

Counting the number of correlated pairs in a nucleus

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We suggest that the number of correlated nucleon pairs in an arbitrary nucleus can be estimated by counting the number of proton-neutron, proton-proton, and neutron-neutron pairs residing in a relative S state. We present numerical calculations of those amounts for the nuclei ^4He , ^9Be , ^{12}C , ^{27}Al , ^{40}Ca , ^{48}Ca , ^{56}Fe , ^{63}Cu , ^{108}Ag , and ^{197}Au . The results are used to predict the values of the ratios of the per-nucleon electron-nucleus inelastic scattering cross section to the deuteron in the kinematic regime where correlations dominate.

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The nucleus is a prototype of a dense quantum liquid with a high packing fraction [1]. Naively one could expect some severe medium effects for the nucleons. Several experimental investigations confirmed the robustness of the nucleons. This is for example reflected in the successful use of the impulse approximation (IA) in nuclear reaction theory. In the IA the bound and free nucleon properties (charges, magnetic moments, form factors) are considered identical. A few experiments, however, found indications for medium-modified nuclear properties. Recent $^4\text{He}(\vec{e}, e' \vec{p})$ measurements [2], for example, could be described after implementing medium-modified proton form factors. Also in comparing deep inelastic scattering cross sections with those on the deuteron, one finds that under some kinematics conditions the naive scaling ratios do not hold. This observation is known as the EMC (European Muon Collaboration) effect [3] and indicates that under selected kinematics the whole of the nucleus appears to be more than the sum of its constituents.

Recently, it was suggested [4] that the magnitude of the EMC effect can be predicted from the knowledge of the measured $a_2(A/D)$ coefficients. The $a_2(A/D)$ coefficients are defined as

$$a_2(A/D)(x_B, Q^2) = \frac{2}{A} \frac{\sigma^A(x_B, Q^2)}{\sigma^D(x_B, Q^2)}, \quad (1)$$

where $\sigma^A(x_B, Q^2)$ is the inclusive (e, e') cross section for the target nucleus A at a particular four-momentum transfer Q^2 and Bjorken $1.4 \leq x_B = \frac{Q^2}{2M\omega} \leq 2$ (M is the nucleon mass, and ω the energy transfer). The observed plateau in the measured x_B dependence of a_2 for $1.4 \leq x_B \leq 2$ is a strong indication for scattering from a correlated nucleon pair [5,6]. As a matter of fact, the a_2 coefficients can be interpreted as a measure for the effect of short-range correlations (SRCs) in the target nucleus A relative to deuteron D . In this Rapid Communication, we suggest a technique that allows one to estimate the number of nucleon pairs prone to SRC in an arbitrary nucleus $A(N, Z)$. We use these estimates to predict the values of the coefficients $a_2(A/D)$.

A time-honored method to quantify the effect of correlations in classical and quantum systems is the use of correlation functions. The latter encode those portions of the system that depart from mean-field behavior. The realistic (correlated) wave functions $|\bar{\Psi}\rangle$ are constructed by applying a many-body

correlation operator to the mean-field Slater determinant $|\Psi\rangle$ [7,8]

$$|\bar{\Psi}\rangle = \frac{1}{\sqrt{\langle \Psi | \hat{\mathcal{G}}^\dagger \hat{\mathcal{G}} | \Psi \rangle}} \hat{\mathcal{G}} |\Psi\rangle. \quad (2)$$

The $\hat{\mathcal{G}}$ reflects the full central, spin, and isospin dependence of the nucleon-nucleon force but is dominated by the central and tensor correlations

$$\begin{aligned} \hat{\mathcal{G}} &\approx \hat{\mathcal{S}} \left\{ \prod_{i < j=1}^A [1 - g_c(r_{ij}) + f_{t\tau}(r_{ij}) \widehat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j] \right\}, \\ &= \hat{\mathcal{S}} \left\{ \prod_{i < j=1}^A [1 - g_c(r_{ij}) + \widehat{t}(i, j)] \right\}, \end{aligned} \quad (3)$$

where $g_c(r_{12})$ and $f_{t\tau}(r_{12})$ are the central and tensor correlation functions, \widehat{S}_{12} the tensor operator, and $\hat{\mathcal{S}}$ the symmetrization operator. The correlation functions g_c and $f_{t\tau}$ determine the radial dependence and magnitude of the correlations. Over the last couple of decades, various many-body calculations adopting a plethora of techniques [7–10] have made predictions for the correlation functions g_c and $f_{t\tau}$. These calculations confirmed the following robust features. First, the two-nucleon correlations represent a local property. This implies that the correlations are universal or only weakly A dependent [11]. This means that g_c and $f_{t\tau}$ are very much confined to the bulk part of the nuclear density and only depend on the internucleon distance. The universality property implies that the $f_{t\tau}(r_{ij}) \widehat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j$ correlation operator in a nucleus A is not very different from the one that mixes the 3D_1 and 3S_1 wave-function components in deuterium [8]. Second, it was observed that for moderate relative pair momenta ($300 \leq k_{12} \leq 600$ MeV), the effect of the tensor correlations is dominant [12,13]. As the \widehat{S}_{12} exclusively affects nucleon pairs in a spin $S = 1$ state, it makes the proton-neutron (pn) correlations to dominate at moderate values of the relative pair momentum. We stress that the universality property does not imply that the correlation functions g_c and $f_{t\tau}$ are insensitive to model assumptions. The correlation functions depend on the choice of the Hamiltonian, for example. Indeed, a softer Hamiltonian (implying less correlated wave functions) will require other correlation functions than a hard Hamiltonian [14].

Upon computing the response of the nucleus to some one-body operator $\widehat{\Omega} = \sum_{i=1}^A \widehat{\Omega}^{[1]}(i)$, into lowest order the effect of the correlations can be implemented by means of an effective transition operator, which includes the effect of the correlations [8,12]

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^\dagger \widehat{\Omega} \widehat{\mathcal{G}} \approx \widehat{\Omega} + \sum_{i<j=1}^A \{[\widehat{\Omega}^{[1]}(i) + \widehat{\Omega}^{[1]}(j)] \times [-g_c(r_{ij}) + \widehat{t}(i, j)] + \text{H.c.}\}. \quad (4)$$

Obviously, through the correlations a typical one-body operator (like the γ^* -nucleus interaction in the IA approximation) receives two-nucleon contributions, which are completely determined by the product of the correlation functions and the one-body operator. The nucleon-nucleon correlations are very local and will only affect nucleon pairs which are close. Accordingly, the correlation operators $-g_c(r_{ij}) + \widehat{t}(i, j)$ act as projection operators and will almost exclusively affect nucleon pairs that reside in a relative S state.

We suggest that the significance of two-nucleon correlations in a certain nucleus $A(N, Z)$ is proportional to the number of relative S states. In order to compute this number, a coordinate transformation from (\vec{r}_1, \vec{r}_2) to $(\vec{r}_{12} = \vec{r}_1 - \vec{r}_2, \vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2})$ is required. The single-particle states in the Slater determinant $|\Psi\rangle$ are denoted by $\alpha_a = (n_a l_a j_a m_a t_a)$, where $t_a = \pm \frac{1}{2}$ is the isospin quantum number. In a harmonic-oscillator (HO) basis the normalized and antisymmetrized two-nucleon wave functions can be written as

$$\begin{aligned} & |\alpha_a \alpha_b; J_R M_R\rangle \\ &= \sum_{LM_L} \sum_{nl} \sum_{N\Lambda} \sum_{SM_S} \sum_{TM_T} \frac{1}{\sqrt{1 + \delta_{\alpha_a \alpha_b}}} [1 - (-1)^{l+S+T}] \\ & \times C(\alpha_a \alpha_b; J_R M_R; (nl N\Lambda) LM_L SM_S TM_T) \\ & \times \left| (nl, N\Lambda) LM_L, \left(\frac{1}{2} \frac{1}{2}\right) SM_S, \left(\frac{1}{2} \frac{1}{2}\right) TM_T \right\rangle, \quad (5) \end{aligned}$$

where T (S) is the total isospin (spin) of the pair. Further, $|nl\rangle$ ($|N\Lambda\rangle$) is the relative [center of mass (c.m.)] pair wave

function. The explicit expression for the coefficient \mathcal{C} can be found in Eq. (20) of Ref. [15]. With the aid of the above expression [Eq. (5)] one can project two-nucleon states in (\vec{r}_1, \vec{r}_2) on nucleon states in (\vec{r}_{12}, \vec{R}) and determine for each pair $(\alpha_a \alpha_b)$ of shell-model states the weight of the various relative (nl) and c.m. ($N\Lambda$) quantum numbers. For two-nucleon states in a non-HO basis, one can obtain the weights of the various ($nlN\Lambda$) combinations by expanding the single-particle wave functions in a HO basis.

We have computed the \mathcal{C} coefficients for all target nuclei A either for which the $a_2(A/D)$ coefficient has been published or for which one may expect data in the foreseeable future [16]. The Slater determinant is constructed by filling the single-particle states as they are determined in the nuclear shell model. We denote the Fermi level for the proton and neutron single-particle states as α_F^p and α_F^n . The quantity

$$\sum_{J_R M_R} \sum_{\alpha_a \leq \alpha_F^p} \sum_{\alpha_b \leq \alpha_F^n} \langle \alpha_a \alpha_b; J_R M_R | \alpha_a \alpha_b; J_R M_R \rangle = NZ, \quad (6)$$

determines exactly the number of proton-neutron pairs. Similar expressions hold for the number of proton-proton $[\frac{Z(Z-1)}{2}]$ and neutron-neutron pairs $[\frac{N(N-1)}{2}]$. After inserting the right-hand side of Eq. (5) in the above expression, one can compute how much of each combination $|(nl, N\Lambda) LM_L, SM_S, TM_T\rangle$ of pair quantum numbers contributes to the total number of pairs. Here, we are particularly interested in the quantum numbers (nl) of the relative wave function. We denote the relative orbital angular momentum $l = 0, 1, 2, \dots$ as S, P, D, \dots . The numerical calculations get increasingly more time consuming as A increases due to the combinatorics of all possible shell-model pairs. The accuracy of the numerical calculations can be checked against the normalization condition of Eq. (6). In Fig. 1 we display the relative contribution of the various l to the pair wave functions $|(nl, N\Lambda) LM_L, (\frac{1}{2} \frac{1}{2}) SM_S, (\frac{1}{2} \frac{1}{2}) TM_T\rangle$ for the nuclei ${}^4\text{He}$, ${}^9\text{Be}$, ${}^{12}\text{C}$, ${}^{27}\text{Al}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$, ${}^{56}\text{Fe}$, ${}^{63}\text{Cu}$, ${}^{108}\text{Ag}$, and ${}^{197}\text{Au}$. It is obvious that with increasing A a smaller fraction of the nucleon pairs resides in a relative

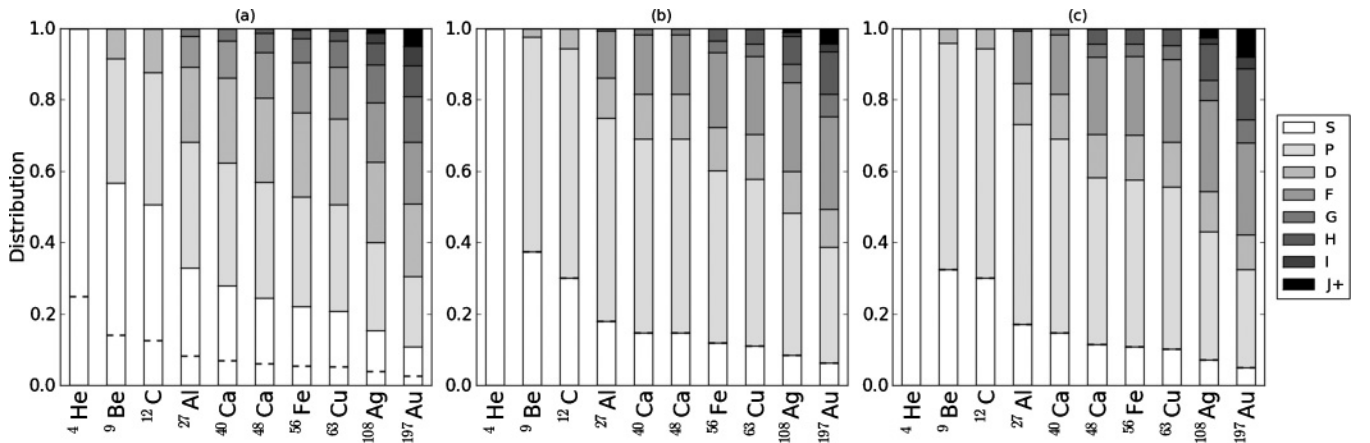


FIG. 1. The distribution of the relative quantum numbers $l = S, P, D, F, G, H, I, \geq J$ for (a) the proton-neutron pairs, (b) the proton-proton pairs, and (c) the neutron-neutron pairs for the various target nuclei. For the proton-neutron pairs there are contributions from ${}^1S_0(T = 1)$ and ${}^3S_1(T = 0)$. The contribution from the ${}^1S_0(T = 1)$ is indicated by the dashed line. Results are obtained in HO basis with $\hbar\omega(\text{MeV}) = 45A^{-\frac{1}{3}} - 25A^{-\frac{2}{3}}$.

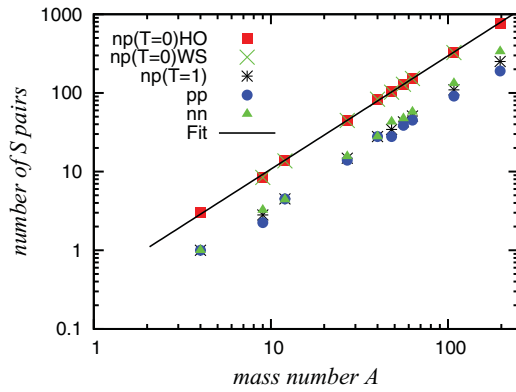


FIG. 2. (Color online) The computed number of pp , nn , and pn pairs with $l = 0$. For pn we discriminate between ${}^3S_1(T = 0)$ and ${}^1S_0(T = 1)$. Unless indicated otherwise the results are for a HO basis. For the ${}^3S_1(T = 0)$ pn pairs also the predictions in a WS basis are shown. The parametrizations for the WS potentials are from Ref. [17].

S state. Whereas, for ${}^{12}\text{C}$ about 50% of the pn pairs has $l = 0$ for the heaviest nucleus ${}^{197}\text{Au}$ this is a mere 10%. Accordingly, with increasing A , a smaller and smaller fraction of the nucleon-nucleon pairs will be prone to correlation effects. In addition, there is a strong isospin dependence as the fraction of the proton-neutron pairs residing in a relative S state is substantially larger than for proton-proton and neutron-neutron pairs.

Naively, one could expect that the number of correlated pn (pp) pairs in a nucleus scales like NZ ($\frac{Z(Z-1)}{2}$) $\sim A^2$. As illustrated in Fig. 2 our calculations rather indicate that the number of pairs that are prone to correlation effects follows a power law $\sim A^{1.44 \pm 0.01}$. As a matter of fact, we find that the power law is very robust. Calculations with Woods-Saxon (WS) wave functions, for example, result in a computed number of S states that is very close (order of one percent) to the HO predictions. The $N-Z$ asymmetry is reflected in an unequal number of pp , nn , and pn ${}^1S_0(T = 1)$ pairs. We stress that the ratio of the nn to pp ${}^1S_0(T = 1)$ pairs can be considerably smaller than predicted by naive $\frac{N(N-1)}{Z(Z-1)}$ combinatorics. For Au, for example, one expects a ratio of 2.24 whereas the data of Fig. 2 lead to 1.77.

Now, we wish to connect the number of pairs with $l = 0$ with the measured values of $a_2(A/D)$. In an inclusive $A(e, e')$ process the correlated part of the electron-nucleus (eA) response [corresponding with the last two terms in Eq. (4)] can be probed by selecting events $1.4 \leq x_B \leq 2$. The magnitude of the response is proportional with a product of two terms. First, the number of pairs that are prone to SRC, and, second, the value of the correlation functions evaluated at the relative momentum of the pair. Indeed, as is pointed out in Refs. [19,20] in the kinematical regime where correlations are probed, the eA response obeys $\sim F(P)\sigma_{eNN}(k_{12})$, where P is the c.m. momentum of the correlated pair on which the absorption takes place and $F(P)$ is the corresponding c.m. distribution (the combination $F(P)\sigma_{eNN}(k_{12})$ is referred to as the decay function in Ref. [19]). The σ_{eNN} stands for the elementary cross section for electron scattering from a correlated NN pair. The σ_{eNN} contains the Fourier-transformed correlation functions $g_c(k_{12})$

and $f_{\tau}(k_{12})$ evaluated at the relative momentum k_{12} of the pair. An analytic expression for σ_{epp} can be found in Ref. [20]. It is worth stressing that given the kinematics, there are two possible values of k_{12} corresponding with photoabsorption on nucleon “1” and photoabsorption on nucleon “2” of the pair. The dominant contribution to the inclusive $A(e, e')$ cross section for $1.4 \lesssim x_B \lesssim 2$ stems from pairs with $k_F \lesssim k_{12} \lesssim 2k_F$, with k_F the Fermi momentum. In that momentum region, the $g_c(k_{12})$ is substantially smaller than $f_{\tau}(k_{12})$, which causes the tensor correlated pn pairs to dominate [21–23].

The universality of the tensor correlations, which translates to the weak A dependence of $f_{\tau}(k_{12})$, allows one to assume that the cross section σ_{epn} for electron scattering from a correlated proton-neutron pair in the nucleus will almost equal the one for electron scattering from the deuteron, provided that the cross sections are evaluated at equal values of the high relative momentum k_{12} of the pair. In a symbolic way, this feature can be expressed through the scaling relation $\sigma_{epn}(k_{12}) \approx \sigma_{eD}(k_{12})$. This property is related to the fact that at high momenta the nuclear momentum distributions $n^A(k)$ are very much like scaled deuteron momentum distributions: $n^A(k) \approx C^A n^D(k)$, where C^A is a measure for the number of pn correlations in A [11,24].

With the above-mentioned scaling relation $\sigma_{epn}(k_{12}) \approx \sigma_{eD}(k_{12})$ valid at high relative momenta, one can transform the ratio of Eq. (1) (the per-nucleon electron-nucleus inelastic scattering cross section to the deuteron) into the form

$$a_2(A/D) = \frac{2}{A} \frac{\int_{\text{PS}} d\vec{k}_{12} d\vec{P} F(P) B_{l=0}^{np}(A) \sigma_{epn}(k_{12})}{\int_{\text{PS}} d\vec{k}_{12} \sigma_{eD}(k_{12})}, \quad (7)$$

$$\approx \frac{2}{A} B_{l=0}^{np}(A) \int_{\text{PS}} d\vec{P} F(P),$$

where the integrations extend over those parts of the phase space (PS) that are compatible with $1.4 \leq x_B \leq 2$. The quantity $B_{l=0}^{np}(A)$ is the number of pn pairs in a relative $|n, l\rangle$ state with the quantum numbers of the

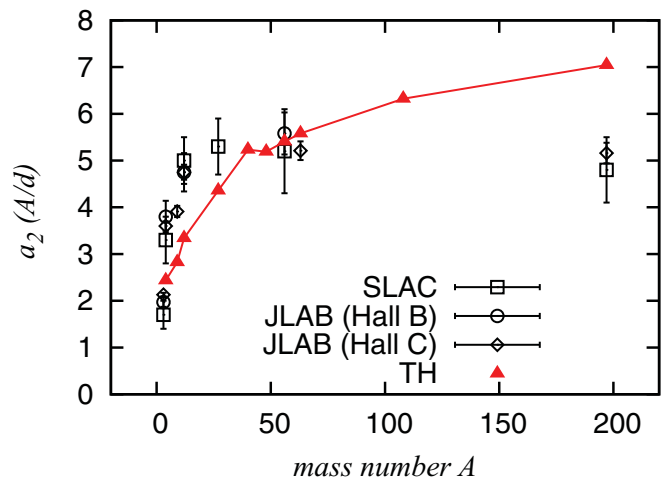


FIG. 3. (Color online) The computed values for the $a_2(A/D)$ for various nuclei. The data are from Refs. [5] (SLAC), [6] (JLAB Hall B) and [18] (JLAB Hall C). The triangles denote the theoretical predictions obtained with the Eq. (7).

TABLE I. The $a_2(A/D)$ values for various nuclei. The data from direct measurements of the nucleus to deuteron cross sections are from Refs. [5] (SLAC), [6] (JLAB Hall B), and [18] (JLAB Hall C). The values of Ref. [4] are phenomenological extractions based on the measured EMC data and the observed linear correlation between the magnitude of the EMC effect and the measured a_2 scaling factor. The quoted values of Ref. [18] are the raw values. Reference [18] also contains corrected values for a_2 which are about 15% smaller.

A	Ref. [5]	Ref. [6]	Ref. [4]	Ref. [18]	Eq. (7)
^4He	3.3 ± 0.5	3.80 ± 0.34		3.60 ± 0.10	2.4
^9Be			4.08 ± 0.60	3.91 ± 0.12	2.8
^{12}C	5.0 ± 0.5	4.75 ± 0.41		4.75 ± 0.16	3.3
^{27}Al	5.3 ± 0.6		5.13 ± 0.55		4.4
^{40}Ca			5.44 ± 0.70		5.2
^{48}Ca					5.2
^{56}Fe	5.2 ± 0.9	5.58 ± 0.45			5.4
^{63}Cu				5.21 ± 0.20	5.6
^{108}Ag			7.29 ± 0.83		6.3
^{197}Au	4.8 ± 0.7		6.19 ± 0.65	5.16 ± 0.22	7.0

deuteron, $^3S_1(T=0)$. One can estimate the $B_{l=0}^{np}(A)$ from Eq. (5) by combining the computed coefficients for all possible $|(n=0, l=0, N\Lambda)LM_L, S=1M_S, T=0M_T=0\rangle$ combinations. In Fig. 2 we have summed over all possible n to obtain the total number of $l=0$ states. For all target nuclei, the $n=0$ contribution dominates, but its relative importance decreases with growing A . The $n=0$ represents 100% of the $l=0$ pn states for ^4He , about 80% for the medium-heavy nuclei (Ca, Fe, Cu), 70% for ^{108}Ag , and 62% for ^{197}Au . Pairs residing in a $|n \neq 0, l=0\rangle$ state have a much smaller chance of being close than their $|n=0, l=0\rangle$ counterparts and are less prone to SRC effects. We assume that only $|n=0, l=0\rangle$ proton-neutron pairs contribute to $B_{l=0}^{np}(A)$.

The c.m. motion of the pair in finite nuclei (absent in the deuteron) and the imposed conditions in x_B make that a fraction of the correlated proton-neutron pairs are not counted in the $A(e, e')$ signal in the numerator of Eq. (1). The $F(P)$ for a nucleus A can be reliably computed in a mean-field model. Indeed, the $^{12}\text{C}(e, e'pp)$ measurements of Ref. [25] determined $F(P)$ over a large P range and observed it to be compatible with a mean-field prediction. We have performed Monte-Carlo simulations in order to determine the correction factor $\int_{\text{PS}} d\vec{P} F(P)$ for all nuclei that are considered here. We find that for $A > 4$ about 25% of the correlated pairs are excluded from the experimentally scanned phase space due to the c.m. motion of the correlated pair. From the simulations we observed that the correction factor is only slightly mass number dependent. With this correction factor, Eq. (7) allows us to make predictions for the $a_2(A/D)$. The predictions are contained in Fig. 3 and Table I and compared with experimental data. One striking observation from our calculations, is that the predicted $a_2(A/D)$ for ^{40}Ca and ^{48}Ca are identical and equal to 5.2. On the basis of naive NZ combinatorics one

may have expected a 30% difference between the two. For heavier target nuclei, the data seem to suggest that the $a_2(A/D)$ coefficient saturates. Our calculations predict a strong linear rise in the A dependence of the a_2 for $A \lesssim 40$. At higher A one enters a second regime with a much softer linear rise with A . Our calculations increase linearly with $\log(A)$ and tend to underestimate the data at low A and overestimate the data for the heavier nuclei. Final state interactions, for example, which are neglected in this work, may induce some additional A dependence in the a_2 ratio [18]. It is clear that more data are needed to establish the situation at large A . The observed phenomenological linear relationship between the scaling factor a_2 and the magnitude of the EMC effect [4] gives $a_2 = 7.29 \pm 0.83$ for Ag and $a_2 = 6.19 \pm 0.65$ for Au, values that are not inconsistent with our results.

In conclusion, we suggest that the number of correlated pairs in a nucleus is proportional with the number of relative S two-nucleon states. We find this number to obey a power law $dA^{1.44 \pm 0.01}$ with $d = 0.39 \pm 0.02$ ($d = 0.13 \pm 0.01$) for $T=0$ ($T=1$) proton-neutron pairs. The power law is robust in that it is independent of the choices made with regard to the single-particle wave functions. We have used the computed number of $T=0$ pn pairs to predict the value of the measured $a_2(A/D)$ coefficients, which provide a measure of the number of correlated pairs in the target nucleus A relative to the deuteron. The observed power law in the number of relative S states translates to a linear increase of a_2 with $\log(A)$. We observe that our predictions are not inconsistent with the trend and magnitude of the data, lending support to our suggestion.

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- [1] S. Wong, *Introductory Nuclear Physics*, 2nd ed. (Wiley, New York, 1998).
 [2] M. Paolone *et al.* (E03-104 Collaboration), *Phys. Rev. Lett.* **105**, 072001 (2010).
 [3] D. F. Geesaman, K. Saito, and A. W. Thomas, *Annu. Rev. Nucl. Part. Sci.* **45**, 337 (1995).
 [4] L. Weinstein *et al.*, *Phys. Rev. Lett.* **106**, 052301 (2011).

- [5] L. L. Frankfurt, M. I. Strikman, D. B. Day, and M. Sargsyan, *Phys. Rev. C* **48**, 2451 (1993).
 [6] K. S. Egiyan *et al.* (CLAS Collaboration), *Phys. Rev. Lett.* **96**, 082501 (2006).
 [7] S. C. Pieper, R. B. Wiringa, and V. R. Pandharipande, *Phys. Rev. C* **46**, 1741 (1992).

- [8] R. Roth, T. Neff, and H. Feldmeier, *Prog. Part. Nucl. Phys.* **65**, 50 (2010).
- [9] M. Alvioli, C. Ciofi degli Atti, and H. Morita, *Phys. Rev. Lett.* **100**, 162503 (2008).
- [10] C. Bisconti, F. A. de Saavedra, and G. Co', *Phys. Rev. C* **73**, 054304 (2006).
- [11] H. Feldmeier, W. Horiuchi, T. Neff, and Y. Suzuki, [arXiv:1107.4956](https://arxiv.org/abs/1107.4956).
- [12] S. Janssen, J. Ryckebusch, W. Van Nespen, and D. Debruyne, *Nucl. Phys. A* **672**, 285 (2000).
- [13] R. Schiavilla, R. B. Wiringa, S. C. Pieper, and J. Carlson, *Phys. Rev. Lett.* **98**, 132501 (2007).
- [14] E. R. Anderson, S. K. Bogner, R. J. Furnstahl, and R. J. Perry, *Phys. Rev. C* **82**, 054001 (2010).
- [15] J. Ryckebusch, D. Debruyne, and W. Van Nespen, *Phys. Rev. C* **57**, 1319 (1998).
- [16] N. Fomin, Ph.D. thesis, University of Virginia, 2008 (unpublished).
- [17] N. Schwierz, I. Wiedenhover, and A. Volya, [arXiv:0709.3525](https://arxiv.org/abs/0709.3525).
- [18] N. Fomin *et al.*, [arXiv:1107.3583](https://arxiv.org/abs/1107.3583).
- [19] L. Frankfurt, M. Sargsian, and M. Strikman, *Int. J. Mod. Phys. A* **23**, 2991 (2008).
- [20] J. Ryckebusch, *Phys. Lett. B* **383**, 1 (1996).
- [21] E. Piasezky, M. Sargsian, L. Frankfurt, M. Strikman, and J. W. Watson, *Phys. Rev. Lett.* **97**, 162504 (2006).
- [22] R. Subedi *et al.*, *Science* **320**, 1476 (2008).
- [23] H. Baghdasaryan *et al.* (CLAS Collaboration), *Phys. Rev. Lett.* **105**, 222501 (2010).
- [24] C. Ciofi degli Atti and C. B. Mezzetti, *Phys. Rev. C* **79**, 051302 (2009).
- [25] K. I. Blomqvist *et al.*, *Phys. Lett. B* **421**, 71 (1998).