Examining the scaling behavior of Delbrück scattering in experimental data

B. Kunwar,[1,2,*] A Bhadra,[1,†] and S. K. Sen Gupta[3,‡]

[1]*High Energy & Cosmic Ray Research Centre, University of North Bengal, Siliguri, West Bengal, India 734013*
[2]*Department of Physics, Sikkim Government College, Gangtok, Sikkim-737102, India*
[3]*Department of Electronics & Communication, Jalpaiguri Government Engineering College, Jalpaiguri, WB, 735101, India*

(Received 14 July 2011; published 28 September 2011)

The conventional perception is that the amplitudes of Delbrück scattering calculated to all orders in the charge number Z of the target nucleus should exhibit a scaling behavior at high energies. To examine this hypothesis the available experimental data of differential cross sections of elastic scattering in the energy range between 140 MeV and 7.11 GeV are analyzed. It is found that the experimental data do not show scaling characteristics. Such a finding, though apparently against the standard notion, is not unexpected because at high energies Delbrück scattering is in very forward direction and the theoretical arguments demand that to observe scaling, not only the energy itself but the product of scattering angles and energy also should be very large.

DOI: 10.1103/PhysRevC.84.034614 PACS number(s): 25.20.−x, 37.10.De, 31.30.J−

I. INTRODUCTION

Delbrück scattering, the elastic scattering of photons in the static Coulomb field of atomic nuclei via virtual electron-positron pairs, is one of the nontrivial predictions of quantum electrodynamics (QED). It is an observable high-order nonlinear process in QED which has allowed testing a single Feynman graph of the order Z^2e^6 with an accuracy of 5%. Delbrück scattering is also of interest because of its interference effect in investigation of nuclear structure through photon scattering.

Theoretically a reasonable understanding of the Delbrück scattering has been achieved so far, though there is still no general solution of the Delbrück scattering problem for arbitrary photon energies and scattering angles [1]. All the theories developed so far have their own ranges of validity. The calculations of lowest-order Delbrück amplitudes based on the Born approximation [2] are found to be valid at lower energies ($\omega < 1.33$ MeV) for all scattering angles [1,3]. For higher photon energies ($\omega > 1.33$ MeV), Coulomb correction terms are to be added to the Born approximation amplitudes [4,5] but a general theoretical prediction of Coulomb correction terms is so far not available. Amongst the available analytical solutions of Delbrück scattering include the results for forward scattering [6] and the high-energy small-angle regime [7–10].

A very useful feature of Delbrück scattering is its scaling behavior at high energies which was first demonstrated theoretically by Cheng *et al*. [11]. They showed that as a consequence of finiteness of Delbrück amplitude in the limit $m \to 0$ where m is the electron mass, the asymptotical expression for Delbrück amplitude exhibits scaling behavior and takes the form $\omega^{-1} f(\theta)$, θ being the scattering angle and $f(\theta)$ an arbitrary function of θ. Though their derivation was restricted to the lowest order scattering process, they argued that the scaling behavior is obeyed by the Delbrück amplitudes even if the higher-order diagrams of multiphoton exchanges (Coulomb correction effect) are taken into account.

Cheng *et al*. [11], however, reported a strong conflict of the experimental data at 7.9 and 10.83 MeV with the scaling behavior. Later Rullhusen *et al*. [12] argued that the violation reported in Ref. [11] is mainly from the presence of interfering (background) nuclear scattering amplitudes. They showed that the predicted scaling pattern is observed to a certain level when only the lowest-order Delbrück amplitudes are considered for evaluating the elastic scattering cross sections. Subsequently from a detailed theoretical calculation they [13] pointed out that the fulfillment of the scaling conditions demands higher photon energies ($\omega > 30$ MeV or so).

The experimental support of the scaling behavior of Delbrück scattering, however, is still lacking. So far only experimental data at energies of few MeV have been considered for testing scaling studies [11,12] but the scaling is expected to show up clearly above 30 MeV or so and for large momentum transfer. Although Delbrück scattering was investigated experimentally over the last eight decades and both the real as well as the imaginary parts of Delbrück amplitudes were detected experimentally [14], there is a paucity of experimental data of elastic scattering at high energies with required kinematic conditions.

In the present work we will show that the available experimental data of Delbrück scattering between 140 MeV and 7.11 GeV energy range do not exhibit scaling behavior. The probable reasons for such nonobservation of the scaling feature will be explored. At relatively lower energies scaling is, however, noticed to a certain extent but in a slightly modified manner than what Cheng *et al*. [11] had originally prescribed.

The organization of the paper is as follows. In the next section the scaling hypothesis of Delbrück scattering amplitudes will be outlined briefly. In Sec. III the available experimental data will be analyzed to examine the scaling behavior of Delbrück scattering. We would discuss the findings and their probable explanations in Sec. IV and finally we will conclude in Sec. V.

*bhaktakunwar@yahoo.co.in
†aru_bhadra@yahoo.com
‡swapan_k_sengupta@yahoo.co.in

II. SCALING HYPOTHESIS OF DELBRÜCK AMPLITUDES

In the Delbrück scattering process an incident photon is assumed to get converted into a pair of electron and positron in the Coulomb field of the scattering nucleus, and then interact with the nucleus via virtual photons and again recombine to form the final photon having the same energy. The amplitude of the Delbrück scattering is usually constructed in the form of multiple integrals in the momentum space adopting the Feynman techniques. But the integrand is so complex that the integrals are insolvable in general. So often simple situations are considered so that the integrand becomes tractable but the solutions obtained thereby are of a limited range of validity.

Cheng *et al.* [11] advanced a simple but general argument to show a scaling behavior of Delbrück scattering amplitudes at high energies. In general the amplitude of an elastic scattering should be a function of ω, Δ, and μ_i, where $\Delta(\equiv 2\omega \mathrm{Sin}(\theta/2))$ denotes the momentum transfer and μ_i $(i = 1, 2, ..)$ are the masses of particles involved. Constructing dimensionless parameters out of ω, Δ, and μ_i and expressing the scattering amplitude in terms of those dimensionless variables one may write

$$\frac{A}{\omega^d} = f(\theta, \mu_i/\omega), \tag{1}$$

where d is the dimension of scattering amplitude which is equal to -1 for Delbrück scattering. As $\omega \to \infty$ with θ fixed, $\mu_i/\omega \to 0$ which can be treated equivalently by taking $\mu_i \to 0$. The authors then considered all possible Feynman diagrams at the lowest order for Delbrück scattering and with the help of the Coleman-Norton theorem [15] they showed that the lowest order Delbrück scattering has no divergence in the limit $\mu_i \to 0$ (for QED electron is the only massive particle, so in this case $\mu = m$). They further argued that Delbrück amplitudes will converge even if the higher-order diagrams of multiphoton exchanges (i.e., Coulomb correction effect) are taken into account. Accordingly the right-hand side of the above equation becomes a finite function as $\omega \to \infty$. Hence in the limit $\omega \to \infty$ Delbrück amplitude should scale as

$$A = \omega^{-1} f(\theta). \tag{2}$$

In terms of the differential cross section for Delbrück scattering ($\frac{d\sigma^D}{d\Omega}$), the scaling pattern may be expressed as

$$\omega^2 \frac{d\sigma^D}{d\Omega} \sim 4\pi |f(\theta)|^2. \tag{3}$$

Additionally the condition of large momentum transfer is usually imposed for validity of scaling [11]. In practical purpose the conditions for scaling may read as

$$\omega \gg m, \ \ \Delta \gg m. \tag{4}$$

Exploiting the finiteness of Delbrück amplitude in the limit $m \to 0$, Cheng *et al.* [11] applied a clever logic to deduce functional form of $f(\theta)$. The finiteness of Delbrück amplitude demands that the terms in the numerators of the integrand of Delbrück amplitude those diverge in the limit $m \to 0$ must cancel one another. This makes the integrand less complex and the authors obtained a very lengthy expression in the form of an integral for Delbrück amplitude [Eq. (2.60) along with Eqs. (2.61)–(2.68) of [11]). In principle, the scaling function $f(\theta)$ should follow from the Eq. (2.60) of [11] when one puts $m = 0$ and $\omega = 1$ but this requires numerical evaluation of Eq. (2.60) of [11]. In an earlier work Cheng *et al.* [10] derived simpler expressions for Delbrück amplitudes for $m \ll \Delta \ll \omega$ which can be written in terms of the cross section as

$$\frac{d\sigma^D}{d\Omega} = \frac{1}{4\pi^2}(Z\alpha)^4 r_o^2 \frac{m^2}{\omega^2 sin^4\theta/2}(|g(Z)|^2 + |h(Z)|^2), \tag{5}$$

where

$$g(Z) = \frac{1}{3}(Z\alpha)^2\left[1 - \frac{2\pi Z\alpha(1-2(Z\alpha)^2)}{\sinh(2\pi Z\alpha)}\right], \tag{6}$$

and

$$h(Z) = 1 - Z\alpha \mathrm{Im}\psi'(1 - iZ\alpha), \tag{7}$$

where $\psi \equiv \frac{dln\Gamma(x)}{dx}$ is the logarithmic derivative of the gamma function and $\psi' = d\psi/dx$. Equation (5) has the same form as Eq. (3) and it clearly suggests scaling behavior.

III. TESTING SCALING HYPOTHESIS IN EXPERIMENTAL DATA

For the present analysis we have considered two independent data sets in the energy range of 140–450 MeV and 960–7110 MeV.

During the early seventies (of the last century) the scientists from the University of Lund and the DESY jointly measured the differential cross section of Delbrück scattering in the GeV energy range both for small and large momentum transfer conditions using collimated bremsstrahlung beam from the DESY electron synchrotron [16]. In those measurements the scattering was in a very forward direction and the scattering angles were restricted to a milliradian level. On the other hand using backward Compton scattered laser photons of the VEPP-4M collider a group of researchers from Budker Institute of Nuclear Physics recently measured differential Delbrück scattering cross section on a bismuth germinate ($Bi_4Ge_3O_{12}$) target in the photon energy range 140–450 MeV and in the scattering angles range 2.6–16.6 mrad [17]. In this work the scaling feature was searched within the individual data sets but could not be tested between the data sets of the two experimental measurements as there is no common scattering angle in the two measurements. There exists a large-angle high-energy data measured by the MAMI A (Germany) group [18] but unfortunately the uncertainties involved in the data are too high to investigate the scaling feature from it.

At energies above a few tens of MeV, the elastic scattering amplitude is mostly dominated by Delbrück amplitudes, particularly at forward direction and therefore the experimental cross sections can be directly used in Eq. (3) unlike the lower energy cases where various contributing amplitudes have to be disentangled first from the experimental data for scaling studies.

The analysis of the experimental data of energies between 960 and 7110 MeV and of angles between 1.05 and 2.75 mrad [16] have been performed with reference to Eq. (3) and the results are shown in Figs. 1(a) and 1(b) for uranium ($Z =$ 92) and gold ($Z =$79) targets, respectively. It is found that the

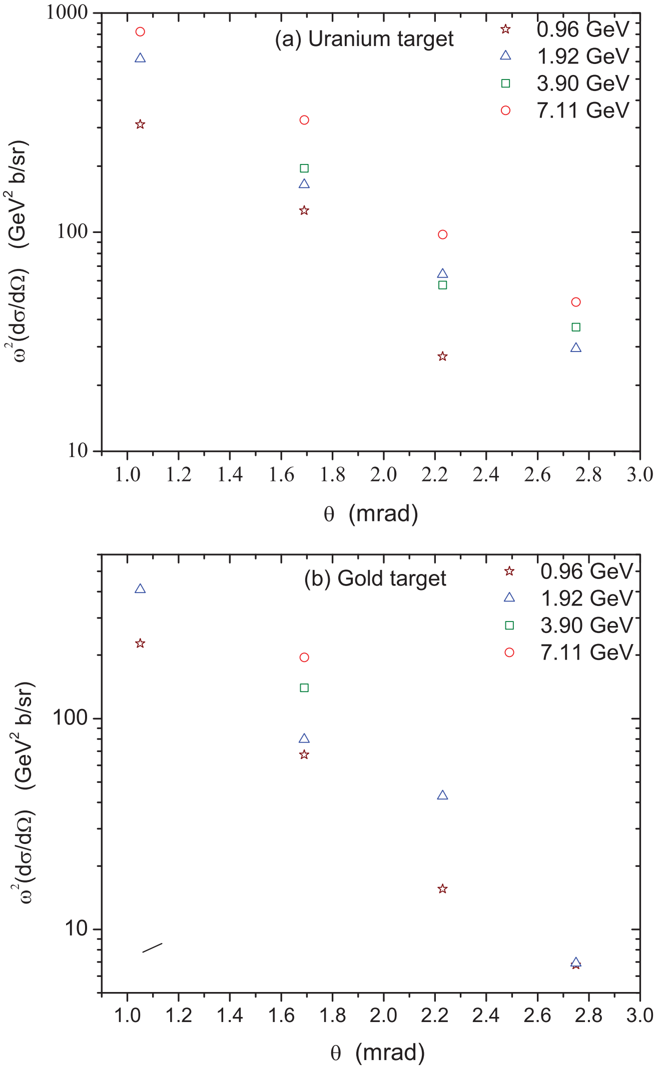

FIG. 1. (Color online) Scaling pattern in the experimental data of Delbrüück scattering between 1 and 7 GeV photons following Eq. (3).

products $\omega^2 \frac{d\sigma^D}{d\Omega}$ at a fixed scattering angle but for different energies do not coincide with each other; a deviation up to a factor of 4 is noticed between the data points of 0.96 and 7.11 GeV energies. Moreover, the deviations are not the same for different scattering angles and also there is no clear sign of a convergence of data points of different energies with increasing scattering angle. Note that although the scattering angles are very small, the conditions laid down in Eq. (4) for scaling are fulfilled by the scattering data to a fair degree except for the lowest energy as shown in Table I.

The Budker Institute group [17] had detected scattered photons within the scattering angles 2.6–16.6 mrad and the measured differential cross sections of Delbrück scattering in the stated range of angles were 5.9 and 1.2 mb at 140 and 450 MeV, respectively. Hence the magnitude of ratio of $\omega^2 d\sigma^D/d\Omega$ between 140 and 450 MeV is 0.48. Thus $\omega^2 d\sigma^D/d\Omega$ at the two energies differs by more than a factor of 2. So the scaling does not reveal in this data set also.

A. A modified form of scaling

It is noticed that the products $\omega^2 d\sigma^D/d\Omega$ systematically remain higher at higher energies for a fixed θ as revealed from Fig. 1 as well as from the Budker Institute group data. To explore whether the experimental measured differential cross section of Delbrück scattering scales differently with energy we consider a slightly modified form of scaling as given by

$$\omega^{2-\beta} \frac{d\sigma^D}{d\Omega} \sim 4\pi |f(\theta)|^2, \quad (8)$$

where β is a parameter, representing departure from the originally predicted scaling formula.

We first look for a value of β for which the deviations between the products $\omega^{2-\beta} \frac{d\sigma^D}{d\Omega}$ for different ω but at a fixed θ are minimum. For DESY data we find that overall $\beta \simeq 0.5$ gives minimum departure between the data points of different energies. The scaling behavior in the DESY data on the basis of the modified scaling formula [Eq. (8)] is plotted for uranium and gold targets in Figs. 2(a) and 2(b), respectively. Although the deviations are found to reduce considerably, the clear signature for scaling is still lacking.

It is interesting to note that a huge improvement toward scaling is also noticed in the measured cross-section data of the Budker Institute group for the same β. With $\beta \sim 0.5$, the ratio of $\omega^{2-\beta} d\sigma^D/d\Omega$ at 140 and 450 MeV becomes 0.85, an improvement by nearly a factor of 1.75.

For the study of scaling of Delbrück amplitude with elastic scattering data at MeV range, interfering contributions from other processes such as other nuclear scatterings need to be disentangled first. The illustration of the scaling property (to a certain extent) in the MeV range [12] was based on the theoretical lowest-order Delbrück amplitude [2] which is almost equivalent to the disentangled experimental data containing the Delbrück amplitude only. Following Papatzacos and Mork [2], the lowest-order Delbrück amplitudes are calculated between 7.9 and 15.1 MeV covering scattering angles from 10° to 150° and the scaling behavior is explored in the data which is depicted in Fig. 3(a). We found that on application of Eq. (8) with the same β (= 0.5), as obtained from the Desy measurement, the deviations appearing in

TABLE I. The momentum transfers at different photon energies and scattering angles.

ω (in GeV)	0.96			1.92				3.9			7.11			
θ (in mrad)	1.05	1.69	2.23	1.05	1.69	2.23	2.75	1.69	2.23	2.75	1.05	1.69	2.23	2.75
Δ (in MeV)	1.01	1.62	2.14	2.02	3.24	4.28	5.28	6.59	8.70	10.72	7.47	12.02	15.86	19.55

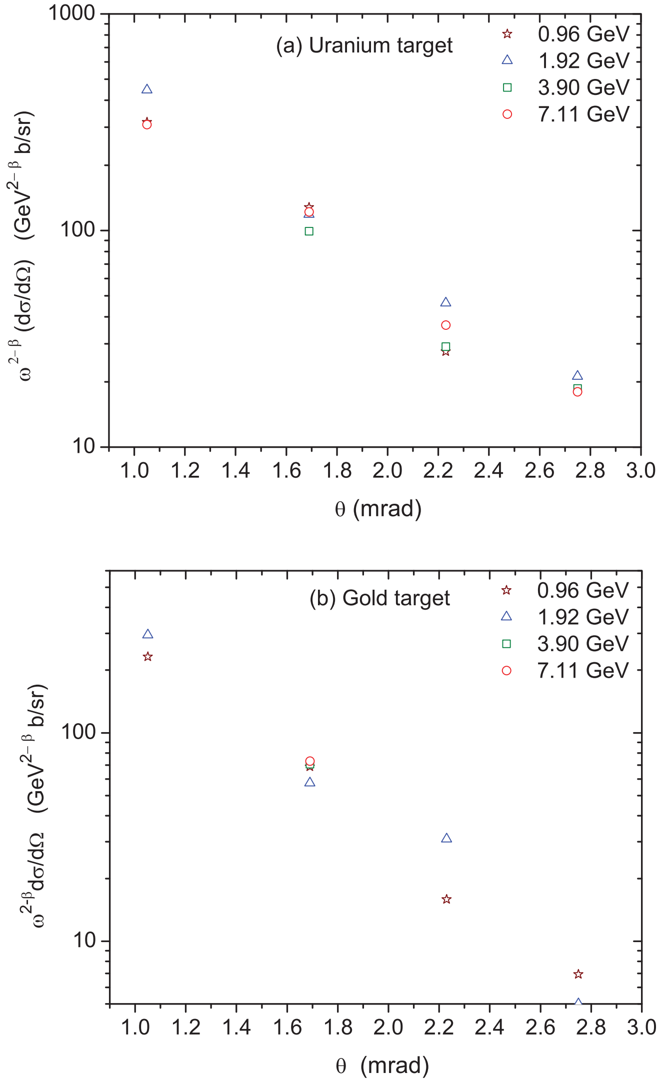

FIG. 2. (Color online) Scaling pattern in the experimental data of Delbrück scattering between 1 and 7 GeV photons according to Eq. (8) with $\beta = 0.5$.

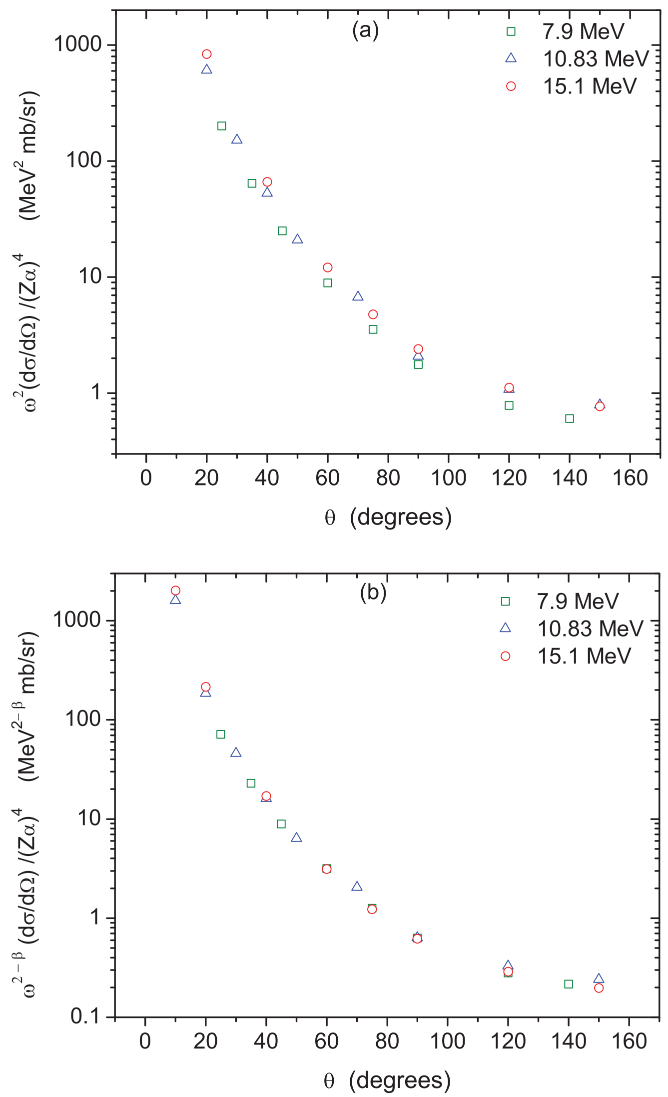

FIG. 3. (Color online) Scaling pattern in the Delbrück scattering data in the MeV range according to (a) Eq. (3) and (b) Eq. (8) with $\beta = 0.5$, respectively.

Fig. 3(a) are reduced drastically and a fair scaling feature emerges as shown in Fig. 3(b).

IV. DISCUSSION

It was believed that the condition $\omega \to \infty$ leads to scaling as the right-hand side of Eq. (1) becomes energy independent for such a condition. But though $\omega \to \infty$ is a necessary condition, it is not the sufficient condition for scaling to take place. This can be understood from the following example. The function $f(\theta, m/\omega)$ may contain a cross term such as $\theta^{-s}(m/\omega)^r = g$ (say), where s and r are positive numbers. Obviously, $g \neq 0$ in general in the limit $\theta \to 0$ even when $\omega \to \infty$. Then $f(\theta, m/\omega) \neq f(\theta)$ and hence scaling should not be expected at very small scattering angles even if the photon energy is very high.

While demonstrating scaling in Delbrück amplitude at high energies often the condition of large momentum transfer ($\Delta \gg m$) is additionally imposed. For small scattering angle this condition implies that $\theta \gg m/\omega$. Even when this condition is fulfilled, scaling behavior may not be essentially reached. For instance, in the example cited in the above paragraph, when $\theta \gg m/\omega$ the scaling of $f(\theta, m/\omega)$ will depend on the numerical values of s and r.

One may argue that because we have the definite expression of $f(\theta)$ for Delbrück amplitude as given in Eqs. (5)–(7) [or the one that will follow from Eq. (2.60) of [11]], we should consider those expressions rather than the *ad hoc* choice of a term $\theta^{-s}(m/\omega)^r$ in $f(\theta)$ as made in the example cited above. Here it is important to point out that the expression (5) was obtained precisely by taking the limit $\Delta \to \infty$ [10]. For very small angles $\Delta \sim \theta\omega$, so in such a situation ω needs to be extremely high to fulfill the limit $\Delta \to \infty$. Hence Eq. (5) does not hold for very small angles. On the other hand, as remarked already, in principle $f(\theta)$ can be extracted from Eq. (2.60) of [11] but in practice it is quite a difficult task and the functional form of $f(\theta)$ has not been obtained so far. So the fact is that we don't have an analytical expression of $f(\theta)$ yet for very small angles at the MeV to GeV range.

Figures 2 (of [16]) and 8 (of [17]), where the numerically obtained differential cross section of Delbrück scattering is plotted against θ for different energies, provide support for our point. The stated figures show that the angular (θ) dependence of the differential cross section of Delbrück scattering at small angles differs significantly with photon energy.

V. CONCLUSION

The scaling behavior of Delbrück scattering as predicted nearly 30 years back was examined in experimental data at very high energies. The available experimental data of the differential cross section of Delbrück scattering in the MeV to GeV energy range, however, do not show a reasonable degree of scaling behavior against the standard expectations. The angular dependence of the differential cross section of Delbrück scattering was found to depend strongly on energy for very small scattering angles.

Usually the limit $\omega \to \infty$ is considered as the condition for attaining the scaling feature. Sometimes an additional condition of $\Delta \gg m$ is imposed. However, we pointed out that these are not sufficient conditions for holding scaling. Mathematically it is not possible to conclude about the scaling pattern when $\theta \to 0$ even if $\omega \to \infty$. At very high energies the scattering is in a very forward direction; the typical scattering angle is of the order of milliradian for photon energy of 1 GeV. Hence the violation of scaling as revealed from the experimental data is not very unexpected. It follows from the present work that the ideal region within which the scaling is expected to show up clearly is the high-energy and large-angle regime. Unfortunately, still there are no sufficient good quality experimental data in this region which fulfilled the stated conditions.

A reasonable scaling behavior was noticed with the introduction of a slightly modified scaling formula as proposed in Eq. (8). But the scaling in the strict sense cannot be attained at small angles with such a modification as the deviations are not the same for all θ. It appears that the data may be described better by a scale-breaking formula but because of scarcity of data we have not adventured on that direction at this moment.

ACKNOWLEDGMENTS

The authors are thankful to an anonymous referee for useful suggestions. B.K. thanks University Grants Commission (India) for providing financial support under the FIP scheme. A.B. thanks the Department of Science and Technology (Government of India) for support under Grant No. SR/S2/HEP-14/2007.

[1] R. H. Pratt, Radiat. Phys. Chem. **74**, 411 (2005).
[2] P. Papatzacos and K. Mork, Phys. Rev. D **12**, 206 (1975).
[3] B. Kunwar, A. Bhadra, S. K. Sen Gupta, J. P. J. Carney, and R. H. Pratt, Phys. Rev. A **71**, 032724 (2005).
[4] B. Kasten, D. Schaupp, P. Rullhusen, F. Smend, M. Schumacher, and L. Kissel, Phys. Rev. C **33**, 1606 (1986).
[5] P. Rullhusen, U. Zurmuhl, F. Smend, M. Schumacher, and H. G. Borner, and S. A. Kerr, Phys. Rev. C **27**, 559 (1983).
[6] F. Rohrlich and R. L. Gluckstern, Phys. Rev. **86**, 1 (1952).
[7] A. Milstein and V. M. Strakhovenko, Phys. Lett. A **95**, 135 (1983).
[8] A. I. Milstein and V. M. Strakhovenko, Sov. Phys. JETP **58**, 8 (1983) [Zh. Eksp. Teor. Fiz. **85**, 14 (1983)].
[9] H. Cheng and T. T. Wu, Phys. Rev. D **2**, 2444 (1970).
[10] H. Cheng and T. T. Wu, Phys. Rev. D **5**, 3077 (1972).
[11] H. Cheng, Er-Cheng Tsai, and X. Zhu, Phys. Rev. D **26**, 908 (1982).
[12] P. Rullhusen, U. Zurmuhl, F. Smend, M. Schumacher, H. G. Borner, and S. A. Kerr, Phys. Rev. D **27**, 1962 (1983).
[13] P. Rullhusen, J. Trube, A. Baumann, K. W. Rose, F. Smend, and M. Schumacher, Phys. Rev. D **36**, 733 (1987).
[14] M. Schumacher, Radiat. Phys. Chem. **56**, 101 (1999).
[15] S. Colemon and R. E. Norton, Nuovo Cimento **38**, 438 (1965).
[16] G. Jarlskog *et al.*, Phys. Rev. D **8**, 3813 (1973).
[17] Sh. Zh. Akhmadaliev *et al.*, Phys. Rev. C **58**, 2844 (1998).
[18] A. Baumann *et al.*, Nucl. Phys. A **536**, 87 (1982).