Probing the density dependence of the symmetry energy via multifragmentation at subsaturation densities

Sanjeev Kumar,[1] Y. G. Ma (马余刚),[1,*] G. Q. Zhang (张国强),[1,2] and C. L. Zhou (周铖龙)[1,2]
[1]*Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China*
[2]*Graduate School of the Chinese Academy of Sciences, Beijing 100080, China*
(Received 18 August 2011; revised manuscript received 22 September 2011; published 31 October 2011)

Symmetry energy for asymmetric nuclear matter at subsaturation densities was investigated in the framework of an isospin-dependent quantum molecular dynamics model. A single ratio of neutrons and protons is compared with the experimental data of Famiano *et al*. [Phys. Rev. Lett. **97**, 052701 (2006)] We have also performed a comparison for the double ratio with experimental as well as different theoretical results of Boltzmann-Uehling-Uhlenbeck in 1997, Isospin-dependent Boltzmann-Uehling-Uhlenbeck in 2004, Boltzmann-Nordeim-Vlasov, and Improved Quantum Molecular Dynamics models. It is found that the double ratio predicts the softness of symmetry energy, which is a little underestimated in the single ratio. Furthermore, the study of the single ratio is extended for different kinds of fragments, while the double ratio is extended for different neutron-rich isotopes of Sn.

DOI: 10.1103/PhysRevC.84.044620 PACS number(s): 21.65.Ef, 21.65.Cd, 25.70.Pq

I. INTRODUCTION

One of the most important challenges in heavy-ion physics is the determination of the isospin dependence of the nuclear equation of state (NEOS), which plays a very important role in low-energy phenomena such as nuclear structure, nuclear astrophysics [1], fusion, and cluster radioactivity [2]; intermediate-energy phenomena such as multifragmentation, stopping, and flow [3–26]; and at last high-energy phenomena such as pion and kaon production [27,28]. The symmetry energy is found to be the prominent candidate to study the isospin dependence of the NEOS. In past years, many studies have been performed on the density dependence of symmetry energy at subsaturation densities by using isotopic scaling [7–11], isobaric ratio [12], single and double ratios [13–17,19,20], isospin diffusion [6,21], isospin distillation and fractionation [6,22], and isospin migration and drift [4,6,23]. Apart from these, transverse and elliptic flows of neutrons and protons are also considered good candidates to emphasize the importance of the density dependence of symmetry energy [24–26]. Even with the help of these studies, the exact determination of symmetry energy is still under way.

In the present work, we only want to address the effect of symmetry energy on the kinetic energy spectra of nucleons as well as the neutron-to-proton ratio parameters. The latter was considered to be the first-ever prominent candidate to extract the density dependence of the symmetry energy.

Before we discuss the contents of the present work, let us discuss some highlights of the single and double ratios in heavy-ion collisions. The single-ratio study in heavy-ion collisions has already been performed by different experimental and theoretical groups [13,14,20]. In the experiments, near Fermi energy, Hilscher *et al*. [14] found that the single ratio of preequilibrium nucleons is consistently higher than that of the projectile-target system and it cannot be explained by the Coulomb effects alone. Another experimental observation is the ratio of free neutrons and protons from two isotopic systems at 26 MeV/nucleon. A lot of interesting observations are made from the data. Schroder *et al*. [29] also systematically studied the spectra of preequilibrium neutrons and protons in both isospin-symmetric and isospin-asymmetric systems. Recently, at the Michigan State University (MSU) National Superconducting Cyclotron Labortory (NSCL), Famiano *et al*. [13] measured the single and double ratios of free neutrons to protons for ^{112}Sn + ^{112}Sn and ^{124}Sn + ^{124}Sn at 50 MeV/nucleon. The results of the double ratio of the above data have also been reproduced by different theoretical models, such as Boltzmann-Uehling-Uhlenbeck in 1997 (BUU97) [15], Isospin-dependent Boltzmann-Uehling-Uhlenbeck in 2004 (IBUU04) [16], Boltzmann-Nordeim-Vlasov (BNV) [17], and Improved Quantum Molecular Dynamics (ImQMD) [19]. Even so, there are a lot of uncertainties in the determination of the symmetry energy in terms of different parameters such as the cross section, the symmetry energy coefficient, the impact parameter, and the method of clusterization.

However, no study exists in the literature where the comparison of a single ratio of neutrons to protons is performed with the experimental data. One step ahead, the single ratio for the fragments is still poorly known in the literature. A few studies existed from the BNV and IBUU04 calculations for the single ratio using the intermediate-mass fragments (IMFs), which is only limited by the small range of the kinetic energy [6,18], however, isospin distillation and fractionation was studied up to higher kinetic energy by Li *et al*. [30]. In extension of the single ratio to the double ratio, no one has tried to compare the double-ratio findings for experiments and theories at one place to see which one is the most appropriate model and symmetry energy form. Also absent from the literature is a study of the effect on the double ratio if we consider the series of isotopes with different isospin contents. From all these gaps, it seems interesting to perform a study on the single and double ratios simultaneously.

In this article, we focus on the comparative study of single and double ratios of neutrons to protons with the experimental

*Author to whom all correspondence should be addressed: ygma@sinap.ac.cn

data of the MSU NSCL collaborations [13]. Moreover, in addition to the comparison with the experimental data, our results of the isospin-dependent quantum molecular dynamics (IQMD) model (initially developed by Hartnack *et al.* [31]) are also compared with other studies of BUU97, IBUU04, BNV, and ImQMD models. The study of the single ratio is extended for different kinds of fragments up to higher kinetic energy, while the double ratio is investigated for different neutron-rich systems having different isospin content.

The article is organized as follows. We discuss the model briefly in Sec. II. Our results and discussions are given in Sec. III and we summarize the results in Sec. IV.

II. FORMALISM: IQMD MODEL

In the IQMD model [5,26,31], nucleons are represented by the wave packets, just like in the quantum molecular dynamics (QMD) model [32]. These wave packets of the target and projectile interact by the full Skyrme potential energy, which is represented by U and is given as

$$U = U_\rho + U_{\rm Coul}. \quad (1)$$

Here $U_{\rm Coul}$ is the Coulomb energy, and U_ρ is originated from the density dependence of the nucleon optical potential and is given as

$$U_\rho = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma+1}\frac{\rho^{\gamma+1}}{\rho_0^\gamma} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}\delta^2\rho, \quad (2)$$

where $\delta = (\frac{\rho_n-\rho_p}{\rho_n+\rho_p})$; $\rho = \rho_n + \rho_p$, and ρ_n and ρ_p are the neutron and proton densities, respectively. The densities ρ, ρ_n, and ρ_p have dimensions of fm^{-3}.

First two of the three parameters of Eq. (2) (α and β) are determined by demanding that, at normal nuclear matter densities, the binding energy should be equal to 16 MeV and the total energy should have a minimum at ρ_0. The third parameter, γ, is usually treated as a free parameter. Its value is given in terms of the compressibility:

$$\kappa = 9\rho^2\frac{\partial^2}{\partial\rho^2}\left(\frac{E}{A}\right). \quad (3)$$

The different values of compressibility give rise to soft and hard equations of state. The soft equation of state is employed in the present study with the parameters $\alpha = -356$ MeV, $\beta = 303$ MeV, and $\gamma = 7/6$ corresponding to isoscalar compressibility of $\kappa = 200$ MeV. In the calculations, we use the isospin-dependent in-medium cross section in the collision term and the Pauli blocking effects just like in the QMD model [32]. The third term in Eq. (2) is the symmetry potential energy for finite nuclear matter. The symmetry energy per nucleon employed in the simulation is the sum of the kinetic and potential terms. So, the total symmetry energy is given as

$$E_{\rm Sym}(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}, \quad (4)$$

where $C_{s,k} = 25$ MeV from the Fermi-Dirac distribution, which is well explained in Ref. [33] and is known as the symmetry kinetic energy coefficient, while $C_{s,p} = 35.19$ MeV is parametrized on the basis of the experimental value of the symmetry energy, known as symmetry potential energy coefficient. On the basis of the γ_i value, the symmetry energy is divided into two types with $\gamma_i = 0.5$ and $\gamma_i = 1.5$, corresponding to the soft and stiff symmetry energies, respectively. Note that the γ used in the determination of the equation of state and γ_i used in the determination of the symmetry energy are different parameters. The interesting feature of symmetry energy is that its value increases with decreasing γ_i at subsaturation densities, whereas the opposite is true at suprasaturation densities. In other words, the soft symmetry energy is more pronounced at subsaturation densities, whereas the stiff symmetry energy is more pronounced at suprasaturation densities.

The cluster yields are calculated by means of the coalescence model, in which particles with relative momentum smaller than $P_{\rm Fermi}$ and relative distance smaller than R_0 are coalesced into a cluster. The values of R_0 and $P_{\rm Fermi}$ for the present work are 3.5 fm and 268 MeV/c, respectively.

III. RESULTS AND DISCUSSION

In the present study, we simulate thousands of events for the isotopes of Sn, namely ^{112}Sn + ^{112}Sn, ^{124}Sn + ^{124}Sn, and ^{132}Sn + ^{132}Sn at incident energy of 50 MeV/nucleon by using the soft and stiff symmetry energy having $\gamma_i = 0.5$ and 1.5, respectively. The collision geometry for the study is from semicentral to semiperipheral, keeping in mind the importance of the impact parameter of the MSU NSCL collaboration's experimental results. As discussed earlier, the soft equation of state with an isospin-dependent nucleon-nucleon cross sections of $\sigma_{\rm med} = (1 - 0.2\frac{\rho}{\rho_0})\sigma_{\rm free}$ is employed. The single and double ratios are considered a point of importance in the present study. The neutron-to-proton ratio is among the first observable that was proposed as a possible sensitive probe for symmetry energy prediction [13,14,20]. This ratio is studied for the free nucleons, light charged particles (LCPs) (having charge number of 1 and 2) and IMFs (having charge between 3 and $Z_{\rm tot}/6$, where $Z_{\rm tot}$ is the total charge of the projectile and target under study). The single ratio is just the ratio of neutrons to protons and is represented in the study by $R_{N/Z}$, while the double ratio is the ratio of the single ratios of any two isotopes of Sn. To study the systematics of the isospin effect, the single ratio of the isotope with more neutrons is always given in the numerator when the double ratio is calculated. Mathematically, the double ratio is represented by $DR_{N/Z}$ and is given as

$$DR_{N/Z} = \frac{R_{N/Z}^{\rm neutron\ rich}}{R_{N/Z}^{\rm neutron\ weak}}. \quad (5)$$

A. Kinetic energy spectra

To go into detail in the results from all the aforementioned ratios, let us understand the kinetic energy spectra of protons and neutrons for all types of fragments in the center-of-mass frame. The spectra of free protons and neutrons are very important experimental observables that can provide useful information about the particle production mechanism and reaction dynamics.

Figure 1 displays the kinetic energy spectra for protons and neutrons at incident energy $E = 50$ MeV/nucleon for

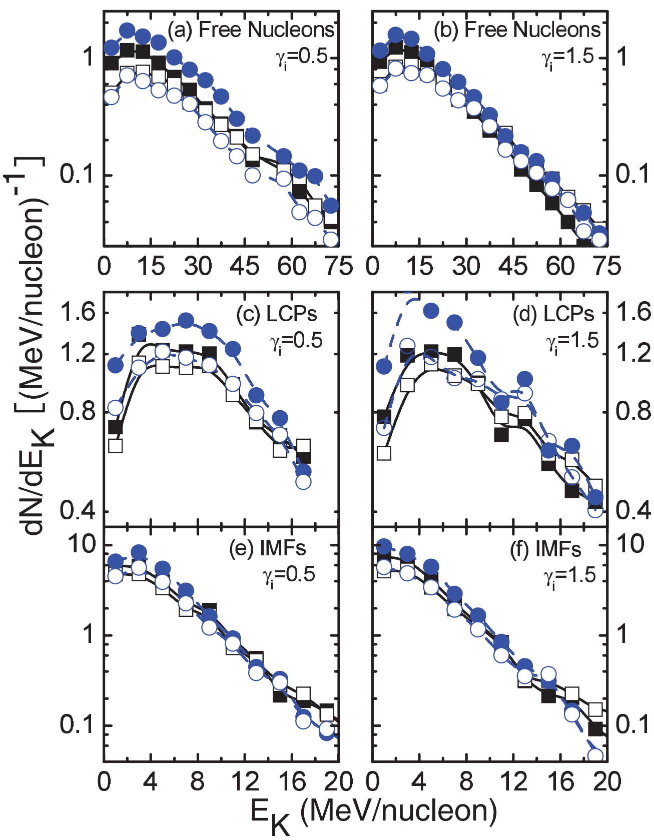

FIG. 1. (Color online) The kinetic energy spectra in the center-of-mass system for neutrons (solid symbols) and protons (open symbols) from (a, b) the free nucleons, (c, d) LCPs, and (e, f) IMFs at semicentral geometry ($b = 2$ fm) of ^{132}Sn + ^{132}Sn (blue circles) and ^{112}Sn + ^{112}Sn (black squares) collisions at $E = 50$ MeV/nucleon by using the (left) soft and (right) stiff symmetry energy, respectively.

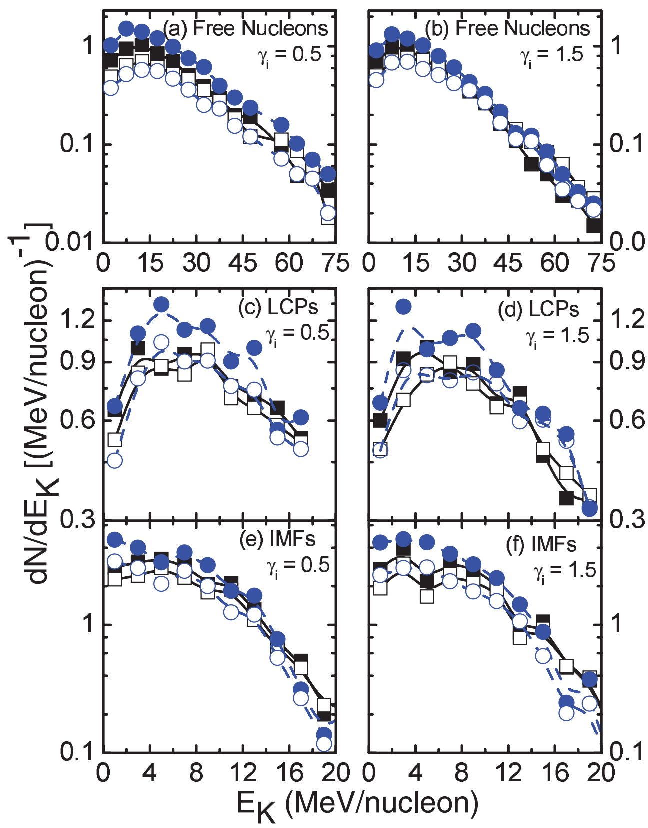

FIG. 2. (Color online) Same as in Fig. 1, but for the semiperipheral geometry ($b = 6$ fm).

semicentral geometry, while Fig. 2 is at semiperipheral geometry. The results are for the neutron-rich system ^{132}Sn + ^{132}Sn and the neutron-weak system ^{112}Sn + ^{112}Sn, using the soft and stiff symmetry energy, respectively. The left- and right-hand panels in both figures are with the soft and stiff symmetry energy, while the top, middle, and bottom panels are for the free nucleons and for bound nucleons inside LCPs and IMFs, respectively.

It is clear from the figures that the production of neutrons is more favorable for a neutron-rich system [34]. This is also true for all types of fragments as well as for the soft and stiff symmetry energy. This is because, in the more neutron-rich system, the symmetry energy is more repulsive (attractive) for neutrons (protons) and hence more neutrons can be produced. However, the difference between yield or content of neutrons and protons decreases with increasing of the kinetic energy. This is due to the Coulomb repulsion, which shifts the protons from low to high kinetic energy. The behavior is the same for all types of fragments. Interestingly, more neutrons can be produced with the soft symmetry energy for free particles as compared to the stiff one. The opposite is true for the LCPs and IMFs up to a certain kinetic energy and after that the same trend is observed just like for the free particles. It is an interesting phenomenon that has unfortunately gone unnoticed. This is because the Coulomb effects are stronger inside LCPs and IMFs as compared to free particles. However, at a sufficiently high kinetic energy, the symmetry energy dominates over the Coulomb interactions and the behavior becomes similar to that of free nucleons. However, this intersection between soft and stiff symmetry energy for the fragments is not so clearly observed from here, so we have extended the study with the single ratio $R_{N/Z}$ in Fig. 3.

Let us move to Fig. 2, which displays semiperipheral geometry. Almost the same spectra are observed at semiperipheral geometry, but for some exceptions. Once again, an interesting point is that the yield of free neutrons at high kinetic energy for the semicentral geometry (Fig. 1) is higher in comparison with the semiperipheral one (Fig. 2). As we already knew, the symmetry energy or potential has two important functions: first, it tends to unbound more neutrons, and second, it makes the neutrons more energetic than protons. Because of this, most of the finally observed neutrons are unbounded in the very early stage of the reaction as a result of the nucleon-nucleon collisions at semicentral collisions. Now, the symmetry energy at preequilibrium time is just shifting more neutrons toward the high kinetic energy. However, at semiperipheral geometry, the emission of neutrons also depends on the symmetry potential or energy due to the relative lack of the nucleon-nucleon collisions. The symmetry energy makes the neutrons unbound, but at relatively low kinetic energy. That is why isospin effects are more pronounced at low kinetic energy for peripheral

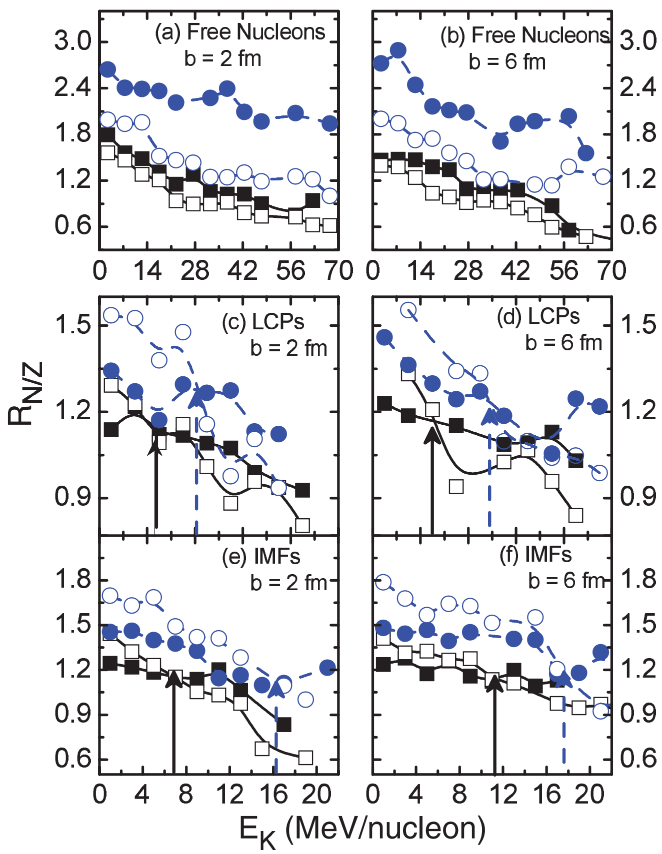

FIG. 3. (Color online) The ratio of neutrons to protons (a,b) at freeze-out time from free nucleons, (c, d) LCPs, and (e, f) IMFs as a function of kinetic energy at (left) semicentral and (right) semiperipheral geometries by using the soft (solid symbols) and stiff symmetry energy (open symbols). The vertical lines in the plots of LCPs and IMFs represent the kinetic energy at the crossover points of the soft and stiff symmetry energies. Blue circles represent ^{132}Sn + ^{132}Sn and black squares represent ^{112}Sn + ^{112}Sn at $E =$ 50 MeV/nucleon.

collisions and at high kinetic energy for central collisions. These results are also consistent with those in Ref. [6]. When one moves from LCPs to IMFs, the content of neutrons for a neutron-rich system is lower than for the neutron-weak system at high kinetic energies for semicentral as well as semiperipheral geometries. This is because at high kinetic energy the yield of free nucleons is higher as compared to the fragments and it will result in more production of free neutrons for a neutron-rich system as compared to the fragments.

B. Single ratio

To make sure about the preceding discussion from Figs. 1 and 2, it is interesting to investigate the single ratio ($R_{N/Z}$) of neutrons to protons for free nucleons, LCPs, and IMFs, which is shown in Fig. 3 for the neutron-rich system ^{132}Sn + ^{132}Sn and the neutron-weak system ^{112}Sn + ^{112}Sn by using the soft and stiff symmetry energy. The left- and right-hand panels are at semicentral and semiperipheral geometries, respectively. As is expected from Figs. 1 and 2, Fig. 3 depicts the following results:

(i) The isospin effects for a more neutron-rich system are stronger and it is consistent with Ref. [6] and with Figs. 1 and 2.

(ii) $R_{N/Z}$ decreases with the kinetic energy for all types of fragments at semicentral as well as semiperipheral geometries.

(iii) For free nucleons, the isospin effects are stronger at high kinetic energy for a semicentral geometry, while the same is true at low kinetic energy for semiperipheral geometries. This result was also explained earlier in Ref. [6].

(iv) The increase in the neutron-to-proton ratio for a neutron-rich system at sufficiently high kinetic energy is due to the repulsive nature of the symmetry energy for neutrons.

(v) With regard to the single ratio for fragments, $R_{N/Z}$ of IMFs was earlier studied by the Catania group using the BNV [18] and by the Texas group using the IBUU04 model [6]. Both models have different approaches for the symmetry energy and hence the results are a little different from each other. In the BNV results, the ratio decreases at low fragment kinetic energy and then increases at high kinetic energy for a neutron-rich system with stiff symmetry energy, but in the IBUU04 calculations, the ratio is found to decrease with fragment kinetic energy. However, both calculations have the same behavior with the soft and stiff symmetry energy. But, both groups have limited their study only to the relative low kinetic energy and were not able to investigate the crossover phenomenon of symmetry energy, which takes place at higher kinetic energies and is discussed in detail in this study.

(vi) The large isospin effects are observed with soft symmetry energy for free nucleons along the whole range of the kinetic energy [20], whereas crossover happens for the LCPs and IMFs at a certain kinetic energy. Below the crossover kinetic energy, the stiff symmetry energy produces a larger neutron-to-proton ratio and, after the crossover, the same is true for the soft symmetry energy, which behaves just like in the case of ratio from free nucleons. Recently, Harmann *et al.* [35] displayed the data for a single ratio from the IMFs below the crossover kinetic energy. These data (not shown here) favor the soft symmetry energy in our studies with the IQMD; however, with the BNV, their data are well explained by the stiff symmetry energy. If one looks carefully, we can find that the soft symmetry energy is more soft and the stiff symmetry energy is less stiff in the BNV as compared to the IQMD and ImQMD models. In other words, the stiff symmetry energy from the BNV model and the soft symmetry energy from the IQMD and ImQMD models lies between the stiff symmetry energy from the IQMD and ImQMD models and the soft symmetry energy from the BNV calculations. It means that the data favor almost the same symmetry energy from both models.

(vii) The crossover kinetic energy is at a higher value for the more neutron-rich system and increases with the size of the fragments (i.e., from LCPs to IMFs).

(viii) The crossover value of the kinetic energy also raises when one moves from semicentral to semiperipheral geometries. This value is more affected for the more neutron-rich system.

(ix) It is also clear from here that the gas phase (free nucleons) is significantly enriched in neutrons relative to the liquid phase or fragments that are represented by the bounding nuclei. The phenomenon is known as isospin distillation and fractionation and is discussed many times in the literature, only in terms of the free and bound nucleons [6]. More interesting results are expected for isospin distillation if one tries to study it in terms of different kinds of fragments.

The theoretical results become more interesting and useful if one compares the results with the experimental data. In Fig. 4, we compare the results of a single ratio of neutrons to protons from free nucleons for the neutron-weak system ^{112}Sn + ^{112}Sn [Fig. 4(a)] and for the neutron-rich system ^{124}Sn + ^{124}Sn [Fig. 4(b)] at 50 MeV/nucleon with the experimental data [13]. The behavior of the single ratio results for free nucleons is explained in Fig. 3. The conclusions from the figure are that (1) greater $R_{N/Z}$ is observed for the more neutron-rich system, which is also predicted by theoretical predictions, and (2) $R_{N/Z}$ shows an increment at higher kinetic energy, especially for ^{124}Sn + ^{124}Sn. It indicates that the theoretical results are consistent with the experimental result.

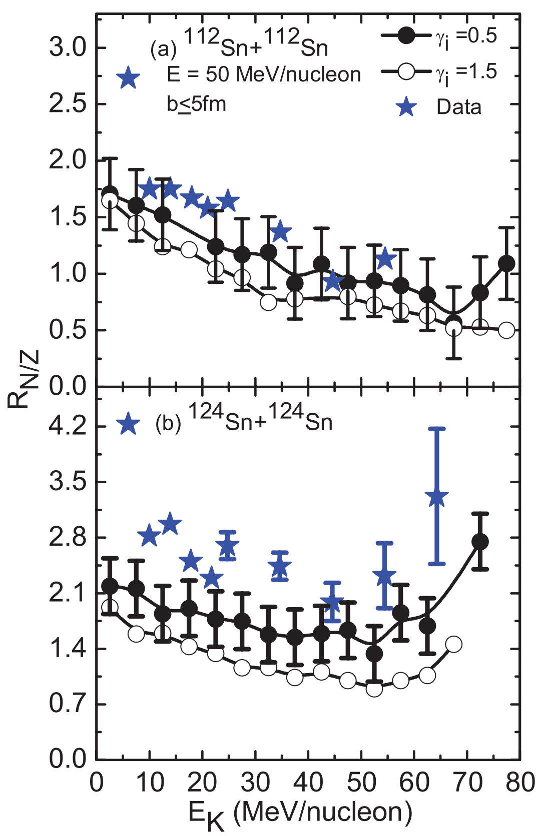

FIG. 4. (Color online) The comparison of neutron-to-proton ratios from free nucleons for the systems (a) ^{112}Sn + ^{112}Sn and (b) ^{124}Sn + ^{124}Sn at $E = 50$ MeV/nucleon and impact parameter $b \leqslant 5$, with the experimental data of the MSU NSCL collaborations [13]. The solid and open circles represent the soft and stiff symmetry energies, respectively.

The results are in good agreement with the soft symmetry energy except at very low and very high kinetic energies. The difference between the soft and stiff symmetry energy results for the neutron-weak system is almost comparable to the error bar, whereas for the neutron-rich system, the difference has a great importance over the error bar. In other words, the error bar of the theoretical results with the soft symmetry energy covers the error bar of the experimental data for both systems under consideration. The difference at high kinetic energy between theoretical and experimental results for the neutron-rich system is due to the large uncertainty in the measurement of $R_{N/Z}$. By using the single-ratio observable, one can reach a partial conclusion that the asymmetric nuclear matter favors the soft symmetry energy at subsaturation densities, which is also consistent with the other findings in the literature [4,15–17,19,20,24].

As we observed, the single ratio mixes the symmetry energy with Coulomb effects throughout the kinetic energy range. To minimize the Coulomb effects and systematic error, it is reasonable to study the double neutron-to-proton ratio for the isotopes of the same element. This is also studied in the literature with only two isotopes [15,16,19,20]. No one has tried to investigate the effect of a double ratio on a series of isotopes in asymmetric nuclear matter so far.

C. Double ratio

In the present study, we consider reactions between three isotopes of Sn and observe the relative effect of these isotopes on the double ratio and symmetry energy. The pairs are as follows: ^{132}Sn + ^{132}Sn and ^{124}Sn + ^{124}Sn, ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn, and ^{132}Sn + ^{132}Sn and ^{112}Sn + ^{112}Sn. The three pairs have differences of 8, 12, and 20 neutrons, respectively. The universal behavior for the double ratio is observed with the kinetic energy; that is, with the increase in kinetic energy, the double ratio is found to increase for all three sets of isotopes, which we plot in Fig. 5. The increase in the double ratio is due to the effect that now energetic nucleons are more affected by the symmetry potential, which are already suppressed by the Coulomb repulsion in the single-ratio results. The effect of symmetry energy on the double ratio is the same as for the single ratio, that is, a larger value with the soft symmetry energy as compared to the stiff one. Moreover, the double ratio continues to increase with the increase of the neutron difference between the pairs discussed earlier, or, in other words, the double ratio from free nucleons continues to increase with the initial-state double ratio of the systems from three different pairs of isotopes of Sn.

This increase is due to the effect that the greater the number of neutrons, the more repulsive is their symmetry energy. The Coulomb effects are already canceled by taking the double ratio. Hence, the results are just as expected. The double ratio is found to be weakly sensitive toward the collision geometry.

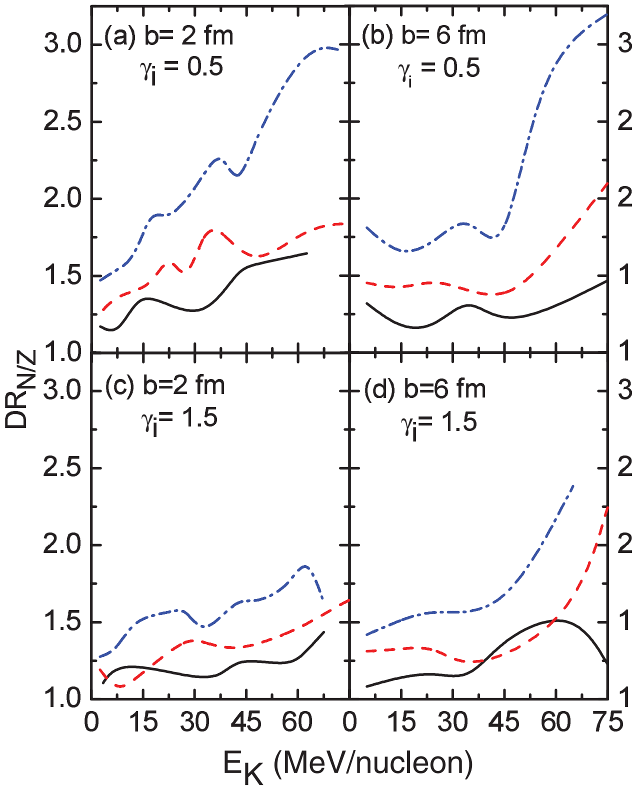

FIG. 5. (Color online) Free neutron-to-proton double ratio, at (a, c) semicentral and (b, d) semiperipheral geometry with the (a, b) soft and (c, d) stiff symmetry energy, as a function of kinetic energy at the incident energy $E = 50$ MeV/nucleon. The different lines are the double ratio from different pairs: solid line, ^{132}Sn + ^{132}Sn and ^{124}Sn + ^{124}Sn; dashed line, ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn; and dash-dotted line, ^{132}Sn + ^{132}Sn and ^{112}Sn + ^{112}Sn.

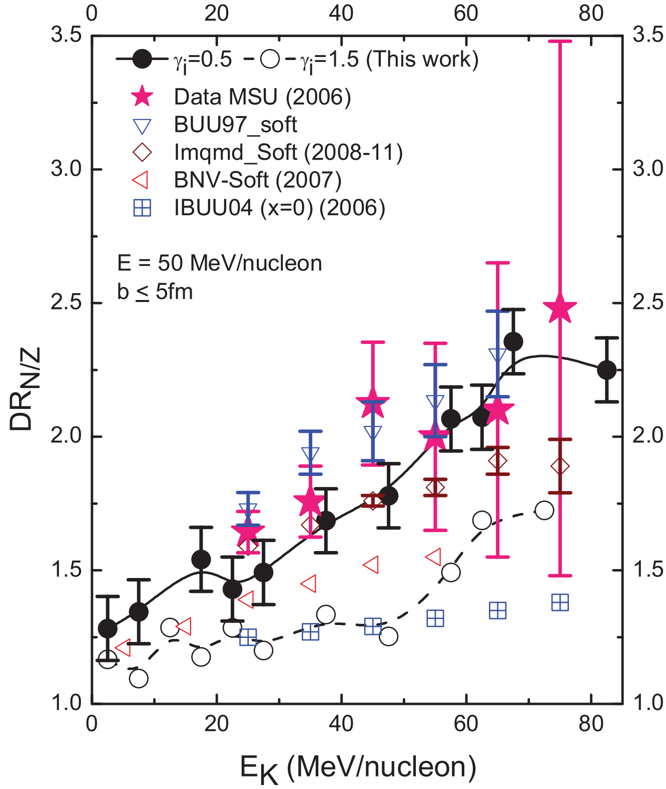

FIG. 6. (Color online) The comparison of free neutron-to-proton double ratio at $E = 50$ MeV/nucleon and $b \leqslant 5$ with the MSU NSCL data and BUU97, IBUU04, BNV, and ImQMD simulations.

However, a little increase is observed at the semiperipheral geometry compared to the semicentral one at high kinetic energy. This is true with the stiff as well as the soft symmetry energies.

The double ratio was studied many times in the past couple of years by different groups with the help of the BUU97, IBUU04, BNV, and ImQMD models and compared with the experimental results. Even so, we are still far away from the exact conclusion about the symmetry energy form. Along with all the possible results in the literature, we compare the double ratio with the IQMD model in Fig. 6. Let us start with very first comparison of the BUU97 model [15]. The results were very close to the experimental one, but the reaction conditions were different. First, in the BUU97 calculations, the incident energy was 40 MeV/nucleon, not 50 MeV/nucleon, just like the experimental result. Second, the data set is only for the transverse emission, whereas in the BUU97 calculations, the nucleons used are emitted in all directions. Moving one step ahead to the IBUU04 results [16], where the symmetry energy is introduced with the help of momentum-dependent interactions, the results are very far from the experimental data. The same is true for the BNV calculations performed by the Catania group in 2007 [17]. The closest agreement between the data and the calculation was observed by the ImQMD model in 2009 [19]. They found that the results with $\gamma_i = 0.75$ are best fit with the experimental data for an impact parameter $b \leqslant 2$ fm. In the present study, we performed simulations for $b \leqslant 5$ fm and for the angular cuts, as mentioned in the experiments, with the soft and stiff symmetry energy and displayed the theoretical results over the whole range of the kinetic energy. Figure 6 clearly indicates that our results are very close to the experimental data.

If we see the comparison of theoretical and experimental results from single- (Fig. 4) and double-ratio (Fig. 6) results, it seems that the single-ratio results require $\gamma_i < 0.5$ to explain the data, whereas the data are well explained by the $\gamma_i = 0.5$ for the double ratio. This is because the single ratio suffers the effect from the Coulomb interactions in addition to symmetry energy. Because our main purpose is to extract the symmetry energy, the double ratio can act as a better candidate than the single ratio. In conclusion, the results of the double ratio can be very well explained by the soft symmetry energy with $\gamma_i = 0.5$ in comparison with the single ratio, where the data are a little underestimated by the theoretical predictions.

IV. SUMMARY

In summary, we performed a detailed analysis of the kinetic energy spectra of free nucleons and bound nucleons inside fragments as well as the ratio parameters for the three reaction channels of Sn isotopes at $E = 50$ MeV/nucleon via multifragmentation. The kinetic energy spectra of protons and neutrons from free nucleons and all types of fragments in the center-of-mass frame show that the content of neutrons is more favorable for neutron-rich systems since the symmetry energy becomes more repulsive (attractive) for neutrons (protons) and hence more neutrons can be produced. In addition, the difference between yields or contents of neutrons and protons decreases with increasing kinetic energy, which can be explained by the Coulomb repulsion shifts, shifting the protons from low to high kinetic energy. Interestingly, more free neutrons can be produced with the soft symmetry energy as compared to the stiff one.

The single ratio of free neutrons to protons decreases with the kinetic energy for all types of fragments at semicentral as well as semiperipheral geometries. However, the increase of the ratio from free nucleons for a neutron-rich system is observed at sufficient high kinetic energy, which can be explained by the repulsive nature of the symmetry energy for neutrons. For single ratios of LCPs and IMFs, we noticed a transition at a certain kinetic energy between the soft and stiff symmetry energies, whereas there was no transition for the free nucleons or gas phase. Below the crossover kinetic energy, the stiff symmetry energy produces a larger ratio of neutrons to protons and after the crossover, the same is true with the soft symmetry energy, which behaves just like the case of free nucleons. This transition is also found to be strongly dependent on the isospin of the colliding partners and the size of the fragment and is weakly dependent on the collision geometry. Moreover, isospin distillation is also observed when one moves from the gas phase to the liquid phase. It is furthermore interesting to study the isospin distillation in terms of the different kinds of fragments, as compared to considering bound fragments as a single liquid phase.

The comparison of the theoretical results of single and double ratios with the experimental data emphasizes the softness of the symmetry energy at subsaturation densities, which is yet uncertain at the suprasaturation densities. However, the single-ratio study underestimates the data a little as compared to the double ratio for the same stiffness of symmetry energy ($\gamma_i = 0.5$), which reflects that the double ratio is a relatively good candidate for the density dependence of symmetry energy at subsaturation densities because of the canceling of the Coulomb effect between the two systems. Of course, the magnitude of the double ratio of neutrons and protons from free nucleons strongly depends on the initial double ratio of the systems. It gives us an indication that it is better to study the isospin physics with the pair ^{132}Sn + ^{132}Sn and ^{112}Sn + ^{112}Sn.

ACKNOWLEDGMENTS

This work is supported in part by the Chinese Academy of Sciences Support Program for young international scientists under Grant No. 2010Y2JB02; the 973 Program under Contract No. 2007CB815004; the National Science Foundation of China under Contracts No. 11035009, No. 11005140, and No. 10979074; and the Knowledge Innovation Project of the Chinese Academy of Sciences under Grant No. KJCX2-EW-N01.

[1] J. M. Lattimer and M. Prakash, Phys. Rep. **333**, 121 (2000).
[2] F. Amorini *et al.*, Phys. Rev. Lett. **102**, 112701 (2009); Z. Q. Feng, G. M. Jin, and F. S. Zhang, Nucl. Phys. A **802**, 91 (2008); I. Dutt and R. K. Puri, Phys. Rev. C **81**, 047601 (2010); **81**, 064609 (2010); Z. Q. Feng and G. M. Jin, *ibid.* **80**, 037601 (2009); R. K. Puri and R. K. Gupta, J. Phys. G **18**, 903 (1992); Z. Q. Feng, G. M. Jin, and J. Q. Li, Nucl. Phys. Rev. **28**, 1 (2011).
[3] Y. G. Ma *et al.*, Phys. Rev. C **71**, 054606 (2005); Y. G. Ma and W. Q. Shen, *ibid.* **51**, 3256 (1995); **51**, 710 (1995); Y. G. Ma, W. Q. Shen, J. Feng, and Y. Q. Ma, *ibid.* **48**, R1492 (1993); G. Q. Zhang *et al.*, *ibid.* **84**, 034612 (2011)
[4] M. D. Toro *et al.*, Int. J. Mod. Phys. E **17**, 1799 (2008).
[5] S. Kumar, S. Kumar, and R. K. Puri, Phys. Rev. C **81**, 014601 (2010); **81**, 014611 (2010); G. Lehaut *et al.*, Phys. Rev. Lett. **104**, 232701 (2010); S. Kumar and S. Kumar, Chin. Phys. Lett. **27**, 062504 (2010).
[6] B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).
[7] H. S. Xu *et al.*, Phys. Rev. Lett. **85**, 716 (2000); M. B. Tsang, W. A. Friedman, C. K. Gelbke, W. G. Lynch, G. Verde, and H. Xu, *ibid.* **86**, 5023 (2001); A. S. Botvina, O. V. Lozhkin, and W. Trautmann, Phys. Rev. C **65**, 044610 (2002); A. Le Fevre *et al.*, Phys. Rev. Lett. **94**, 162701 (2005).
[8] M. Colonna and M. B. Tsang, Eur. Phys. J. A **30**, 165 (2006).
[9] J. B. Natowitz *et al.*, Phys. Rev. Lett. **104**, 202501 (2010); S. Kowalski *et al.*, Phys. Rev. C **75**, 014601 (2007); Z. Chen *et al.*, *ibid.* **81**, 064613 (2010); R. Wada *et al.*, Phys. Rev. Lett. **58**, 1829 (1987).
[10] D. V. Shetty, S. J. Yennello, and G. A. Souliotis, Phys. Rev. C **75**, 034602 (2007); G. A. Souliotis, A. S. Botvina, D. V. Shetty, A. L. Keksis, M. Jandel, M. Veselsky, and S. J. Yennello, *ibid.* **75**, 011601 (2007); S. Wuenschel *et al.*, *ibid.* **79**, 061602 (2009).
[11] Y. G. Ma *et al.*, Phys. Rev. C **69**, 064610 (2004); **72**, 064603 (2005); Y. Fu *et al.*, Chin. Phys. Lett. **26**, 082503 (2009); P. Zhou *et al.*, Phys. Rev. C **84**, 037605 (2011); W. D. Tian *et al.*, *ibid.* **76**, 024607 (2007); Int. J. Mod. Phys. E **17**, 1719 (2008); D. Q. Fang *et al.*, J. Phys. G **34**, 2173 (2007); Q. M. Su *et al.*, Chin. Phys. Lett. **25**, 2000 (2008).
[12] M. Huang *et al.*, Phys. Rev. C **81**, 044620 (2010); C. W. Ma, F. Wang, Y. G. Ma, and C. Jin, *ibid.* **83**, 064620 (2011).
[13] M. A. Famiano *et al.*, Phys. Rev. Lett. **97**, 052701 (2006).
[14] D. Hilscher *et al.*, Phys. Rev. C **36**, 208 (1987).
[15] B. A. Li, C. M. Ko, and Z. Z. Ren, Phys. Rev. Lett. **78**, 1644 (1997).
[16] B. A. Li, L. W. Chen, G. C. Yong, and W. Zuo, Phys. Lett. B **634**, 378 (2006).
[17] H. H. Wolter, *Investigation of Low-Density Symmetry Energy via Nucleon and Fragment Observables*, Exotic States of Nuclear Matter, edited by Marcello Baldo, Fiorella Burgio, Hans-Josef Schulze, and Umberto Lombardo (Word Scientific, INFN, Sezione di Catania, Italy), pp. 71-76.
[18] M. Colonna, V. Baran, M. Di Toro, and H. H. Wolter, Phys. Rev. C **78**, 064618 (2008).
[19] Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, and M. B. Tsang, Phys. Lett. B **664**, 145 (2008); M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. **102**, 122701 (2009).
[20] M. B. Tsang *et al.*, Prog. Part. Nucl. Phys. **66**, 400 (2011).

[21] M. B. Tsang *et al*., Phys. Rev. Lett. **92**, 062701 (2004).
[22] J. Y. Liu, W. J. Guo, Y. Z. Xing, and H. Liu, Nucl. Phys. A **726**, 123 (2003).
[23] V. Baran, M. Colonna, M. Di Toro, V. Greco, M. Zielinska-Phabe, and H. H. Wolter, Nucl. Phys. A **703**, 603 (2002); V. Baran, M. Colonna, M. Di Toro, M. Zielinska-Pfabe, and H. H. Wolter, Phys. Rev. C **72**, 064620 (2005); I. Lombardo *et al*., *ibid*. **82**, 014608 (2010).
[24] P. Russotto *et al*., Phys. Lett. B **697**, 471 (2011); M. D. Cozma, *ibid*. **700**, 139 (2011).
[25] B. A. Li, Phys. Rev. Lett. **85**, 4221 (2000).
[26] S. Gautam, A. D. Sood, R. K. Puri, and J. Aichelin, Phys. Rev. C **83**, 034606 (2011); S. Gautam, R. Chugh, A. D. Sood, R. K. Puri, C. Hartnack, and J. Aichelin, J. Phys. G **37**, 085102 (2010); S. Gautam and A. D. Sood, Phys. Rev. C **82**, 014604 (2010); S. Kumar, Rajni, and S. Kumar, *ibid*. **82**, 024610 (2010); S. Kumar and S. Kumar, Cent. Eur. J. Phys. **9**, 986 (2011).
[27] B. A. Li, Nucl. Phys. A **734**, 593c (2004); G. C. Yong, B. A. Li, L. W. Chen, and W. Zuo, Phys. Rev. C **73**, 034603 (2006); Z. Q. Feng and G. M. Jin, *ibid*. **82**, 044615 (2010); Phys. Lett. B **683**, 140 (2010); M. Zhang, Z. G. Xiao, B. A. Li, L. W. Chen, G. C. Yong, and S. J. Zhu, Phys. Rev. C **80**, 034616 (2009).
[28] C. Hartnack, H. Oeschler, and J. Aichelin, J. Phys. G **35**, 044021 (2008); Phys. Rev. Lett. **96**, 012302 (2006).
[29] *Isospin Physics in Heavy-Ion Collisions at Intermediate Energies*, edited by B.-A. Li and W. Udo Schroder (Nova Science, New York, 2001), Chap. 3.
[30] B. A. Li, L. W. Chen, H. R. Ma, J. Xu, and G. C. Yong, Phys. Rev. C **76**, 051601(R) (2007).
[31] C. Hartnack, R. K. Puri, J. Aichelin, J. Konopka, S. A. Bass, H. Stöcker, and W. Greiner, Eur. Phys. J. A **1**, 151 (1998); C. Hartnack, H. Oeschler, Y. Leifels, E. L. Bratkovskaya, and J. Aichelin, Phys. Rep. (2011), doi:10.1016/j.physrep.2011.08.004.
[32] J. Aichelin, Phys. Rep. **202**, 233 (1991).
[33] [http://th.physik.uni-frankfurt.de/∼liqf/thesis.doc]
[34] Y. G. Ma *et al*., Phys. Rev. C **60**, 024607 (1999).
[35] [http://www.ectstar.eu/Meetings/ConfsWksAndCollMeetings/confwksdocument/2011/talks/workshop_13_06_2011/wolte.ppt].