proton-nucleus and antiproton-nucleus reactions [13]. Accordingly, we are going to analyze the chiral sum rules in nuclear matter.

Chiral symmetry is explicitly broken by nonzero quark masses, but quantum chromodynamics is still approximately invariant under transformations which are restricted to the light quark sector. The associated conserved currents are the well-known light quark vector and axial-vector currents. Their

mixing under chiral transformation and Wigner's nondegeneracy of the hadron spectrum is interpreted as the spontaneous breakdown of chiral symmetry. Although currents, related to mesons which are represented in the quark model as composed of a light quark and a heavy quark, are neither conserved nor associated with a symmetry transformation, vector and axial-vector currents still mix under a transformation which is restricted to the light quark sector. Consider, for example, the infinitesimal rotation in flavor space $\psi \rightarrow e^{-it^a \Theta^a \Gamma} \psi \approx (1 - it^a \Theta^a \Gamma)\psi$, where $t^a \in \mathrm{SU}(n_f)$, $\Gamma = \mathbb{1}$ for the vector transformations and $\Gamma = \gamma_5$ for the axial transformations, and Θ^a denote a set of infinitesimal rotation parameters (rotation angles). The vector current transforms as

$$j_\mu^{V,\tau} = \bar{\psi}\gamma_\mu \tau \psi \rightarrow j_\mu^{V,\tau} \pm i\bar{\psi}\Gamma\gamma_\mu \Theta^a [t^a, \tau]_- \psi, \quad (1.1)$$

where $+ (-)$ refers to the vector (axial) transformations and τ denotes a matrix in flavor space. In the three-flavor sector $\psi = (\psi_1, \psi_2, \psi_3)$, where the first two quarks are light, the choice $\Theta = (\Theta_1, \Theta_2, \Theta_3, 0, \dots, 0)$ and $2t^a = \lambda^a$, the well-known Gell-Mann matrices, clearly leaves the QCD Lagrangian invariant. To be specific, let us consider $\tau = (\lambda^4 + i\lambda^5)/2$, where the corresponding current transforms as

$$j_\mu^{V,\tau} = \bar{\psi}_1 \gamma_\mu \psi_3 \rightarrow j_\mu^{V,\tau'} = j_\mu^{V,\tau}$$
$$-\frac{i}{2}[\bar{\psi}_1 \Gamma\gamma_\mu \psi_3 \Theta_3 + (\Theta_1 + i\Theta_2)\bar{\psi}_2 \Gamma\gamma_\mu \psi_3]. \quad (1.2)$$

Obviously, the heavy-light vector current mixes with heavy-light axial-vector currents if an axial transformation, with $\Gamma = \gamma_5$, is applied. Analog expressions hold for spin-0 mesons, and the result is the same as in the spin-1 case, but with the replacements $\gamma_\mu \rightarrow \mathbb{1}$ for the vector transformation and $i\gamma_\mu[t^a, \tau]_- \rightarrow [t^a, \tau]_+$ for the axial transformation. Hence, a symmetry and its spontaneous breakdown in the light quark sector must also be reflected in the spectrum of mesons composed of a heavy and a light quark. In particular, the splitting of the spectral densities between heavy-light parity partners must be driven by order parameters of spontaneous chiral symmetry breaking only.

Probing the chiral symmetry restoration via the change of order parameters requires a reliable extraction of their medium dependence. As has been long known for the vacuum case, the

chiral condensate is numerically suppressed in QCD sum rules in the light quark sector due to the tiny light quark mass but occurs amplified by the large heavy quark mass in QCD sum rules involving a light and a heavy quark [14]. Despite the amplification in terms of $m_c \langle \bar{u}u \rangle$, with m_c and $\langle \bar{u}u \rangle$ to be evolved to the appropriate scale, in case of the D mesons, the dependence of the in-medium D meson spectrum on the chiral condensate is not as direct as anticipated. This is clear insofar as there are always particle and antiparticle contributions to the spectrum of pseudoscalar states and one has to deal with both pieces. This accounts for inherent suppressions and amplifications of different types of condensates due to the generic structure of the sum rule in that case. Indeed, a precise analytic and numerical investigation points to competitive numerical influences of various condensates for the mass splitting of particle and antiparticle, whereas the determination of the mass center rather depends on the modeling of the continuum threshold [15].

In the difference of chiral partner spectra (Weinberg-type sum rules) the dependence on chirally symmetric condensates drops out. At the same time, the amplification of the chiral condensate by the heavy quark mass is still present. This makes Weinberg-type sum rules of mesons composed of heavy and light quarks an interesting object of investigation.

Our paper is organized as follows. The operator product expansion (OPE) and a suitable projection are elaborated in Sec. II for differences of current-current correlators. These are spelled out for chiral partners of open-charm mesons in the form of Weinberg-type sum rules with the focus on the OPE sides in Sec. III; a remark on heavy-quark symmetry is included as well. Numerical examples are exhibited in Sec. III D. The summary can be found in Sec. IV. In Appendix A, some relations between current-current correlators are considered. Appendices B and C list projection coefficients and prove a necessary relation for the quark propagator.

II. DIFFERENCES OF CURRENT-CURRENT CORRELATORS AND THEIR OPERATOR PRODUCT EXPANSION

We consider the currents

$$j^S(x) := \bar{q}_1(x)\, q_2(x), \tag{2.1a}$$

$$j^P(x) := \bar{q}_1(x)\, i\gamma_5\, q_2(x), \tag{2.1b}$$

$$j^V_\mu(x) := \bar{q}_1(x)\, \gamma_\mu\, q_2(x), \tag{2.1c}$$

$$j^A_\mu(x) := \bar{q}_1(x)\, \gamma_5 \gamma_\mu\, q_2(x), \tag{2.1d}$$

and the corresponding causal correlators

$$\Pi^{(S,P)}(q) = i \int d^4x\, e^{iqx} \langle T[j^{(S,P)}(x) j^{(S,P)\dagger}(0)] \rangle, \tag{2.2a}$$

$$\Pi^{(V,A)}_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle T[j^{(V,A)}_\mu(x) j^{(V,A)\dagger}_\nu(0)] \rangle, \tag{2.2b}$$

where $T[\ldots]$ denotes time ordering and $\langle \ldots \rangle$ means Gibbs averaging. Retarded and advanced correlators are related via analytic continuation to the causal correlator [16].

In the rest frame of the nuclear medium, i.e., $n = (1, \vec{0})$, and for mesons at rest, i.e., $q = (1, \vec{0})$, the (axial-) vector correlator can be decomposed as

$$\Pi^{(V,A)}_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi^{(V,A)}_T(q) + \frac{q_\mu q_\nu}{q^2} \Pi^{(V,A)}_L(q), \tag{2.3}$$

with $\Pi^{(V,A)}_T(q) = \frac{1}{3}(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}) \Pi^{(V,A)}_{\mu\nu}(q)$ and $\Pi^{(V,A)}_L(q) = \frac{1}{q^2} q^\mu q^\nu \Pi^{(V,A)}_{\mu\nu}(q)$ written explicitly covariant. Π_T contains the information about (axial-) vector degrees of freedom. Π_L refers to (pseudo-) scalar states and can be related to the (pseudo-) scalar correlator (see Appendix A for details).

We now proceed with the OPE's for $\Pi^X \in \{\Pi^{(S,P)}, \Pi^{(V,A)} \equiv g^{\mu\nu}\Pi^{(V,A)}_{\mu\nu}\}$. We do not include the α_s corrections which would arise from inserting the next-to-leading order interaction term in the time-ordered product of the current-current correlator. Such terms account, e.g., for four-quark condensates. They are of mass dimension 6 and beyond the scope of this investigation. According to standard OPE techniques (see, e.g., Refs. [17–19]), the time-ordered product can be expanded into normal-ordered products multiplied by Wick-contracted quark-field operators. Dirac indices can be projected onto elements of the Clifford algebra, $\Gamma \in \{\mathbb{1}, \gamma_\mu, \sigma_{\mu\nu}, \gamma_5\gamma_\mu, \gamma_5\}$, which provides an orthonormal basis in the space of 4×4 matrices with the scalar product $(A, B) \equiv \frac{1}{4}\text{Tr}_D[AB]$. Color indices can be projected onto an analogously appropriate basis. Thereby, color and Dirac traces of the quark propagator occur. Using the background field method in fixed-point gauge (cf. Refs. [17,18]) for the gluon background field $A_\mu(x)$, $x_\mu A^\mu(x) = 0$, one can expand the nonlocal quark field operators by a covariant expansion $q(x) = \sum_n \frac{x^{\alpha_1} \ldots x^{\alpha_n}}{n!}(D_{\alpha_1} \ldots D_{\alpha_n} q)|_{x=0}$ to arrive, after a Fourier transformation, at

$$\Pi^X(q) = -i^3 \int \frac{d^4p}{(2\pi)^4} \langle : \text{Tr}_{C,D}[S_1(p)\Gamma^X S_2(q+p)\Gamma^X] : \rangle$$

$$+ i^2 \sum_\Gamma \frac{1}{4} \sum_{n=0}^\infty \frac{1}{n!}(-i)^n \partial^{\bar{\alpha}_n}_q \langle : \bar{q}_1 \Gamma \text{Tr}_D[\Gamma\Gamma^X S_2(-q)\Gamma^X] D_{\bar{\alpha}_n} q_1 + (-1)^n \bar{q}_2 \Gamma \text{Tr}_D[\Gamma\Gamma^X S_1(q)\Gamma^X] D_{\bar{\alpha}_n} q_2 : \rangle$$

$$= \Pi^{X(0)}(q) + \Pi^{X(2)}(q), \tag{2.4}$$
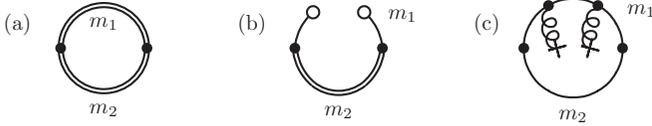
FIG. 1. Diagrammatical representation of (a) $\Pi^{(0)}$ and (b) $\Pi^{(2)}$. (c) is for $\Pi^{(0)}$ with two gluon lines attached to the free propagator of quark 1. Double lines stand for the complete perturbative series of the quark propagator, wiggly lines are for gluons, circles denote nonlocal quark condensates, and crosses symbolize local quark or gluon condensates.

where $D_{\vec{\alpha}_n} = D_{\alpha_1} \ldots D_{\alpha_n}$ (with an analog notation for the partial derivative) and quark fields and their derivatives are taken at $x = 0$. For X denoting vector and axial-vector states, $(\Gamma^X)_{ij}(\Gamma^X)_{kl} \equiv (\Gamma^X_\mu)_{ij}(\Gamma^{X\mu})_{kl}$ is understood. $\Pi^{(0)}(q)$ denotes the fully contracted [depicted in Fig. 1(a)] and $\Pi^{(2)}(q)$ the two-quark term (see Fig. 1(b) for $\langle \bar\psi \ldots \psi \rangle$), and $\mathrm{Tr}_{C,D}$ means trace with respect to color and Dirac indices. In doing so, we introduce the momentum-space perturbative quark propagator in a gluonic background field as $S(q) = \int d^4x \, e^{iqx} S(x,0) = \int d^4x \, e^{-iqx} S(0,x)$ and $S(x,y) = -i\langle 0|T[q(x)\bar{q}(y)]|0\rangle$. To have the same structures in both quantities $\Pi^{(0)}$ and $\Pi^{(2)}$, we also project the matrix product $\Gamma^X_\mu S \Gamma^\mu_X$ onto this basis (see Appendix B):

$$\Pi^{(P,S)(0)}(q) = -i \int \frac{d^4p}{(2\pi)^4} \Big\langle : \frac{1}{4}\mathrm{Tr}_C\left[\mathrm{Tr}_D[S_2(p+q)]\mathrm{Tr}_D[S_1(p)]\right.$$
$$+ (-1)^{(P,S)}\mathrm{Tr}_D[S_2(p+q)\gamma_\mu]\mathrm{Tr}_D[S_1(p)\gamma^\mu]$$
$$+ \frac{1}{2}\mathrm{Tr}_D[S_2(p+q)\sigma_{\mu\nu}]\mathrm{Tr}_D[S_1(p)\sigma^{\mu\nu}]$$
$$+ (-1)^{(P,S)}\mathrm{Tr}_D[S_2(p+q)\gamma_5\gamma_\mu]\mathrm{Tr}_D[S_1(p)\gamma_5\gamma^\mu]$$
$$+ \mathrm{Tr}_D[S_2(p+q)\gamma_5]\mathrm{Tr}_D[S_1(p)\gamma_5]] : \Big\rangle, \quad (2.5a)$$

$$\Pi^{(V,A)(0)}(q) = i \int \frac{d^4p}{(2\pi)^4} \langle : \mathrm{Tr}_C\left[\mathrm{Tr}_D[S_2(p+q)]\mathrm{Tr}_D[S_1(p)]\right.$$
$$+ (-1)^{(V,A)}\mathrm{Tr}_D[S_2(p+q)\gamma_\mu]\mathrm{Tr}_D[S_1(p)\gamma^\mu]$$
$$- (-1)^{(V,A)}\mathrm{Tr}_D[S_2(p+q)\gamma_5\gamma_\mu]\mathrm{Tr}_D[S_1(p)\gamma_5\gamma^\mu]$$
$$- \mathrm{Tr}_D[S_2(p+q)\gamma_5]\mathrm{Tr}_D[S_1(p)\gamma_5]] : \rangle, \quad (2.5b)$$

where $(-1)^{(P,V)} = -1$ for pseudoscalar and vector mesons and $(-1)^{(S,A)} = 1$ for scalar and axial-vector mesons. $\Pi^{X(2)}(q)$ in Eq. (2.4) may be simplified using Table II in Appendix B.

To obtain the OPE's for the difference of chiral partners $\Pi^{P-S} \equiv \Pi^P - \Pi^S$ and $\Pi^{V-A} \equiv \Pi^V - \Pi^A$, one can use Eq. (2.5) or directly project the occurring anticommutators. The result reads

$$\Pi^{P-S(0)}(q)$$
$$= -i \int \frac{d^4p}{(2\pi)^4} \Big\langle : \frac{1}{2}\mathrm{Tr}_C\{\mathrm{Tr}_D[S_2(p+q)]\mathrm{Tr}_D[S_1(p)]$$
$$+ \frac{1}{2}\mathrm{Tr}_D[S_2(p+q)\sigma_{\mu\nu}]\mathrm{Tr}_D[S_1(p)\sigma^{\mu\nu}]$$
$$+ \mathrm{Tr}_D[S_2(p+q)\gamma_5]\mathrm{Tr}_D[S_1(p)\gamma_5]\} : \Big\rangle, \quad (2.6a)$$

$$\Pi^{P-S(2)}(q)$$
$$= \sum_n \frac{(-i)^n}{n!} \frac{1}{2} \sum_\Gamma^{\{1,\sigma_{\alpha<\beta},\gamma_5\}} \langle : \bar{q}_1 \overleftarrow{D}_{\vec{\alpha}_n} \Gamma \partial^{\vec{\alpha}_n} (\mathrm{Tr}_D[\Gamma S_2(q)]) q_1$$
$$+ \bar{q}_2 \Gamma \partial^{\vec{\alpha}_n} (\mathrm{Tr}_D[\Gamma S_1(-q)]) \overrightarrow{D}_{\vec{\alpha}_n} q_2 : \rangle, \quad (2.6b)$$

$$\Pi^{V-A(0)}(q)$$
$$= i \int \frac{d^4p}{(2\pi)^4} \langle : 2\mathrm{Tr}_C\{\mathrm{Tr}_D[S_2(p+q)]\mathrm{Tr}_D[S_1(p)]$$
$$- \mathrm{Tr}_D[S_2(p+q)\gamma_5]\mathrm{Tr}_D[S_1(p)\gamma_5]\} : \rangle, \quad (2.6c)$$

$$\Pi^{V-A(2)}(q)$$
$$= -\sum_n \frac{(-i)^n}{n!} 2 \sum_\Gamma^{\{1,i\gamma_5\}} \langle : \bar{q}_1 \overleftarrow{D}_{\vec{\alpha}_n} \Gamma \partial^{\vec{\alpha}_n} (\mathrm{Tr}_D[\Gamma S_2(q)]) q_1$$
$$+ \bar{q}_2 \Gamma \partial^{\vec{\alpha}_n} (\mathrm{Tr}_D[\Gamma S_1(-q)]) \overrightarrow{D}_{\vec{\alpha}_n} q_2 : \rangle, \quad (2.6d)$$

where the Clifford basis is modified now by the imaginary unit in front of γ_5 for the vector–axial-vector difference. The advantage is that we are left with three different types of Dirac traces for the quark propagators in the P−S case and only two in the V−A case.

The perturbative series for a momentum-space quark-propagator in a gluon background field in fixed-point gauge is presented in Appendix C. Its derivative is given by the Ward identity $\partial^\mu S(q) = -S(q)\Gamma^\mu(q,q;0)S(q)$, where $\Gamma^\mu(q,q;0)$ denotes the exact quark-gluon vertex function at vanishing momentum transfer. $\Gamma^\mu(q,q;0) = \gamma^\mu$ holds for a classical background field, meaning that the Ward identity for the complete perturbation series has the same form as for free quarks [20].

For the limit of a massless quark flavor attributed to q_1, $m_1 \to 0$, one can show (see Appendix C) that $\mathrm{Tr}_D[\Gamma S_1(q)] = 0$ for $\Gamma \in \{1, \sigma_{\mu\nu}, \gamma_5\}$ and $q^2 \neq 0$. In this limit only the diagram in Fig. 1(b) gives a contribution to $\Pi^{(2)}$. The corresponding diagram for $1 \longleftrightarrow 2$ vanishes, because the sum over Dirac matrices in Eq. (2.6) covers such elements where the corresponding traces vanish. Hence, for chiral partner sum rules the often-used approximation of a static quark, which results in vanishing heavy-quark condensates, is not necessary as their Wilson coefficients vanish in the limit of the other quark being massless. This means that, if one is seeking the occurrence of quark condensates in lowest (zeroth) order of the strong coupling α_s, at least one quark must have a nonzero mass. (Higher-order interaction term insertions cause the occurrence of further quark condensates proportional to powers of α_s, cf. Refs. [9,21,22] for examples.) Hence, the structure of the OPE side of the famous Weinberg sum rules for light quarks with respect to quark condensates is shown in all orders of the quark propagators and quark fields.

For the completely contracted term $\Pi^{(0)}$ the situation is somewhat more involved. In the case of two light quarks, i.e., $m_{1,2} \to 0$, the limiting procedure and the momentum integration commute and, hence, $\Pi^{(0)} = 0$ is obvious, because all nonvanishing terms drop out in the chiral difference. If one of the quarks has a nonzero mass, the limiting procedure and the momentum integration do not commute due to the

occurrence of infrared divergences [23–27], e.g., for the term depicted in Fig. 1(c) which is proportional to the gluon condensate. As the integration domains in Eq. (2.5) and Eqs. (2.6a) and (2.6c) involve momenta $p = 0$, $\mathrm{Tr}[\Gamma S(p, m)]$ does not converge uniformly for $m \to 0$. Hence, the integration and the limit $m \to 0$ cannot be interchanged. In fact, a direct calculation of the perturbative contribution to chiral partner OPE's shows that the infrared divergences are the only remaining terms. All finite terms cancel out in the chiral difference. These divergences have to be absorbed by introducing condensates which are not normal ordered. We demonstrate here explicitly their cancellation. Up to mass dimension 5, the only product of traces which contributes to $\Pi^{(0)}$ is $\mathrm{Tr}_D[S_1]\mathrm{Tr}_D[S_2]$. This is true for arbitrary quark masses. Indeed, up to order α_s^1 one can show that the contribution $-\int d^4 p \, \mathrm{Tr}_D[S_2^{(0)}(p + q)]\langle : \mathrm{Tr}_D[S_1^{(2)}(p)] :\rangle$, which is the remaining term of Eq. (2.5) after the limit $m_1 \to 0$ has been taken, is canceled by $\int d^4 p \, \mathrm{Tr}_D[S_2^{(0)}(q)]\langle \mathrm{Tr}_D[S_1^{(2)}(p)]\rangle$. The latter quantity is introduced by the definition of non-normal ordered condensates

$$\langle \Omega | \bar{q} \hat{O}[D_\mu] q | \Omega \rangle = \langle \Omega | : \bar{q} \hat{O}[D_\mu] q : | \Omega \rangle$$
$$- i \int \frac{d^4 p}{(2\pi)^4} \langle \Omega | \mathrm{Tr}_{C,D}[\hat{O}[-ip_\mu - i\tilde{A}_\mu]S_q(p)] | \Omega \rangle, \quad (2.7)$$

where $\hat{O}[D_\mu]$ is an arbitrary operator depending on D_μ and $\tilde{A}_\mu$ denotes the Fourier transform of A_μ, resulting in a factor $\int \langle \mathrm{Tr}_D[S_1^{(2)}(p)]\rangle$ to the Wilson coefficient $\mathrm{Tr}_D[S_2^{(0)}(q)]$ of the chiral condensate in Eq. (2.4). Hence, in the case of heavy-light systems, first, the integration has to be performed and then one has to introduce non-normal ordered condensates according to Eq. (2.7) and, afterward, the limit $m_1 \to 0$ can be taken. In case of equal masses, $m_1 = m_2$, the divergences cancel each other with virtue to

$$\int d^4 p \{ \mathrm{Tr}_D[S_1^{(2)}(p + q)]\mathrm{Tr}_D[S_2^{(0)}(p)]$$
$$+ \mathrm{Tr}_D[S_1^{(0)}(p + q)]\mathrm{Tr}_D[S_2^{(2)}(p)] \} = 0. \quad (2.8)$$

If two heavy (static) quarks are considered, only $\Pi^{(0)}$ gives a contribution to the chiral OPE, whereas for two massless quarks, both $\Pi^{(0)}$ and $\Pi^{(2)}$ vanish.

Putting everything together this means that for light quarks $(m_{1,2} \ll \Lambda_{\mathrm{QCD}})$ in the chiral-difference OPE the corresponding traces, and, therefore, the corresponding Wilson coefficients, vanish, whereas for heavy quarks $(m_{1,2} \gg \Lambda_{\mathrm{QCD}})$ the condensates vanish. To obtain quark condensates in order α_s^0 the two flavors must be of different mass scales, i.e., $q_1 \in \{u, d(, s)\}$ is a light quark and $q_2 \in \{c, b, t\}$ is a heavy quark. Hence, to seek for condensates which are connected to chiral symmetry breaking as possible order parameters in order α_s^0, a natural choice is to consider chiral partner mesons composed of a light and a heavy quark. Given the above-mentioned experimentally envisaged research programs at FAIR [11–13], we focus on open-charm mesons. The presented formulas may be directly transferred to open bottom mesons by $m_c \to m_b$.

III. CHIRAL PARTNERS OF OPEN-CHARM MESONS

We now consider a light $(q_1 \equiv q)$ and a heavy $(q_2 \equiv q_c)$ quark entering the currents in Eq. (2.1).

A. The case of P−S

For the P−S case we consider the pseudoscalar $D(0^-)$ $(D^\pm, D^0, \text{ and } \bar{D}^0)$ and its scalar partners $D_0^*(0^+)$. Of course, all the results also account for other heavy-light (pseudo-) scalar mesons. (For open-charm mesons D_s which contain a strange quark, however, the limit $m_s \to 0$ may not be a good approximation and terms $\propto m_s$ should be taken into account as well.)

Up to and including mass dimension 5, after absorbing the divergences in non-normal ordered condensates, the OPE gets the following compact form:

$$\Pi^{P-S}(q) \equiv \Pi^P(q) - \Pi^S(q) = \Pi^{P(2)}(q) - \Pi^{S(2)}(q)$$
$$= \sum_n \frac{(-i)^n}{n!} \sum_\Gamma^{\{\mathbb{1}, \sigma_{\alpha < \beta}\}} \langle \bar{q} \overleftarrow{D}_{\vec{\alpha}_n} \Gamma \partial^{\vec{\alpha}_n} \mathrm{Tr}_D[\Gamma S_c(q)] q(0) \rangle, \quad (3.1)$$

where the sum over the elements of the Clifford algebra no longer contains γ_5 to this mass dimension. To evaluate the condensates in Eq. (3.1) in terms of expectation values of scalar operators, Lorentz indices have to be projected onto $g_{\mu\nu}$ and $\epsilon_{\mu\nu\kappa\lambda}$ in vacuum, whereas the medium four-velocity n_μ provides an additional structure at finite densities and/or temperatures [28]. Hence, new condensates must be introduced which vanish in the vacuum. Thereby, temperature and density dependencies stem from Gibbs averages of medium specific operators. The evaluation in the nuclear matter rest frame $n^\mu = (1, \vec{0})$ for mesons at rest yields

$$\Pi^{P-S}(q_0) = 2\langle \bar{q}q \rangle \frac{m_c}{q_0^2 - m_c^2} - \langle \bar{q}g\sigma\mathscr{G}q \rangle \frac{m_c q_0^2}{(q_0^2 - m_c^2)^3}$$
$$+ [\langle \bar{q}g\sigma\mathscr{G}q \rangle - 8\langle \bar{q}D_0^2 q \rangle] \frac{m_c q_0^2}{(q_0^2 - m_c^2)^3}, \quad (3.2)$$

where we separated a medium-specific term (last line, $\langle \bar{q}g\sigma\mathscr{G}q \rangle - 8\langle \bar{q}D_0^2 q \rangle \equiv \langle \Delta \rangle$) vanishing in vacuum, $g^2 = 4\pi\alpha_s$, $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, $\mathscr{G}_{\mu\nu} = i[D_\mu, D_\nu]/g$ and q denotes either d or u quark field operators. Equation (3.2) reduces to the vacuum result at zero density and temperature [14,29,30]. The condensates $\langle \bar{q}q \rangle$, $\langle \bar{q}g\sigma\mathscr{G}q \rangle$, and $\langle \bar{q}D_0^2 q \rangle$ may have different medium dependencies.

An odd part of the OPE, i.e., $\Pi(q_0) = \Pi^{\mathrm{even}}(q_0) + q_0\Pi^{\mathrm{odd}}(q_0)$ with $\Pi^{\mathrm{even}}(q_0) = [\Pi(q_0) + \Pi(-q_0)]/2$ and $\Pi^{\mathrm{odd}}(q_0) = [\Pi(q_0) - \Pi(-q_0)]/(2q_0)$, where both functions are even with respect to q_0, does not appear up to this mass dimension. Although there is no γ_μ projection of the condensates for difference OPE's of chiral partner, it may arise from an odd number of derivatives in Eq. (3.1).

As usual, within a QCD sum-rule analysis [17–19], a Wick rotation to Euclidean momenta, $q_0 = iQ$, has to be performed. The OPE in the deep-space-like region based on asymptotically free quark degrees of freedom can be expected

to converge. The OPE of the current-current correlation function, in turn, is related to the hadronic spectral density by an in-medium dispersion relation. To evaluate the correlator at arbitrary values of the energy q_0 off the real axis, $\mathrm{Im}\,q_0 \neq 0$, e.g., $q_0^2 = -Q^2 \to -\infty$, an integral over its discontinuities (spectral densities) along the real axis $\Delta\Pi(q_0, \vec{q} = 0) = \frac{1}{2i}\lim_{\epsilon\to 0}[\Pi(q_0 + i\epsilon) - \Pi(q_0 - i\epsilon)] = \mathrm{Im}\,\Pi(q_0)$ is performed. After a Borel transformation the result is

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, e^{-\omega^2/M^2}\, \omega\, \Delta\Pi^{P-S}(\omega)$$

$$= e^{-m_c^2/M^2}\left[-2m_c\langle\bar{q}q\rangle + \left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\langle\bar{q}g\sigma\mathscr{G}q\rangle\right.$$

$$\left.-\langle\Delta\rangle\left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\right], \tag{3.3}$$

where M is the Borel mass. In a medium, the integral over the spectral densities runs over positive and negative frequencies, ω, covering both particles and their antiparticles of given quantum numbers.

It is instructive to cast Eq. (3.3) in the form of Weinberg-type sum rules [7,9]. This can be accomplished by expanding the exponential on both sides and comparing the coefficients of inverse powers of the Borel mass. In such a way we can relate moments of the spectral P−S differences to condensates via

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega\,\Delta\Pi^{P-S}(\omega) = -2m_c\langle\bar{q}q\rangle, \tag{3.4a}$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^3\,\Delta\Pi^{P-S}(\omega)$$

$$= -2m_c^3\langle\bar{q}q\rangle + m_c\langle\bar{q}g\sigma\mathscr{G}q\rangle - m_c\langle\Delta\rangle, \tag{3.4b}$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^5\,\Delta\Pi^{P-S}(\omega)$$

$$= -2m_c^5\langle\bar{q}q\rangle + 3m_c^3\langle\bar{q}g\sigma\mathscr{G}q\rangle - 3m_c^3\langle\Delta\rangle + \dots, \tag{3.4c}$$

where the dots denote the neglected contribution of mass dimension 7. This generalizes the OPE side of Weinberg-type sum rules to scalar and pseudoscalar mesons in the heavy-light quark sector for the first time.

If one attributes chiral symmetry to the degeneracy of chiral partners [i.e., the left-hand side of Eq. (3.4) vanishes], then the vanishing of $\langle\bar{q}q\rangle$ and $\langle\bar{q}D_0^2q\rangle = \frac{1}{8}(\langle\bar{q}g\sigma\mathscr{G}q\rangle - \langle\Delta\rangle)$ on the right-hand side is required. In this spirit, these condensates may be considered as possible order parameters of chiral symmetry. Note that Eqs. (3.4) also allow us to consider the omitted mass dimension 7 condensate as an order parameter. Remarkably, the chiral condensate $\langle\bar{q}q\rangle$ of light quarks figures here in conjunction with the heavy quark mass as parameter for chiral symmetry breaking in each of the moments (for vacuum, cf. Ref. [29]). Of next importance is $\langle\bar{q}D_0^2q\rangle$, again in combination with the heavy quark mass. The right-hand side quantities must be taken at a proper renormalization scale. Formally, in the chiral (i.e., strictly massless) limit for *all* quarks the right-hand sides of Eqs. (3.4) would vanish.

B. The V−A case

In the same manner we proceed in the V−A case. From Eqs. (2.6c) and (2.6d) we obtain, up to and including mass dimension 5,

$$\Pi^{V-A}(q) \equiv \Pi^V(q) - \Pi^A(q) = \Pi^{V(2)}(q) - \Pi^{A(2)}(q)$$

$$= -\sum_n \frac{(-i)^n}{n!} 2 \sum_{\Gamma}^{\{\mathbb{1}, i\gamma_5\}} \langle\bar{q}_1\overleftarrow{D}_{\vec{\alpha}_n}\Gamma\partial^{\vec{\alpha}_n}\mathrm{Tr}_D[\Gamma S_2(q)]q_1\rangle, \tag{3.5}$$

where only the $\mathbb{1}$-projection survives. The in-medium evaluation results in

$$\Pi^{V-A}(q) = -8\langle\bar{q}q\rangle\frac{m_c}{q_0^2 - m_c^2} + 4\langle\bar{q}g\sigma\mathscr{G}q\rangle\frac{m_c^3}{(q_0^2 - m_c^2)^3}$$

$$-4\langle\Delta\rangle\frac{m_c q_0^2}{(q_0^2 - m_c^2)^3} \tag{3.6}$$

and, together with (A14) and (3.2), we obtain for the correlator containing information about the vector and axial-vector degrees of freedom

$$\Pi_T^{V-A}(q) = \Pi^{P-S} + \frac{m_c}{(q_0^2 - m_c^2)^2}\langle\bar{q}g\sigma\mathscr{G}q\rangle$$

$$+\frac{1}{3}\frac{m_c}{(q_0^2 - m_c^2)^2}\langle\Delta\rangle. \tag{3.7}$$

For vacuum, the result of Ref. [14,30] is recovered. The Borel transformed sum rule is given by

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, e^{-\omega^2/M^2}\, \omega\, \Delta\Pi_T^{V-A}(\omega) = e^{-m_c^2/M^2}\left[-2m_c\langle\bar{q}q\rangle\right.$$

$$\left.+\frac{m_c^3}{2M^4}\langle\bar{q}g\sigma\mathscr{G}q\rangle - \langle\Delta\rangle\left(\frac{m_c^3}{2M^4} - \frac{4}{3}\frac{m_c}{M^2}\right)\right]. \tag{3.8}$$

The corresponding moments, therefore, are

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega\,\Delta\Pi_T^{V-A}(\omega) = -2m_c\langle\bar{q}q\rangle, \tag{3.9a}$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^3\,\Delta\Pi_T^{V-A}(\omega) = -2m_c^3\langle\bar{q}q\rangle - \frac{4}{3}m_c\langle\Delta\rangle, \tag{3.9b}$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^5\,\Delta\Pi_T^{V-A}(\omega) = -2m_c^5\langle\bar{q}q\rangle + m_c^3\langle\bar{q}g\sigma\mathscr{G}q\rangle$$

$$-\frac{11}{3}m_c^3\langle\Delta\rangle + \dots \tag{3.9c}$$

with the same meaning of "…" as above. This second set of Weinberg-type sum rules contains the same condensates as the first set in Eq. (3.4) but in different combinations. Again, addressing chiral symmetry to the left-hand side, one may consider the chiral condensate $\langle\bar{q}q\rangle$ and $\langle\Delta\rangle$ as possible order parameters. As the P−S case allowed us to identify $\langle\bar{q}D_0^2q\rangle$ as an order parameter, $\langle\bar{q}g\sigma\mathscr{G}q\rangle$ also qualifies as an order parameter.

In addition,

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \omega \Delta \tilde{\Pi}_T^{V-A}(\omega) = 0 \qquad (3.10)$$

follows from Eq. (2.3) but for the decomposition $\Pi_{\mu\nu} = (q_\mu q_\nu - q^2 g_{\mu\nu})\tilde{\Pi}_T + q_\mu q_\nu \tilde{\Pi}_L$. This corresponds to Weinberg's first sum rule [7]. Note that, in contrast to Weinberg's original sum rule [7], no Goldstone boson properties appear on the right-hand side of (3.10) because the heavy-light currents involved in our case are generally not conserved. The Borel transformed sum rule for $\tilde{\Pi}_T = \Pi_T/q^2$ reads

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega e^{-\omega^2/M^2} \omega \Delta \tilde{\Pi}_T^{V-A}(\omega)$$

$$= \langle \bar{q}q \rangle \frac{2}{m_c} [1 - e^{-m_c^2/M^2}] - \langle \bar{q}g\sigma\mathscr{G}q \rangle \frac{1}{m_c^3}$$

$$\times \left[1 - e^{-m_c^2/M^2} \left(1 + \frac{m_c^2}{M^2} + \frac{m_c^4}{2M^4} \right) \right] + \langle \Delta \rangle \frac{1}{m_c^3}$$

$$\times \left\{ \frac{7}{3} \left[1 - e^{-m_c^2/M^2} \left(1 + \frac{m_c^2}{M^2} \right) \right] - e^{-m_c^2/M^2} \frac{m_c^4}{2M^4} \right\}$$

$$(3.11)$$

and reproduces Eqs. (3.9). Note that using Π_T, instead of $\tilde{\Pi}_T$, is more appropriate for the heavy-quark limit.

C. Heavy-quark symmetry

In the heavy-quark limit, $m_2^2 \gg |q^2|$, the leading contributions in Eq. (3.7) are

$$\Pi_T^{V-A}(q)\big|_{m_2^2 \gg |q^2|} \approx \Pi^{P-S}(q)\big|_{m_2^2 \gg |q^2|} \approx -\frac{2}{m_2} \langle \bar{q}q \rangle,$$

$$(3.12)$$

where $\frac{1}{q^2 - m_2^2} = -\frac{1}{m_2^2} \sum_{n=0}^{\infty} (\frac{q^2}{m_2^2})^n$ has been exploited. This result is in agreement with the expected degeneracy of vector and pseudoscalar mesons and of axial-vector and scalar mesons in the heavy-quark limit [31]. Actually the $\langle \bar{q}q \rangle$ parts of Π^{P-S} and Π_T^{V-A} agree, as one can check by comparing (3.3) with (3.8).

D. Numerical examples

Using the condensates from Ref. [15] the right-hand side of Eqs. (3.2), (3.8), and (3.11), i.e., the OPE sides, are exhibited in Fig. 2 as a function of the Borel mass for vacuum and cold nuclear matter. The Borel curves are significantly modified by changes of the entering condensates due to their density dependencies, i.e., their magnitudes are lowered by approximately 30% of their vacuum values. It should be emphasized, however, that the density dependence is estimated in linear approximation. For the chiral condensate $\langle \bar{q}q \rangle$ it is known [32,33] that the actual density dependence at zero temperature is not so strong. One may expect, accordingly, a somewhat weaker impact of the medium effects.

For further evaluations, the hadronic spectral functions or moments thereof must be specified, e.g., by using suitable moments as in Refs. [34,35].
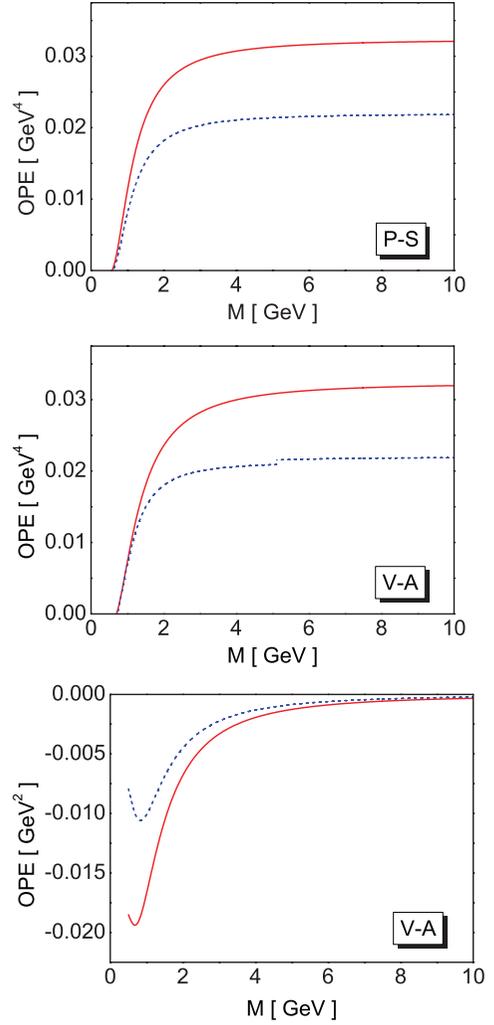
FIG. 2. (Color online) The OPE sides of Eqs. (3.2) (upper panel, P−S), (3.8) (middle panel, V−A) and (3.11) (lower panel V−A) as a function of the Borel mass in vacuum (solid red line) and at nuclear saturation density $n = 0.15$ fm^{-3} (dashed blue line). The employed condensates are $\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 + 45n/11$, $\langle \bar{q}g\sigma\mathscr{G}q \rangle = 0.8\,\text{GeV}^2 \langle \bar{q}q \rangle_0 + 3n\,\text{GeV}^2$, $\langle \bar{q}q \rangle_0 = (-0.245\,\text{GeV})^3$, and $\langle \Delta \rangle = 8 \times 0.3\,n\,\text{GeV}^2$.

IV. SUMMARY

In summary, we present difference QCD sum rules for chiral partners of mesons with a simple quark structure. Focusing on the operator product expansion of the current current correlator in lowest (zeroth) order of an interaction term insertion, only the combination of a light and a heavy (massive) quark yields a nontrivial result: Differences of spectral moments between pseudoscalar and scalar as well as vector and axial-vector mesons for condensates up to mass dimension $\leqslant 5$ are determined by the combinations $m_c\langle \bar{q}q \rangle$, $m_c\langle \bar{q}g\sigma\mathscr{G}q \rangle$, and $m_c(\langle \bar{q}g\sigma\mathscr{G}q \rangle - 8\langle \bar{q}D_0^2 q \rangle)$ (to be taken at an appropriate scale) which may be considered as elements of "order parameters" of chiral symmetry breaking (see Ref. [36] for a lattice evaluation of the mixed quark-gluon condensate at finite temperature). Vanishing of these condensates at high baryon density and/or temperature would mean chiral

restoration, i.e., the degeneracy of spectral moments of the considered chiral partners. Chiral partners of mesons with light-light or heavy-heavy quark currents are nondegenerate in higher orders of α_s, as exemplified by the Kapusta-Shuryak sum rule. The famous Weinberg sum rules generalized to a hot medium in Ref. [9] refer to lower mass dimension moments where the OPE side vanishes in the chiral limit.

Our results show a significant change of the OPE side when changing from vacuum to normal nuclear matter density. This implies that also the hadronic spectral functions may experience a significant reshaping in the medium.

The open-charm meson sector will be investigated in precision experiments by two collaborations in near future at FAIR in proton and antiproton reactions at nuclei as well as in heavy-ion collisions. These experiments will shed some light on the mass spectrum of open-charm mesons and its medium modifications.

APPENDIX A: RELATIONS BETWEEN CURRENT-CURRENT CORRELATORS

We denote the currents in Eq. (2.1) by

$$j^{X,\tau} = \bar{\psi}\,\Gamma^X \tau\,\psi\,, \qquad \Gamma^X \in \{\mathbb{1}, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}. \quad \text{(A1)}$$

where $\psi = (q_1, \ldots, q_{n_f})$ collects the fields of n_f quark flavors and τ is an arbitrary matrix in flavor space. Due to the special choice of Γ^X the current fulfills $[j^{X,\tau}(x)]^\dagger = j^{X,\tau^\dagger}(x) \equiv \bar{\psi}\Gamma^X \tau^\dagger \psi$. Lorentz indices are suppressed here for V and A currents.

The currents j^S and j^P are supposed to carry the quantum numbers of scalar and pseudoscalar mesons, respectively. The issue is more complicated for j^V and j^A. To see this, we consider the causal correlator of two vector-field valued, mesonic (bosonic) Heisenberg operators A_μ and B_ν [to be identified with $j_\mu^{(V,A),\tau}(x)$ and $j_\nu^{(V,A),\tau^\dagger}(y)$]

$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle T\left[A_\mu(x)B_\nu(0)\right]\rangle. \quad \text{(A2)}$$

A system at nonzero temperature and/or (baryon) density is characterized by a heat-bath vector n_μ [encoded in the Gibb's averages in Eq. (2.2)] which can be normalized by $n^2 = 1$. The heat bath is at rest in a reference frame with $\vec{n} = 0$. For a system with a conserved current j_μ^B, e.g., the baryon current, one can use $n_\mu \propto j_\mu^B$ (Eckart choice). For a thermal system without conserved currents one must lock the flow with local energy density (Landau choice, cf. Ref. [37]).

A rank-2 tensor in four-dimensional Minkowski space-time can be decomposed into six independent algebraic invariants $A, \ldots, F$ being functions of q^2, $n \cdot q$ as follows:

$$\Pi_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) A + q_\mu q_\nu B + n_\mu^q n_\nu^q C$$
$$+ \left(q_\mu n_\nu^q + n_\mu^q q_\nu\right) D + \left(q_\mu n_\nu^q - n_\mu^q q_\nu\right) E$$
$$+ \epsilon_{\mu\nu}{}^{\alpha\beta} q_\alpha n_\beta^q F, \quad \text{(A3)}$$

where $n_\mu^q := n_\mu - \frac{n\cdot q}{q^2} q_\mu$. From invariance with respect to parity one obtains $F = 0$. Imposing additionally time reversal invariance together with translational invariance of the considered medium, e.g. nuclear matter, gives rise to the symmetry property $\Pi_{\mu\nu} = \Pi_{\nu\mu}$, hence, $E = F = 0$. If the correlator (causal, advanced, retarded correlator, or spectral density) was four-transversal in both indices, i.e., $q^\mu \Pi_{\mu\nu} = q^\nu \Pi_{\mu\nu} = 0$, $E = 0$ would follow without additional symmetry constraints. Current conservation alone is not a sufficient requirement for the transversality of the causal correlator.

The coefficients A and C carry the information about the four-transversal degrees of freedom, referring to vector or axial-vector components which can be decomposed into three-momentum transversal and three-momentum longitudinal states [38]. The coefficient B is related to the four-longitudinal states referring to a scalar or pseudo-scalar component. The coefficient D encodes the mixing between three-momentum longitudinal (axial-) vector states and (pseudo-) scalar states occurring due to broken rotational invariance for an excitation (hadron) moving with nonzero velocity in the medium [39,40]. Thus, the spin is no longer a conserved quantum number.

In the special case $n = (1, \vec{0})$ and $q = (q_0, \vec{0})$, one has $n_\mu^q = 0$ and, therefore, C and D drop out. Hence, the correlator can be decomposed into a transverse ($\Pi_T = -A$) and a longitudinal part ($\Pi_L = q^2 B$) [14], see Eq. (2.3). At zero density and zero temperature, Π_L would vanish for B_ν being conserved, as demonstrated below.

To relate the four-longitudinal part of the (axial-) vector current to the (pseudo-) scalar current let us consider the correlator

$$\Pi_{\mu\nu}(q, p) = i^2 \int d^4x\, d^4y\, e^{iqx} e^{-ipy} \langle T[A_\mu(x)B_\nu(y)]\rangle. \quad \text{(A4)}$$

Translation invariance allows us to factor out a momentum dependence

$$\Pi_{\mu\nu}(q, p) = i(2\pi)^4 \delta^{(4)}(p - q)\Pi_{\mu\nu}(q). \quad \text{(A5)}$$

Contracting with $q_\mu p_\nu$, performing a suitable number of partial integrations in each term, and using $\partial_x^0 A_0(x) = \partial_x^\mu A_\mu(x) - \partial_x^i A_i(x)$ gives the nonanomalous Ward identity relating the longitudinal part of the causal correlator of two Lorentz vectors to the correlator of their divergences

$$q^\mu p^\nu \Pi_{\mu\nu}(q, p)$$
$$= -i^2 \int d^4x\, d^4y\, e^{iqx} e^{-ipy} \langle T\left[i\partial_x^\mu A_\mu(x) i\partial_y^\nu B_\nu(y)\right]$$
$$+ i\delta(x_0 - y_0)\{q^\mu [A_\mu(x), B_0(y)]_{x_0=y_0}$$
$$+ \left[A_0(x), i\partial_y^\nu B_\nu(y)\right]_{x_0=y_0}\}\rangle. \quad \text{(A6)}$$

The first and third terms on the right-hand side of this equation vanish if $\partial^\nu B_\nu = 0$ holds.

Identifying A_μ and B_ν with the currents above, $j_\mu^{V,\tau}(x) := \bar{\psi}(x)\gamma_\mu \tau\,\psi(x)$, $j_\mu^{A,\tau}(x) := \bar{\psi}(x)\gamma_5\gamma_\mu \tau\,\psi(x)$, and, using the equations of motion for the quark fields, their divergences are

given by

$$i\partial^\mu j_\mu^{(V,A),\tau} = \bar{\psi}(\gamma_5)[\tau, M]_\mp \psi = (-i)^{(S,P)} j^{(S,P),[\tau,M]_\mp}, \tag{A7}$$

where the upper (lower) sign corresponds to the vector (axial-vector) current and $M = \text{diag}(m_1, \ldots, m_{n_f})$ is the quark mass matrix.

The equal-time commutators in Eq. (A6) can be obtained from the field anticommutators or, equivalently, from current algebra considerations for the first commutator as the second involves a divergence. The equal-time current commutation relations are given by

$$\left[j_\mu^{(V,A),\tau}(x), \left(j_0^{(V,A),\tau'}(y) \right)^\dagger \right]_{x_0=y_0}$$
$$= \delta^{(3)}(\vec{x} - \vec{y}) j_\mu^{V,[\tau,\tau'^\dagger]}(x), \tag{A8a}$$

$$\left[j_0^{(V,A),\tau}(x), i\partial^\nu \left(j_\nu^{(V,A),\tau'}(y) \right)^\dagger \right]_{x_0=y_0}$$
$$= \mp\delta^{(3)}(\vec{x} - \vec{y}) j^{S,[\tau,[M,\tau'^\dagger]_\mp]_\mp}(x). \tag{A8b}$$

We here consider commutators of currents with the same parity. Setting $A_\mu(x) = j_\mu^{(V,A),\tau}(x)$ and $B_\nu(y) = j_\nu^{(V,A),\tau'^\dagger}(y)$ in Eq. (A6), inserting the equal time commutators (A8), using translational invariance, and comparing with Eq. (A5), we obtain the nonanomalous Ward identity which relates the causal correlator of (axial-) vector currents to the causal correlator of their divergences

$$q^\mu q^\nu \Pi_{\mu\nu}^{(V,A)}(q|\tau,\tau') = \Pi^{(S,P)}(q|[\tau, M]_\mp, [\tau', M]_\mp^\dagger)$$
$$- \langle q^\mu j_\mu^{V,[\tau,\tau'^\dagger]} \rangle \mp \langle j^{S,[\tau,[M,\tau'^\dagger]_\mp]_\mp} \rangle. \tag{A9}$$

We turn now to an $n_f = 2$ flavor system. Let heavy-light meson currents be given by $\tau = \tau' = (\sigma^1 + i\sigma^2)/2$, where $\sigma^i = 2t^i$ are the Pauli matrices, which gives (2.1). The diagonal mass matrix can be written as $M = (C_V\sigma_3 + C_A\mathbb{1})/2$ and the commutators fulfill $[\tau, M]_\mp = \mp C_{(V,A)}\tau$, $[\tau, \tau^\dagger]_\mp = \tau^{(V,A)}$, $[\tau, [M, \tau^\dagger]_\mp]_\mp = \mp C_{(V,A)}\tau^{(V,A)}$, where we have defined $C_V = m_1 - m_2$, $C_A = m_1 + m_2$ and $\tau^V = \sigma^3$, $\tau^A = \mathbb{1}$. Note that q_1 is attributed to a light-quark field (e.g., up or down) and q_2 to a heavy-quark field (e.g., charm or bottom).

With these relations the longitudinal part of (2.3) is given as

$$q^2\Pi_L^{(V,A)}(q) = q^\mu q^\nu \Pi_{\mu\nu}^{(V,A)}(q)$$
$$= C_{(V,A)}^2 \Pi^{(S,P)}(q) - \langle \bar{\psi}\hat{q}\sigma^3\psi \rangle$$
$$+ C_{(V,A)} \langle \bar{\psi}\tau^{(V,A)}\psi \rangle \tag{A10}$$

with $\hat{q} = \gamma_\mu q^\mu$. [Equation (A10) also holds in a three-flavor system with two light quarks and one massive quark.] Furthermore, Π_L is zero if the current $j_\nu^{X',\tau'}(y)$ [or B_ν in Eq. (A2)] is conserved, i.e., $m_1 = m_2$ for vector currents or $m_1 + m_2 = 0$ for axial-vector currents [the first and third terms on the right-hand side of Eq. (A10) vanish], and the difference of light and heavy net quark currents being zero, i.e., $\langle \bar{\psi}\hat{q}\sigma^3\psi \rangle = 0$. Current conservation alone is not sufficient for a vanishing longitudinal projection. Indeed, the transversality in the case of current conservation generally reads $q^\mu\Pi_{\mu\nu}^{(V,A)}(q|\tau,\tau') = -\langle j_\nu^V(0|[\tau,\tau'^\dagger]_-)\rangle$, which vanishes only in a medium which is symmetric with respect to the quark flavors of the meson current. If Π_L is zero, the transversal projection $\Pi_T^{(V,A)}(q)$ is proportional to the trace of the correlator $\Pi_T^{(V,A)}(q) = -g^{\mu\nu}\Pi_{\mu\nu}^{(V,A)}(q)/3$. Otherwise, the trace $\Pi^{(V,A)} \equiv g^{\mu\nu}\Pi_{\mu\nu}^{(V,A)}(q)$ contains pieces of (axial-) vectors and (pseudo-) scalars. The pure (axial-) vector information is encoded in $\Pi_T^{(V,A)}(q)$ for which one obtains

$$3\Pi_T^{(V,A)} = \frac{C_{(V,A)}^2}{q^2}\Pi^{(S,P)} + \frac{1}{q^2}\langle \bar{\psi}\hat{q}\sigma^3\psi \rangle$$
$$+ \frac{C_{(V,A)}}{q^2}\langle \bar{\psi}\tau^{(V,A)}\psi \rangle - g^{\mu\nu}\Pi_{\mu\nu}^{(V,A)} \tag{A11}$$

which relates $\Pi_T^{(V,A)}$ to the trace of the correlator and $\Pi^{(S,P)}$. For m_1 and m_2 being arbitrary quark masses, the chiral difference $\Pi^{P-S}(q)$ and the sum $\Pi^{S+P}(q)$ enter the chiral difference $q^\mu q^\nu \Pi_{\mu\nu}^{V-A}(q)$ and, hence, Π_T^{V-A}. The expectation value of the quark current cancels out in any case:

$$\Pi_T^{V-A} = -\frac{1}{3}\left(\frac{m_1^2 + m_2^2}{q^2}\Pi^{P-S} + 2\frac{m_1 m_2}{q^2}\Pi^{P+S} \right.$$
$$\left. + \frac{2}{q^2}(m_1\langle\bar{q}_2 q_2\rangle + m_2\langle\bar{q}_1 q_1\rangle) + \frac{\Pi^{V-A}}{q^2} \right). \tag{A12}$$

If one quark mass is zero, $m_1 \to 0$, one obtains the relation

$$q^\mu q^\nu \Pi_{\mu\nu}^{V-A}(q) = -m_2^2\Pi^{P-S}(q) - 2m_2\langle\bar{q}_1 q_1\rangle \tag{A13}$$

TABLE I. Coefficients of the projection of $\{S, \gamma_5\}$.

$A_{\hat{\partial}}$	$\{S, \gamma_5\}$	$\{\gamma_\mu S\gamma^\mu, \gamma_5\}$
$A_{\mathbb{1}}$	$\frac{1}{2}\text{Tr}_D[S\gamma_5]$	$-2\text{Tr}_D[S]$
A_-^ν	0	0
$A^{\mu<\nu}$	$\frac{1}{2}\text{Tr}_D[S\gamma_5\sigma^{\mu<\nu}]$	0
A_+^ν	0	0
A_5	$\frac{1}{2}\text{Tr}_D[S]$	$2\text{Tr}_D[S]$

TABLE II. Coefficients of the projection of $\Gamma_\mu^C S\Gamma_C^\mu$.

Γ^C	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$
$A_{\mathbb{1}}$	$\frac{1}{4}\text{Tr}_D[S]$	$\frac{1}{4}\text{Tr}_D[S]$	$\text{Tr}_D[S]$	$-\text{Tr}_D[S]$
A_-^ν	$\frac{1}{4}\text{Tr}_D[S\gamma_\nu]$	$-\frac{1}{4}\text{Tr}_D[S\gamma_\nu]$	$-\frac{1}{2}\text{Tr}_D[S\gamma_\nu]$	$-\frac{1}{2}\text{Tr}_D[S\gamma_\nu]$
$A^{\mu<\nu}$	$\frac{1}{4}\text{Tr}_D[S\sigma^{\mu<\nu}]$	$\frac{1}{4}\text{Tr}_D[S\sigma^{\mu<\nu}]$	0	0
A_+^ν	$\frac{1}{4}\text{Tr}_D[S\gamma_5\gamma_\mu]$	$-\frac{1}{4}\text{Tr}_D[S\gamma_5\gamma_\mu]$	$\frac{1}{2}\text{Tr}_D[S\gamma_5\gamma_\mu]$	$\frac{1}{2}\text{Tr}_D[S\gamma_5\gamma_\mu]$
A_5	$\frac{1}{4}\text{Tr}_D[S\gamma_5]$	$\frac{1}{4}\text{Tr}_D[S\gamma_5]$	$-\text{Tr}_D[S\gamma_5]$	$-\text{Tr}_D[S\gamma_5]$

and, therefore,

$$\Pi_T^{V-A}(q) = -\frac{m_2^2}{3q^2}\Pi^{P-S}(q) - \frac{1}{3}\Pi^{V-A}(q) - \frac{2}{3}\frac{m_2}{q^2}\langle\bar{q}_1 q_1\rangle.$$

(A14)

The last term contains the interesting combination of light-quark condensate and heavy-quark mass.

APPENDIX B: DIRAC PROJECTION

Consider the trace over Dirac indices in $\text{Tr}_D[S_2\gamma_5\{S_1, \gamma_5\}]$. To write this as a product of two simpler traces, one can project the commutator $\{S, \gamma_5\} = \sum_{\hat{O}} A_{\hat{O}}\hat{O}$ with coefficients $A_{\hat{O}} = \frac{1}{4}\text{Tr}_D[\{S, \gamma_5\}\hat{O}]$, $\hat{O}$ being an element of the Clifford algebra. A similar expression applies for $\text{Tr}_D[S_2\gamma_5\{\gamma_\mu S_1\gamma^\mu, \gamma_5\}]$ with coefficients listed in Table I. Alternatively, the coefficients for projecting $\Gamma_\mu^C S\Gamma_C^\mu = \sum_{\hat{O}} A_{\hat{O}}\hat{O}$ are given in Table II.

Equation (2.5) can then be obtained by using $\gamma_5\sigma^{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$, leading to $\text{Tr}_D[S_1\gamma_5\sigma_{\mu\nu}]\text{Tr}_D[S_2\gamma_5\sigma^{\mu\nu}] = \text{Tr}_D[S_1\sigma_{\mu\nu}]\text{Tr}_D[S_2\sigma^{\mu\nu}]$.

APPENDIX C: THE PERTURBATIVE QUARK PROPAGATOR

The perturbative quark propagator in momentum space in a weak (classical) gluonic background field can be written as $S(p) = \sum_{n=0}^{\infty} S^{(n)}(p)$ with $S^{(n)}(p) = (-1)^n S^{(n-1)}(p)(\gamma\tilde{A})S^{(0)}(p) = (-1)^n S^{(0)}(p)(\gamma\tilde{A})S^{(n-1)}(p)$. $\tilde{A}$ denotes a derivative operator which arises due to the Fourier transform of the perturbation series for the quark propagator in coordinate space from the gluonic background field A_μ; $\tilde{A}_\mu = \sum_{n=0}^{\infty} \tilde{A}_\mu^{(n)}$ with $\tilde{A}_\mu^{(n)} = -\frac{(-i)^{n+1}g}{n!(n+2)}[D_{\alpha_1}\dots D_{\alpha_n}\mathscr{G}_{\mu\nu}(0)]\partial^\nu\partial^{\alpha_1}\dots\partial^{\alpha_n}$. The trace of each term $S^{(n)}(p)$ can be written as

$$\text{Tr}_D[\Gamma S^{(n)}(p)] = (-1)^n\left(\frac{1}{4}\right)^n \sum_{k_1,\dots,k_n}\left(\tilde{D}_{\alpha\vec{k}_n}\mathscr{G}_{\mu_n\nu_n}\right)\dots\left(\tilde{D}_{\alpha\vec{k}_1}\mathscr{G}_{\mu_1\nu_1}\right)$$

$$\times \sum_{\Gamma_1,\dots,\Gamma_n}\text{Tr}_D[\Gamma_n\Gamma S^{(0)}(p)\gamma^{\mu_n}][(\partial^{\nu_n}\partial^{\alpha\vec{k}_n}\text{Tr}_D[\Gamma_{n-1}\Gamma_n S^{(0)}(p)\gamma^{\mu_{n-1}}]\dots\{\partial^{\nu_1}\partial^{\alpha\vec{k}_1}\text{Tr}_D[\Gamma_1 S^{(0)}(p)]\})]. \quad (C1)$$

The sum runs over elements of the Clifford basis. For the sake of a concise notation we have defined $\tilde{D}_{\alpha\vec{k}}\mathscr{G}_{\mu\nu} \equiv -g\frac{(-i)^{k+1}}{k!(k+2)}(D_{\alpha_1}\dots D_{\alpha_k}\mathscr{G}_{\mu\nu}|_{x=0})$ and $\partial^{\alpha\vec{k}} \equiv \partial^{\alpha_1}\dots\partial^{\alpha_k}$. With these reduction formulas it is only necessary to consider traces of the free propagator $S^{(0)} = (\hat{p} + m)/(p^2 - m^2)$. Moreover, the single traces form a chain making the sum over Γ_i dependent on traces surviving from the sum over Γ_{i-1}. In the limit $m \to 0$ and $p^2 \neq 0$ we have $\text{Tr}_D[\Gamma_1 S^{(0)}(p, m = 0)] = 0 \,\forall\Gamma_1 \neq \gamma_\mu$. Hence, from the last trace in (C1) only

γ_μ is passed to the next trace. For the latter one we therefore have $\text{Tr}_D[\Gamma_1\Gamma_2 S^{(0)}(p, m = 0)\gamma_\mu] = 0 \,\forall\Gamma_2 \notin \{\gamma_\alpha, \gamma_5\gamma_\alpha\}$. As $\text{Tr}_D[\gamma_5\gamma_\alpha\Gamma_n S^{(0)}(p, m = 0)\gamma_\mu] = 0 \,\forall\Gamma_n \notin \{\gamma_\alpha, \gamma_5\gamma_\alpha\}$ also holds, each sum merely covers γ_μ and $\gamma_5\gamma_\mu$. Finally, we obtain for the first trace in the second line of (C1) $\text{Tr}[\Gamma_n\Gamma S^{(0)}(p, m = 0)\gamma_\mu] = 0 \,\forall\Gamma \notin \{\gamma_\alpha, \gamma_5\gamma_\alpha\}$, which means that $\text{Tr}_D[\Gamma S^{(n)}(p, m = 0)] = 0 \,\forall\Gamma \notin \{\gamma_\mu, \gamma_5\gamma_\mu\}$. As this is true for all orders of the perturbative sum, we can conclude that $\text{Tr}_D[\Gamma S(p, m = 0)] = 0 \,\forall\Gamma \notin \{\gamma_\mu, \gamma_5\gamma_\mu\}$.

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