Transition and static moments of octupole-deformed heavy nuclei

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We studied the properties of even-even actinide nuclei using the binary cluster model. Band-mixing calculation of the $K^\pi = 0^-$ negative parity bands arising from the core nucleus having two different states and with the core and cluster having different effective charges enabled a simultaneous interpretation of the electromagnetic transitions.

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I. INTRODUCTION

Many papers have reported the existence of broken reflection symmetry in the rare-earth and heavy nuclei [1–8]. The experimental signatures of the asymmetry have been shown to include, but are not limited to, the observed low-lying bandhead of the $K^\pi = 0^-$ band in the light actinide nuclei, the alternating parity at high spin of the even-A nuclei, and the enhanced dipole transition between the opposite parity states. All of these features have been associated with the equilibrium quadrupole and octupole deformations of the heavy nuclei, and as a result the theoretical studies and the interpretation of the experimental data are mostly done within the collective model and deformed shell model. For example, self-consistent mean-field techniques and Nilsson-Strutinsky deformed potential plus shell correction methods have been widely used to investigate the nuclear reflection asymmetry [1].

The appearance of cluster structures and its connection with nuclear deformations in light nuclei [9–11] and the relationship between the equilibrium shape, nuclear clustering, and the observed exotic decay of a number of heavy actinide nuclei [12–17] naturally suggest the possibility of describing the above experimental features within the cluster model. This idea has led to a number of theoretical investigations on the connection between nuclear clustering and the reflection asymmetric properties of heavy nuclei [18–22]. A particularly simple and intuitive cluster model interpretation of the properties of heavy nuclei is the exotic-cluster model of Buck *et al*. [18,19,23,24]. This variant of the cluster model seeks to describe the positive parity ground state band $K^\pi = 0^+$ and the negative parity band $K^\pi = 0^-$ of the even-even actinide nuclei in terms of the core-cluster relative motion with both systems restricted to their respective ground states. However, the negative parity band is usually placed at too high an excitation energy relative to the positive parity ground state band. Furthermore the corresponding dipole $E1$ transitions turns out to be much larger than the experimental values [19]. A similar marked deviation of the model prediction from experimental data is also found for the dipole moments of the even-even heavy nuclei [15]. Recently, the exotic-cluster model description of the negative parity bands was extended in order to explain the salient experimental features common to a number of heavy nuclei (see Refs. [25,26] for details). This approach assumes that the low-lying negative parity bands of even-even heavy nuclei are more appropriately described by coupling the core at its lowest $I^\pi = 3^-$ state with the relative motion angular momentum L^π of the core-cluster system. This formalism generates a set of negative parity bands which can be labeled as $K^\pi = 0^-, 1^-, 2^-, 3^-$ bands. It was found that the lowest-energy odd-spin states forming the negative parity $K^\pi = 0^-$ band do not decay via $E1$ transition to the positive parity ground state band. A band-mixing calculation of the two $K^\pi = 0^-$ negative parity bands obtained with $I^\pi = 0^+$ and with $I^\pi = 3^-$ produces $E1$ transition strengths which gave satisfactory description of the electromagnetic data of ^{238}U [27]. Higher multipolarities, such as the octupole transition strength of heavy nuclei, have also been described to a good degree of accuracy within the alternative excited core formalism [26]. Furthermore application of the excited-core model to a ^{212}Po nucleus treated as an α cluster plus excited ^{208}Pb core has been shown to reproduce the recently observed experimental data [28].

The purpose of this work is to investigate the electromagnetic properties of the negative parity bands of octupole-deformed nuclei within the excited-core cluster model formalism. We are particularly interested in the dipole moments and the enhanced $E1$ transitions between members of alternating parity bands. The existence of these observables in the light even-even actinides is indicative of nuclear asymmetric shape [1–3] and of cluster structures with differing charge-to-mass ratios [29]. We used here the proposed formalism [27] to calculate the electromagnetic transition strengths. We extend our calculations to include the static multipole moments in the excited core model, and by using the radial wave functions of the cluster-core relative motion we give a more explicit quantitative description of nuclei with octupole correlations.

The remainder of this paper is organized as follows. We give the theoretical formalism in Sec. II. The results and discussion of our calculations are presented in Sec. III, followed by our conclusions in Sec. IV.

II. THEORETICAL FRAMEWORK

In the exotic-cluster model the ground state of the parent nucleus is described as an exotic cluster in its ground state orbiting a core nucleus. The core-cluster splitting is such that the cluster nucleons occupy states above the Fermi level of the core in order to satisfy the Pauli principle. The orbit of the cluster can be described by a large value of the quantum number $G = 2n + L$, where n is the number of nodes of the relative motion and L is the orbital angular momentum. The core-cluster relative motion characterized by an even value of G generates the positive parity states $L^{\pi}_{even} = 0^+, 2^+, 4^+, \ldots, G^+$, which, when coupled with the core nucleus at its ground state $I^{\pi} = 0^+$, results in the positive parity ground state band of the parent nucleus $J^{\pi} = 0^+, 2^+, 4^+, \ldots, G^+$. Similarly, an excited negative parity band $L^{\pi}_{odd} = 1^-, 3^-, 5^-, \ldots, (G+1)^-$ corresponding to an odd-G relative motion can be coupled with the core spin $I^{\pi} = 0^+$ to generate the $K^{\pi} = 0^-$ [i.e., $J^{\pi} = 1^-, 3^-, 5^-, \ldots, (G+1)^-$] negative parity band. Previous studies have, however, shown that the observed low-lying negative parity bands found in most even-even heavy nuclei are not adequately described by the relative motion of the spinless core and cluster systems characterized by odd-G quantum number.

An alternative view of these bands, recently proposed, is that the core nucleus could be found in either the positive parity ground state $I^{\pi} = 0^+$ or the lowest negative parity excited state $I^{\pi} = 3^-$. Coupling the excited state $I^{\pi} = 3^-$ to the even-G core-cluster relative motion yields a rich spectrum of negative parity states, which has given a good representation of the experimental situation in a number of heavy actinide nuclei [25,26,30].

Here we employ the alternative approach to obtain the low-lying negative parity bands by solving the Schrödinger equation of the form

$$[H(\mathbf{r}, \boldsymbol{\xi}) - E]\Psi_{JM}(\mathbf{r}, \boldsymbol{\xi}) = 0, \tag{1}$$

where $H(\mathbf{r}, \boldsymbol{\xi})$ is the sum of the relative motion Hamiltonian with coordinate $\boldsymbol{r}$, the core Hamiltonian with internal coordinate $\boldsymbol{\xi}$, and a non-central interaction $V'(\mathbf{r}, \boldsymbol{\xi})$, which couples both the relative and the internal motions. The total wave function $\Psi_{JM}(\mathbf{r}, \boldsymbol{\xi})$ may then be expanded in terms of basis states involving the product of the relative motion wave function and the core eigenfunction, which, in compact notation, may be written as

$$|JM\rangle = \sum_{L'} \alpha^J_{L'} |L'I'; JM\rangle, \tag{2}$$

where $\alpha^J_{L'}$ is the expansion coefficient.

Following Refs. [25,26,30], we assume the relative motion component of $H(\mathbf{r}, \boldsymbol{\xi})$ is not affected by the core state I^{π} so that the positive parity excitation energies arising from the relative motion correspond to the ground state band, here represented by $E(L)$, and that these energies may be taken from the experimental spectrum. Given the nature of the problem, we may proceed to solve Eq. (1) by using a standard diagonalization technique.

The Schrödinger equation above may simply be cast into the form

$$\sum_{L'} H^J_{LL'} \alpha^J_{L'} = E_J \alpha^J_L, \tag{3}$$

where the Hamiltonian matrix $H^J_{LL'}$ contains the coupling matrix elements

$$V^J_{LL'} = \langle LI; JM|V'(\mathbf{r}, \boldsymbol{\xi})|L'I; JM\rangle. \tag{4}$$

By using a quadrupole-quadrupole form for the non-central interaction, $V'(\mathbf{r}, \boldsymbol{\xi}) = V_2(r)Y_2(\hat{\mathbf{r}}) \cdot Y_2(\hat{\boldsymbol{\xi}})$, the coupling matrix element may be rewritten as [25,26,31]

$$V^J_{LL'} = (-)^{J+L-L'} 7\beta \hat{L}\hat{L}' W(LL'33; 2J)\langle L0L'0|20\rangle \times \langle 3030|20\rangle. \tag{5}$$

In Eq. (5), β is the radial integral involving the non-central interaction strength $V_2(r)$ and the relative motion wave functions, the symbol $\hat{N} = \sqrt{2N+1}$, $W(abcd; ef)$ is a standard Racah coefficient, and $\langle a\alpha b\beta|c\gamma\rangle$ is the Clebsch-Gordan coefficient.

Implicit in Eq. (5) is the assumed similarity of the relative motion radial wave functions, thus giving a constant radial integral β, which we considered to be an adjustable strength parameter. This assumption has been shown to hold especially for heavy nuclei with core-cluster systems having a large quantum number G [23,32]. In arriving at Eq. (5) we have adopted a phase convention that is different from the one employed in earlier works [25,26].

A. Transition strength

The in-band quadrupole transition strength for the $K^{\pi} = 0^-$ band involving an initial state J_i and a final state J_f, each of which has two components with different sets of coordinates, takes the form

$$[B(E2; J_i \longrightarrow J_f)]^{\frac{1}{2}} = \sum_{L_i L_f \text{even}} (-1)^{J_f+L_i-1} \times \sqrt{\frac{5}{4\pi}} \alpha^{J_i}_{L_i} \alpha^{J_f}_{L_f} \beta_2 \hat{J}_f \hat{L}_i W(L_f L_i J_f J_i; 23) \times \langle L_i 020|L_f 0\rangle \langle L_f|r^2|L_i\rangle, \tag{6}$$

where the symbol $\hat{N} = \sqrt{2N+1}$. The quantities $\alpha^{J_i}_{L_i}$ and $\alpha^{J_f}_{L_f}$ are the expansion coefficients of the initial and final states, and β_2 is a factor arising from the multipole operator whose general form is given by [23]

$$\beta_\ell = \frac{Z_1 {A_2}^\ell + (-1)^\ell Z_2 {A_1}^\ell}{(A_1 + A_2)^\ell}, \tag{7}$$

where ℓ is the multipolarity and Z_1, Z_2 and A_1, A_2 are the core and cluster charge and mass numbers, respectively. As can be seen, Eq. (6) is consistent with Eq. (12) of [27]. The matrix elements of the squared radial separation may be calculated using wave functions from a numerical solution of the radial Schrödinger wave equation (RSWE) for the relative motion.

For the dipole transition strength $B(E1)$ we consider the mixing of the negative parity states of angular momenta $1^-, 3^-, 5^-, \ldots$ belonging to the $G+1$ band with the negative parity states of the $K^{\pi} = 0^-$ band arising from the core excitation [27,30]. An initial mixed negative parity state of angular momentum J_i can then decay through an electric dipole transition to a final positive parity state of angular momentum $J_f = J_i \pm 1$ belonging to the positive parity ground state band. In this model the decay is essentially that from a pure $G+1$ state to a pure G state, moderated by the mixing coefficient C_J of the form

$$C_J = \frac{\gamma}{4\pi \Delta E_J} \sum_{L=|J-3|_{\text{even}}}^{J+3} \alpha_L^J \hat{L} \langle J0L0|30\rangle, \tag{8}$$

where we have assumed an octupole-octupole mixing interaction of the type $V''(\mathbf{r}, \boldsymbol{\xi}) = g(r) Y_3(\hat{\mathbf{r}}) \cdot Y_3(\hat{\boldsymbol{\xi}})$. The strength γ is the radial integral involving the unknown strength function $g(r)$ and the even-G and odd-G relative motion wave functions. The energy denominator ΔE_J is the energy difference between two admixed negative parity states having the same spin J. The dipole transition strength is then given by

$$\begin{aligned} B(E1; J_i \longrightarrow J_f) &= \frac{3}{4\pi} C_{J_i}^2 \beta_1^2 |\langle J_i 010|J_f 0\rangle \langle J_f|r|J_i\rangle|^2 \\ &= \frac{3}{64\pi^3} \left(\frac{\gamma \beta_1}{\Delta E_{J_i}}\right)^2 \langle J_i 010|J_f 0\rangle^2 \langle J_f|r|J_i\rangle^2 \\ &\quad \times \left[\sum_{L=|J_i-3|_{\text{even}}}^{J_i+3} \alpha_L^{J_i} \hat{L} \langle J_i 0L0|30\rangle\right]^2, \end{aligned} \tag{9}$$

where the radial matrix elements $\langle J_f|r|J_i\rangle$ involve the relative motion wave functions with odd spin J_i and even spin J_f and the factor β_1 is the dipole form of Eq. (7).

Within the exotic-cluster model, the octupole transition strength $B(E3)$ from a $J^\pi = 3^-$ state in any of the K bands to the $J^\pi = 0^+$ ground state of the parent nucleus is essentially given by the octupole transition strength of the Pb core from its 3^- state to its 0^+ ground state moderated by the squared modulus of their respective expansion coefficients for the $L=0$ component of the wave functions of the $J^\pi = 3^-$ states of the parent nucleus [25,26]. For the admixed $K^\pi = 0^-$ band we have, in addition to the dominant transition from the 3^- state of the core, a non-negligible contribution from the 3^- state of the excited $G+1$ band to the 0^+ ground state of the parent nucleus moderated also by the mixing coefficient C_J. In general the octupole transitions from an initial negative parity state J_i to a final positive parity state J_f may be determined from the squared modulus of the sum of contributions from the excited 3^- state of the core nucleus to its ground state moderated by the expansion coefficient for components of the wave functions of J_i having the same L as the final positive parity state J_f and from the 3^- state of the excited $G+1$ band to the final state J_f moderated by the mixing coefficient of the state J_i.

B. Static moment

We define the intrinsic multipole moment $Q_{\ell m'}$ as [33]

$$Q_{\ell m'} = \sqrt{\frac{16\pi}{5}} \langle \phi_{I,\lambda=I}(\boldsymbol{\xi}) | \hat{O}'_{\ell m'} | \phi_{I,\lambda=I}(\boldsymbol{\xi}) \rangle, \tag{10}$$

where $\phi_{I,\lambda=I}(\boldsymbol{\xi})$ is the internal eigenfunction, λ is the projection of the core spin I onto the quantization axis, and $\hat{O}'_{\ell m'}$ is the multipole operator in the body-fixed axes.

In the cluster model the intrinsic quadrupole moments can be obtained from the transition strength $B(E2)$ by using the wave functions of Eq. (2) and the transformation

$$\hat{O}_{\ell,m} = \sum_{m'} D^{\ell}_{mm'} \hat{O}'_{\ell m'} \tag{11}$$

connecting the space-fixed $\hat{O}_{\ell m}$ and the body-fixed $\hat{O}'_{\ell m'}$ operators. Thus after some angular momentum recoupling the $B(E2)$ can be rewritten as

$$\begin{aligned} &[B(E2; J_i \longrightarrow J_f)]^{\frac{1}{2}} \\ &\quad = \sqrt{\frac{5}{16\pi}} Q_{20} \sum_{L_f L_i} (-)^{J_f+L_i-1} \alpha_{L_f}^{J_f} \alpha_{L_i}^{J_i} \hat{J}_f \hat{L}_i \\ &\qquad \times W(L_f L_i J_f J_i; 23) \langle L_f|L_i\rangle \langle L_i 020|L_f 0\rangle \end{aligned} \tag{12}$$

where Q_{20} is the the static quadrupole moment. The moment can therefore be deduced from a combination of Eqs. (6) and (12). The intrinsic dipole moment Q_{10} can similarly be derived from the transition strength $B(E1)$ by using instead the admixed wave function containing the factor C_J,

$$B(E1; J_i \longrightarrow J_f) = \left| \sqrt{\frac{3}{4\pi}} C_{J_i} Q_{10} \langle J_f|J_i\rangle \langle J_i 010|J_f 0\rangle \right|^2. \tag{13}$$

This gives a dipole moment

$$Q_{10} = \frac{\beta_1 \langle J_f|r|J_i\rangle}{\langle J_f|J_i\rangle}. \tag{14}$$

With the formalism described above and by using an effective charge ϵ, via a transformation $Z_i = Z_i + \epsilon A_i$ [19,23], the experimental electromagnetic quadrupole transitions can be reproduced to within a very good accuracy. However, the same cannot be said for the dipole transitions within the binary cluster model. The reason is partly because the proton polarization charge e_p has been set equal to the effective charge of the neutrons e_n. This is common practice, which may not necessarily hold [34–36]. In fact, the difference between the nucleon effective charges is important and necessary for a simultaneous description of both the quadrupole and the dipole transitions in the exotic-cluster model. The overall effect is that the core effective charge ϵ_1 is slightly different from the cluster effective charge ϵ_2. If, for example, the difference $e_p - e_n$ is not negligible, one has

$$\beta_\ell = \frac{\left(1 + \frac{e_p}{e} - \frac{e_n}{e}\right)[Z_1 {A_2}^\ell + (-1)^\ell Z_2 {A_1}^\ell] + \left[A_2^{\ell-1} + (-1)^\ell A_1^{\ell-1}\right] A_1 A_2 \frac{e_n}{e}}{(A_1 + A_2)^\ell}, \tag{15}$$

TABLE I. The experimental level scheme of ^{226}Ra [2] and the calculated spectra obtained with SW + SW^3 potential parameter values of Ref. [19].

J^π	E_{expt} (MeV)	E_{th} (MeV)
0^+	0.000	0.000
2^+	0.068	0.073
4^+	0.212	0.217
6^+	0.417	0.419
8^+	0.669	0.674
10^+	0.960	0.975
12^+	1.282	1.318
14^+	1.629	1.699
16^+	1.999	2.114
18^+	2.389	2.559
20^+	2.801	3.031
22^+	3.233	3.525
24^+	3.686	4.037
26^+	4.158	4.562
28^+	4.651	5.098

where e is the electron charge. Equation (15) gives the same factor $\beta_{\ell=1}$ for the dipole operator as in [37]. In fact, by using

TABLE II. The experimental and the calculated excitation energies of the $K^\pi = 0^-$ and $K^\pi = 1^-$ negative parity bands of ^{226}Ra (in MeV units). The predicted $K^\pi = 2^-$ and $K^\pi = 3^-$ negative parity bands (in MeV) are given in the sixth and seventh columns, respectively.

J^π	$K^\pi = 0^-$		$K^\pi = 1^-$		$K^\pi = 2^-$	$K^\pi = 3^-$
	E_{expt}	E_{th}	E_{expt}	E_{th}	E_{th}	E_{th}
1^-	0.254	0.261	1.040	1.045		
2^-			1.070	1.078	3.393	
3^-	0.322	0.326		1.159	3.453	7.304
4^-				1.205	3.531	7.388
5^-	0.447	0.450		1.349	3.624	7.488
6^-				1.394	3.732	7.600
7^-	0.627	0.631		1.596	3.852	7.725
8^-				1.631	3.983	7.860
9^-	0.858	0.861		1.885	4.124	8.004
10^-				1.909	4.274	8.157
11^-	1.134	1.132		2.207	4.431	8.317
12^-				2.220	4.596	8.484
13^-	1.448	1.436		2.556	4.767	8.657
14^-				2.558	4.944	8.836
15^-	1.797	1.768		2.929	5.128	9.020
16^-				2.920	5.316	9.210
17^-	2.175	2.125		3.324	5.510	9.405
18^-				3.304	5.709	9.605
19^-	2.579	2.503		3.741	5.914	9.810
20^-				3.709	6.123	10.020
21^-	3.007	2.903		4.178	6.339	10.235
22^-				4.134	6.558	10.456
23^-	3.455	3.322		4.636	6.785	10.681
24^-				4.580	7.013	10.911
25^-	3.922	3.762		5.114	7.250	11.147
26^-				5.047	7.489	11.387

TABLE III. Reduced matrix elements for the $E2$ transitions.The experimental data are taken from Ref. [3].

	$\langle J_f \vert\vert M(E2) \vert\vert J_i \rangle$ (e fm^2)	
$J_i^\pi \to J_f^\pi$	Experiment	Theory
$1^- \to 3^-$	366^{+6}_{-12}	287
$3^- \to 5^-$	409^{+5}_{-10}	392
$5^- \to 7^-$	407^{+3}_{-7}	473
$7^- \to 9^-$	545^{+3}_{-17}	541
$9^- \to 11^-$	677^{+30}_{-70}	599
$11^- \to 13^-$	1000^{+110}_{-70}	651
$13^- \to 15^-$	970^{+300}_{-220}	696
$15^- \to 17^-$		737
$17^- \to 19^-$		772
$19^- \to 21^-$		799

an effective charge $e^{eff} = e(1+\chi)$, where χ corresponds to $\chi = \frac{e_p}{e} - \frac{e_n}{e}$ in Eq. (15), the $E1$ transitions in the Ba-Sm region were satisfactorily described in an α cluster model [37]. The transition is interpreted as the result of a coupling between the cluster mode and the giant dipole vibrations [37]. For a typical heavy actinide nuclei, Eq. (9) with Eq. (15) gave a good description of the $E1$ transitions. However, we find that the expected quadrupole transitions of the positive parity ground state band are strongly underestimated. Alternatively, if we use different core and cluster effective charges and take into consideration the difference given by

$$\epsilon_1 - \epsilon_2 = \left(\frac{e_p}{e} - \frac{e_n}{e}\right)\frac{Z_1A_2 - Z_2A_1}{A_1A_2}, \tag{16}$$

we can give a good account of the quadrupole and the dipole transitions as well as the static moments. The difference

TABLE IV. Reduced matrix elements for the $E1$ transitions. The experimental data are taken from Ref. [3].

	$\langle J_f \vert\vert M(E1 \vert\vert J_i \rangle$ (e fm)	
$J_i^\pi \to J_f^\pi$	Experiment	Theory
$0^+ \to 1^-$	0.050 ± 0.009	0.096
$1^- \to 2^+$	0.068 ± 0.010	0.065
$2^+ \to 3^-$	0.061 ± 0.007	0.032
$3^- \to 4^+$	0.057 ± 0.005	0.007
$4^+ \to 5^-$	0.093 ± 0.007	0.053
$5^- \to 6^+$		0.097
$6^+ \to 7^-$	0.157 ± 0.010	0.156
$7^- \to 8^+$	0.21 ± 0.08	0.198
$8^+ \to 9^-$	0.24 ± 0.02	0.276
$9^- \to 10^+$	0.27 ± 0.05	0.308
$10^+ \to 11^-$	0.33 ± 0.04	0.410
$11^- \to 12^+$	0.35 ± 0.08	0.423
$12^+ \to 13^-$	0.63 ± 0.08	0.556
$13^- \to 14^+$	$0.65^{+0.15}_{-0.05}$	0.541
$14^+ \to 15^-$		0.709
$15^- \to 16^+$	$0.55^{+0.07}_{-0.14}$	0.658
$16^+ \to 17^-$		0.870
$17^- \to 18^+$		0.773

TABLE V. Ratios of the electric dipole to quadrupole moments ($|Q_{10}/Q_{20}| \times 10^{-4}$) and the corresponding ratios of the reduced transition strengths of ^{226}Ra. The experimental data are taken from Refs. [2,19,43].

J_i^π	$\lvert\frac{Q_{10}}{Q_{20}}\rvert^{\text{expt}}$ (fm^{-1})	$\lvert\frac{Q_{10}}{Q_{20}}\rvert^{\text{th}}$ (fm^{-1})	$\frac{B(E1\downarrow)}{B(E2\downarrow)}^{\text{expt}}$ (fm^{-2})	$\frac{B(E1\downarrow)}{B(E2\downarrow)}^{\text{th}}$ (fm^{-2})
3^-		2.60		1.37×10^{-9}
5^-	$\geqslant 1.0$	2.52		1.69×10^{-7}
7^-	2.30 ± 0.14	2.44		5.76×10^{-7}
9^-	2.56 ± 0.18	2.37	$(2.21 \pm 0.19) \times 10^{-7}$	1.20×10^{-6}
11^-	2.50 ± 0.14	2.30	$(1.83 \pm 0.44) \times 10^{-7}$	2.04×10^{-6}
13^-	1.96 ± 0.16	2.24	$(1.25 \pm 0.17) \times 10^{-7}$	3.09×10^{-6}
15^-		2.18		4.33×10^{-6}
17^-	2.49 ± 0.51	2.13	$(2.04 \pm 0.83) \times 10^{-7}$	5.76×10^{-6}
19^-		2.08		7.37×10^{-6}

between the effective charges ϵ_1 and ϵ_2 of the core and the cluster therefore depends on the difference in the charge to mass ratios of the core and the cluster and on the difference in the proton and neutron effective charges. The existence of the core and cluster effective charges may be explained by the following. First the fluctuations produced by the clusterization process i.e., the transient formation and dissolution of the clusters in the parent nuclei, may result in the existence of a mixture of different core-cluster configurations. Second, the overall polarization of the core may be very different from that of the cluster, probably due to the excess neutron of the core [38].

III. RESULTS AND DISCUSSIONS

In this section we first present the results of our calculations for ^{226}Ra treated as a ^{14}C exotic cluster plus a ^{212}Pb core. For both systems in their respective ground states the excitation energies and the wave functions of members of the positive parity ground state band of ^{226}Ra are obtained by solving the relative motion RSWE with Woods-Saxon potential (SW + SW^3). The potential parameter values and the global quantum number G are taken from Ref. [19]. The experimental and the calculated energies are listed in Table I for comparison. Our results agree with those of Ref. [19] with a further extension of the level scheme to the recently observed $J^\pi = 28^+$ state.

The quadrupole transition strength $B(E2; J_i \longrightarrow J_f)$ from an initial state of angular momentum J_i to a final state of angular momentum J_f of the positive parity band can be obtained from the generalized expression given in Refs. [19,30]. Our calculated values $B(E2; 2^+ \longrightarrow 0^+) = 115$ W.u. and $B(E2; 4^+ \longrightarrow 2^+) = 164$ W.u. are satisfactory when compared with the corresponding measured values of 123 ± 5 W.u. and $\sim$212 W.u., respectively. These results are obtained with a core effective charge $\epsilon_1 = 0.33$, and the cluster effective charge ϵ_2 is determined from Eq. (16).

The negative parity bands are generated by considering an excited core plus a cluster in its ground state. The core excitation energy $E(3^-)$ and the coupling strength β are treated as adjustable parameters whose values may be defined following the procedure outlined in Refs. [25,26]. However, due to the nonavailability of the required experimental data for the low-spin members of $K^\pi = 1^-$ and $K^\pi = 2^-$ bands, we choose to fit the parameters to the observed states of the $K^\pi = 0^-$ and the $K^\pi = 1^-$ bands. The values obtained

TABLE VI. The experimental and the calculated excitation energies of the $K^\pi = 0^+$ positive parity bands of ^{222}Ra, ^{226}Th, and ^{232}U (in MeV units).

	^{222}Ra		^{226}Th		^{232}U	
J^π	E_{expt}	E_{th}	E_{expt}	E_{th}	E_{expt}	E_{th}
0^+	0.000	0.000	0.000	0.000	0.000	0.000
2^+	0.111	0.078	0.072	0.069	0.048	0.045
4^+	0.302	0.234	0.226	0.208	0.157	0.150
6^+	0.550	0.448	0.447	0.407	0.323	0.300
8^+	0.843	0.718	0.722	0.660	0.541	0.493
10^+	1.173	1.039	1.040	0.959	0.806	0.726
12^+	1.537	1.406	1.395	1.305	1.112	0.996
14^+	1.933	1.815	1.782	1.695	1.454	1.301
16^+	2.359	2.262	2.196	2.124	1.828	1.640
18^+	2.811	2.743	2.635	2.590	2.232	2.010
20^+	3.288	3.255	3.097	3.091	2.659	2.411
22^+					3.113	2.840
24^+					(3.590)	3.297

TABLE VII. The experimental and calculated excitation energies of the $K^\pi = 0^-$ negative parity bands of ^{222}Ra, ^{226}Th, and ^{232}U (in MeV units).

	^{222}Ra		^{226}Th		^{232}U	
J^π	E_{expt}	E_{th}	E_{expt}	E_{th}	E_{expt}	E_{th}
1^-	0.242	0.327	0.230	0.270	0.563	0.567
3^-	0.317	0.391	0.308	0.338	0.629	0.632
5^-	0.474	0.542	0.451	0.470	0.747	0.748
7^-	0.703	0.758	0.658	0.663	0.915	0.914
9^-	0.992	1.025	0.923	0.911	1.131	1.127
11^-	1.331	1.334	1.238	1.205	1.391	1.384
13^-	1.710	1.677	1.596	1.539	1.689	1.680
15^-	2.125	2.054	1.989	1.906	2.023	2.012
17^-	2.570	2.461	2.413	2.303	2.385	2.376
19^-	3.041				2.775	2.769
21^-					3.187	3.189

TABLE VIII. Ratios of the electric dipole to quadrupole moments ($|Q_{10}/Q_{20}| \times 10^{-4}$) and the corresponding ratios of the reduced transition strengths of ^{222}Ra. The experimental data are taken from Ref. [2].

J_i^π	$\lvert\frac{Q_{10}}{Q_{20}}\rvert^{\text{expt}}$ (fm^{-1})	$\lvert\frac{Q_{10}}{Q_{20}}\rvert^{\text{th}}$ (fm^{-1})	$\frac{B(E1\downarrow)}{B(E2\downarrow)}^{\text{expt}}$ (fm^{-2})	$\frac{B(E1\downarrow)}{B(E2\downarrow)}^{\text{th}}$ (fm^{-2})
3^-		2.10		9.42×10^{-10}
5^-		2.04		1.15×10^{-7}
7^-	4.51 ± 0.28	1.98	$(7.05 \pm 1.94) \times 10^{-7}$	3.95×10^{-7}
9^-	3.78 ± 0.21	1.92	$(4.86 \pm 1.43) \times 10^{-7}$	8.29×10^{-7}
11^-	3.94 ± 0.20	1.87	$(5.22 \pm 134) \times 10^{-7}$	1.41×10^{-6}
13^-	5.22 ± 0.69	1.82	$(9.08 \pm 4.60) \times 10^{-7}$	2.13×10^{-6}
15^-	4.67 ± 0.98	1.77	$(7.23 \pm 6.50) \times 10^{-7}$	2.98×10^{-6}
17^-		1.73		3.96×10^{-6}

from a least-squares fit are $E(3^-) = 3.2821$ MeV and $\beta = -2.3731$ MeV. These values are seen to deviate markedly from the values obtained for heavier nuclei in earlier works [25,26]. The experimental ground state band $E(L)$ of the ^{226}Ra isotope is taken from [2]. We obtained a series of negative parity states that, following Refs. [25,26,31,39], were grouped into four $K^\pi = 0^-, 1^-, 2^-, 3^-$ negative parity bands. The classification into the different K bands is consistent with their separations in the strong-coupling limit [25,26,30,31]. A comparison of the calculated $K^\pi = 0^-$ and $K^\pi = 1^-$ bands with experimental data is presented in Table II. The agreement of the level spectra with the observed data is good. The characteristic doublets of the low-spin states of the $K^\pi = 1^-$ band are in agreement with theoretical results obtained in Refs. [26,40]. The inverted doublets of the high-spin members of the band are an interesting feature with no experimental counterpart. We note, however, that the bandheads of the predicted $K^\pi = 2^-$ and $K^\pi = 3^-$ excited bands are too high when compared with, for example, the theoretical and experimental band assignments in the even-even actinide nuclei (see Refs. [26,40,41]). The upward shift of these bands is understood as an effect of the strong-coupling strength β.

Tables III and IV compare the calculated quadrupole and dipole transition strengths with experimental data. Following Refs. [3,19], we have converted the reduced transition strengths to the corresponding reduced matrix elements for direct comparison with experiment. We calculate the wave functions of the excited $G + 1$ negative parity band using the same SW + SW^3 parameter values. The strength γ of the mixing coefficient C_{J_i} is parametrized as $\gamma = A + BJ_i$ [42], where the parameters A and B together with the difference between the proton and neutron polarization charges $\chi = \frac{e_p}{e} - \frac{e_n}{e}$ have been fitted to the experimental dipole transition matrix element $\langle J_f||M(E1||J_i\rangle$. The parameter values $A = -44.15$ MeV, $B = 15.67$ MeV, and $\chi = -0.948$ are obtained. The overall agreement of our results with experimental data is satisfactory. We find that the absolute dipole moment $|Q_{10}| \simeq 0.20$ e fm averaged over a spin range $I = 3\hbar$–$19\hbar$ compares favorably with the experimental result $|Q_{10}| = 0.19 \pm 0.03$ e fm quoted in Ref. [2] for the spin range $I = 9\hbar$–$18\hbar$.

The absolute values of the ratios of the dipole to quadrupole moments and of the reduced transition strengths are listed in Table V. Our ratios of the intrinsic moments agree well with the data. We note, however, that our calculated ratio of the reduced transition strength differs by an order of magnitude from the experiment. The disagreement is due to the increasing nature of the spin-dependent strength γ, indicating larger contributions from the intermediate and high spin wave functions of the excited $G + 1$ band to the corresponding $K^\pi = 0^-$ negative parity states.

The reduced transition strength $B(E3; 0^+ \longrightarrow 3^-)$ for the $K^\pi = 0^-$ band is calculated from the squared modulus of the sum of contributions from the excited 3^- state of the core nucleus to its ground state and from the 3^- state of the excited $G + 1$ band to the ground state. We use a previous estimate $Q_3 = 1.129$ e^2 b^3 for the octupole transition strength of the

TABLE IX. Ratios of the electric dipole to quadrupole moments ($|Q_{10}/Q_{20}| \times 10^{-4}$) and the corresponding ratios of the reduced transition strengths of ^{226}Th. The experimental data are taken from Ref. [43].

J_i^π	$\lvert\frac{Q_{10}}{Q_{20}}\rvert^{\text{expt}}$ (fm^{-1})	$\lvert\frac{Q_{10}}{Q_{20}}\rvert^{\text{th}}$ (fm^{-1})	$\frac{B(E1\downarrow)}{B(E2\downarrow)}^{\text{expt}}$ (fm^{-2})	$\frac{B(E1\downarrow)}{B(E2\downarrow)}^{\text{th}}$ (fm^{-2})
3^-		3.59		2.50×10^{-9}
5^-		3.52		3.09×10^{-7}
7^-		3.45		1.06×10^{-6}
9^-	3.8 ± 0.2	3.33	$(4.91 \pm 1.36) \times 10^{-7}$	2.17×10^{-6}
11^-	3.9 ± 0.2	3.26	$(5.11 \pm 1.34) \times 10^{-7}$	3.72×10^{-6}
13^-		3.19		5.77×10^{-6}
15^-	3.9 ± 0.3	3.12	$(5.04 \pm 1.98) \times 10^{-7}$	8.19×10^{-6}
17^-	3.6 ± 0.3	3.05	$(4.28 \pm 1.98) \times 10^{-7}$	1.10×10^{-5}
19^-	3.9 ± 0.5		$(5.00 \pm 3.28) \times 10^{-7}$	

Pb core [25,26], and the remaining contribution resulting from the band mixing is estimated using Eq. (4) of Ref. [19]. Our calculated value of 0.43 e^2 b^3 is a factor of $\sim$3 away from the measured value 1.10 ± 0.11 e^2 b^3 [3].

We next apply the formalism described above to ^{222}Ra, ^{226}Th, and ^{232}U treated as ^{208}Pb core plus ^{14}C, ^{18}O, and ^{24}Ne exotic clusters, respectively. Our choice of the core-cluster system is guided by the experimentally observed exotic-cluster decay [44]. The parameter values are fixed as for the ^{226}Ra calculations. The only exception is that we use the SW + SW^3 potential parameter values of Eq. (4) in Ref. [23] for ^{226}Th. Table VI shows good agreement between experimental energies of the positive parity ground state band and the calculated spectra of the Ra, Th, and U isotopes. We see a marked compression in the calculated positive parity band of the ^{232}U. The quadrupole transition strengths $B(E2; 2^+ \longrightarrow 0^+) = 124$ W.u. and $B(E2; 4^+ \longrightarrow 2^+) = 177$ W.u. obtained for ^{222}Ra are to be compared with 111 ± 9 and $\geqslant 12$ W.u., respectively. For ^{226}Th the transition strength $B(E2; 2^+ \longrightarrow 0^+) = 153$ obtained with core effective charge $\epsilon_1 = 0.25$ also compares favorably with the measured value $B(E2; 2^+ \longrightarrow 0^+) = 164 \pm 10$ W.u. Similarly, the $B(E2; 2^+ \longrightarrow 0^+) = 231$ W.u. obtained for ^{232}U agrees with the measured value 241 ± 21 using a smaller core effective charge $\epsilon_1 = 0.20$.

In Table VII we present only the results for the $K^\pi = 0^-$ negative parity bands. The experimental energies are well reproduced by the theory except for the slight underbinding in the low-spin states of ^{222}Ra. We have used a core energy $E(3^-) = 5.8404$ MeV and strength $\beta = -4.0360$ MeV, obtained from the fit to ^{232}U data [45], to generate the negative parity band excitation energies of the nucleus. It is remarkable to find that our predictions for the previously unknown 13^- state and the intermediate-spin states of the ^{232}U are in good agreement with the recently measured values [46].

The results for the high-K negative parity bands (not shown) are seen to present similar features observed in ^{226}Ra. The average dipole moments $|Q_{10}| = 0.16$ and 0.30 e fm obtained for ^{222}Ra and ^{226}Th are to be compared with experimental values $|Q_{10}| = 0.27 \pm 0.04$ e fm and 0.30 ± 0.03 e fm [1,2], respectively. The $|Q_{10}/Q_{20}|$ and $B(E1\downarrow)/B(E2\downarrow)$ ratios for ^{222}Ra and ^{226}Th are compared with experimental data, where available, in Tables VIII and IX. The experimental $B(E1\downarrow)/B(E2\downarrow)$ ratios are obtained from the corresponding $|Q_{10}/Q_{20}|$ ratios quoted in Refs. [2,43] using the strong-coupling limit of the rotational model. We see that the calculated $|Q_{10}/Q_{20}|$ ratio is a factor of $\sim$2 lower than the experimental data for ^{222}Ra, whereas a remarkable agreement between theory and experiment is achieved for ^{226}Th. The ratios of the transition strengths are found to be an order of magnitude larger than the experimental data, as for ^{226}Ra. Our theoretical estimates of the $B(E3\uparrow)$ for the ^{222}Ra and ^{226}Th are 0.48 and 0.46 e^2 b^3, respectively.

The calculated average of the absolute dipole moment of ^{232}U is 0.18 e fm, and the weighted average from a recent measurement is 0.11 ± 0.03 e fm [46]. The octupole transition strength $B(E3\uparrow) = 0.45$ e^2 b^3 is obtained for the nucleus. Apparently, both the predicted and the observed values of the dipole moments are larger than the expected value for an octupole vibrational nuclei. We note also that the large differences in the values of $E(3^-)$ and β used to generate the $K^\pi = 0^-$ band of ^{232}U and the fixed values used for the Ra and Th isotopes probably indicate the important difference in the structure of the U isotope and those of the light actinide nuclei. The important point here is that the cluster model is able to give a good account of the properties of nuclei with different structures.

IV. CONCLUSIONS

The properties of the low-lying positive- and negative-parity bands of heavy nuclei have been described within the exotic-cluster model. The positive parity band is obtained from the relative motion of the spinless core and cluster systems. The negative parity bands are generated by coupling the core nucleus at $I^\pi = 3^-$ with the core-cluster relative motion. Application of the model to some nuclei is found to generate energies in good agreement with the experimental spectra. It is shown that perturbations from an excited negative parity band obtained with a core and cluster in their respective ground states together with a different core and cluster effective charges is necessary to account for a simultaneous description of the quadrupole and the dipole transitions and moments.

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