Stability of triaxial shapes in ground and excited states of even-even nuclei in the $A \sim 70$ region

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Total-Routhian-surface calculations by means of the pairing-deformation-frequency self-consistent cranked shell model have been carried out for even-even germanium and selenium isotopes to search for possible stable triaxial deformations of nuclear states. The maximum triaxiality of $|\gamma| \approx 30^\circ$ is found in the ground and excited rotational states of the nuclei ^{64,74}Ge. The calculations are compared with available experimental data, giving a general agreement with observed triaxiality in the isotopes.

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I. INTRODUCTION

The study of deformed shapes of atomic nuclei suggests that 85% of nuclei in the nuclear chart have prolate shapes [1]. The semiclassical periodic-orbit theory addresses that closed orbits predominate at prolate shapes as compared with oblate shapes [2–4], pointing in particular to the role of the flat bottom of the nuclear potential and the spin-orbit interaction. Related effects originate from the shell filling of high-j orbits. As seen in the deformed diagrams of single-particle levels, high-j low-Ω orbits have a strong prolate-driving force, while high-j high-Ω orbits have an oblate-driving force. When sufficient oblate-driving orbits are occupied, the nucleus can develop into a stable oblate shape in its ground state (g.s.). For example, it has been found that neutron-deficient mercury in which the outermost valence neutrons occupy the high-j high−Ω orbits of the $i_{13/2}$ subshell have oblate shapes [5–7]. Certain particle-hole excitations can also drastically modify the nuclear shape, as evidenced in the excited 0^+ states of certain Pb isotopes, see Ref. [8] and references therein. Spherical and oblate shapes make the next two largest groups of nuclei, though other shapes (such as octupole deformations) are possible.

There is an interesting question why the nonaxial γ deformation is not favored in the ground states of even-even (e-e) nuclei. In the mathematical expression of the nuclear shape, the quadrupole γ deformation seems to be equally important as the quadrupole β_2 parameter that describes axially symmetric shapes. Evidence for nonaxial γ deformations has been widely found in collective rotational states. The γ deformation has led to very interesting characteristics of nuclear motions, such as wobbling [9], chiral band [10], and signature inversion in rotational states [11]. There is no doubt that the γ softness and Coriolis coupling belong to the most important mechanisms that break the axial symmetry of the system dynamically [12]. The quest for stable triaxial shapes in the ground states of e-e nuclei, with a maximum triaxial deformation of $|\gamma| \approx 30^\circ$, is still a major theme in nuclear structure [13]. An early work using the Skyrme-Hartree-Fock (SHF) model with the neutron-proton interaction predicted the coexistence of prolate and oblate shapes in the ground states of e-e germanium isotopes [14]. However, their calculations were restricted to axially symmetric shapes. In a recent investigation employing the SHF and Gogny Hartree-Fock-Bogoliubov (HFB) approaches, it was pointed out that most of the germanium isotopes have soft triaxial deformations in their ground states, but the results are sensitive to the choices of model parameters [15]. Studies of the general evolution of shapes in this mass region were presented previously in Ref. [16].

II. MODEL

In the present work, we use the cranked Woods-Saxon (WS) shell model to investigate possible triaxial shapes in ground and collective rotational states. The cranked WS model with the parameters [17] used in our calculations reproduces rather well at the mean-field level the oblate-prolate shape coexistence and the evolution of the two shapes with neutron number in light Kr isotopes as well as in neutron-deficient Pb isotopes. One may therefore render some confidence in the predictions of our work.

Experimentally, it is difficult to determine the value of the triaxial parameter for a nucleus. When the axial symmetry breaks, one cannot determine at the same time the quadrupole deformation parameters of both β_2 and γ from the experimental $B(E2)$ value. Andrejtscheff *et al.* used the sum-rule method to estimate the asymmetry from experimental electromagnetic transition matrix elements [18,19]. In the method, the deviation from the axial symmetry is written approximately as [18]

$$\langle \cos 3\delta_{\text{g.s.}} \rangle \approx -\sqrt{\frac{7}{10}} \langle Q^2_{\text{g.s.}} \rangle^{-3/2} [\langle 0^+||E2||2^+_1\rangle^2 \langle 2^+_1||E2||2^+_1\rangle + 2\langle 0^+||E2||2^+_1\rangle \langle 2^+_1||E2||2^+_2\rangle \langle 2^+_2||E2||0^+\rangle], \quad (1)$$

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where $\langle Q^2_{\rm g.s.}\rangle \approx \sum_{j=1,2} |\langle 0^+||E2||2^+_j\rangle|^2$ and $\langle i||E2||j\rangle$ is the experimental reduced $E2$ matrix element between the ith and jth states. The parameter of

$$\delta_{\rm eff} = \tfrac{1}{3}\arccos(\langle\cos 3\delta_{\rm g.s.}\rangle) \qquad (2)$$

gives an effective asymmetry of the nucleus [18,19]. However, the asymmetry parameter $\delta_{\rm eff}$ includes contributions from both the static (rigid) and dynamic (soft) γ deformations [18,19]. In the rigid case, the parameter $\delta_{\rm eff}$ corresponds to the usual geometric-model parameter of the γ deformation [19]. In the sum-rule method, the symmetric quadrupole deformation is defined by [18,19]

$$\beta_{\rm rms} = \frac{4\pi}{3ZR_0^2}\sqrt{\langle Q^2_{\rm g.s.}\rangle}. \qquad (3)$$

Again, such a definition includes contributions from both static and dynamic symmetric deformations, which corresponds to the usual β_2 parameter in the rigid case [19].

With the above approximation, the asymmetry of about 70 e-e nuclei in $A \approx 50$–80 [19] and 90–190 [18] were estimated with the available experimental $E2$ matrix elements. The most pronounced triaxiality in ground states was found in the nuclei of $^{70-76}$Ge and $^{74-82}$Se [19]. Indeed, the maximum triaxiality is very rare in the ground state. The sum-rule method provides an approximation to deduce the value of the triaxial parameter from the experimental data. However, such an analysis is unable to determine whether the triaxiality described by $\delta_{\rm eff}$ is caused by the γ softness (dynamic) or static γ deformation [18,19]. For a γ-soft deformed nucleus, the γ-vibrational effect on collective rotation is pronounced, and one expects low-lying γ-vibrational states to be observed experimentally. According to the works of Refs. [20,21], one should not expect rigid triaxiality in the ground state of any nucleus.

In this paper, we investigate the possible maximum triaxiality of $|\gamma| \sim 30^\circ$ in the ground states of e-e nuclei. The total-Routhian-surface (TRS) method (see, e.g., Refs. [22,23]) has been used to determine the stability of the triaxiality with rotational frequency. The TRS at zero frequency corresponds to the usual potential energy surface (PES) for the ground state. The TRS calculations are carried out by means of the pairing-deformation-frequency self-consistent cranked shell model [22,23]. In this model, pairing and deformation changing as a function of rotational frequency are determined self-consistently. In practical calculations, for a given frequency and deformation, pairing is treated self-consistently by solving the cranked pairing-correlated Hamiltonian and then the shape of a given state is determined by minimizing the calculated TRS (for details, see Refs. [22,23]). The single-particle energies are obtained with the nonaxial deformed WS potential [17]. Particle number projection is approximated by the Lipkin-Nogami (LN) approach [22,24,25]. Both monopole and quadrupole pairings [26–28] are considered with the monopole pairing strength G determined by the average-gap method [29] and quadrupole strengths obtained by restoring the Galilean invariance of the paired many-body Hamiltonian [26,27]. The quadrupole pairing has an important influence on the collective rotation [23,30]. In the cranked model calculation [22,23], the nuclear energy at zero frequency is calculated

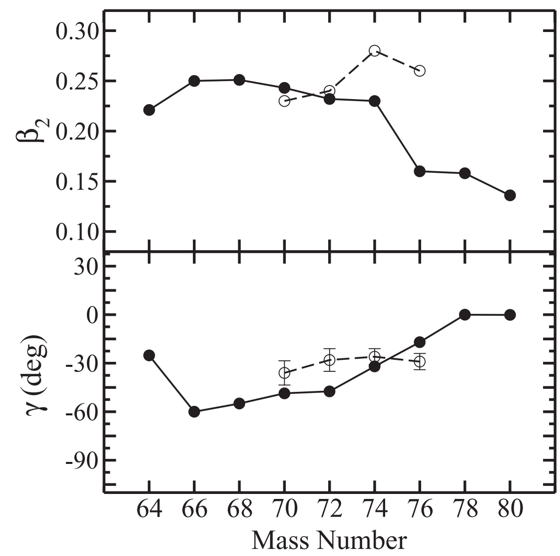

FIG. 1. Deformation parameters β_2 and γ obtained from calculated PESs for the ground states of even-mass $^{64-80}$Ge, indicated by filled circles with solid lines. Open circles with dashed lines are for $\beta_{\rm rms}$ (in upper panel) and $\delta_{\rm eff}$ (in lower panel) values given in Ref. [19] using the sum-rule method with available experimental reduced $E2$ matrix elements. For $\delta_{\rm eff}$, the authors of Ref. [19] estimated experimental errors which are indicated by error bars.

by the Strutinsky method [31] with the standard liquid-drop energy [32]. Calculations are performed in the lattice of quadrupole (β_2, γ) deformations with the hexadecapole β_4 variation.

III. CALCULATIONS AND DISCUSSIONS

Calculated deformation values deduced from the PES calculations for the ground states of even-mass $^{64-80}$Ge are shown in Fig. 1. At each grid point of the PES quadrupole deformation (β_2, γ) lattice, the calculated energy has been minimized with respect to the hexadecapole deformation β_4. A prolate (oblate) shape corresponds to $\gamma = 0^\circ$ ($\pm 60^\circ$). For the ground states of germanium isotopes, we see the shape transitions from a triaxial shape in ^{64}Ge to nearly oblate shapes in $^{66-72}$Ge, and to a $\gamma = -30^\circ$ triaxial shape again in ^{74}Ge, and toward weakly deformed prolate shapes in ^{78,80}Ge (note that, for a ground state, the PES is reflection symmetric about $\gamma = 0^\circ$). This is in agreement with the possible existence of a shape transition around $N = 40$ as predicted by Lecomte *et al.* [33].

It is a striking observation that the stable nucleus ^{74}Ge is calculated to have a deformed $|\gamma| = 30^\circ$ triaxial shape in the ground state, which would be a unique case in the whole nuclear chart. To determine the stability of the triaxiality in ^{74}Ge, we performed the TRS calculations presented in Fig. 2 at the rotational frequencies of $\hbar\omega = 0.0$, 0.3, 0.9, and 1.2 MeV, corresponding to a spin range of $I \sim (0$–$24)\hbar$. It appears that the triaxial minimum at $\beta_2 \approx 0.23$ and $\gamma \approx -30^\circ$ exists starting from the ground state ($\hbar\omega = 0$) up to a rotational frequency of $\hbar\omega \approx 1.1$ MeV (correspondingly $I \approx 18$). The triaxial minimum keeps yrast up to $\hbar\omega \approx 0.9$ MeV ($I \approx 14$)

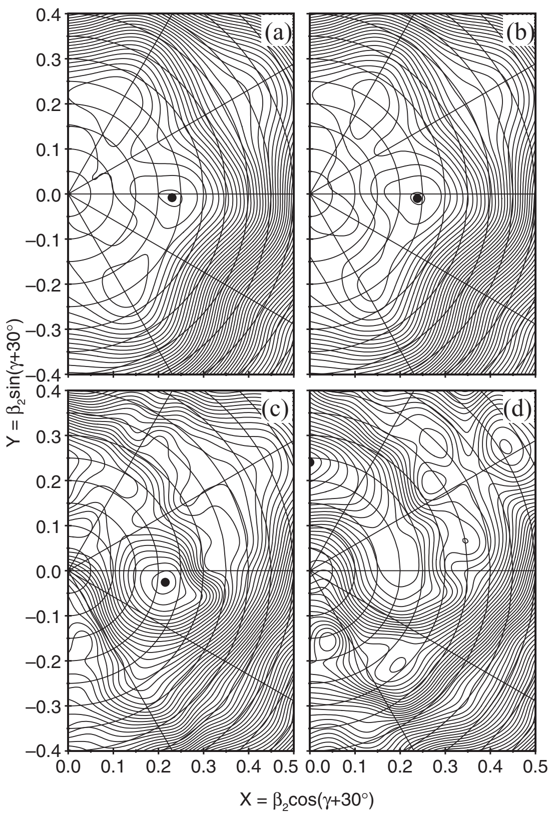

FIG. 2. Calculated TRSs for the lowest positive-parity rotational states in ^{74}Ge, at (a) $\hbar\omega = 0.0$, (b) 0.3, (c) 0.9, and (d) 1.2 MeV. The black dot indicates the lowest minimum. The energy difference between neighboring contours is 200 keV.

and disappears at $\hbar\omega \approx 1.2$ MeV. A noncollective oblate minimum at $\beta_2 \approx 0.24$ and $\gamma = 60^\circ$ becomes yrast at $\hbar\omega \approx 1.0$ MeV. One should also note that the depth of the triaxial minimum increases with increasing the rotational frequency in the range of $0.0 \leqslant \hbar\omega \leqslant 0.9$ MeV. Our calculations show that a rotational alignment of a pair of neutron $g_{9/2}$ orbits takes place at $\hbar\omega \approx 0.5$ MeV and again one pair of proton $g_{9/2}$ orbits align at $\hbar\omega \approx 1.0$ MeV, resulting in the decrease of the collectivity of the triaxial minimum and the appearance of the noncollective oblate minimum.

Experimentally, 0_2^+, 2_2^+, 4_1^+ states in ^{74}Ge were observed to have almost degenerate energies at about twice the energy of the 2_1^+ state [34]. These three excited states were viewed as vibrational states [34]. However, the work by Toh *et al.* [35] disagrees with the vibrational interpretation. Instead it was argued that the 0_1^+, 2_1^+, and 4_1^+ states form a rotational band, while the 0_2^+ level is an intruder spherical state [35]. Based on the analysis of the Coulomb excitation, a similar suggestion was put forward also for ^{72}Ge [36]. The present TRS calculations do not reproduce reasonably the experimental energies of the observed excited states. The calculated TRSs in Fig. 2 for the spin range of $I \approx 0$–4 are, in particular, soft toward spherical deformation, having only about a 300 keV difference between the spherical shape and the triaxial minimum in ^{74}Ge. This implies that the low-energy excitation spectrum in the spins of $0 \leqslant I \leqslant 4$ will be dominated by the vibrational motion that is not included in the TRS model. In addition, at triaxial deformations, one should invoke three-dimensional cranking calculations to better describe the energies of collective rotational states. The present TRS calculations show that the triaxial shape becomes more stable at $I \geqslant 6$, which can form a triaxial deformed rotational band with $I = 6$–18, providing a useful prediction for the possible experimental observation of the band including wobbling sequences. The nuclei ^{72,76}Ge are calculated to be very soft in both γ and β_2 deformations, implying an appreciable effect from triaxiality on the vibrational excitation spectrum but no static triaxial deformation.

In Fig. 1, we also displayed the deformation parameters given in Ref. [19] for even-even $^{70-76}$Ge, which were deduced from the available experimental reduced $E2$ matrix elements using the sum-rule method and considered as experimental quadrupole-deformation values [18,19]. In the ^{74,76}Ge cases, the $\beta_{\rm rms}$ values estimated by the sum-rule approximation are clearly larger than the β_2 values obtained in calculated PESs. However, our calculations show that these two nuclei are specially soft in both β_2 and γ deformations in their ground and lowly excited states (see, e.g., Fig. 2 for ^{74}Ge), and therefore the larger $\beta_{\rm rms}$ values will indicate the large contribution from the softness (dynamic).

For selenium isotopes, shown in Fig. 3, the PES calculations give large oblate deformations in the ground states of $^{68-76}$Se, and a change into a weakly deformed prolate shape at ^{78}Se. This agrees with the experimental suggestions of well-deformed oblate shapes for the ground states of ^{68}Se [37], ^{70,72}Se [38], and ^{74}Se [39]. The asymmetry feature in $^{74-82}$Se was given in Ref. [19] using the available experimental $E2$ matrix elements. However, as has been mentioned, the sum-rule approximation employed in Ref. [19] cannot distinguish whether the γ deformation (static) or the γ softness (dynamic) causes the asymmetry feature. Our PES calculations show that $^{74-82}$Se have soft quadrupole deformations that can result in significant dynamical deformation effects. Recently, the authors of Ref. [40] calculated low-lying states in ^{68,70,72}Se

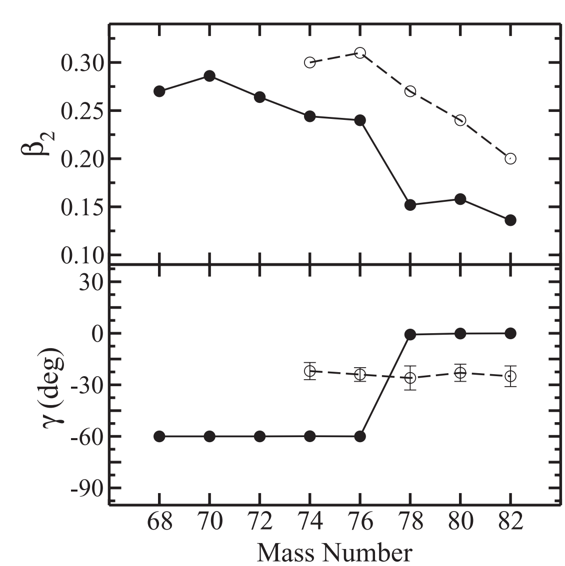

FIG. 3. Similar to Fig. 1, but for even-mass $^{68-82}$Se.

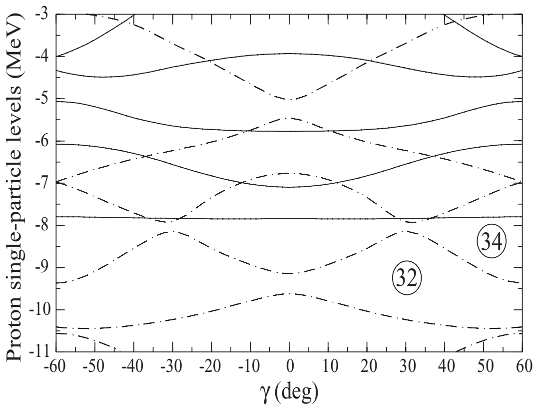

FIG. 4. Calculated WS single-proton levels versus the γ deformation. Solid (dot-dashed) curves stand for positive-parity (negative-parity) levels. The calculation is done with fixing $\beta_2 = 0.23$ and $\beta_4 = 0.0$ (corresponding to the deformation values of the ground state of ^{74}Ge). The proton numbers of $Z = 32$ and 34 are indicated. The neutrons have similar single-particle levels.

using the local quasiparticle-random-phase approximation (local QRPA) which includes large-amplitude shape mixing. Their calculations showed remarkable dynamic triaxiality in the selenium isotopes [40].

Single-particle level diagrams can give a further understanding of the origin of the triaxiality. In Ref. [41], a deformed $\gamma \approx 30^\circ$ shell gap at $Z(N) = 32$ was revealed already. Figure 4 displays the calculated WS diagram against the triaxial deformation γ, which gives a similar level structure to the Nilsson diagram of Ref. [41]. Indeed, the PES calculations show that the nucleus ^{64}Ge ($Z = N = 32$) has a triaxial shape with $\gamma \approx -25^\circ$ (see Fig. 1). At $N = 34$, however, an oblate shell gap appears, which results in an oblate shape in ^{66}Ge ($N = 34$). With increasing neutron number, the effect from the $N = 34$ oblate gap decreases, and hence the deformations of heavier germanium isotopes change toward the triaxiality (or prolate). Single-particle spectra for protons and neutrons are rather similar, having shell gaps at the same particle numbers. The $Z = 34$ oblate gap, combined with the effect from the neutron $N = 34$ oblate gap, will be the reason for the e-e selenium isotopes lighter than ^{78}Se to have oblate shapes (see Fig. 3).

IV. SUMMARY

In summary, using the self-consistent total-Routhian-surface model, we investigated the shapes of ground and excited rotational states in even-mass $^{64-80}$Ge and $^{68-82}$Se isotopes. The shape phase transition from oblate, through triaxiality, to prolate deformations has been found in germanium isotopes. Especially the ^{74}Ge nucleus has the most pronounced triaxiality starting from the ground state. Still, the Ge and Se isotopes have γ-soft shapes, resulting in significant dynamical triaxial effects. There is no evidence in the calculations pointing toward rigid triaxiality in ground states. In $6 \leqslant I \leqslant 18$, however, ^{74}Ge has a stable $\gamma \sim -30^\circ$ ($\beta_2 \approx 0.22$) triaxial shape, which can lead to a well-deformed triaxial rotational band, providing a useful prediction for a future experimental search of the band. The quantitative descriptions of the experimental energies of the excited states can be improved by the inclusion of the vibrational dynamics, such as in the local QRPA calculation [40].

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