Influence of statistical sequential decay on isoscaling and symmetry energy coefficient in a GEMINI simulation

P. Zhou,[1,2] W. D. Tian,[1,*] Y. G. Ma,[1,†] X. Z. Cai,[1] D. Q. Fang,[1] and H. W. Wang[1]
[1]*Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China*
[2]*Graduate School of the Chinese Academy of Sciences, Beijing 100080, China*
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Extensive calculations on isoscaling behavior with the sequential-decay model *gemini* are performed for the medium-to-heavy nuclei in the mass range $A = 60$–120 at excitation energies up to 3 MeV/nucleon. The comparison between the products after the first-step decay and the ones after the entire-steps decay demonstrates that there exists a strong sequential decay effect on the final isoscaling parameters and the apparent temperature. Results show that the apparent symmetry energy coefficient γ_{app} does not reflect the initial symmetry energy coefficient C_{sym} embedded in the mass calculation in the present GEMINI model.

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One of main goals of isospin physics is to determine the isospin dependence of the in-medium nuclear effective interactions and the equation of state (EOS) of isospin asymmetric nuclear matter or finite nuclei, particularly its isospin-dependent term, i.e., the density dependence of the nuclear symmetry energy. Knowledge of nuclear symmetry energy is essential for understanding not only many problems in nuclear physics, such as the dynamics of heavy-ion collisions induced by radioactive beams and the structure of exotic nuclei, but also a number of important issues in astrophysics, such as supernova simulation and neutron-star models, which require inputs of the nuclear equation of state at extreme values of density and asymmetry [1,2]. Recently, impressive progress was made both experimentally and theoretically; a number of earlier reviews on isospin physics with heavy-ion reactions can be found in several references [3–5].

Symmetry energy could be extracted from heavy-ion collisions using the isoscaling approach [6–15]. The isoscaling law means that the ratio of isotope yields $R_{21}(N, Z) = Y_2(N, Z)/Y_1(N, Z)$ from two similar reactions, denoted as reaction 1 and 2, which are different only in their isospin asymmetry, is found to exhibit an exponential relationship as a function of the neutron number N and proton number Z [6]; that is,

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C \exp(\alpha N + \beta Z), \tag{1}$$

where $Y_2(N, Z)$ and $Y_1(N, Z)$ are the fragment yields from the neutron-rich and the neutron-deficient reaction, respectively, C is an overall normalization factor, and α and β are fitted parameters. The isoscaling parameter α is related to the symmetry energy coefficient C_{sym} of the EOS in microcanonical and canonical frames by the following relations [6]:

$$\alpha = \frac{4C_{sym}}{T}\left[\left(\frac{Z}{A}\right)^2_{s1} - \left(\frac{Z}{A}\right)^2_{s2}\right] \equiv \frac{4C_{sym}}{T}\Delta\left(\frac{Z}{A}\right)^2_s, \tag{2}$$

and

$$\beta = \frac{4C_{sym}}{T}\left[\left(\frac{N}{A}\right)^2_{s1} - \left(\frac{N}{A}\right)^2_{s2}\right] \equiv \frac{4C_{sym}}{T}\Delta\left(\frac{N}{A}\right)^2_s, \tag{3}$$

where Z_{s1}, Z_{s2}, N_{s1}, N_{s2}, and A_{s1}, A_{s2} are the charge numbers, neutron numbers, and mass numbers of the sources from the two systems; T is their temperature; and C_{sym} is the symmetry energy coefficient. This relation was also evidenced in other model frameworks [7–15]. A great deal of effort has been devoted to investigate the nuclear symmetry energy and its density and temperature dependencies [3,5,16–19].

Ideally, primary fragments should be detected right after emission to extract information about the collisions, and Eq. (2) is derived based on the primary reaction products bypassing secondary decays. However, the detected experimental data are for cold products after the secondary decays from hot products. Isoscaling was also reasonably reproduced in the sequential decay codes [15,20]. However, there are still arguments on the sequential decay effect on isoscaling; some models show that the effect from sequential decays on isoscaling is negligible, but some efforts show that sequential decay affects the isoscaling parameters and then distorts the extraction of the symmetry energy coefficient C_{sym} [21,22]. There are some issues still unsolved or unclear, such as the sequential decay effect on isoscaling parameters, derived apparent temperature T_r from the experimental measurement, and the isospin evolution of the decaying sources; these factors affect the extraction of the symmetry energy coefficient C_{sym}.

The statistical GEMINI model [23] calculates the decay of compound nuclei by modes of sequential binary decays. The model employs a Monte Carlo technique to follow the decay chains of individual compound nuclei through sequential binary decays until the resulting products are unable to undergo further decay. GEMINI has been widely used to simulate the hot equilibrium source deexcitation, or as an "afterburner" code to analyze the hot fragment decay after dynamical simulation [24–26]. Isoscaling was investigated via the statistical sequential secondary decay code GEMINI [23], in which only the first-step sequential decay was simulated and Eq. (2) was confirmed for the fragments that are decayed

*tianwendong@sinap.ac.cn
†ygma@sinap.ac.cn

directly from the initial sources [15]. In the present work, we investigate the entirely decayed fragments from excited sources, comparing with the first-decay-only fragments from the same source. The influences of sequential decays on isoscaling parameters α, β, and the apparent temperature T_r are discussed, and the apparent symmetry energy coefficient γ_{app} is extracted.

The detailed description of the GEMINI code can be found in Refs. [15,23]; the same configuration and parameters of the GEMINI code were adopted as in Ref. [15]. Several pairs of equilibrated sources are considered at various initial excitation energies $E_{\text{ex}} = 1.0$, 1.4, 2.0, 2.4, and 3.0 MeV/nucleon. We selected source pairs with the same proton number Z_s but different mass number A_s to systematically study the isoscaling behavior. In this case, possible effects of different magnitudes of Coulomb interaction on isotopic distributions are avoided. The equilibrated source pairs are chosen in different mass regions and system isospin asymmetry N/Z. Two groups of source pairs were used: (1) $Z_s = 50$ with $A_s = 100$, 105, 110, and 115; and (2)$Z_s = 30$ with $A_s = 60$, 63, 66, and 69, respectively. Following the literature, the index 2 denotes a more neutron-rich system as widely used in convention, and the index 1 denotes the more neutron-deficient system. In our previous work [15], the statistical decay stops after one particle is emitted from the source, which is called the "first-step″ decay in this Brief Report, which is a simple picture, and the decay procedure can be expressed definitely and clearly, isoscaling was confirmed for the first-step decay products in detail, and the reasonability of extracting the symmetry energy coefficient C_{sym} from the simulation results via experimental analysis technique. But the first-step decay was not the real case. Experiments measure the final products after multistep decays until no fragments are produced or γ rays emitted, which is called the "entire-steps″ decay in this Brief Report.

Isoscaling is analyzed from the emitted light fragments in both of the aforementioned cases, namely first-step decay only and entire-steps decay chains for all the simulated systems. As an example shown in Fig. 1, the comparison of the isoscaling parameters α and β is plotted as a function of emitted light fragment proton number Z and neutron number N, between the first-step decay and entire-steps decay fragments for the source pairs of $Z_s = 50$. As we can see, isoscaling parameters α and β are essentially independent of Z or N of fragments, for both first-step and entire-steps decay fragments, especially for $Z \leqslant 4$ or $N \leqslant 5$. It was evidenced that, in the first-step decay case, the probability of producing a cluster with a given Z and A at T depends exponentially on the free energy of that cluster, $F(Z, A, T)$; in the GEMINI simulation, the cluster-free energies depend on the strength of the symmetry term in the liquid-drop energy through Eq. (2) with $C_{\text{sym}} \approx 24$ MeV [15]. If the entire-steps sequential secondary decay is included, the source isospin N/Z and temperature T as well as the isoscaling parameters vary after each step of decay. Similarly, in a previous study, the time dependence of isoscaling parameters was discussed in a molecular dynamics model [14], indicating that the final values of these parameters could be related to the last part of the reaction where the fragments finish cooling by particle evaporation. In the present GEMINI model,

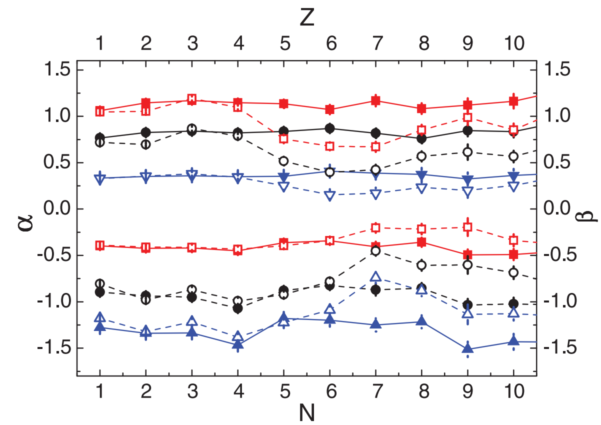

FIG. 1. (Color online) Comparisons of isoscaling parameters α (positive values) or β (negative values) as a function of Z or N from source pairs of $Z_s = 50$ at excitation energies $E_{\text{ex}} = 2.4$ MeV/nucleon. All solid symbols represent the results for only the first-step secondary decay products, and open symbols represent entire-step secondary decay products. Symbols in the figure correspond to $Y_{A_s=115}/Y_{A_s=100}$ (upward triangles), $Y_{A_s=110}/Y_{A_s=100}$ (circles), and $Y_{A_s=110}/Y_{A_s=105}$ (downward triangles), respectively.

fragment yields are strongly expected to be degraded after the entire-steps decay, as for the isotopic yield ratios. It is not surprising that in Fig. 1 that parameters α and β extracted from isotopic yield ratios of the final emitted light fragments show a discrepancy, especially for the intermediate mass fragments like $Z \geqslant 5$ in some cases, since those heavier fragments experience strong multistep decay and feeding-down effects. Finally, α and β change a lot compared to the first-step-only decay case, and the fluctuation increases in the entire-steps decay case. Average α and β values over fragments Z and N are used in the following discussions to observe the overall properties.

In Fig. 2 we show the comparison of isoscaling parameters α and β as a function of the excitation energy of sources from different source pairs. If the entire-steps decay chains are included, which are depicted by the open symbols in Fig. 2, α and $|\beta|$ values show significant decrease. This reduction is about 20% on average, consistent with the result in Refs. [20,21] for the $C_{\text{sym}} \approx 25$ MeV case, but the excitation-energy-dependent trend of α and β does not change.

As already discussed, the isoscaling parameter α is related to the symmetry energy coefficient C_{sym} of the nuclear binding energy through Eqs. (2) and (3). This relation provides a direct link between the measurable quantities and the nuclear symmetry energy coefficient. It should be noticed that the parameters α and β refer to the hot primary fragments, which have to undergo sequential decays into cold fragments. It was assumed that the secondary decay on the yield of a specific isotope is similar for the two reactions; thus, the effect of the sequential decays on $R_{21}(N, Z)$ is small and $R_{21}(N, Z)$ reflects the properties of the primary source. In our present study, the first-step statistical sequential decay process stems from a fixed initial source with definite excitation energy (temperature) and isospin asymmetry, and it was verified by Eqs. (2) and (3) to

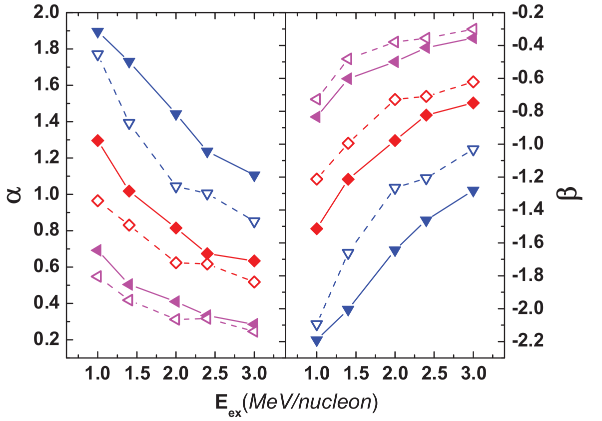

FIG. 2. (Color online) Comparisons of isoscaling parameters α (left) and β (right) as a function of the source excitation energy from source pairs of $Z_s = 30$. All solid symbols represent the first-step-only secondary decay products; open symbols represent the entire-steps secondary decay products. Symbols in the figure correspond to $Y_{A_s=69}/Y_{A_s=60}$ (downward triangles), $Y_{A_s=69}/Y_{A_s=63}$ (diamonds), and $Y_{A_s=69}/Y_{A_s=66}$ (left-pointing triangles).

reflect the link between the measurable quantity $R_{21}(N, Z)$ and the symmetry energy coefficient C_{sym}.

It was found in Fig. 1 that the isoscaling behavior is still present after considering the entire-steps decay chains. But in this case, the isoscaling parameters α and β decrease compared to the first-step-only decay case as seen in Fig. 2. In the statistical sequential decay, the source temperature T and isospin asymmetry N/Z also change after each step of the decays; thus, the parameters T and $\Delta(Z/A)_s^2$ [or $\Delta(N/A)_s^2$] vary during the sequential decay process, where many intermediate sources are different from the initial source.

To explore the validity of Eqs. (2) and (3) in the entire-steps statistical sequential decay, we plot αT as a function of $\Delta(Z/A)_s^2$ and βT as a function of $\Delta(N/A)_s^2$ in Fig. 3. For the first-step-only decay, data points are depicted by the solid symbols, using the initial source temperature T_i, which was calculated by input excitation energy as shown in Table I and isoscaling parameters after the first-step decay. The linear fit (the solid line) gives a symmetry energy coefficient $C_{\mathrm{sym}} = 24.2 \pm 0.3$ MeV. To investigate the decay effect on the symmetry energy coefficient, data points from the entire-steps decays are also plotted in Fig. 3 as shown by the open symbols. As we mentioned for the entire-steps decay chains, there are many intermediate-state sources with different temperature T and isospin asymmetry Z/A or N/A. Nevertheless, the final isoscaling parameters α and β still show similar rules as the first-step-only decay case; that is, αT (βT) and $\Delta(Z/A)_s^2$ [$\Delta(Z/A)_s^2$] can be still fitted by another linear function, namely

$$\alpha = \frac{4\gamma_{\mathrm{app}}}{T}\left[\left(\frac{Z}{A}\right)_{s1}^2 - \left(\frac{Z}{A}\right)_{s2}^2\right] \equiv \frac{4\gamma_{\mathrm{app}}}{T}\Delta\left(\frac{Z}{A}\right)_s^2, \quad (4)$$

and

$$\beta = \frac{4\gamma_{\mathrm{app}}}{T}\left[\left(\frac{N}{A}\right)_{s1}^2 - \left(\frac{N}{A}\right)_{s2}^2\right] \equiv \frac{4\gamma_{\mathrm{app}}}{T}\Delta\left(\frac{N}{A}\right)_s^2, \quad (5)$$

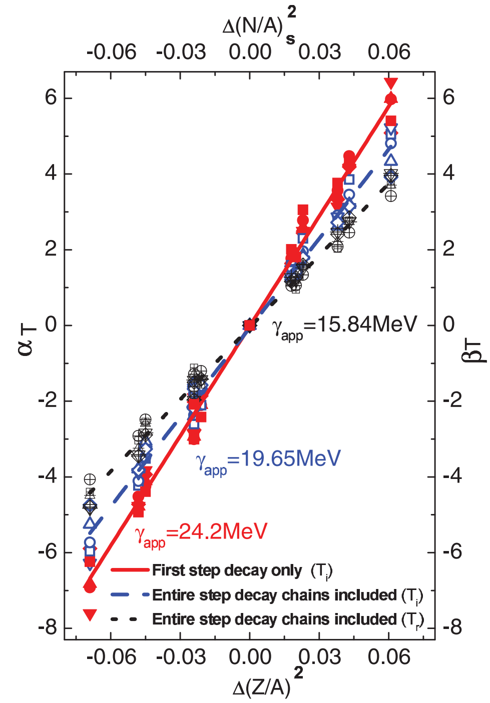

FIG. 3. (Color online) αT (βT) as a function of the source isospin difference $\Delta(Z/A)_s^2$ [$\Delta(N/A)_s^2$] from source pairs of $Z_s = 30$. All solid symbols represent the first-step-only secondary decay case, open symbols without or with a cross represent the entire-steps secondary decay products with T_i and T_r, respectively. Symbols in the figure correspond to excitation energies $E_{\mathrm{ex}}/A = 1.0$ (squares), 1.4 (circles), 2.0 (upward triangles), 2.4 (downward triangles) and 3.0 MeV (diamonds), respectively. The solid, dashed, and dotted lines are the linear fitting for the above three cases, which gives the apparent symmetry energy coefficients $\gamma_{\mathrm{app}} = 24.2$, 19.65, and 15.84 MeV, respectively.

which gives the apparent symmetry energy coefficient $\gamma_{\mathrm{app}} = 19.65 \pm 0.25$ MeV if the same T_i are used.

In fact, for the entire-steps source decays, the intermediate-state sources have different isospin asymmetry N/Z ranging from initial isospin asymmetry to the stable line or the evaporation attraction line [27] and different temperature T ranging from initial temperature to zero. In this case, principally both T and $\Delta(Z/A)_s^2$ [$\Delta(Z/A)_s^2$] need to be corrected to reflect the intermediate sources. In the simulation,

TABLE I. Initial temperature (T_i) and final-state apparent temperature (T_r) in different systems ($Z_s = 30$, $A_s = 60$ or 63, 66 or 69).

	$A_s = 60$ or 63		$A_s = 66$ or 69	
E_{ex}	T_i (MeV)	T_r (MeV)	T_i (MeV)	T_r (MeV)
1.0	2.8/2.9	2.3/2.4	2.9/2.9	2.4/2.4
1.4	3.4/3.5	2.8/2.9	3.5/3.5	2.9/3.0
2.0	4.1/4.2	3.5/3.6	4.2/4.2	3.6/3.6
2.4	4.5/4.6	3.8/3.9	4.6/4.7	3.9/4.0
3.0	5.1/5.1	4.3/4.4	5.2/5.3	4.4/4.5

the initial source temperature T can be calculated, and the intermediate source tracing the sequential decay chains can also be performed. From an experimental point of view, the temperature and isospin asymmetry of the intermediate source can be extracted from evaporation products, which reflects the entire-decay chains.

Traditionally, temperature can be extracted from the measurements of spectral slopes or double isotopic ratios at lower energies [28,29]. In the present work, the initial temperatures T_i are calculated directly in the GEMINI code by the input excitation energy [15], and the final-state temperature T_r can be obtained by the neutron and proton spectra fitting when the entire-steps decay chains are included in the GEMINI calculation. The results are displayed in Table I.

When the temperature T_r is used in Eqs. (4) and (5) to fit the linear slope parameter $\gamma_{\rm app}$, it leads to the reduction of the parameter $\gamma_{\rm app}$ as shown in Fig. 3 (short-dashed line). Its slope gives an apparent symmetry energy coefficient $\gamma_{\rm app} = 15.84 \pm 0.18$ MeV, which is one-third the reduction when comparing with the first-step-only decay case. In this context, we should be careful to use the apparent symmetry energy derived directly from the final fragments, which could be distorted because of the multistep sequential decays. Of course, the present results are specific for the use of GEMINI to describe the secondary decay; i.e., they may depend on the details of the sequential decay code.

In summary, we performed the isoscaling analysis for both light fragments from the first-step-only decay and the entire-steps decay chains with the GEMINI code. It was found that isoscaling can still be observed and Eqs. (4) and (5), which are used to extract the symmetry energy coefficient, also work after the entire-steps decay is taken into account. However, the statistical sequential decay leads to the decreasing of isoscaling parameters α and β as well as temperature. Therefore, the reduced (apparent) source temperature together with the reduced isoscaling parameters leads to a smaller symmetry energy parameter $\gamma_{\rm app}$ in comparison with the initial symmetry energy coefficient $C_{\rm sym}$, which is constrained from the first-step-only statistical decay calculation. From the present GEMINI model calculations, we carefully consider the multistep sequential decay effect on the extraction of the symmetry energy coefficient via the final cold products.

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