Time reversal invariance violating and parity conserving effects in neutron-deuteron scattering

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(Received 6 May 2011; published 25 August 2011)

Time reversal invariance violating and parity conserving effects for low-energy elastic neutron-deuteron scattering are calculated for meson exchange and effective field theory type potentials in a distorted wave-born approximation using realistic hadronic wave functions, obtained by solving three-body Faddeev equations in configuration space.

DOI: 10.1103/PhysRevC.84.025501 PACS number(s): 24.80.+y, 25.10.+s, 11.30.Er, 13.75.Cs

I. INTRODUCTION

We consider phenomenological time reversal invariant violating (TRIV) and parity conserving (PC) interaction, which we refer to as TVPC interaction in order to distinguish it from TVPV interactions, which violate both time reversal invariance and parity. TVPC interaction was introduced in Ref. [1] as a possible explanation of *CP* violation in K^0-meson decay. According to *CPT* theorem, the violation of *CP* invariance implies the TRIV. In spite of the fact that almost all known possible mechanisms of *CP* violation also violate parity, TVPC interactions have been a subject of experimental and theoretical studies for decades (see, for example, Refs. [2–18] and references therein) because they directly manifest phenomena beyond the standard model. Most experimental constraints for this interaction were obtained by using low-energy nuclear physics processes, which cover a large variety of nuclear reactions and nuclear decays. There are a number of advantages of the search for TRIV in those processes: for example, the possibility of enhancement of T-violating observables in neutron-induced reactions by many orders of a magnitude due to complex nuclear structure (see, e.g., Ref. [10] and references therein), similar to the enhancement observed for parity violating effects. Another advantage to be mentioned is the existence of observables that cannot be imitated by final-state interactions [19–21]. Then, the measurement of nonzero values for these observables directly indicates TRIV, similar to the case of neutron electric dipole measurements.

A promising process for the search for TRIV in nuclear reactions is a measurement of TVPC effects in the transmission of polarized neutrons through a polarized target [3,4]. These effects can be enhanced [9,18] by a factor of 10^6 and therefore could be measured at new spallation neutron facilities, such as the Spallation Neutron Source at the Oak Ridge National Laboratory or the J-SNS at the Japan Proton Accelerator Research Complex (J-PARC).

However, despite the advantage of the enhancement, the complexity of nuclear system makes it difficult to directly relate observations of TRIV effects to nucleon TVPC coupling constants. Therefore, it is interesting to compare the calculations of TVPC effects in complex nuclei with the calculations of these effects in the simplest few-body systems, which could be useful for the clarification of the influence of nuclear structure on values of TVPC effects. Thus, as a first step to many-body nuclear effects, we study TRIV and parity violating effects in one of the simplest available nuclear processes, namely, elastic neutron-deuteron scattering. The calculations of these effects for a specific type of TVPC interaction in a short-range approximation [7] show strong dependence of TVPC observables on neutron energy, which gives the opportunity to improve existing constrains on TVPC interactions using simple few-body systems. Therefore, it is desirable to calculate these effects for the general case of TVPC interactions to clarify this opportunity.

In this paper we treat TVPC nucleon-nucleon interactions as a perturbation, while nonperturbed three-body wave functions are obtained by solving Faddeev equations for a realistic strong interaction Hamiltonian, based on AV18 + UIX interaction model. For description of TRIV potentials, we use both a meson-exchange model and an effective field theory (EFT) approach. For different cases of symmetry violation we use the following notation: $\not{T}P$ for TVPC, $\not{T}\not{P}$ for TVPV, and $\not{P}$ for parity violating (PV) cases.

II. OBSERVABLES

We consider TVPC effects related to $\boldsymbol{\sigma}_n \cdot [\boldsymbol{p} \times \boldsymbol{I}](\boldsymbol{p} \cdot \boldsymbol{I})$ correlation, where $\boldsymbol{\sigma}_n$ is the neutron spin, $\boldsymbol{I}$ is the target spin, and $\boldsymbol{p}$ is the neutron momentum, which can be observed in the transmission of polarized neutrons through an aligned (or tensor polarized) target. This correlation leads to the difference [3,4] between the total neutron cross sections for neutrons polarized perpendicular to the neutron momentum and averaged over elongated target states,

$$\Delta\sigma^{\not{T}P} = \frac{4\pi}{p}\mathrm{Im}(f_+ - f_-), \tag{1}$$

and neutron spin rotation angle [5,9] ϕ around the axis $[\boldsymbol{p} \times \boldsymbol{I}](\boldsymbol{p} \cdot \boldsymbol{I})$,

$$\frac{d\phi^{\not{T}P}}{dz} = -\frac{2\pi N}{p}\mathrm{Re}(f_+ - f_-). \tag{2}$$

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Here $f_{+,-}$ are the zero-angle scattering amplitudes for neutrons polarized parallel and antiparallel to the $[\boldsymbol{p} \times \boldsymbol{I}](\boldsymbol{p} \cdot \boldsymbol{I})$ axis, respectively, z is the target length, and N is the number of target nuclei per unit volume. It should be noted that for a nonzero value of this fivefold correlation, the spin I must be larger or equal to 1; i.e., these TVPC effects require a tensor-polarized target. Therefore, this correlation cannot be observed in nucleon-nucleon scattering.

The scattering amplitudes can be represented in terms of matrix $\hat{R}$, which is related to scattering matrix $\hat{S}$ as $\hat{R} = \hat{1} - \hat{S}$. In partial-wave basis, we define $R^J_{l'\mathcal{S}',l\mathcal{S}} = \langle l'\mathcal{S}'|R^J|l\mathcal{S}\rangle$, where unprimed and primed parameters correspond to initial and final states, respectively, l is an orbital angular momentum between neutron and deuteron, $\mathcal{S}$ is a sum of neutron spin and deuteron total angular momentum, and J is the total angular momentum of the neutron-deuteron system. Since we are interested in low-energy neutrons, one can consider only s, p, and d partial waves with mixing only between s and d waves and p and p waves. Then, one can write the TVPC parameters as

$$\frac{1}{N}\frac{d\phi^{TP}}{dz} = \frac{1}{2}\frac{\pi}{10p^2}\mathrm{Re}\Big[5\sqrt{2}R^{\frac{1}{2}}_{0\frac{1}{2},2\frac{3}{2}} - 5\sqrt{2}R^{\frac{1}{2}}_{2\frac{3}{2},0\frac{1}{2}} + 5\sqrt{2}R^{\frac{1}{2}}_{1\frac{1}{2},1\frac{3}{2}} - 5\sqrt{2}R^{\frac{1}{2}}_{1\frac{3}{2},1\frac{1}{2}} + 10R^{\frac{3}{2}}_{0\frac{3}{2},2\frac{1}{2}} - 10R^{\frac{3}{2}}_{2\frac{1}{2},0\frac{3}{2}} - 2\sqrt{5}R^{\frac{3}{2}}_{1\frac{1}{2},1\frac{3}{2}} + 2\sqrt{5}R^{\frac{3}{2}}_{1\frac{3}{2},1\frac{1}{2}}\Big], \tag{3}$$

$$\Delta\sigma^{TP} = -\frac{1}{2}\frac{\pi}{5p^2}\mathrm{Im}\Big[5\sqrt{2}R^{\frac{1}{2}}_{0\frac{1}{2},2\frac{3}{2}} - 5\sqrt{2}R^{\frac{1}{2}}_{2\frac{3}{2},0\frac{1}{2}} + 5\sqrt{2}R^{\frac{1}{2}}_{1\frac{1}{2},1\frac{3}{2}} - 5\sqrt{2}R^{\frac{1}{2}}_{1\frac{3}{2},1\frac{1}{2}} + 10R^{\frac{3}{2}}_{0\frac{3}{2},2\frac{1}{2}} - 10R^{\frac{3}{2}}_{2\frac{1}{2},0\frac{3}{2}} - 2\sqrt{5}R^{\frac{3}{2}}_{1\frac{1}{2},1\frac{3}{2}} + 2\sqrt{5}R^{\frac{3}{2}}_{1\frac{3}{2},1\frac{1}{2}}\Big]. \tag{4}$$

The symmetry violating $\hat{R}$-matrix elements can be calculated with a high level of accuracy in distorted wave-born approximation (DWBA) as

$$R^J_{l'\mathcal{S}',l\mathcal{S}} \simeq 4i^{-l'+l+1}\mu p\ {}^{(-)}\langle\Psi,(l'\mathcal{S}')JJ^z| \times V^{TP}|\Psi,(l\mathcal{S})JJ^z\rangle^{(+)}, \tag{5}$$

where μ is a neutron-deuteron reduced mass, V^{TP} is the TVPC nucleon-nucleon potential, and $|\Psi,(l'\mathcal{S}')JJ^z\rangle^{(\pm)}$ are solutions of three-body Faddeev equations in configuration space for a strong interaction Hamiltonian satisfying the outgoing (incoming) boundary condition. The factor $i^{-l'+l}$ in this expression is introduced to match the R-matrix definition in the modified spherical harmonics convention [22] with the wave functions expressed in spherical harmonics convention. The matrix elements of the TVPC potential in spherical harmonics convention and the R matrix in modified spherical harmonics convention are antisymmetric under the exchange between initial and final states.

For calculations of wave functions, we used the jj coupling scheme instead of the $l\mathcal{S}$ coupling scheme. We can relate R-matrix elements in the $l\mathcal{S}$ coupling scheme to the jj coupling scheme using unitary transformation (see, for example, Ref. [23]):

$$|[l_y\otimes(s_k\otimes j_x)_{\mathcal{S}}]_{JJ_z}\rangle = \sum_{j_y}|[j_x\otimes(l_y\otimes s_k)_{j_y}]_{JJ_z}\rangle(-1)^{j_x+j_y-J}(-1)^{l_y+s_k+j_x+J} \times[(2j_y+1)(2\mathcal{S}+1)]^{\frac{1}{2}}\begin{Bmatrix} l_y & s_k & j_y\\ j_x & J & \mathcal{S}\end{Bmatrix}, \tag{6}$$

where j_x is a spin (total angular momentum of the target, $j_x = 1$ for the deuteron) and s_k is a spin of the projectile ($s_k = \frac{1}{2}$ for the neutron). In the $l\mathcal{S}$ coupling scheme spins of the projectile and the target are added, giving partial spin $\mathcal{S}$ to which relative projectile-target angular momentum l is added to obtain total angular momentum J of the system. On the contrary, in the jj coupling scheme, the relative projectile-target angular momentum l is added to the projectile spin s_k, giving intermediate angular momentum j before coupling it with target spin j_x in order to obtain total angular momentum J of the system.

III. TIME REVERSAL VIOLATING AND PARITY CONSERVING POTENTIALS

The most general form of the time reversal violating and parity conserving part of the nucleon-nucleon Hamiltonian in the first order of relative nucleon momentum can be written as [24]

$$\begin{aligned}H^{TP} &= [g_1(r)+g_2(r)\tau_1\cdot\tau_2+g_3(r)T^z_{12}+g_4(r)\tau_+]\hat{r}\cdot\frac{\bar{\boldsymbol{p}}}{m_N} + [g_5(r)+g_6(r)\tau_1\cdot\tau_2+g_7(r)T^z_{12}+g_8(r)\tau_+]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\hat{r}\cdot\frac{\bar{\boldsymbol{p}}}{m_N}\\ &+[g_9(r)+g_{10}(r)\tau_1\cdot\tau_2+g_{11}(r)T^z_{12}+g_{12}(r)\tau_+]\left(\hat{r}\cdot\boldsymbol{\sigma}_1\frac{\bar{\boldsymbol{p}}}{m_N}\cdot\boldsymbol{\sigma}_2+\hat{r}\cdot\boldsymbol{\sigma}_2\frac{\bar{\boldsymbol{p}}}{m_N}\cdot\boldsymbol{\sigma}_1-\frac{2}{3}\hat{r}\cdot\frac{\bar{\boldsymbol{p}}}{m_N}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)+[g_{13}(r)\\ &+g_{14}(r)\tau_1\cdot\tau_2+g_{15}(r)T^z_{12}+g_{16}(r)\tau_+]\left[\hat{r}\cdot\boldsymbol{\sigma}_1\hat{r}\cdot\boldsymbol{\sigma}_2\hat{r}\cdot\frac{\bar{\boldsymbol{p}}}{m_N}-\frac{1}{5}\left(\hat{r}\cdot\frac{\bar{\boldsymbol{p}}}{m_N}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\hat{r}\cdot\boldsymbol{\sigma}_1\frac{\bar{\boldsymbol{p}}}{m_N}\cdot\boldsymbol{\sigma}_2+\hat{r}\cdot\boldsymbol{\sigma}_2\frac{\bar{\boldsymbol{p}}}{m_N}\cdot\boldsymbol{\sigma}_1\right)\right]\\ &+g_{17}(r)\tau_-\hat{r}\cdot\left(\boldsymbol{\sigma}_\times\times\frac{\bar{\boldsymbol{p}}}{m_N}\right)+g_{18}(r)\tau^z_\times\hat{r}\cdot\left(\boldsymbol{\sigma}_-\times\frac{\bar{\boldsymbol{p}}}{m_N}\right),\end{aligned} \tag{7}$$

where the exact form of $g_i(r)$ depends on the details of a particular theory of TVPC.

One should note that pions, being spin-zero particles, do not contribute to TVPC on-shell interaction [25]. Therefore to describe TVPC nucleon-nucleon interactions in the meson-exchange-potential model by assuming *CPT* conservation, one should consider the contribution from heavier mesons: $\rho(770)$, $I^G(J^{PC}) = 1^+(1^{--})$, and $h_1(1170)$, $I^G(J^{PC}) = 0^-(1^{+-})$ (see, for example, Refs. [9,11,12] and references therein). For example, Lagrangians for ρ and h_1 are

$$\mathcal{L}^{st} = -g_\rho \bar{N}\left(\gamma_\mu \rho^{\mu,a} - \frac{\kappa_V}{2M}\sigma_{\mu\nu}\partial^\nu \rho^{\mu,a}\right)\tau^a N - g_h \bar{N}\gamma^\mu\gamma_5 h_\mu N, \tag{8}$$

$$\mathcal{L}^{TP} = -\frac{\bar{g}_\rho}{2m_N}\bar{N}\sigma^{\mu\nu}\epsilon^{3ab}\tau^a\partial_\nu\rho^b_\mu N + i\frac{\bar{g}_h}{2m_N}\bar{N}\sigma^{\mu\nu}\gamma_5\partial_\nu h_\mu N, \tag{9}$$

where we neglected terms such as $\bar{N}\gamma_5\partial^\mu h_\mu N$, which are small at low energy, and g and $\bar{g}$ represent strong and TVPC meson-nucleon couplings, respectively. Then, one can obtain TVPC potentials

$$V^{TP}_\rho = \frac{g_\rho\bar{g}_\rho m_\rho^2}{8\pi m_N} Y_1(m_\rho r)\tau^z_\times \hat{r}\cdot\left(\boldsymbol{\sigma}_- \times \frac{\bar{\boldsymbol{p}}}{m_N}\right),$$
$$V^{TP}_{h_1} = -\frac{g_h\bar{g}_h m_h^2}{2\pi m_N} Y_1(m_h r) \times\left(\boldsymbol{\sigma}_1\cdot\frac{\bar{\boldsymbol{p}}}{m_N}\boldsymbol{\sigma}_2\cdot\hat{r} + \boldsymbol{\sigma}_2\cdot\frac{\bar{\boldsymbol{p}}}{m_N}\boldsymbol{\sigma}_1\cdot\hat{r}\right), \tag{10}$$

where $Y_1(x) = (1+\frac{1}{x})\frac{e^{-x}}{x}$, $x_a = m_a r$. Comparing these potentials with Eq. (7), one can see that in this model, all $g_i(r)^{ME} = 0$, except for

$$g_5^{ME}(r) = \left(-\frac{4g_h\bar{g}_h}{3m_N}\right)\left(\frac{m_h^2}{4\pi}Y_1(m_h r)\right) = C^{TP}_{5,h} f^{TP}_{5,h}(r,\mu=m_h),$$
$$g_9^{ME}(r) = \left(-\frac{2g_h\bar{g}_h}{m_N}\right)\left(\frac{m_h^2}{4\pi}Y_1(m_h r)\right) = C^{TP}_{9,h} f^{TP}_{9,h}(r,\mu=m_h),$$
$$g_{18}^{ME}(r) = \left(\frac{g_\rho\bar{g}_\rho}{2m_N}\right)\left(\frac{m_\rho^2}{4\pi}Y_1(m_\rho r)\right) = C^{TP}_{18,\rho} f^{TP}_{18,\rho}(r,\mu=m_\rho), \tag{11}$$

where we introduced dimensional constants C_n^{TP} and scalar function $f_n^{TP}(\mu) = \frac{\mu^2}{4\pi}Y_1(\mu r)$, so that $g_n(r)$ can be written as

$$g_n(r) = \sum_a C^{TP}_{n,a} f^{TP}_{n,a}(r). \tag{12}$$

If we include isovector $J=1$ a_1 and b_1 mesons, whose masses are close to the value of the h_1 mass, functions g_6 and g_{10} will also contribute to the TVPC potential.

In the EFT approach we consider Eq. (7) as a leading order of the TVPC potential. The final result must not depend on the particular form of the $g_n(r)$ functions as long as they are localized, such as the δ function or its derivative. Therefore, using EFT, one can estimate the contribution of each term of the potential by substituting $g_n(r)$ functions by the corresponding products of low-energy constants (LECs) and Yukawa functions $Y_1(\mu r)$. The mass scale μ represents a regularization scale of EFT. For example, at low energy we can assume that $\mu \simeq m_\pi$ for the pionless EFT approach.

IV. RESULTS AND DISCUSSIONS

For the calculation of TRIV amplitudes in the DWBA approach, we used the nonperturbed (time reversal invariance conserving) three-body wave functions for neutron-deuteron scattering obtained by solving Faddeev equations in configuration space [26,27]. The detailed procedure for these calculations is described in Refs. [23,28]. As before, we employed the AV18 nucleon-nucleon potential in conjunction with UIX three-nucleon force. The obtained contribution of each TVPC operator from Eq. (7) to the matrix element of Eq. (5) is summarized in Tables I and II, which give the real and imaginary parts, respectively. The matrix elements are evaluated using the jj coupling scheme for the neutron-deuteron center of mass energy $E_{cm} = 100$ keV and the regularization scale set by $\mu = m_\rho$, which is equal to the mass of the lightest meson contributing to TVPC interaction. It should be noted that each matrix element presented in Tables I and II contains a sum of contributions from different Faddeev components of wave functions with a large number of partial waves. Therefore, the values of the matrix elements are strongly dependent on the detailed behavior of exact wave functions. However, despite the possibility of a numerical suppression of matrix elements for some operators, the calculated values are stable enough to be used for estimations of TRIV effects.

The contributions from each TVPC operator to the difference of scattering amplitudes $f_{+,-}$ are summarized in Table III, where we distinguish three columns representing results with a different choice of the characteristic mass scale for $g_i(r)$ functions. Thus, for example, the $\frac{\Delta f^\pi}{p}$ column corresponds to the description of the TVPC potential in pionless EFT. (As mentioned above, π-meson exchange cannot lead to TVPC interaction.)

It should be noted that in spite of the fact that all the results of the calculations are presented only for neutron energy $E_{cm} = 100$ keV, they can be easily extrapolated for any value of neutron energy below 1 MeV since they have a simple dependence on neutron energy E as

$$\mathrm{Re}\frac{\Delta f^{TP}}{p} \sim \sqrt{E}, \quad \mathrm{Im}\frac{\Delta f^{TP}}{p} \sim E. \tag{13}$$

This is because these TVPC observables are the result of a mixing of initial and final p waves, or s and d waves, by TVPC interactions in scattering amplitudes.

To have insight into the structure of TVPC scattering amplitudes, one can compare them with strong, PV, and TVPV amplitudes at the same energy $E_{cm} = 100$ keV ($p = 0.567 \times 10^{-1}$ fm^{-1}) and $\mu = m_\pi$, which corresponds to pionless EFT

TABLE I. Representative contribution of each TVPC potential term to the real part of the matrix element ($\mathrm{Re}\frac{\langle (l'j'),J|V^{TP}|(lj),J\rangle}{p^2}$). The results are presented using the jj coupling scheme for wave functions obtained using AV18 + UIX interaction at $E_{cm} = 100$ keV. For all operators a scalar function $\frac{m_\rho^2}{4\pi}Y_1(m_\rho r)$ has been used. Operators 3, 7, 11, and 15 are null due to the isospin selection rules. All data are in fm^2.

n	$\langle 2\frac{3}{2}\|v^{\frac{1}{2}}\|0\frac{1}{2}\rangle/p^2$	$\langle 1\frac{3}{2}\|v^{\frac{1}{2}}\|1\frac{1}{2}\rangle/p^2$	$\langle 2\frac{3}{2}\|v^{\frac{3}{2}}\|0\frac{1}{2}\rangle/p^2$	$\langle 1\frac{3}{2}\|v^{\frac{3}{2}}\|1\frac{1}{2}\rangle/p^2$	$\langle 2\frac{5}{2}\|v^{\frac{3}{2}}\|0\frac{1}{2}\rangle/p^2$
1	-0.278×10^{-06}	0.219×10^{-06}	-0.710×10^{-06}	-0.148×10^{-07}	0.307×10^{-05}
2	0.107×10^{-04}	0.876×10^{-05}	0.415×10^{-05}	-0.470×10^{-05}	-0.423×10^{-05}
4	-0.329×10^{-05}	-0.314×10^{-05}	-0.673×10^{-06}	0.158×10^{-05}	-0.166×10^{-05}
5	-0.108×10^{-04}	-0.939×10^{-05}	-0.216×10^{-05}	0.487×10^{-05}	-0.146×10^{-05}
6	0.664×10^{-07}	-0.371×10^{-06}	0.256×10^{-05}	-0.682×10^{-06}	-0.897×10^{-05}
8	0.108×10^{-04}	0.951×10^{-05}	0.131×10^{-05}	-0.465×10^{-05}	0.445×10^{-05}
9	0.609×10^{-04}	-0.407×10^{-04}	-0.223×10^{-03}	0.186×10^{-04}	-0.103×10^{-03}
10	-0.180×10^{-03}	0.122×10^{-03}	0.672×10^{-03}	-0.557×10^{-04}	0.308×10^{-03}
12	-0.823×10^{-06}	-0.140×10^{-06}	-0.992×10^{-06}	0.804×10^{-08}	0.749×10^{-07}
13	-0.490×10^{-05}	0.157×10^{-05}	0.133×10^{-04}	-0.602×10^{-06}	0.662×10^{-05}
14	0.149×10^{-04}	-0.468×10^{-05}	-0.401×10^{-04}	0.189×10^{-05}	-0.199×10^{-04}
16	-0.682×10^{-07}	-0.844×10^{-08}	0.248×10^{-07}	-0.266×10^{-07}	0.269×10^{-07}
17	0.182×10^{-06}	-0.434×10^{-05}	-0.355×10^{-06}	-0.784×10^{-05}	0.853×10^{-06}
18	0.547×10^{-06}	-0.384×10^{-05}	-0.407×10^{-06}	-0.700×10^{-05}	0.108×10^{-06}

potential. Then, the strong s-wave scattering amplitude f^{st} is

$$\frac{f^{st}}{p} = (72.2 + i24.8)\ \mathrm{fm}^2, \tag{14}$$

giving the total cross section $\sigma_{\rm tot} = \frac{4\pi}{p}\mathrm{Im}\, f^{st}(p) = 3.11$ b. The PV difference of scattering amplitudes in EFT is [23]

$$\frac{1}{m_N C_n^P}\frac{\Delta f^P(\mu = m_\pi)}{p} = [(-1.93\cdots 2.42) + i(-0.22\cdots 0.67)]\ \mathrm{fm}^2. \tag{15}$$

The difference of TRIV amplitudes with parity violation in EFT is [28]

$$\frac{1}{m_N C_n^{TP}}\frac{\Delta f^{TP}(\mu = m_\pi)}{p} = [(-1.63\cdots 0.66) + i(-0.063\cdots 0.22)]\ \mathrm{fm}^2, \tag{16}$$

and for TVPC ones, it is

$$\frac{1}{m_N C_n^{TP}}\frac{\Delta f^{TP}(\mu = m_\pi)}{p} = [(-0.01\cdots 0.03) + i(-0.0013\cdots 0.0004)]\ \mathrm{fm}^2. \tag{17}$$

Here C_n^P, C_n^{TP}, and C_n^{TP} are low-energy constants for PV, TVPV, and TVPC interactions, respectively, with a separated factor $1/m_N$ to match dimensions of the final result and retain the dimension of LECs C_n in [fm]. The range of values for the real and imaginary parts of PV and TRIV amplitudes is defined by a value of possible contribution from each PV, TVPV, or TVPC operator.

The aforementioned amplitudes follow the simple kinematic rule of the suppression $\sim(pR_{\rm nuc})$ for an additional p wave involved in PV and TVPV amplitudes and $\sim(pR_{\rm nuc})^2$ for two p waves or one d wave for the case of TVPC amplitudes. [Here $R_{\rm nuc}$ is an effective range of strong

TABLE II. Same as in Table I but for the imaginary part of the matrix element ($\mathrm{Im}\frac{\langle (l'j'),J|V^{TP}|(lj),J\rangle}{p^2}$).

n	$\langle 2\frac{3}{2}\|v^{\frac{1}{2}}\|0\frac{1}{2}\rangle/p^2$	$\langle 1\frac{3}{2}\|v^{\frac{1}{2}}\|1\frac{1}{2}\rangle/p^2$	$\langle 2\frac{3}{2}\|v^{\frac{3}{2}}\|0\frac{1}{2}\rangle/p^2$	$\langle 1\frac{3}{2}\|v^{\frac{3}{2}}\|1\frac{1}{2}\rangle/p^2$	$\langle 2\frac{5}{2}\|v^{\frac{3}{2}}\|0\frac{1}{2}\rangle/p^2$
1	-0.541×10^{-05}	-0.168×10^{-04}	-0.193×10^{-05}	0.909×10^{-06}	0.839×10^{-05}
2	0.209×10^{-03}	-0.672×10^{-03}	0.113×10^{-04}	0.288×10^{-03}	-0.115×10^{-04}
4	-0.642×10^{-04}	0.241×10^{-03}	-0.184×10^{-05}	-0.968×10^{-04}	-0.454×10^{-05}
5	-0.211×10^{-03}	0.720×10^{-03}	-0.590×10^{-05}	-0.298×10^{-03}	-0.399×10^{-05}
6	0.129×10^{-05}	0.284×10^{-04}	0.696×10^{-05}	0.417×10^{-04}	-0.245×10^{-04}
8	0.210×10^{-03}	-0.730×10^{-03}	0.358×10^{-05}	0.284×10^{-03}	0.122×10^{-04}
9	0.119×10^{-02}	0.312×10^{-02}	-0.609×10^{-03}	-0.114×10^{-02}	-0.281×10^{-03}
10	-0.351×10^{-02}	-0.939×10^{-02}	0.183×10^{-02}	0.341×10^{-02}	0.842×10^{-03}
12	-0.160×10^{-04}	0.107×10^{-04}	-0.271×10^{-05}	-0.493×10^{-06}	0.203×10^{-06}
13	-0.955×10^{-04}	-0.120×10^{-03}	0.364×10^{-04}	0.369×10^{-04}	0.181×10^{-04}
14	0.290×10^{-03}	0.359×10^{-03}	-0.109×10^{-03}	-0.116×10^{-03}	-0.545×10^{-04}
16	-0.133×10^{-05}	0.647×10^{-06}	0.678×10^{-07}	0.163×10^{-05}	0.734×10^{-07}
17	0.355×10^{-05}	0.333×10^{-03}	-0.967×10^{-06}	0.480×10^{-03}	0.233×10^{-05}
18	0.107×10^{-04}	0.295×10^{-03}	-0.111×10^{-05}	0.429×10^{-03}	0.294×10^{-06}

TABLE III. Difference of scattering amplitudes $(f_+ - f_-)/p$ for TVPC potential from each operator and mass scale at $E_{cm} = 100$ keV. Note that pion mass scale does not corresponds to the physical meson-exchange potential. Operators with $n = 3, 7, 11, 15$ are equal to zero due to isospin selection rules. All data are in fm^2.

n	$\frac{\Delta f^{\pi}}{p}$	$\frac{\Delta f^{\rho}}{p}$	$\frac{\Delta f^{h_1}}{p}$
1	$0.49 \times 10^{-5} - i0.25 \times 10^{-5}$	$0.82 \times 10^{-5} - i0.88 \times 10^{-6}$	$0.58 \times 10^{-5} - i0.55 \times 10^{-6}$
2	$0.46 \times 10^{-2} + i0.49 \times 10^{-4}$	$0.16 \times 10^{-3} + i0.64 \times 10^{-5}$	$0.37 \times 10^{-4} + i0.27 \times 10^{-5}$
4	$-0.15 \times 10^{-2} - i0.14 \times 10^{-4}$	$-0.62 \times 10^{-4} - i0.12 \times 10^{-5}$	$-0.18 \times 10^{-4} - i0.35 \times 10^{-6}$
5	$-0.50 \times 10^{-2} - i0.16 \times 10^{-3}$	$-0.18 \times 10^{-3} - i0.51 \times 10^{-5}$	$-0.49 \times 10^{-4} - i0.16 \times 10^{-5}$
6	$0.23 \times 10^{-2} - i0.13 \times 10^{-4}$	$-0.76 \times 10^{-5} + i0.25 \times 10^{-5}$	$-0.14 \times 10^{-4} + i0.16 \times 10^{-5}$
8	$0.43 \times 10^{-2} + i0.17 \times 10^{-3}$	$0.18 \times 10^{-3} + i0.42 \times 10^{-5}$	$0.54 \times 10^{-4} + i0.11 \times 10^{-5}$
9	$-0.43 \times 10^{-1} + i0.19 \times 10^{-2}$	$-0.14 \times 10^{-2} + i0.71 \times 10^{-4}$	$-0.43 \times 10^{-3} + i0.21 \times 10^{-4}$
10	$0.13 \times 10^{+0} - i0.62 \times 10^{-2}$	$0.44 \times 10^{-2} - i0.21 \times 10^{-3}$	$0.13 \times 10^{-2} - i0.65 \times 10^{-4}$
12	$0.56 \times 10^{-3} + i0.79 \times 10^{-4}$	$0.90 \times 10^{-6} - i0.11 \times 10^{-6}$	$0.11 \times 10^{-9} - i0.21 \times 10^{-7}$
13	$-0.99 \times 10^{-3} + i0.41 \times 10^{-4}$	$0.72 \times 10^{-4} - i0.50 \times 10^{-5}$	$0.25 \times 10^{-4} - i0.17 \times 10^{-5}$
14	$0.24 \times 10^{-2} - i0.92 \times 10^{-4}$	$-0.22 \times 10^{-3} + i0.15 \times 10^{-4}$	$-0.77 \times 10^{-4} + i0.51 \times 10^{-5}$
16	$0.18 \times 10^{-3} - i0.10 \times 10^{-4}$	$0.47 \times 10^{-6} - i0.27 \times 10^{-7}$	$0.30 \times 10^{-7} - i0.25 \times 10^{-8}$
17	$-0.19 \times 10^{-3} - i0.17 \times 10^{-4}$	$-0.78 \times 10^{-5} - i0.14 \times 10^{-7}$	$-0.23 \times 10^{-5} + i0.10 \times 10^{-8}$
18	$-0.26 \times 10^{-3} + i0.21 \times 10^{-4}$	$-0.88 \times 10^{-5} + i0.31 \times 10^{-6}$	$-0.22 \times 10^{-5} + i0.74 \times 10^{-7}$

interaction, which leads to $\sim(pR_{\rm nuc}) \sim 0.1$ for neutron energy $E_{cm} = 100$ keV.] In addition to this kinematic factor, TVPC scattering amplitudes are suppressed, as compared to PV or TVPV ones, by a factor $\frac{\bar{p}}{m_N} \sim 0.1$, which results from an extra momentum dependence of all operators in the TVPC potential. By increasing neutron energy, one can easily increase the kinematic factor up to 1. Then, the only suppression of TVPC matrix elements in the amplitude will be left due to $\frac{\bar{p}}{m_N} \sim 0.1$. It should be noted that this suppression factor is well known [8–10,18] for TVPC matrix elements in nuclei.

It is noteworthy that our calculations are in good agreement with results [7], obtained using zero-range force approximation for calculations of TVPC effects in n-d scattering. For example, using Eq. (8) of Ref. [7], one can obtain for $E_{cm} = 100$ keV

$$\frac{\Delta f^{TP}}{p} = g'(0.0004 + i0.0013)\ \text{fm}^2, \tag{18}$$

where g' is an unknown TVPC nucleon-nucleon coupling constant.

The results of Table III can also be used to express TVPC parameters in terms of the meson-exchange model. Since the TVPC meson-exchange model does not allow pion exchanges, the lightest mesons to be considered are ρ and h_1 mesons. Then, assuming only contributions from these mesons, one can obtain for $E_{cm} = 100$ keV

$$\begin{aligned}\Delta\sigma^{TP} &= 10^{-6}[g_h\bar{g}_h(-1.09) + g_\rho\bar{g}_\rho(4.20\times\ 10^{-3})]\ \text{b},\\ \frac{1}{N}\frac{d\phi^{TP}}{dz} &= -10^{-3}[g_h\bar{g}_h(1.24)\\ &\quad - g_\rho\bar{g}_\rho(5.81\times 10^{-3})]\ \text{rad fm}^2.\end{aligned} \tag{19}$$

Finally, we conclude that neutron-deuteron scattering is a promising process to improve current experimental constraints on TVPC interactions. The TVPC observables can be large enough to be measured at neutron energy of hundreds of keV due to strong energy dependence. On the other hand, they can be precisely calculated, providing the possibility to extract the TVPC nucleon coupling constants from the experiment.

ACKNOWLEDGMENTS

This work was supported by DOE Grant No. DE-FG02-09ER41621. This work was performed with access to the HPC resources of IDRIS granted under Allocation No. 2009-i2009056006 made by GENCI (Grand Equipement National de Calcul Intensif). We thank the staff members of IDRIS for their constant help.

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