Gyromagnetic factors in $^{144-150}$Nd

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The U(5) to SU(3) evolution of the nuclear structure in the even $^{144-156}$Nd isotopes has been investigated in the framework of the interacting boson approximation (IBA-2) model, taking into account the effect of the partial $Z = 64$ subshell closure on the structure of the states of a collective nature. The analysis, which led to a satisfactory description of excitation energy patterns, quadrupole moments, and decay properties of the states (even when important $M1$ components were present in the transitions), is extended to the available data on g factors, in $^{144-150}$Nd. Their values are reasonably reproduced by the calculations.

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I. INTRODUCTION

Magnetic moments are almost insensitive to the nuclear deformation, so that g factors are expected to be constant along an isotopic chain. Deviations from such a behavior can provide clues on particular features related to the microscopic structure of the wave functions of the relevant states.

The measurements of g factors of the 2_1^+ state [$g(2_1^+)$] in nuclei belonging to the $N \approx 90$ region have provided one of the most striking examples of such a possibility. Indeed, their values increase as N increases from 84 to 90 [1], displaying an intriguing trend for the standard nuclear models.

The correlation between the growth of $g(2_1^+)$ in Nd, Sm, and Gd isotopes, as neutron pairs are added outside the $N = 82$ closed shell, and the transition from near-spherical vibrators to well-deformed rotors (with an abrupt increase in collectivity at $N \approx 90$) was first pointed out by Casten *et al.* [2]. They interpreted such a behavior as due to the isospin $T = 0$ component of the p-n interaction involving chiefly the spin-orbit partners $\pi h_{11/2}$ and $\nu h_{9/2}$. As the neutrons begin to fill the $\nu h_{9/2}$ orbit, the occupancy of the $\pi h_{11/2}$ orbit becomes more favored. This gives rise to an increase of deformation and to a reduction of the effective single-particle energy of the $h_{11/2}$ proton orbit, which leads to the obliteration of the proton gap at $Z = 64$ for $N \geqslant 90$.

The interacting boson approximation (IBA) model [3], in the version that distinguishes between proton and neutron bosons (IBA-2) [4–6], establishes a relation between the collective motion and the underlying shell model structure. Therefore, it is particularly well suited to investigate possible effects of subshell closure near $Z = 64$.

In the early 1980s, Wolf *et al.* [7,8] studied, in the framework of the IBA-2 model, the Ba-Dy region. The anomalous values of $g(2_1^+)$ in the $Z = 64$ subshell region were accounted for by changes in the effective number of proton bosons N_π^{eff} taking part in the collective motion. Their values, for $N \leqslant 88$, were deduced by keeping the effective proton and neutron gyromagnetic ratios, g_π and g_ν, fixed to the values obtained from the analysis of the nuclei with $N \geqslant 90$ and by comparing experimental and calculated values of $g(2_1^+)$.

In the same years, a phenomenological analysis of g factors of the 2_1^+ states in even-even rare-earth nuclei was carried out by Sambataro *et al.* [9] to investigate whether the available experimental information could be described by the IBA-2 model. To allow for possible major shell effects, g_π and g_ν were assumed to vary smoothly as a function of N_π and N_ν in the $Z = 50$–82 and $N = 50$–82 or 82–126 shells, respectively.

Very recently, the evolution from a spherical [U(5)] to a symmetrically deformed [SU(3)] structure in $^{144-156}$Nd isotopes [$Z = 60$] has been studied [10] in the framework of the IBA-2 model. The proton number N_π was counted in the usual way, but to take into account the partial subshell closure at $Z = 64$, the proton effective charge e_π (kept fixed for $N \geqslant 90$) was allowed to vary for $N < 90$. The value of g_π was estimated by assuming its proportionality to e_π. In the analysis, excitation energy patterns, quadrupole moments, and decay properties of collective states have been taken into account. States of possible mixed-symmetry character (i.e., nonsymmetric in the proton and neutron degrees of freedom) have also been considered. A peculiar property of these states is their possible decay to fully symmetric states through $M1$ transitions. A test of the calculated $M1$ transition strengths, which are related to the values of g_π and g_ν, is provided by the experimental $B(M1)$ values, $E2/M1$ mixing ratios, and by branching ratios in case that one of the de-exciting transitions has a large $M1$ component. Since the predictions compare rather well with the experimental data (including $M1$ transitions), the analysis has been extended to the g factors in $^{144-150}$Nd isotopes, where precise measurements, not limited to the 2_1^+ state, are now available for a comparison.

II. DATA ANALYSIS

The measurements of the magnetic moments of the 2_1^+ state in $^{144-150}$Nd [11–14] and 4_1^+ state in ^{150}Nd [14] were performed in the two decades from the 1970s to the 1990s by using the transient field technique in reactions where Nd targets were Coulomb excited by lighter beams.

In a new kind of experiment, where the transient field technique was applied in an inverse kinematic reaction, with Nd projectiles Coulomb excited by a light target, Holden *et al.* [15,16] remeasured the $g(2_1^+)$ in $^{144-150}$Nd and $g(4_1^+)$ in ^{150}Nd and extended the measurements to the 4_1^+ state in

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$^{144-148}$Nd, 6_1^+ states in ^{144,148,150}Nd, and $8_1^+$and 10_1^+ states in ^{150}Nd.

All these data allow us to extend the comparison between the predictions of the IBA-2 model, performed with the parameters adopted in Ref. [10], and the experimental values of the g factors in $^{144-150}$Nd as a function of A as well as of J. The procedure followed in Ref. [10] is summarized hereafter. The $^{144-150}$Nd isotopes have $N_\pi = 5$ and N_ν ranging from 1 to 4 with respect to the $Z = 50$ and $N = 82$ closed shells, respectively. In the adopted Hamiltonian,

$$H = \varepsilon(\hat{n}_{d_\pi} + \hat{n}_{d_\nu}) + \kappa\, \hat{Q}_\pi^{(\chi_\pi)} \cdot \hat{Q}_\nu^{(\chi_\nu)} + \hat{M}_{\pi\nu}(\xi_1, \xi_2, \xi_3), \quad (1)$$

the parameters χ_π and χ_ν were kept fixed to the value $-\sqrt{7}/2$, which characterizes the SU(3)$_{\pi\nu}$ symmetry. It was also checked that a different choice of χ_ν does not lead to any improvement with respect to the results obtained using the adopted value.

In the Majorana operator $\hat{M}_{\pi\nu}$, which properly accounts for states not fully symmetric in the proton and neutron degrees of freedom, the parameters ξ_1, ξ_2, and ξ_3 can take independent values [17]. The Hamiltonian was diagonalized by using the NPBOS code [18].

The values of χ_ρ ($\rho = \pi, \nu$) in the $E2$ operator, $\hat{T}(E2) = e_\nu \hat{Q}_\nu^{(\chi_\nu)} + e_\pi \hat{Q}_\pi^{(\chi_\pi)}$, are the same as in the Hamiltonian (consistent-Q formalism [19,20]).

The parameters g_ρ, which appear in the expression of the $M1$ operator,

$$\hat{T}(M1) = \sqrt{\frac{3}{4\pi}}(g_\nu \hat{L}_\nu + g_\pi \hat{L}_\pi), \quad \hat{L}_\rho = \sqrt{10}(d_\rho^\dagger \times \tilde{d}_\rho)^{(1)}, \quad (2)$$

are related to the g factors via the expression [3]

$$g(J) = \frac{\langle J \parallel g_\nu \hat{L}_\nu + g_\pi \hat{L}_\pi \parallel J\rangle}{\sqrt{J(J+1)(2J+1)}}. \quad (3)$$

Since the angular momentum operator $\hat{L}$ ($\hat{L} = \hat{L}_\pi + \hat{L}_\nu$) is diagonal in every basis, the values of the reduced transition matrix elements, $\langle J_f \parallel \hat{L}_\nu \parallel J_i\rangle$ and $\langle J_f \parallel \hat{L}_\pi \parallel J_i\rangle$, are opposite. As a consequence, the $B(M1; J_i \to J_f)$ value is proportional to $(g_\nu - g_\pi)^2$, and its measurement provides information on the difference of g_ν and g_π. The proton and neutron gyromagnetic factors contribute instead separately to determine the value of the diagonal reduced matrix element, so that the knowledge of $g(J)$ factors allows us to perform a more significant test of the values adopted for g_ν and g_π

The standard outputs of the NPBOS code [18] for the calculation of $g(J)$, reported in columns (7) and (8), represent the quantities $\langle J \parallel \hat{L}_\nu \parallel J\rangle$ and $\langle J \parallel \hat{L}_\pi \parallel J\rangle$, respectively, divided by $\sqrt{2 \times 3 \times 5}$. To obtain $g(J)$ for $J \neq 2$ one has to multiply these values by $\sqrt{2 \times 3 \times 5}/\sqrt{J(J+1)(2J+1)}$.

The procedure followed to select the states to be included in the analysis and to deduce the Hamiltonian parameters is described in detail in Ref. [10]. It is just worth mentioning that, for the parameters in the $M1$ operator, reference was made to the IBA-2 analysis of $g(2_1^+)$ factors in $N \geqslant 90$ nuclei, performed by Wolf *et al.* [8]. In their work N_π was counted from $Z = 50$, and the $g(2_1^+)$ factors were calculated via [21]

$$g(2_1^+) = g_\pi N_\pi/(N_\pi + N_\nu) + g_\nu N_\pi/(N_\pi + N_\nu), \quad (4)$$

which is exact in the limit of the IBA symmetries [22], assuming that the 2_1^+ state is fully symmetric. Through a fit to the experimental data the values $g_\pi = 0.63$ (4) and $g_\nu = 0.05$ (5) ($\chi^2 = 0.8$) were deduced [8]. In Ref. [10] the value $g_\nu = 0.05$ was adopted for all the isotopes, while $g_\pi = 0.63$ was used in the analysis of ^{150}Nd. In ^{144,146,148}Nd g_π was deduced by assuming a proportionality relation between g_π and e_π, based on the fact that, in the simplest scenario, the g factors of collective states depend on the number of protons taking part in the nuclear collective motion [23]. As mentioned before, in Ref. [10] e_π was allowed to vary as a function of A to take into account possible effects induced by the subshell closure at $Z = 64$.

TABLE I. Parameters appearing in the Hamiltonian and $E2$ and $M1$ transition operators, varied in the analysis performed in Ref. [10]. The parameters ε, κ, ξ_2, and ξ_3 are given in MeV and e_π is in eb. The parameters e_ν and g_ν were kept fixed at the values 0.097 eb and 0.05, respectively.

A	ε	κ	ξ_2	ξ_3	e_π	g_π
144	0.900	−0.100	0.350	−0.350	0.105	0.367
146	0.850	−0.108	0.150	−0.350	0.110	0.385
148	0.750	−0.092	0.080	−0.300	0.140	0.490
150	0.500	−0.065	0.060	0.300	0.180	0.630

The values of the parameters used in the analysis are shown in Table I. The Majorana parameters are reported for completeness; however, their values essentially do not affect the properties of the states of the ground-state band (the only ones relevant to this work), which turn out to have a quite pure fully symmetric character.

III. RESULTS AND DISCUSSION

The difficulties encountered by nuclear models to reproduce the trend of g factors as a function of A and J in the $N \leqslant 90$ region were clearly pointed out by Holden *et al.* [16], whose results allow very stringent tests.

As shown in Fig. 1 [line (a)], the problems are evident already when comparing the experimental $g(2_1^+)$ factors in $^{144-150}$Nd to the predictions of the standard collective model [23], where protons and neutrons are assumed to take part equally in the motion, so that a monotonic, slow decrease ($g = Z/A$) is predicted along an isotopic chain.

The $g(2_1^+)$ factors were evaluated in the even $Z = 56$–78 isotopic chains by Sambataro *et al.* [9], in the framework of the IBA-2 model, via Eq. (4), allowing $g_\pi(N_\pi)$ and $g_\nu(N_\nu)$ to vary smoothly as a function of A inside the relevant major shells. They found that in the lower half of the $Z = 50$–82 shell g_π is constant and close to unity, whereas in the lower half of the $N = 82$–126 shell g_ν increases, as a function of N, from ≈−0.4 to 0. An overall good agreement was obtained with the experimental data available at that time, apart from the notable deviations observed for the $N = 86$ and $N = 88$ isotopes of Nd and Sm. The comparison concerning neodymium isotopes is shown in Fig. 1 [line (b)], where the

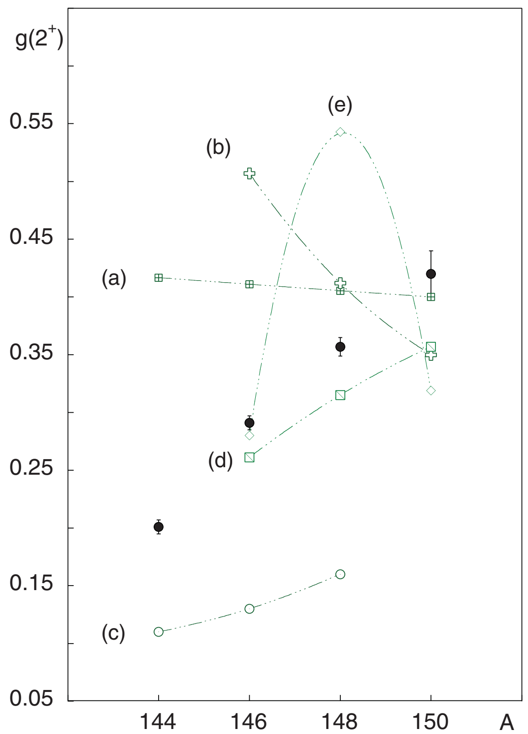

FIG. 1. (Color online) Experimental (solid circles) $g(2_1^+)$ factors are compared to the values calculated (open symbols, connected by a dashed line) in (a) the standard collective model [23], (b) the IBA-2 model [9], (c) the framework of BCS theory [24], (d) the Migdal approximation [25], and (e) the cranked HFB approach [26]. The experimental values are from Ref. [16] and are the averages of the values obtained in Refs. [12–14,16].

calculated values have been obtained by using in Eq. (4) the values of g_π and g_ν deduced from Fig. 2 of Ref. [9]. It is seen that, even though protons and neutrons are treated separately, the predictions are incompatible with the experimental data. Indeed, the calculated trend is opposite to the experimental one. A similar trend is obtained by using in Eq. (4) the values $g_\pi = 0.62$ and $g_\nu = 0.07$ deduced in Ref. [14] from a fit to the experimental $g(2_1^+)$ in $N \geqslant 90$ isotopes of Nd and Sm isotopes. In both IBA-2 analyses [9,14] it was remarked that the disagreement for $N \leqslant 88$ could be related to the presence of a $Z = 64$ subshell.

The g factors of the 2_1^+ state in $^{144-148}$Nd have been calculated [24] in the framework of the Bardeen-Cooper-Schrieffer (BCS) model, with a pairing plus quadrupole force residual interaction between nucleons. The increasing trend of the g factors is correctly reproduced, whereas the values are underestimated [Fig. 1, line (c)].

A better agreement [Fig. 1, line(d)] is obtained [25] for the $g(2_1^+)$ factors in $^{146-150}$Nd using the Migdal approximation for the moment of inertia and taking into account the single-particle motion through microscopic calculations of deformations and pair gaps. However, the calculations are limited to the data reported in Fig. 1, so the comparison cannot be extended to the g factors of the states with $J > 2$.

The values calculated [26] for $^{146-150}$Nd in the framework of the cranked Hartree-Fock-Bogoliubov (HFB) formalism with the inclusion of hexadecapole deformation in the cranking Hamiltonian are not able to reproduce the trend of $g(2_1^+)$ as a function of A [Fig. 1, line (e)] or that of the g factors as a function of J for a given A.

In the paper by Holden *et al.* [16] two possible reasons are considered for the discrepancies between the predictions of the different models and the experimental values of the g factors. The first one would be the interplay between single-particle configurations and collective excitations, which substantially alters the structure of the low-lying states in the lighter isotopes, causing the drop in g factors as the closed shell is approached. This explanation is preferred to the second one, which would ascribe the discrepancies to the $Z = 64$ subshell closure.

In the IBA-2 study of $^{144-156}$Nd isotopes of Ref. [10], states clearly displaying a noncollective structure were excluded from the analysis, and at the same time, effects due to a possible $Z = 64$ subshell closure were taken into account. This led to a satisfactory agreement between experimental and predicted quantities (including $M1$ transitions), which implies

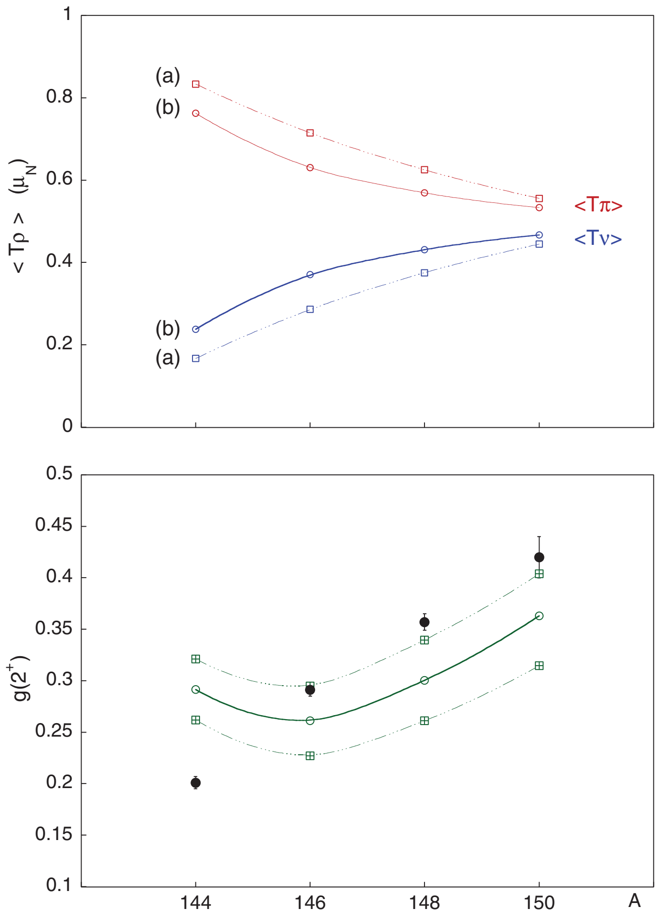

FIG. 2. (Color online) (top) Reduced matrix elements $\langle J \parallel T(M1)_{\pi,\nu} \parallel J\rangle$ of the 2_1^+ state in $^{144-150}$Nd isotopes evaluated via Eq. (4) [lines labeled (a)] and using the parameters reported in Table I [lines labeled (b)]. (bottom) The experimental (solid circles) values of $g(2_1^+)$ factors are compared to the values calculated (open symbols) referring to the values of g_π and g_ν reported in Ref. [8] (solid line) and to the values obtained when both parameters given in Ref. [8] are increased or reduced by one standard deviation (dashed lines) (see text).

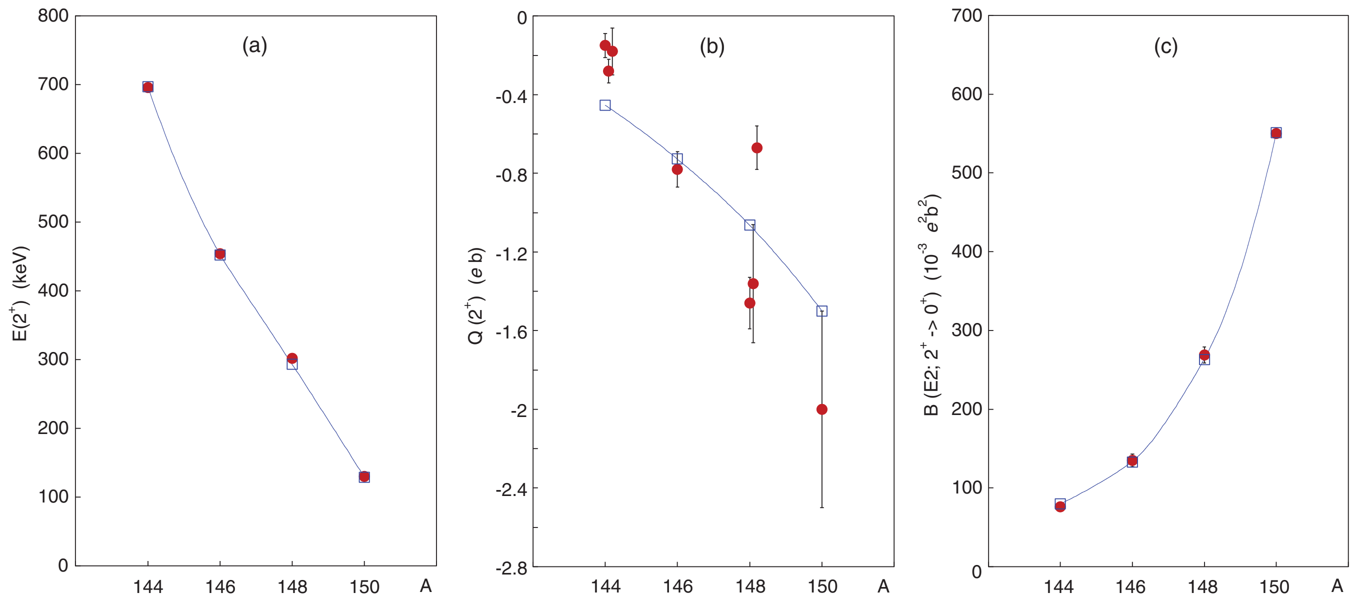

FIG. 3. (Color online) The experimental (solid circles) values of (a) excitation energies and (b) quadrupole moment of the 2_1^+ state and (c) of $B(E2;2_1^+ \rightarrow 0_1^+)$ reduced transition strengths, reported as a function of A, are compared to the calculated (open squares) ones. In (c) the error bars, when not visible, are smaller than the dimensions of the symbols. The experimental values are from Ref. [27].

a reasonable description of the states and a proper choice of the effective gyromagnetic ratios. The results obtained in the present work, where the analysis (performed with the parameters given in Table I) is extended to the $g(J)$ factors available in the heavy neodymium isotopes ($^{144-150}$Nd), are shown in Fig. 2 for the 2_1^+ state (the overall comparison is reported in Fig. 4).

In the top panel of Fig. 2 the values of the $M1$ reduced matrix elements $\langle \| T(M1)_\rho \| \rangle$ calculated via Eq. (4) [lines labeled (a)] and in the present work [lines labeled (b)] are shown. It is seen that the two calculations give rather close values for both $\langle \| T(M1)_\pi \| \rangle$ and $\langle \| T(M1)_\nu \| \rangle$ terms. For small A, $\langle \| T(M1)_\pi \| \rangle$ is much larger than $\langle \| T(M1)_\nu \| \rangle$, so that, as long as $g_\pi \gg g_\nu$, the value of $g(2_1^+)$ is basically determined by that of g_π.

The values of $g(2_1^+)$ calculated in the present work are shown in the bottom panel of Fig. 2 (solid line) together with the predictions obtained when the parameters g_π and g_ν, given in Ref. [8], are both increased or reduced by one standard deviation. It is seen that, with the increased values, the agreement for $^{146-150}$Nd is notably improved.

By comparing the present results with those of Fig. 1 [line (b)], concerning the IBA-2 analysis performed by Sambataro *et al.* [9], the importance of taking into account, for $N \leqslant 88$, the effects of the subshell closure at $Z = 64$ is evident. This leads to a noticeable reduction of the g_π values (see Table I) with respect to the value (close to 1) obtained in Ref. [9] from the analysis of a large region of nuclei where only effects related to the major shells were considered.

It is, however, apparent from Fig. 2 that the value of $g(2_1^+)$ in ^{144}Nd is not reproduced, even though the g_π value is still decreasing from ^{146}Nd to ^{144}Nd. A more general picture of the structure of the 2_1^+ state in $^{144-150}$Nd can be obtained considering also the comparison on excitation energies, quadrupole moments, and $B(E2;2_1^+ \rightarrow 0_1^+)$ reduced transition strengths, reported in Fig. 3. It is seen that in going from ^{144}Nd to ^{150}Nd the evolution from a near-spherical to a symmetrically deformed nuclear structure gives rise to a strong decrease of the excitation energy and to a large increase of the absolute value of the quadrupole moment of the 2_1^+ state and of the $B(E2;2_1^+ \rightarrow 0_1^+)$ values. The predicted values match well the energies and $B(E2)$ transition strengths and reproduce satisfactorily the trend of the quadrupole moment, even though, in this case, the large errors prevent a stringent comparison. On this basis, one could conclude that a description of the 2_1^+ state in these isotopes as a state of a collective nature is quite correct. However, the value predicted for the $g(2_1^+)$ factor, which is close to the experimental ones in $^{146-150}$Nd, is largely overestimated in ^{144}Nd. This suggests the presence of non-negligible single-particle components in the wave function of the 2_1^+ level in ^{144}Nd.

The importance of g factor measurements to determine the structure of a state is even more apparent when extending the comparison to the g factors of the 4_1^+ and 6_1^+ states in ^{144}Nd. The predicted values of $g(4_1^+)$ (0.309) is much larger than the experimental one [0.131(36)], and that of $g(6_1^+)$ (0.318) is incompatible with the experimental one [−0.56(22)]. This supports the conclusions drawn in Ref. [10] that (i) the excitation energy of the 4_1^+ level and the $B(E2;4_1^+ \rightarrow 2_1^+)$ strength reveal the presence of important single-particle components in the wave function of this state and (ii) the 6_1^+ level is outside the framework of the IBA-2 model (as such it was not included in the analysis of Ref. [10]).

In Fig. 4 the comparison between calculated and experimental g factors includes all the available data in $^{144-150}$Nd isotopes, with the exception of the $g(4_1^+)$ and $g(6_1^+)$ factors in ^{144}Nd, mentioned above. The values of the g factors are shown as a function of A and, for each A, as a function of J. For a

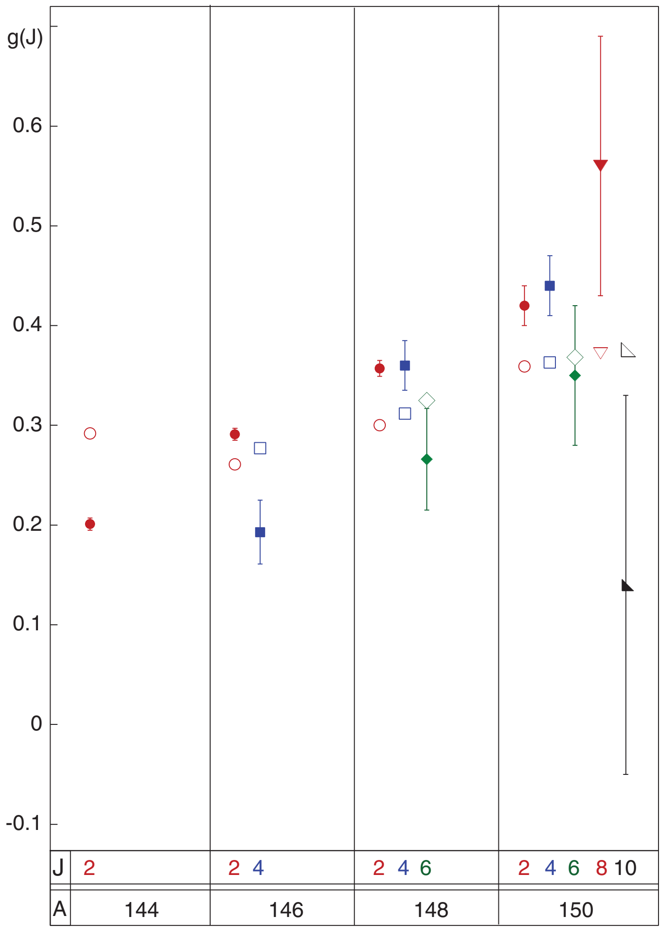

FIG. 4. (Color online) Comparison of experimental (solid symbols) and calculated (open symbols) g factors as a function of A and J. A different symbol was used for every J. The experimental values for $J = 2$ are the average of those obtained in Refs. [12–14,16], those for $J \geqslant 4$ are from Ref. [16], except for the $g(4_1^+)$ in ^{150}Nd, which is the average of the values obtained in Refs. [14,16]. The error reported for $g(4_1^+)$ in ^{146}Nd includes the statistical one (0.26) and that (0.06) due to problems in the analysis of the spectra [16].

given A, the calculations predict a small monotonic increase, as a function of J, for the g factors of the states shown in Fig. 4, with a slope that diminishes from ^{146}Nd to ^{150}Nd. In the latter isotope the variation is predicted to be ≈5% from $J = 2$ to $J = 10$, and the average $g(J)$ value is close to that (0.4) of the standard collective model [23].

It is seen that, except for the $g(2_1^+)$ in ^{144}Nd, the general trend, as a function of A, is correctly reproduced. As to the behavior of the g factors as a function of J, the uncertainties on the experimental data do not allow us to perform a stringent test of the predictions; however, the available data do not exclude the correctness of the trend predicted. In particular, the experimental data in ^{150}Nd are compatible with the predictions of very close values for $g(J)$, $J = 2, 4, 6, 8, 10$.

The difference between experimental and calculated $g(J)$ for the states reported in Fig. 4 is, on average, 16%. This level of agreement supports the correctness of the assumptions underlying the calculations, i.e., that the trend of the g factors as a function of A can be related to the $Z = 64$ subshell closure and that g_π can be assumed to be proportional to e_π.

The conclusions one can draw from the present analysis are in agreement with those of Ref. [16] about the relevant levels of ^{148,150}Nd, i.e., that they are well described by collective excitations. As to the interpretation given in Ref. [16] of a low-excitation structure of ^{144,146}Nd, dominated by the $2f_{7/2}$ neutron configuration, the present analysis, joined to that performed in Ref. [10], leads to the conclusion that, in ^{144}Nd, single-particle components contribute significantly to determine the structure of the 2_1^+ state. Their importance increases in the 4_1^+ state and becomes predominant in the 6_1^+ state. The comparison of the g factors of the 2_1^+ and 4_1^+ states in ^{146}Nd does not provide strong evidence against their interpretation as collective states.

IV. CONCLUSIONS

Very recently, the spectroscopic properties (not including g factors) of even $^{144-156}$Nd isotopes have been studied [10] in the framework of the IBA-2 model, considering the effect of a partial subshell closure at $Z = 64$ on the structure of the collective states and using effective gyromagnetic factors deduced with no fit to the experimental data. The calculations reproduce satisfactorily the experimental data even when $M1$ transitions are considered.

In the present study the analysis has been extended to the available data on $g(J)$ factors in even heavy neodymium isotopes ($^{144-150}$Nd). The contribution that g factors can provide to test model predictions and to investigate the structure of the states is pointed out.

It has been found that the experimental $g(J)$ values are reasonably matched by the predictions. One can therefore conclude that, on the whole, the IBA-2 calculations are able to provide a rather good description of all the spectroscopic properties of the collective states of the heavy neodymium isotopes.

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