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Effects of isospin mixing in the $A = 32$ quintet

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For the $A = 32$, $T = 2$ quintet we provide a unified theoretical description for three related aspects of isospin mixing: the necessity of more than three terms in the isobaric mass multiplet equation, isospin-forbidden proton decay in ^{32}Cl, and a correction to the allowed Fermi β^+ decay of ^{32}Ar. We demonstrate that all three effects observed in experiment can be traced to a common origin related to isospin mixing of the $T = 2$ states with $T = 1$ states.

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Recent measurements of the lowest $T = 2$ states in ^{32}Cl and ^{32}S have made the $A = 32$ multiplet the most precisely measured $T = 2$ quintet [1,2]. In first-order perturbation theory the masses in an isobaric multiplet are given by the isobaric mass multiplet equation (IMME):

$$M(T_z) = a + bT_z + c(T_z)^2, \tag{1}$$

where $T_z = (N - Z)/2$ [3]. Multiplets with $T > 1$ may require terms of higher order in T_z that enter in second-order perturbation theory along with isospin mixing. The $A = 32$ multiplet requires a small but nonzero higher-order term, proportional to either T_z^3 [1] or T_z^4 [2].

Isospin-forbidden proton decay, another signature of isospin mixing [4], has been observed from the $T = 2$ state in ^{32}Cl to the low-lying $T = 1/2$ states in ^{31}S with a decay width of 20(5) eV [5].

Finally, the superallowed Fermi β^+ decay of ^{32}Ar has been measured recently. Superallowed 0^+ to 0^+ Fermi decay is measured very precisely for many nuclei and provides the critical data for extracting the weak mixing angle v_{ud} for the Cabibbo-Kobayashi-Maskawa (CKM) matrix [6]. The extraction of v_{ud} from the data requires a small but important correction δ_C due to isospin mixing. The correction is smallest for nuclei near stability, typically 0.5% or less, but can be larger for nuclei far from stability. A correction of 2% was recently measured [5].

These three isospin-mixing effects, usually studied and theoretically treated independently, are derived from the same origin, as evidenced by the $A = 32$ quintet. Calculations for energy levels, spectroscopic factors, gamma decay, isospin-mixing matrix elements, and one-body transition densities for the multiplet were obtained in the *sd*-shell model space with the universal sd-shell (USD), USDA, and USDB interactions [7–9]. The shell model code OXBASH [10] was utilized for a full diagonalization of the Hamiltonians. The USDB interaction will primarily be used for explanation and illustration in this paper, with reference to the other interactions for meaningful comparisons.

All calculations were carried out in the proton-neutron formalism. The isospin-mixing interaction is taken from the work of Ormand and Brown [11] where, in addition to the Coulomb potential, charge-independence-breaking (CIB) and charge-symmetry-breaking (CSB) interactions were added to the USD Hamiltonian. The CIB strength was obtained from a one-parameter fit to the experimental c coefficient of the $T = 1$ IMME and is consistent with the np vs pp scattering data [4]. The CSB strength was obtained from a one-parameter fit to the experimental b coefficients.

All 0^+ states for the $A = 32$ quintet were calculated. The dominant isospin of each state was determined by calculating overlaps with the isospin-conserving part of the interactions. The lowest $T = 2$ state was the ground state for ^{32}Si and ^{32}Ar, the third 0^+ state for ^{32}P and ^{32}Cl, and the tenth (eleventh, tenth) 0^+ state for ^{32}S using the USDB (USDA, USD) interaction.

The experimental masses are given in Table I. The masses for ^{32}S [2,17], ^{32}Cl [1,18], and ^{32}Ar [19] are identical to those given in Table I of [1]. We have combined the two values for ^{32}P, $-19\,232.46\,(15)$ [1,13,14] and $-19\,232.78\,(20)$ [15,16], into a reduced value based on a χ^2 fit to a constant. We use the recent direct measurement at the National Superconducting Cyclotron Laboratory (NSCL) for the ^{32}Si mass [12]. The circles with error bars in the bottom panel of Fig. 1 show the differences in keV between the experimental masses of the $T = 2$ states and those obtained from a fit to Eq. (1). The crosses display the differences obtained for the $T = 2$ energies as calculated with USDB. The differences for both experiment and theory obtained when d (e) terms proportional to T_z^3 (T_z^4) are added as shown in the middle (top) panel. For the experimental data, the χ^2 value of each fit is given in the figure, as well as the best-fit d and e parameters.

In Fig. 1, it can be seen that a much better fit occurs for both USDB and experiment when a d coefficient is used. We repeated the procedure with $M_{\text{expt}} = -24\,080.92\,(5)$ for ^{32}Si, combining the two indirect ^{32}Si masses [1,20] from Table I of [1] into a reduced value. While the parameters of the fit and the mass differences change, the conclusions are identical. Again, a d coefficient is necessary for a reasonable fit to data, producing $\chi^2_{\text{expt}} = 0.58$ with $d_{\text{expt}} = 0.53\,(11)$.

The most significant difference between theory and experiment in the bottom panel of Fig. 1 corresponds to the quality of the fit for ^{32}Ar ($T_z = -2$). A reduction in the error bar of ^{32}Ar, at least to the level of ^{32}Cl, would better constrain the fit and therefore the d parameter. If we exclude the $T_z = -2$ point, the d term can be solved algebraically

TABLE I. Mass excesses of $T = 2$ states in the $A = 32$ quintet.

Isobar	T_z	M_{expt} (keV)	References
^{32}Si	2	−24 077.68 (30)	[12]
^{32}P	1	−19 232.58 (12)	[1,13–16]
^{32}S	0	−13 967.57 (28)	[2,17]
^{32}Cl	−1	−8288.34 (70)	[1,18]
^{32}Ar	−2	−2200.2 (18)	[19]

to give $d_{expt} = 0.95\,(37)$ or $d_{expt} = 0.41\,(33)$ with the indirect ^{32}Si mass. The evidence strongly suggests that the three-term IMME does not fit the data or the USDB calculations. The uncertainty in the theoretical calculations can be assessed by comparing the results for the three different interactions. The USD interaction result patterns the USDB behavior but with larger deviations than seen in Fig. 1, resulting in a greater value of the necessary coefficient $d_{theor} = 0.39$. The USDA interaction cannot be corrected solely by a d coefficient, as large shifts in both ^{32}Cl and ^{32}S occur due to isospin mixing. Both the sign and the magnitude of the necessary coefficient give information about the shifts of the $T = 2$ states, which can be determined theoretically.

Figure 2 shows the first twenty 0^+ levels in ^{32}S with the USDB interaction, categorized by their values of T. The sum of the ground-state energy and the excitation energy of the

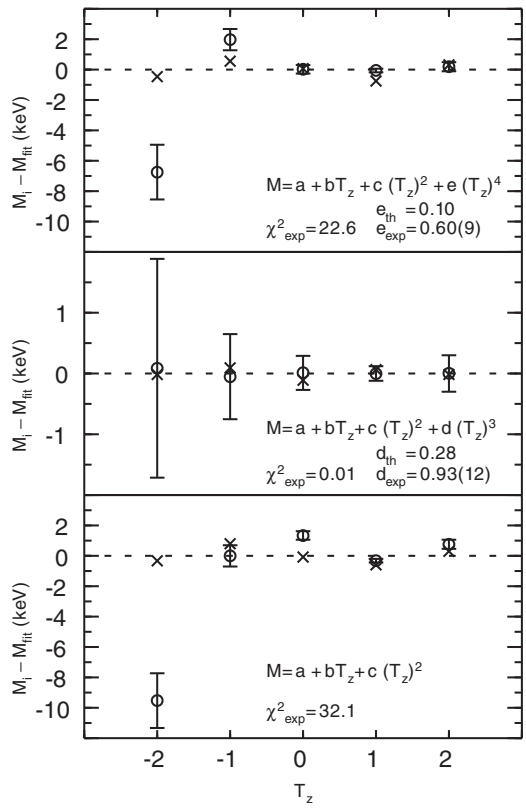

FIG. 1. Accuracy of the IMME in the $A = 32$ quintet with three terms (bottom panel) and with an additional cubic (middle) or quartic (top) term. Circles correspond to experimental data, with error bars from Table I. Crosses correspond to theoretical calculations with the USDB interaction. Note the reduction in scale for the middle panel.

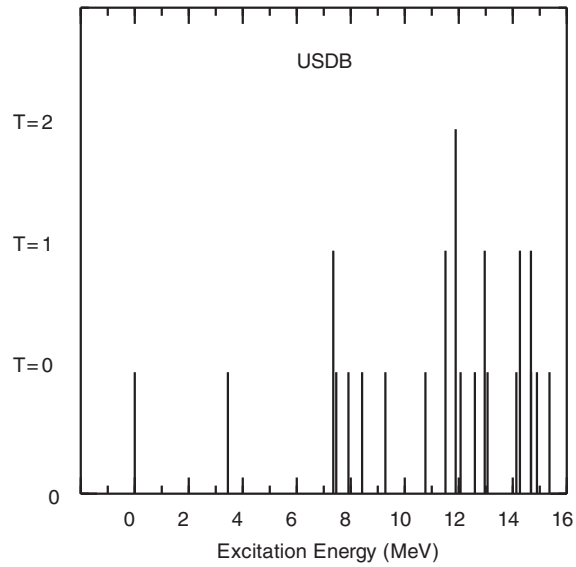

FIG. 2. 0^+ levels in ^{32}S given by T vs E_x. For ^{32}P and ^{32}Cl, the $T = 0$ levels can be ignored to give an approximate distribution of the 0^+ states.

$T = 2$ state gives the energy to be used in the fit for ^{32}S. As seen from the graph, the $T = 2$ state has nearby states that repel it, shifting its energy. The theoretical treatment is restricted to a two-level mixing scheme throughout this work. For large energy differences, the shift of the $T = 2$ state, labeled by "a," is approximately given by

$$\Delta E_a = -\sum_{i \neq a} \frac{\langle i|V|a\rangle^2}{E_i - E_a}, \tag{2}$$

where V denotes the interaction that causes isospin mixing. E_x, the excitation energy of the state, is given by $E_a + \Delta E_a$. The closest states generally have the greatest effect, and therefore most terms in the sum can be ignored. The $T = 2$ state in ^{32}S can be shifted by both $T = 0$ and $T = 1$ states, while the states in ^{32}P and ^{32}Cl can only be shifted by $T = 1$ states ($T = 0$ levels do not exist in these nuclei). Table II shows the significant contributing levels to energy shifts in the $A = 32$ quintet for all three interactions.

The excitation energy of the $T = 2\,0^+$ state in ^{32}S is 11.885 (11.867, 12.011) MeV with the isospin-nonconserving USDB (USDA, USD) interaction, in reasonable agreement with the experimental value of 12.048 MeV. However, the nearest $T = 0\,0^+$ state is 182 (252) keV higher in energy for USDB (USD) but 33 keV lower in energy for USDA. There are no known experimental 0^+ states above the $T = 2$ state at 12.048 MeV; the nearest known experimental 0^+ level is a $T = 0$ state 118 keV below the $T = 2$ state. There is also an experimental 0^+ state 180 keV below without an assigned T value which could correspond to the $T = 1$ state seen 377 (302, 195) keV below for USDB (USDA, USD). The proximity of these two states as calculated via the different empirical interactions has no deep underlying cause, but is rather an incidental effect due to the configurations of the states. Because the shift varies inversely with the energy, the proximity of the mixing state determines the size of the shift and the observed deviation from the three-term IMME. This energy difference (in conjunction with

TABLE II. Energy shifts in the $A = 32$ quintet for the three interactions, where ΔE_i, d_i, and e_i are the contributions to the energy shift, d, and e coefficients, respectively, from state i (i.e., $\Delta E_a = \sum_i \Delta E_i$). The algebraic solution to the five-term IMME gives exact values of d and e for each interaction.

Interaction	Isobar	T	$\langle i\|V\|n\rangle$ (keV)	$E_i - E_a$ (keV)	ΔE_i (keV)	d_i (keV)	e_i (keV)
USDB	^{32}P	1	13.14	1020	−0.163	0.027	0.027
USDB	^{32}P	1	22.48	2387	−0.215	0.036	0.036
USDB	^{32}Cl	1	20.92	−439	1.084	0.181	−0.181
USDB	^{32}S	1	17.26	−377	0.633	0	0.158
USDB	^{32}S	0	7.03	182	−0.294	0	−0.073
Exact						0.28	−0.07
USDA	^{32}P	1	12.73	1025	−0.154	0.026	0.026
USDA	^{32}P	1	20.39	2643	−0.159	0.027	0.027
USDA	^{32}Cl	1	20.54	−352	1.324	0.221	−0.221
USDA	^{32}S	1	12.26	−302	0.405	0	0.101
USDA	^{32}S	0	19.32	−33	4.939	0	1.235
Exact						0.30	1.40
USD	^{32}P	1	13.32	969	−0.174	0.029	0.029
USD	^{32}P	1	19.97	1544	−0.262	0.044	0.044
USD	^{32}Cl	1	19.10	−262	1.606	0.268	−0.268
USD	^{32}S	1	18.18	−195	1.082	0	0.271
USD	^{32}S	0	13.42	475	−0.399	0	−0.100
Exact						0.39	0.03

the size of the isospin-mixing matrix element) determines the size of the necessary d or e coefficient. In an algebraic solution of an isobaric quintet to the five-term IMME [including both $d(T_z)^3$ and $e(T_z)^4$ terms], the USDB and USD calculations, as well as the experimental data, result in a small ($\leqslant$0.1 keV) e coefficient. The USDA calculations require an e coefficient of 1.4 keV, even larger than the necessary d coefficient. Since the small energy difference in the USDA level scheme results in strong mixing in ^{32}S, in opposition to the experimental data, we will favor the USDB result.

Alternatively, the energy difference needed for a single state to reproduce the experimental d coefficient can be determined. The $T = 1$ state in Table II for ^{32}Cl would need to be 73 keV below the $T = 2$ state to reproduce $d_{\rm expt} = 0.93\,(12)$ using an average of the matrix elements for the three interactions. With $d_{\rm expt} = 0.53\,(13)$ from the other mass for ^{32}Si, an energy difference of 128 keV is required. The nearest experimental state below the $T = 2$ state with unassigned J^π has a 267 keV energy difference.

The decay from the $T = 2$ state in ^{32}Cl occurs via two processes: γ decay and proton emission. The primary channel of γ decay is an $M1$ transition to the first excited 1^+ state in ^{32}Cl with a branching ratio of 94%, using the USDB interaction and free gyromagnetic factors. The calculated width Γ_γ is 1.11 eV, in comparison to the experimental value of $\Gamma_\gamma = 1.8\,(5)$ eV. The isospin-forbidden proton transition decays to ^{31}S. The reaction has $Q = 3.45$ MeV for both USDB and experiment. Since the transition is from a $J^\pi = 0^+$ level, and the proton has $j = 1/2^+, 3/2^+, 5/2^+$ in the sd shell, decay can only occur to levels in ^{31}S with those values of J. Five such levels have $E_x \leqslant Q$, all with $T = 1/2$ using the isospin-conserving part of the Hamiltonians. With the inclusion of the CIB and CSB interactions, isospin mixing occurs in both the parent ^{32}Cl and daughter ^{31}S nuclei. The small spectroscopic factors for the isospin-forbidden transitions are shown in Table III. If isospin mixing is only included for ^{32}Cl, the spectroscopic factors are larger. It is therefore important to include mixing in both nuclei to account for interference effects in the wave functions. The decay widths to states in ^{31}S were determined by

$$\Gamma_p = \sum_j C^2S\ \Gamma_{\rm sp}, \tag{3}$$

where C^2S are spectroscopic factors and $\Gamma_{\rm sp}$ is the single-particle width of the resonance peak in the reaction $^{31}{\rm S} + p \rightarrow {}^{32}{\rm Cl}$. The single-particle widths were calculated from scattering phase shifts in a Woods-Saxon potential [21,22] with the potential depths chosen to reproduce the resonance energies. The results for the five levels are shown in Table III, but only the $1/2^+$ ground state and first-excited state ($3/2^+$) contribute to the decay. Therefore, $\Gamma_p = 41.4$ eV, in comparison to the experimental value of 20 (5) eV from [5]. There is a large uncertainty in the $1/2^+$ level, the most important contribution, for two reasons: (i) the calculation of $\Gamma_{\rm sp}$ was determined by

TABLE III. Widths of the isospin-forbidden proton decay for ^{32}Cl using the USDB interaction.

J^π	E_x	C^2S	$\Gamma_{\rm sp}$ (keV)	Γ (eV)
$1/2^+$	0.00	0.000 03	1002	33.82
$3/2^+$	1.19	0.000 41	18.4	7.54
$5/2^+$	2.30	0.000 01	0.3	0.00
$1/2^+$	3.20	0.000 05	$\approx 10^{-6}$	0.00
$5/2^+$	3.32	0.000 22	$\approx 10^{-8}$	0.00

doubling the width at half maximum on the low-energy side of the resonant peak, due to the large tail in the resonant reaction at high energy and (ii) the spectroscopic factor changes by a factor of four depending on the interaction used, with C^2S values of 0.000 03 (0.000 08, 0.000 12) for the USDB (USDA, USD) interactions. Using the same single-particle widths for the USDA interaction gives $\Gamma_p = 92.2$ eV. The isospin-mixing matrix element between the $T = 2$ state in ^{32}Cl and the $T = 1$ state below it is approximately the same for all three interactions, as seen in Table II. The energy-change difference varies from 262 to 439 keV, however, where the range is on the order of the 150 keV global root-mean-square (rms) energy deviation for the three interactions. However, a difference of 100 keV in the energy denominator can significantly affect the width of proton decay due to the amount of mixing. The USDB energy difference is greatest and has the smallest proton decay width, in better agreement with experiment.

In the β^+ decay from the ground state of ^{32}Ar to the $T = 2$, $J^\pi = 0^+$ state in ^{32}Cl (both members of the $T = 2$ multiplet), the ft value differs slightly from the expected value due to the isospin mixing in ^{32}Cl. The only significant contribution comes from the $T = 1$, $J^\pi = 0^+$ state below, which is the same state that is influential in the isospin-forbidden proton decay to ^{31}S and in the deviation from the three-term IMME. The calculated value of δ_C is 0.27% (0.40%, 0.63%) for the USDB (USDA, USD) interactions, allowing us to quote a theoretical value of 0.43 (20)% from an average of the three calculations. Again, the energy difference results in a range of results for an isospin-mixing effect. From [6], δ_C should be a sum of a charge-dependent mixing contribution (calculated here) and a radial overlap component (1.4% from [5]). The sum of these two contributions, or 1.8%, agrees with the experimental values of 2.1(8)% [5] and, based on a new mass of ^{32}Cl, 1.8 (8)% [18]. In all three calculations, nearly the entire remaining strength feeds to the 0^+_2 state in ^{32}Cl, the $T = 1$ state shown in Table II. The transition to this state from the ground state of ^{32}Ar might be accessible experimentally.

The experimental data for the $A = 32$ multiplet differs from the IMME fit with three terms, requiring another term for an adequate fit. Using the new direct measurement of ^{32}Si [12], a d coefficient of 0.93 (12) is necessary. With the indirect mass of ^{32}Si [1,20], the necessary d coefficient is 0.53 (11). The three-term IMME similarly does not reproduce the behavior of the masses using three different sd interactions. The calculated d coefficient is 0.28 (0.30, 0.39) for the USDB (USDA, USD) interactions. The USDA calculations also result in a large e coefficient due to the proximity of a $T = 0$ state to the $T = 2$ state in ^{32}S. There is an inherent uncertainty in our method regarding the shift due to isospin mixing on account of the global rms deviation of 150 keV of empirical interactions. We gain information from using multiple interactions, but rely on experiment to constrain our choice of interaction for comparison. With the USDB interaction, the decay of the $T_z = -1$ state in the multiplet occurs primarily by proton emission with $\Gamma_p = 41.4$ eV, but the gamma decay with $\Gamma_\gamma = 1.11$ eV cannot be neglected. The proton decay width is approximately double the experimental value, while the gamma decay width is in relatively good agreement with experiment. The theoretical Γ_p result varies significantly with the interaction used, suggesting a large uncertainty in the calculated value. Regardless of the interaction chosen, the mixing of the $T = 1$ and $T = 2$ states in ^{32}Cl causes a nonzero isospin-forbidden proton decay to $T = 1/2$ states in ^{31}S. The mixing of these same states also accounts for the deviation in the ft value of the β^+ decay of the ground state of ^{32}Ar. The isospin-breaking correction δ_C = 1.8% agrees with the experimental value.

These observed aspects of isospin mixing occur in relation to the proximity of the levels, separate from the correlation between the configurations of the states. In the event of small energy differences and non-negligible isospin-mixing matrix elements, the effects described above will be seen. The commonness of the fulfillment of these two requirements in other multiplets, such that effects of isospin mixing occur, cannot be determined without more accurate theoretical energies or more complete experimental level schemes.

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