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Effective valence proton numbers for nuclei with $Z \sim 64$

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The subshell effect for nuclei with proton number $Z \sim 64$ has been known for many years. The most economic way to consider this effect is to use the effective valence proton number. In this Brief Report we extract effective valence proton numbers for nuclei in this region by using the systematics of the first 2^+ energies ($E_{2_1^+}$) of even-even nuclei, the ratios of the first 4^+ and 6^+ state energies with respect to $E_{2_1^+}$ (R_4 and R_6), the $B(E2)$ values, the quadrupole deformation parameters e_2, and anomalous g factors of the 2_1^+ state for even-even nuclei. It is noticed that these physical quantities saturate when N_pN_n, the product of the valence proton number and the valence neutron number, is large enough; on the other hand, they go to saturation at different "speeds." We show that the subshell effect is more evident for $E_{2_1^+}$ and yrast state energy ratios (R_4 and R_6), and relatively less for other quantities.

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The subshell of proton number $Z = 64$ was discovered by Ogawa *et al.* in 1978 by measuring the excitation energy of the first 2^+ state (denoted by $E_{2_1^+}$ in this Brief Report) in the ^{146}Gd nucleus [1]. They found that the $E_{2_1^+}$ in ^{146}Gd is 1971 keV, which is much higher than in any other $N = 82$ isotones. In Ref. [2], Casten and collaborators studied the deformation for nuclei with proton numbers $Z \sim 64$ and suggested the relation between the $Z = 64$ subshell closure and the onset of deformation at $N = 88$–90 by using the Talmi rule [3].

The most economic way to consider the $Z = 64$ subshell effect is to use "proper" effective proton numbers for nuclei that are affected by this subshell closure or, alternatively, to use effective proton boson numbers in the interacting boson model (IBM) [4]. In the IBM, the number of bosons is taken to be half of the valence nucleons, while this counting scheme becomes ambiguous for nuclei near the subshell closure. In Ref. [5], Scholten suggested a method to calculate the effective boson number for nuclei affected by the $Z = 64$ subshell. In Refs. [6,7], the effective proton numbers are taken to be an adjustable parameter in the IBM calculations. It was found that the agreement between calculated IBM results and the experimental data is improved by considering a smooth variation of the effective proton boson numbers.

There are two other approaches to extract the effective proton numbers for nuclei around the $Z = 64$ subshell closure. One was suggested by Wolf, Warner, and Benczer-Koller in Ref. [8], where these authors extracted the effective proton boson numbers by using the anomalous values of the g factor of the first 2^+ state for nuclei close to the $Z = 64$ subshell. The other was given by Zhao, Arima, and Casten in Ref. [9], where the effective proton number was extracted by using systematics of the quadrupole deformation parameter e_2 calculated by Möller and Nix in Ref. [10].

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The purpose of this Brief Report is to revisit the effective proton numbers for nuclei with $Z \sim 64$ by using $E_{2_1^+}$, e_2, $B(E2, 2_1^+ \rightarrow 0_1^+)$ [in this Brief Report we use the abbreviation $B(E2)$ for short], and the ratios of $E_{4_1^+}$ and $E_{6_1^+}$ with respect to $E_{2_1^+}$ (denoted by R_4 and R_6, respectively) in the N_pN_n scheme and the $g(2_1^+)$ factor within the IBM as in Ref. [8]. We make comparisons between the results by using different methods as well as previous results given in Refs. [6,7]. It is shown that the subshell effect is more pronounced for $E_{2_1^+}$, R_4, and R_6 and is less on other quantities [e_2 and $B(E2)$ values].

The N_pN_n scheme [11,12] was invented in 1985 by Casten. He suggested that the N_pN_n scheme is a simple evaluation of the residual proton-neutron interaction, which plays a key role in the development of nuclear deformation [13–15]. It was demonstrated that N_pN_n is an excellent scaling factor that allows one to study the evolution of the deformation in different regions [15]. Many physical quantities exhibit remarkable regularities in the N_pN_n scheme [7,9,12,16–20].

In Figs. 1(a)–1(e) we present the correlation for $E_{2_1^+}$, R_4, R_6, e_2, and $B(E2)$ versus N_pN_n. Here $E_{2_1^+}$, $E_{4_1^+}$, and $E_{6_1^+}$ values are taken from Ref. [21], the $B(E2)$ values are from Ref. [22], and the e_2 values are from calculated results of Ref. [10]. It is noted here that we consider only nuclei whose masses have been experimentally measured [23] and take absolute values of all e_2. The reason for this restriction is that the calculated results of e_2 are used as surrogates for directly measured observables. Of course, the importance of residual interactions changes (as does the mean field itself) and care should be taken in extending these results to new regions. In Fig. 1(f) we plot the $g(2_1^+)(N_p + N_n)/N_n$ values versus N_p/N_n, where the experimental data of $g(2_1^+)$ are taken from Refs. [22,24,25]. The solid black circles in Fig. 1 correspond to nuclei that are believed to be unaffected by the $Z = 64$ subshell. It is easily seen that there exist nice correlations between these quantities and N_pN_n for the unaffected cases. The results in color in Fig. 1 correspond to those that are

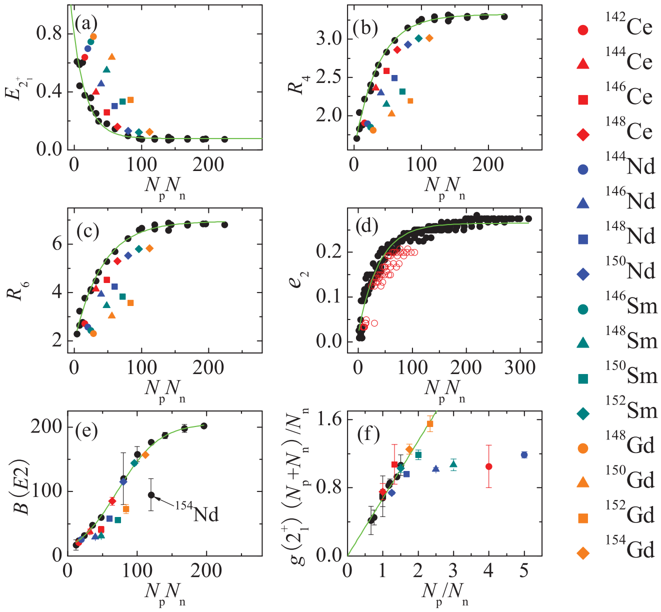

FIG. 1. (Color online) The systematics of (a) $E_{2_1^+}$, (b) R_4, (c) R_6, (d) e_2, and (e) $B(E2)$ in the N_pN_n scheme and (f) the correlation between $g(2_1^+)(N_p+N_n)/N_n$ and N_p/N_n for nuclei in the ($50 < Z < 66$, $82 < N < 104$) region. The black solid circles correspond to nuclei that are believed to be unaffected by the $Z = 64$ subshell, and the results in color correspond to those that are affected. In (d) the red open circles correspond to those anomalous cases including nuclei with both odd and even values of N_p and N_n. In (e) one sees a very small value of $B(E2)$ for the ^{154}Nd nucleus, and the reason of this hinderance is not discussed here. In (f) the straight line is plotted according to the linear relation $g(2_1^+)(N_p+N_n)/N_n = g_\nu + g_\pi N_p/N_n$ (in unit of μ_N), where $g_\nu = 0.00(11)$ and $g_\pi = 0.67(9)$ are obtained by the χ^2 fitting of results for nuclei that are not affected by the $Z = 64$ subshell closure. Here $E_{2_1^+}$, $E_{4_1^+}$, and $E_{6_1^+}$ values are taken from Ref. [21], the e_2 values are from calculated results of Ref. [10], the $B(E2)$ values are from Ref. [22], and the $g(2_1^+)$ values are from Refs. [22,24,25].

believed to be affected by this subshell and are seen to deviate from the simple correlations. In Figs. 1(a)–1(c), one sees that the deviations of $E_{2_1^+}$, R_4, and R_6 for the Gd isotopes are the most pronounced, then the Sm isotopes, and, finally, the Nd, Ce, and Ba isotopes. The deviations decrease as the neutron number N increases from 84 to 86 to 88 and to 90 and gradually vanish for $N \geqslant 90$. Similarly, in Fig. 1(f) large anomalies of the $g(2_1^+)$ factors for ^{142}Ce, ^{144,146}Nd, and ^{148}Sm are easily noticed.

Before we go to the effective proton numbers extracted from Fig. 1, let us point out two very interesting features exhibited here. (1) The $B(E2)$ values and excitation energies of nuclei in this region go to saturation at different N_pN_n. It is well known that both the $B(E2)$ values and excitation energies $E_{2_1^+}$ (also R_4 and R_6) saturate as N_pN_n becomes large enough. However, they saturate at different values of N_pN_n. For $E_{2_1^+}$, R_4, and R_6, the critical value of N_pN_n at which these quantities tend to saturate is $\sim$100; for the $B(E2)$ values, the critical value of N_pN_n is about 170. This means that the $B(E2)$ values and excitation energies go to their saturation values at very different "speeds" in terms of N_pN_n. (2) For *all* nuclei that are assumed to be affected by this subshell, the values of $E_{2_1^+}$, R_4, and R_6 exhibit very large deviations from the normal correlation for nuclei that are not affected by the $Z = 64$ subshell closure. However, many of the $B(E2)$ values and $g(2_1^+)(N_p+N_n)/N_n$ for these "anomalous" cases are well overlapped with the "normal" correlations. This means that the $Z = 64$ subshell affects these physical quantities to very different degrees. Therefore, the subshell effect is more tangible and evident for $E_{2_1^+}$ and yrast state energy ratios (R_4 and R_6) than for other quantities [e_2, $g(2_1^+)$, and $B(E2)$ values]. Because of this fact, it is not surprising that these different methods present us with different effective proton numbers, as will be shown below.

The procedure from which we extract our effective proton numbers by using Figs. 1(a)–1(e) in this Brief Report is described as follows. We first fit the results for nuclei that are not affected by the $Z = 64$ subshell by very simple curves (such as exponential function). Then we ask which values of N_pN_n on the curves correspondingly give the same values of these anomalous $E_{2_1^+}$, R_4, R_6, e_2, and $B(E2)$. Here $N_n = N - 82$, and we thus obtain values of N_p that are assumed to be our effective proton numbers, denoted by $N_p^{\rm eff}$.

Similarly, we extract $N_p^{\rm eff}$ by using $g(2_1^+)$ [see Fig. 1(f)]. This procedure was given in Ref. [8]. In IBM-2 [4], the g factor of the first 2^+ state for an even-even nucleus is described by $g(2_1^+) = g_\pi N_\pi/N_t + g_\nu N_\nu/N_t$, where g_π (g_ν) is the valence proton (neutron) g factor, N_π (N_ν) is the valence proton (neutron) boson number, and $N_t = N_\pi + N_\nu$. We rewrite this relation as follows: $g(2_1^+)(N_p+N_n)/N_n = g_\nu + g_\pi N_p/N_n$. We first obtain the linear correlation between $g(2_1^+)(N_p+N_n)/N_n$ and N_p/N_n for those nuclei that are not affected by this subshell via the χ^2 fitting. Then we solve $g(2_1^+)(N_p+N_n)/N_n = g_\nu + g_\pi N_p/N_n$ by using the experimental data of anomalous $g(2_1^+)$, the values of g_π and g_ν

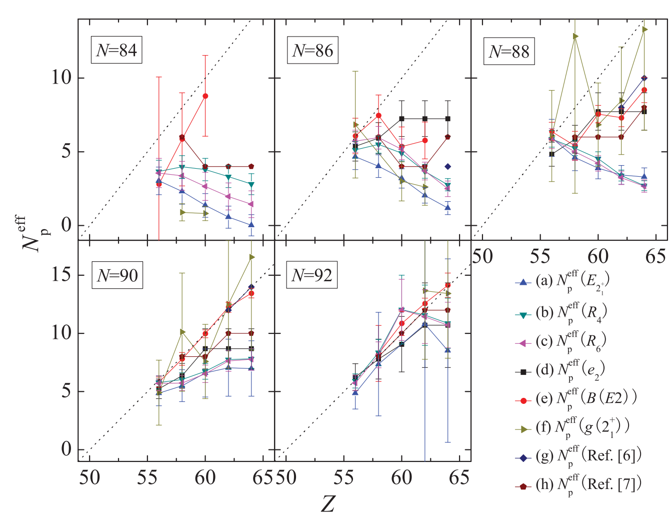

FIG. 2. (Color online) The effective valence proton numbers extracted from Fig. 1. Results (a)–(f) are based on Figs. 1(a)–1(f), respectively. Results (g) and (h) are taken from Refs. [6] and [7]. The dashed line corresponds to valence proton numbers assuming the conventional magic numbers (here 50 and 82), i.e., neglecting the $Z = 64$ subshell.

obtained for nuclei that are not affected by this subshell [$g_\pi = 0.67(9)$ and $g_\nu = 0.00(11)$], and here $N_n = N - 82$. We take the resultant N_p to be our $N_\mathrm{p}^\mathrm{eff}$.

The effective valence proton numbers $N_\mathrm{p}^\mathrm{eff}$ such obtained are summarized in Fig. 2. One sees that these $N_\mathrm{p}^\mathrm{eff}$ increase with N_n and decrease as $Z \to 64$. Scrutinizing more closely, one sees that the values of $N_\mathrm{p}^\mathrm{eff}$ extracted by using systematics of $E_{2_1^+}$, R_4, and R_6 follow this pattern very well. The values of $N_\mathrm{p}^\mathrm{eff}$ obtained by using the $g(2_1^+)$ factor follow this pattern for Nd and Sm isotopes and have very large uncertainties otherwise. In contrast, the values of $N_\mathrm{p}^\mathrm{eff}$ extracted by using $B(E2)$ and calculated e_2 for $N = 84$–88 isotones are larger than those using $E_{2_1^+}$, R_4, and R_6. $N_\mathrm{p}^\mathrm{eff}$ obtained by the IBM calculations [6,7] are between those obtained by using the excitation energy $E_{2_1^+}$ (and R_4 and R_6) and those obtained by using the $B(E2)$ values. This is partly because all quantities should be equally considered in the IBM calculations.

To summarize, in this Brief Report we present systematics of $E_{2_1^+}$, the ratios of $E_{4_1^+}$ and $E_{6_1^+}$ with respect to $E_{2_1^+}$, and e_2 versus $N_\mathrm{p}N_\mathrm{n}$, the product of valence proton number and valence neutron number with respect to the nearest doubly closed nucleus, and $g(2_1^+)(N_\mathrm{p} + N_\mathrm{n})/N_\mathrm{n}$ versus $N_\mathrm{p}/N_\mathrm{n}$ for nuclei near the $Z = 64$ subshell. We find that the $B(E2)$ values and excitation energy $E_{2_1^+}$ (and energy ratios R_4 and R_6) go to their saturation values in terms of $N_\mathrm{p}N_\mathrm{n}$ at different speeds. $E_{2_1^+}$ values (and R_4 and R_6) saturate at a much smaller value of $N_\mathrm{p}N_\mathrm{n}$ than the $B(E2)$ values.

By using these systematics, we obtain effective valence proton numbers $N_\mathrm{p}^\mathrm{eff}$ for nuclei affected by the $Z = 64$ subshell. The values of $N_\mathrm{p}^\mathrm{eff}$ increase with N_n and decrease as $Z \to 64$. The values of $N_\mathrm{p}^\mathrm{eff}$ using systematics of $B(E2; 2_1^+ \to 0_1^+)$ and e_2 for $N = 84$–88 isotones are larger than those using $E_{2_1^+}$, R_4, and R_6, and those previously obtained in the IBM calculations are in between. The $Z = 64$ subshell effect is seen to be more tangible and evident for $E_{2_1^+}$ and yrast state energy ratios (R_4 and R_6) than for other quantities [e_2, $g(2_1^+)$, and $B(E2)$ values].

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[1] M. Ogawa, R. Broda, K. Zell, P. J. Daly, and P. Kleinheinz, Phys. Rev. Lett. **41**, 289 (1978).
[2] R. F. Casten, D. D. Warner, D. S. Brenner, and R. L. Gill, Phys. Rev. Lett. **47**, 1433 (1981).
[3] I. Talmi, Rev. Mod. Phys. **34**, 704 (1962).
[4] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975); Adv. Nucl. Phys. **13**, 139 (1984); F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).
[5] O. Scholten, Phys. Lett. B **127**, 144 (1983).
[6] D. S. Chuu, C. S. Han, S. T. Hsie, and M. M. King Yen, Phys. Rev. C **30**, 1300 (1984); C. S. Han, D. S. Chuu, and S. T. Hsieh, *ibid.* **42**, 280 (1990).
[7] Y. M. Zhao and Y. Chen, Phys. Rev. C **52**, 1453 (1995).
[8] A. Wolf, D. D. Warner, and N. Benczer-Koller, Phys. Lett. B **158**, 7 (1985).
[9] Y. M. Zhao, R. F. Casten, and A. Arima, Phys. Rev. Lett. **85**, 720 (2000).
[10] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[11] R. F. Casten, Nucl. Phys. A **443**, 1 (1985).
[12] R. F. Casten, Phys. Rev. Lett. **54**, 1991 (1985).
[13] A. de Shalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953).
[14] P. Federman and S. Pittel, Phys. Lett. B **69**, 385 (1977).
[15] R. F. Casten and N. V. Zamfir, J. Phys. G **22**, 1521 (1996).
[16] R. F. Casten and N. V. Zamfir, Phys. Rev. Lett. **70**, 402 (1993).
[17] Y. M. Zhao, A. Arima, and R. F. Casten, Phys. Rev. C **63**, 067302 (2001).
[18] J. H. Yoon, E. Ha, and D. W. Cha, J. Phys. G **34**, 2545 (2007).
[19] G. Jin, J. H. Yoon, and D. W. Cha, J. Phys. G **35**, 035105 (2008).
[20] Y. Byun, G. Jin, J. H. Yoon, and D. W. Cha, J. Korean Phys. Soc. **54**, 792 (2009).
[21] LBNL Isotopes Project Nuclear Data Dissemination [http://ie.lbl.gov/toi.html].
[22] ENSDF [http://www.nndc.bnl.gov/ensdf/].
[23] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003) [http://www.nndc.bnl.gov/masses/mass.mas03].
[24] N. J. Stone, At. Data Nucl. Data Tables **95**, 75 (2005).
[25] C. Goodin, J. R. Stone, N. J. Stone, A. V. Ramayya, A. V. Daniel, J. H. Hamilton, K. Li, J. K. Hwang, G. M. Ter-Akopian, and J. O. Rasmussen, Phys. Rev. C **79**, 034316 (2009).