PHYSICAL REVIEW C **84**, 037606 (2011)

Correlation between balance energy and transition energy for symmetric colliding nuclei

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(Received 10 August 2011; published 29 September 2011)

We study the correlation between balance energy and transition energy of fragments in heavy-ion collisions for different systems at incident energies between 40 and 1200 MeV/nucleon using an isospin-dependent quantum molecular dynamics model. With increasing incident energy, the elliptic flow shows a transition from positive (in-plane) to negative (out-of-plane) flow. This transition energy is found to depend on the size of the fragments, composite mass of the reacting system, and the impact parameter of the reaction. It has been observed that a reduced cross section can explain the experimental data. There is a correlation between transition energy and balance energy as their difference decreases with an increase in the total mass of colliding nuclei.

DOI: 10.1103/PhysRevC.84.037606 PACS number(s): 25.70.−z, 25.75.Ld

Introduction. The elliptic flow is a measure that quantifies the azimuthal anisotropy of the momentum distribution. Specifically, we fit the azimuthal distribution of nucleons about the reaction plane with a Fourier expansion of the form

$$\frac{dN}{d\phi} = v_0(1 + 2v_1 \cos\phi + 2v_2 \cos 2\phi + \cdots), \qquad (1)$$

where v_0 is for normalization only, v_1 characterizes the directed in-plane flow, $v_2 > 0$ indicates in-plane enhancement, $v_2 < 0$ characterizes the squeeze-out perpendicular to the reaction plane, and $v_2 = 0$ shows an isotropic distribution of nucleon momentum in the transverse plane. Hence, the ellipticity coefficient v_2 depends on the in-plane and out-of-plane flow amplitudes. The elliptical flow parameters $\langle \cos 2\phi \rangle$ at energies from tens to hundreds of MeV per nucleon are determined by the complex interplay among expansion, rotation, and the shadowing of spectators. Both the mean-field and two-body-collision parts play important roles in this energy region. The mean field plays a dominant role at low energies, and then, gradually, the two-body collision becomes dominant with the increase in energy. Two colliding nuclei create a stopped overlap region. At higher bombarding energies ($E_{\text{lab}} \geqslant 1$ GeV/nucleon) the spectator leaves the interaction zone rapidly. The remaining interaction zone expands almost freely, where the surface is such that in-plane emission is preferred. It is therefore also the interplay between the time scales of the passing time of spectators and the expansion time of the dense, stopped interaction zone that determines the time-integrated elliptic flow signal. Experimentally observed out-of-plane emission, termed squeeze, was first observed by at Saturne (France) by the DIOGENE Collaboration [1]. The Plastic Ball group at the Bevalac in Berkeley quantified the squeeze-out in symmetric systems [2]. Recently, elliptic flow has been measured at the Relativistic Heavy Ion Collider (RHIC) in Au + Au collision at $\sqrt{s} = 130$ GeV/nucleon [3]. At top Alternating-Gradient Synchrotron (AGS) and Super Proton Synchrotron (SPS) energies, elliptic flow is inferred to be a relative enhancement of emission in the plane of the reaction. Elliptic flow is developed mostly in the first several fm/c (of the order of the size of nuclei) after the collision and thus provides information about the early-time thermalization achieved in the collision [4]. A large elliptic flow of all charged particles near midrapidity was reported by the STAR Collaboration [4]. The FOPI, INDRA, and Plastic Ball Collaborations [5,6] are actively involved in measuring the excitation function of the elliptic flow from Fermi energies to relativistic energies. Interestingly, at intermediate energies ($E_{\text{lab}} \approx 100$ MeV/nucleon) the change from in-plane emission (rotation-like behavior) to squeeze-out is predicted [7,8] whereas at relativistic energies ($E_{\text{lab}} \approx 5$ GeV/nucleon) the opposite change from the squeeze-out to in-plane enhancement is observed. Elliptic flow requires reinteractions within the produced matter as a mechanism for transferring the initial spatial deformation of the reaction zone in noncentral collision onto momentum space. It is thus plausible to expect that the largest elliptic flow signal is produced in the hydrodynamic limit and an almost linear increase in its value with the particle transverse momentum below 1.5 GeV/c. In the hybrid model of combining the hydrodynamic model with the relativistic quantum molecular dynamics (RQMD) transport model [9] and choosing certain effective equations of state, it is possible to obtain an elliptic flow that is comparable to the measured ones in heavy-ion collisions at both SPS and RHIC energies [10]. The experimental result shows that elliptic flow first increases with particle transverse momentum and then levels off. The dependence of elliptic flow on both the charged-particle multiplicity [11,12] and the particle pseudorapidity [12] has also been measured. A complete study of the excitation function of transverse momentum and the energy dependence of elliptic flow in the entire energy region can provide useful information about nucleon-nucleon interaction related to the nuclear equation of state. In the literature, many attempts have already been made with hard equations of state (EOS) with free NN cross sections and soft EOS with a reduced nucleon-nucleon cross section with and

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without momentum-dependent interactions, and studies have also tried to explore different aspects of directed sideward flow. This study is a continuation of our previous study [13], in which we showed that experimental balance energies can be explained well with a reduced isospin-dependent NN cross section with a hard equation of state. In the present study our aim is to pin down the relation between balance energy and transition energy, if there is any relation between these two energies and whether there is any mass dependence or not. For the present study, the isospin-dependent quantum molecular dynamics (IQMD) model is used to generate the phase space of nucleons [14].

Results and Discussion. We study the elliptic flow using a stiff equation of state along with isospin-dependent reduced cross sections ($\sigma = 0.9\sigma_{NN}$) by simulating various reactions. The time evolution of the reaction is followed up to 200 fm/c. This is the time at which flow saturates for lighter as well as for heavier systems. For this study, the reactions of $^{40}\mathrm{Ar}_{18} + {}^{45}\mathrm{Sc}_{21}$ ($\hat{b} = 0.4,\ L = 0.5$ L), $^{93}\mathrm{Nb}_{41} + {}^{93}\mathrm{Nb}_{41}$($\hat{b} = 0.3,\ L = 0.7$ L), $^{139}\mathrm{La}_{57} + {}^{139}\mathrm{La}_{57}$($\hat{b} = 0.3,\ L = 0.8$ L), and $^{197}\mathrm{Au}_{79} + {}^{197}\mathrm{Au}_{79}$ ($b = 2.5$ fm, $L =$ L) are simulated, where L is the Gaussian width. As mentioned in Ref. [14], in IQMD the value of Gaussian width L depends on the mass of the system. For Au nuclei $L = 8.66\ \mathrm{fm}^2$ and for Ca nuclei $L = 4.33\ \mathrm{fm}^2$. $\hat{b}$ is the scaled impact parameter, defined as $\hat{b} = \frac{b}{b_{\max}}$ [where b is particular impact parameter in fermi and $b_{\max} = 1.12(A_T^{1/3} + A_P^{1/3})$], and A_T and A_P are the mass of the target and projectile, respectively. The choice of impact parameter is guided by the experimentally extracted information [15–17]. These reaction have been performed at their corresponding balance energies. The above reactions were simulated between 40 and 1200 MeV/nucleon using the hard equation of state along with isospin-dependent reduced cross sections. The phase space generated by the IQMD model has been analyzed using the minimum spanning tree (MST) [18] method. The MST method binds two nucleons in a fragment if their distance is less than 4 fm. In recent years, several improvements have also been suggested. One of the improvements is to also imply a momentum cut of the order of Fermi momentum. This method is dubbed the MSTM method [19]. The entire calculations are performed at $t = 200$ fm/c, i.e., saturation time.

The elliptical flow is defined as the average difference between the square of the x and y components of the particle's transverse momentum. Mathematically, it can be written as

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle, \tag{2}$$

where p_x and p_y are the x and y components of the momentum; p_x is in the reaction plane, while p_y is perpendicular to the reaction plane.

In Fig. 1, we display the transverse momentum dependence of elliptical flow for the free particles and light-charged particles. A Gaussian-type behavior is observed in all cases. Note that this elliptical flow is integrated over the entire rapidity range. It is also evident from Fig. 1 that the peaks of the Gaussian shift toward lower values of P_t for heavier fragments. This is due to the fact that the free and light-charged particles feel the mean field directly, while heavy fragments have weaker sensitivity [20]. In Fig. 2, we display the system size dependence of the transverse momentum at which v_2 becomes zero for different systems and different fragments. The value of transverse momentum decreases with system mass because in heavier systems there are more Coulomb repulsions than in lighter systems. This dependence is again fitted with a power law of the kind

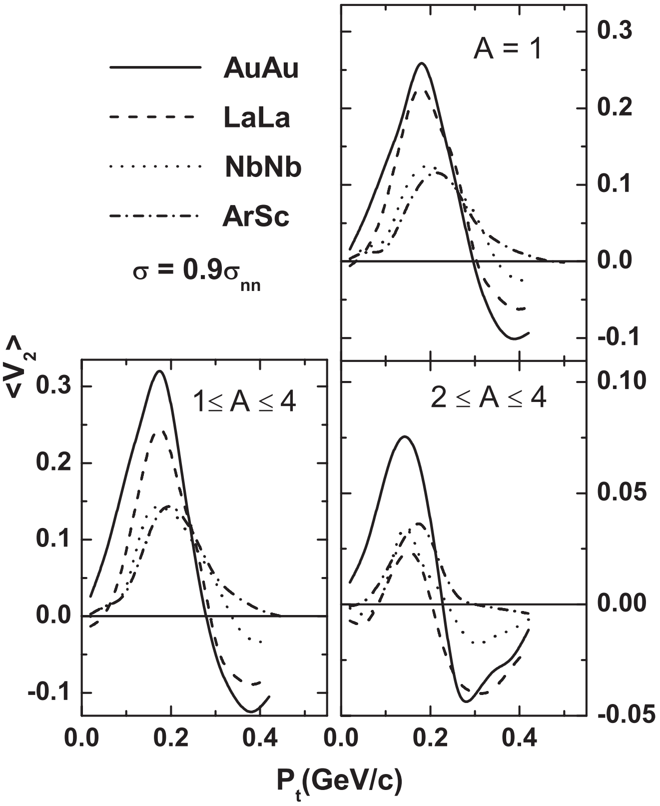

FIG. 1. Transverse momentum dependence of the elliptical flow at $E = 200$ MeV/nucleon. The different lines show the variation with different system masses, and different panels show the fragments of different mass ranges.

$$P_{\mathrm{Trans}} = C\left(A_{\mathrm{tot}}^{-\tau}\right). \tag{3}$$

The value of τ is found to increase with an increase in fragment mass range. In the case of $A = 1$ it is -0.30 ± 0.03, for $1 \leqslant A \leqslant 4$ it is -0.29 ± 0.02, and for $2 \leqslant A \leqslant 4$ it is -0.17 ± 0.07. In Fig. 3, we display the variation of the elliptic flow v_2 for free nucleons and light-charged particle (LCP's) over the midrapidity region as a function of energy. The free particles and LCP's, which originate from the participant zone, show a systematic behavior with the beam energy and with the composite mass of the system as well as with the fragments of different mass ranges. The elliptical flow for these particles is found to become more negative with the increase in the composite mass of system and with the increase in the beam energy as well as with the fragments of different mass ranges. The heavier the system is, the greater the Coulomb repulsion is and the more negative the elliptical flow is.

The elliptical flow is found to show a transition from in-plane to out-of-plane at a certain beam energy known as the transition energy E_{trans} for the midrapidity region [14]. This

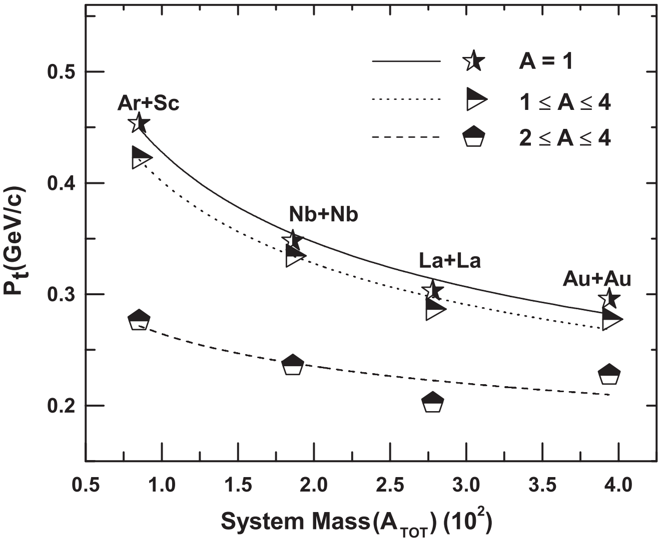

FIG. 2. Transverse momentum dependence P_t for elliptic flow as a function of the combined system mass for different systems with fragments of different mass ranges.

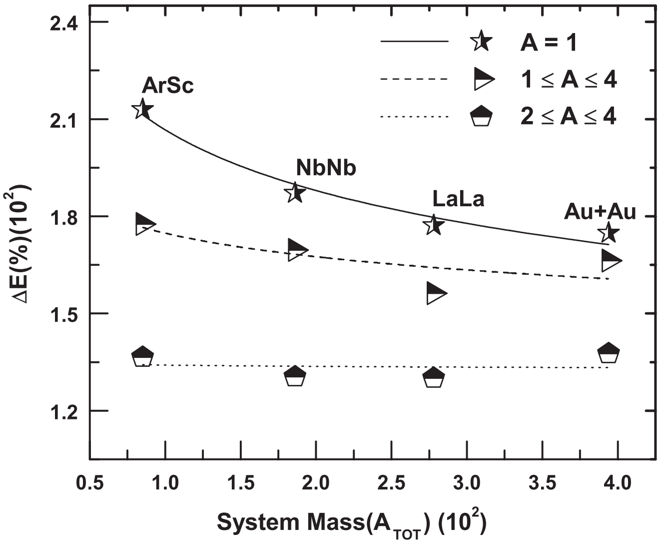

FIG. 4. Difference of transition energy and balance energy as a function of mass of system.

is due to the change in the rotational behavior into expansion with an increase in the incident energy.

In Fig. 4, we display the system size dependence of the difference of transition energy $\Delta E(\%) = \frac{E_{\text{trans}}-E_{\text{bal}}}{E_{\text{bal}}} \times 100$ extracted from Fig. 3 for different fragments and the balance energy E_{bal} ($\sigma = 0.9\sigma_{NN}$) studied in Ref. [13]. For a fragment with low mass range this difference shows a slight decrease as we increase the system mass, and then other effects come into play (i.e., expansion of participant and shadowing of spectator matter), whereas for light charge particle's very small variation is obtained, which means the additional effect is independent of system mass. This shows that the transition energy E_{trans} and balance energy E_{bal} are closer in the heavy mass system compared to lighter systems. This happens because as the number of neutrons increases, the number of collision increases and hence leads to a decrease in transition energy. This dependence is again fitted with the power law. The value of τ is found to increase with an increase in fragment mass range. In the case of $A = 1$ it is -0.14 ± 0.017, for

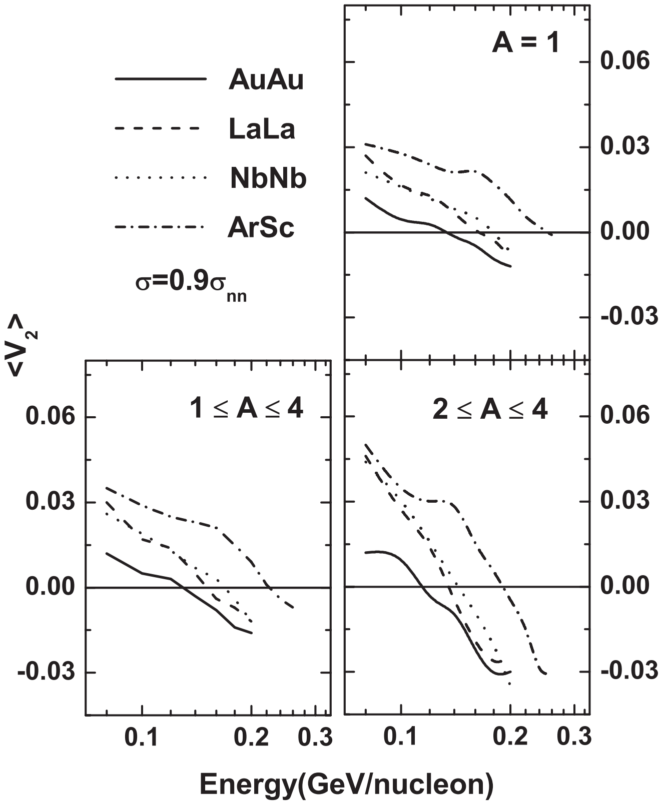

FIG. 3. Variation of the elliptic flow, with beam energy at $|y| = |\frac{y_{\text{cm}}}{y_{\text{beam}}}| \leqslant 0.1$ for different reactions.

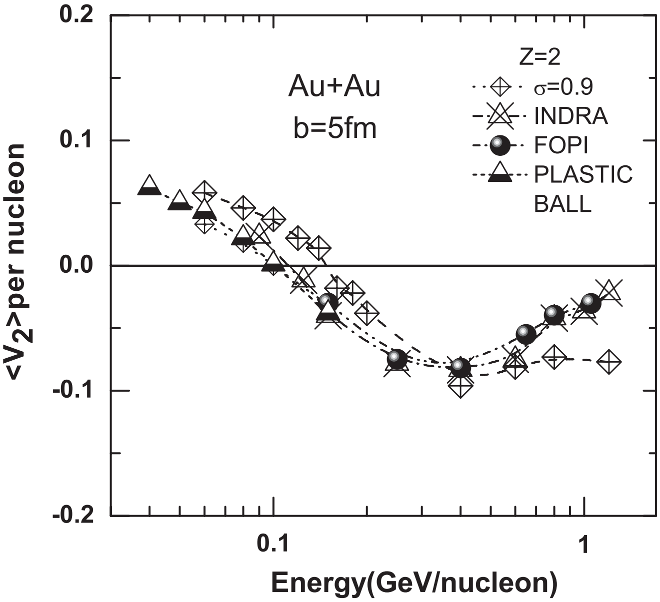

FIG. 5. Energy dependence of the elliptic flow for Au + Au systems and its comparison with the experimental data.

$1 \leqslant A \leqslant 4$ it is -0.061 ± 0.036, and for $2 \leqslant A \leqslant 4$ it is -0.0043 ± 0.031.

In Fig. 5, we show v_2 at midrapidity $|y| = |\frac{y_{\text{cm}}}{y_{\text{beam}}}| \leqslant 0.1$ for $Z = 2$ as a function of incident energy. The rapidity cut is in accordance with the experimental findings. The theoretical results are compared with the experimental data extracted by the INDRA, FOPI, and Plastic Ball Collaborations [5,6]. With the increase in the incident energy, the elliptical flow v_2 changes from positive to negative values, exhibiting a transition from the in-plane to out-of-plane emission of nucleons. This is because the mean field, which contributes to the formation of a rotating compound system, becomes less important and the collective expansion process based on the nucleon-nucleon scattering starts to be predominant. The maximal negative value of v_2 is obtained around $E = 500$ MeV/nucleon with a reduced isospin-dependent cross section. This out-of-plane emission decreases again toward higher incident energies. This happens due to faster movement of the spectator matter after v_2 reaches the maximal negative value [6]. This trend is in agreement with experimental findings. A close agreement with data is obtained in the presence of the hard equation of state and with a reduced isospin-dependent cross section for $Z = 2$ particles. Similar results and trends have also been reported by Zhang *et al.* in their recent communication [21].

Conclusion. By using the IQMD model, we have studied the correlation between transition energy E_{trans} and balance energy E_{bal}. We have investigated the elliptical flow of fragments for different reacting systems at incident energies between 40 and 1200 MeV/nucleon using the isospin-dependent quantum molecular dynamics model. The elliptical flow is found to show a transition from in-plane to out-of-plane at a certain beam energy in the midrapidity region. Our calculation with a stiff equation of state and a reduced isospin-dependent nucleon-nucleon cross section ($\sigma = 0.9\sigma_{NN}$) is in good agreement with the experimental findings. The difference between the balance and transition energy decreases with the increase in the composite mass of colliding nuclei. This tells us that due to the increase of neutrons to colliding nuclei, the difference between the two energies decreases.

Acknowledgments. The authors are grateful to J. Aichelin for discussions during his visit to Panjab University Chandigarh under the Indo-French joint international project.

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