Breakdown of partial conservation of axial current in diffractive neutrino interactions

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We test the hypothesis of partially conserved axial current in high-energy diffractive neutrino production of pions. Since the pion pole contribution to the Adler relation (AR) is forbidden by conservation of the lepton current, the heavier states, like the a_1 pole, ρ-π cut, etc., control the lifetime of the hadronic fluctuations of the neutrino. We evaluate the deviation from the AR in diffractive neutrino production of pions on proton and nuclear targets. At high energies, when all the relevant time scales considerably exceed the size of the target, the AR explicitly breaks down on an absorptive target, such as a heavy nucleus. In this regime, close to the black disk limit, the off-diagonal diffractive amplitudes vanish, while the diagonal one, $\pi \to \pi$, which enters the AR, maximizes and saturates the unitarity bound. At lower energies, in the regime of short lifetime of heavy hadronic fluctuations the AR is restored, i.e., it is not altered by the nuclear effects.

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I. INTRODUCTION

In the chiral limit of massless quarks, isovector components of both vector and axial quark currents are conserved. Though the hadrons acquire masses via the mechanism of spontaneous symmetry breaking, hadronic currents are still conserved. While it is rather straightforward for the vector current, conservation of the axial current looks nontrivial and is possible only due to the presence of a pseudoscalar term in the current, which has to be singular at $Q^2 = 0$ [1]. This singularity is associated with massless Goldstone particles [2] or pions, which appear due to spontaneous chiral symmetry breaking.

Beyond the chiral limit, the pions acquire a small mass and the axial current conservation is not exact, so one can consider a partial conservation of the axial current (PCAC),

$$\partial_\mu J^A_\mu = m_\pi^2 \, f_\pi \, \phi_\pi, \tag{1}$$

where m_π and $f_\pi \approx 0.93 m_\pi$ are the pion mass and decay coupling, and ϕ_π is the pion field.

A beautiful manifestation of PCAC is the Goldberger-Treiman relation [3], which bridges weak and strong interaction. It miraculously connects the pion decay constant with the pion-nucleon coupling, which seem to have very little in common. Indeed, the former depends on the pion wave function, while the latter is controlled by the wave function of the nucleon. Nevertheless, data on β decay and muon capture confirm this relation between very different physical quantities. Since this astonishing relation between the pion pole (suppressed in β decay due to conservation of the lepton current) and heavier states had no natural explanation, except PCAC, it was called the Goldberger-Treiman conspiracy [4] (see more below).

Another intensive source of axial current is high-energy neutrino interactions. In this case PCAC leads to the Adler relation (AR) between the cross sections of processes initiated by neutrinos and pions [5],

$$\left.\frac{d^2\sigma(\nu p \to lX)}{dQ^2\, d\nu}\right|_{Q^2=0} = \xi^2(E,\nu)\sigma(\pi p \to X). \tag{2}$$

Here the kinematic factor is

$$\xi^2(E,\nu) = \frac{G^2}{2\pi^2} f_\pi^2 \frac{E-\nu}{E\nu}; \tag{3}$$

E is the neutrino energy; $G = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the electroweak Fermi coupling; $Q^2 = -q_\mu^2$, where $q_\mu = k_\mu - k'_\mu$ and $\nu = E - E'$ are the four-momentum and energy transfer in the $\nu \to l$ transition (the same notation as for neutrinos should not cause confusion). For the sake of concreteness the target is the proton, but it may be any hadron or a nucleus.

A high-energy neutrino exposes hadronic properties interacting via its hadronic fluctuations [6]. Similar to the Goldberger-Treiman relation, the AR (2) should not be interpreted as pion pole dominance. Neutrino cannot fluctuate to a pion, $\nu \not\to \pi l$, because the pion pole in the dispersion relation in Q^2 for the axial current does not contribute to the interaction of the neutrino at high energies [4,7–9]. Indeed, the axial current $J^A_\mu(Q^2)$ can be presented as

$$J^A_\mu(Q^2) = \frac{q_\mu\, f_\pi}{Q^2 + m_\pi^2}\, T(\pi p \to X) + \frac{f_{a_1}}{Q^2 + m_{a_1}^2}\, T_\mu(a_1 p \to X) + \cdots. \tag{4}$$

Here the second and following terms represent the contributions of the a_1 meson and (implicitly) other heavier axial-vector states.

The first term in (4), corresponding to the pion pole, contains the factor q_μ, which then terminates its contribution to the cross section, Eq. (2). Indeed, the amplitude of the reaction is

$$A(\nu p \to lX) \propto L_\mu J^A_\mu, \tag{5}$$

where $L_\mu = \bar{l}(k')\gamma_\mu(1+\gamma_5)\nu(k)$ is the lepton current, which is transverse, i.e., $q_\mu L_\mu = 0$ (for simplicity hereafter we entirely neglect the lepton mass). Therefore, the pion term in (4) does not contribute to the amplitude Eq. (5), and this is true at any Q^2.

Thus, although it is tempting to interpret the AR Eq. (2) as a manifestation of the pion pole dominance, this is not correct. PCAC connects the contribution of heavy axial states [the second line in Eq. (4)] with the nonexistent pion contribution at $Q^2 = 0$ [4,7–9]. Such a fine tuning, which is very similar to the Goldberger-Treiman conspiracy, looks miraculous, and the PCAC hypothesis for neutrino interactions should be tested thoroughly.

A simple way to see in data whether light or heavy states dominate the dispersion relation for the axial current is to measure the Q^2 dependence of the neutrino cross section at small Q^2. Extrapolating the cross section Eq. (2) with the parametrization $(Q^2 + M_{\rm eff}^2)^{-2}$, one can find the position $Q^2 = -M_{\rm eff}^2$ of the essential singularity in the dispersion relation (4). It is easy to disentangle the effective masses that are as small as the pion mass and heavy singularities, like the ρ-π cut, a_1 meson, etc. Data clearly prefer the latter, $M_{\rm eff} \gtrsim 1$ GeV [9].

II. DIFFRACTIVE NEUTRINO PRODUCTION OF PIONS

This reaction offers probably the most stringent test of PCAC in neutrino interactions. Indeed, the analysis performed by Piketty and Stodolsky [8] revealed a potential problem related to the above dispersion representation for the AR. They made use of the relation between the pion pole and heavy-state contribution in Eq. (4) imposed by PCAC, complemented with few assumptions. That is, the assumed dominance of the axial-vector a_1 meson neglected the higher terms implicitly contributing in (4). It also assumed a smooth Q^2 dependence of the hadronic amplitudes in (4) and related the lepton coupling f_{a_1} to that for the ρ meson, relying on the Weinberg sum rules [10]. Eventually, they arrived at a relation between the elastic and diffractive pion-nucleon cross sections, $\sigma(\pi p \to a_1 p) \approx \sigma(\pi p \to \pi p)$. This relation strongly contradicts data: diffractive production of a_1 mesons is more than an order of magnitude suppressed compared with the elastic cross section.

This puzzle was solved by the authors of [9,11] by pointing out its shaky point, namely, the a_1 pole cannot dominate in the axial current, since it is quite a weak singularity compared to the ρ pole in the vector current. In fact, the main contribution to the expansion Eq. (4) comes from the ρ-π cut, related to diffractive pion excitations. The invariant mass distribution for diffractive $\pi \to 3\pi$ excitations peaks at $M_{3\pi} \approx 1.3$ GeV and is well explained by the so-called Deck mechanism [12] of diffractive excitation $\pi \to \rho\pi$. The interpretation of the observed peak has been a long-standing controversy, until a phase-shift amplitude analysis (see references in [13]) eventually revealed the presence of the very weak a_1 resonance having a similar mass. Moreover, it was found in [11] that even the contribution of the ρ-π cut in the dispersion relation for the diffractive amplitude has a Q^2 dependence similar to that of the a_1 pole. Summing up all diffractive excitations (excluding large invariant masses corresponding to the triple-Pomeron term), one concludes that the magnitudes of single-diffractive and elastic pion-proton cross section are indeed similar. This helps to resolve the Piketty-Stodolsky puzzle.

Based on these observations, in what follows we employ the simple two-channel model, replacing all heavy singularities contributing to the AR, by one effective pole a representing a_1, $\rho - \pi$, etc. We assume that

$$\sigma_{sd}^{\pi p}(\pi p \to ap) = \sigma_{\rm el}^{\pi p}, \tag{6}$$

and this allows the AR to hold. Notice that in applying the AR to neutrino production of the effective state a, $\nu + p \to l + a + p$, we should also conclude that

$$\sigma_{\rm tot}^{ap} = \sigma_{\rm tot}^{\pi p}. \tag{7}$$

We also assume the same impact parameter dependences of the elastic $\pi \to \pi$ and $a \to a$ and diffractive $\pi \to a$ amplitudes.

At this point we do not pursue a high accuracy of the dispersion approach, which needs much more model-dependent information about many singularities contributing to the AR. Our objective here is to highlight the importance of absorptive corrections that affect differently the diagonal and off-diagonal terms in the hadronic current (4), which results in an unavoidable breakdown of the AR. The proposed simple model, which may not be accurate numerically, provides an excellent playground for study of the effects of absorptive corrections, keeping the physics transparent, and also allows estimation of the magnitude of the absorptive corrections.

III. DIAGONAL VS OFF-DIAGONAL DIFFRACTION

The relation (6) between off-diagonal and diagonal diffractive cross sections cannot be universal and independent of energy and target. This can be understood within the general quantum-mechanical interpretation of diffraction [14–19]. A hadron has a composite structure, and its light-cone wave function consists of different hadronic components, the Fock states, which interact with the target differently, leading to a modification of their weights. Such a modified wave packet is no longer orthogonal to other hadrons, which makes possible production of new hadrons.

It turns out that the off-diagonal diffractive amplitudes can be expressed in terms of the diagonal ones. Let us consider two different sets of states, one consisting of the mass matrix eigenstates, $|h\rangle$, and the other of the states $|\alpha\rangle$, which are eigenstates of the interaction Hamiltonian, i.e., satisfy the condition $\hat{f}_{\rm el}|\alpha\rangle = f_\alpha\,|\alpha\rangle$, where $\hat{f}_{\rm el}$ is the elastic amplitude operator.

Both sets of states are assumed to be complete, so one of them can be expanded over the full basis of states in the alternative representation,

$$|h\rangle = \sum_{\alpha=1} C_\alpha^h\,|\alpha\rangle. \tag{8}$$

Because of the completeness and orthogonality of each set of these states, the coefficients C_α^h in (8) satisfy the relations

$$\langle h'|h\rangle = \sum_{\alpha=1} \left(C_\alpha^{h'}\right)^* C_\alpha^h = \delta_{hh'},$$
$$\langle \beta|\alpha\rangle = \sum_{h'} \left(C_\beta^{h'}\right)^* C_\alpha^{h'} = \delta_{\alpha\beta}. \tag{9}$$

The elastic and single-diffraction amplitudes can therefore be expressed via the eigenamplitudes as

$$f_{el}^{h\to h} = \sum_{\alpha=1} \left|C_\alpha^h\right|^2 f_\alpha, \qquad f_{sd}^{h\to h'} = \sum_{\alpha=1} \left(C_\alpha^{h'}\right)^* C_\alpha^h\, f_\alpha. \tag{10}$$

These relations show that diagonal and off-diagonal diffractive amplitudes are affected by the unitarity (or absorption) corrections quite differently. For instance, in the black disk limit, which is expected to be reached in the Froissart regime at very high energies (or in central collisions with a heavy nucleus), all the partial eigenamplitudes reach the unitarity bound, $\mathrm{Im}\, f_\alpha = 1$. Then, according to the completeness and orthogonality conditions Eqs. (10), the diffractive amplitudes in the black disk limit read

$$f_{\rm el}^{h\to h} \Rightarrow \sum_{\alpha=1} \left|C_\alpha^h\right|^2 = 1, \qquad f_{sd}^{h\to h'} \Rightarrow \sum_{\alpha=1} \left(C_\alpha^{h'}\right)^* C_\alpha^h = 0. \tag{11}$$

Off-diagonal diffraction is impossible within a black disk and can only happen on its periphery, $b \sim R$. Since in the Froissart regime the interaction radius rises with energy as $R \propto \ln(s)$, the elastic and diffractive cross sections, which are the amplitudes squared integrated over impact parameter, acquire a different energy dependence,

$$\sigma_{\rm el} \propto \ln^2(s), \qquad \sigma_{sd} \propto \ln(s), \tag{12}$$

i.e., $\sigma_{sd}/\sigma_{\rm el} \propto 1/\ln(s)$, apparently breaking relation (6). Similarly, off-diagonal diffraction on a heavy nucleus is also suppressed,

$$\frac{\sigma_{sd}^A}{\sigma_{\rm el}^A} \propto A^{-1/3}. \tag{13}$$

Thus, we conclude that the relation (6), which follows from AR, can be strongly broken either at high energies or on nuclear targets.

IV. ABSORPTIVE CORRECTIONS TO THE ADLER RELATION ON THE PROTON

As was argued above, the absorptive corrections break down the validity of Eq. (6). In order to estimate the magnitude of deviation from the AR on a proton target, we rely on the simple Regge model proposed in Sec. II, with two channels in the axial current, the pion pole and the effective axial-vector pole a representing the a_1 pole and other singularities producing the bump at $M \approx 1.3$ GeV in the invariant mass distribution of the 3π diffractive excitation of a pion. As a starting point, we assume that in the single-Pomeron approximation the AR holds, i.e., the diffractive and elastic amplitudes are equal and Eq. (6) holds. Now we introduce absorptive corrections related to the multiple-Pomeron exchanges in the initial and final states and see how much the relation (6) is broken for the output amplitudes. This can be considered as an estimate for the magnitude of deviation from the AR on a proton.

We rely on the same two-channel model for multi-Pomeron corrections. The unitarized elastic cross section reads

$$\sigma_{\rm el}^{\pi p} = \int d^2b \left[1 - e^{-\mathrm{Im}\, f_{\rm el}^{\pi p}(b)}\right]^2. \tag{14}$$

Although this expression looks like the conventional single-channel Glauber model, it remains unchanged within the two-channel model under consideration. For the sake of simplicity we neglect the real part of the amplitude.

Similarly, the cross section of the diffractive excitation $\pi p \to ap$, corrected for absorption, reads

$$\sigma_{sd}^{\pi p} = \int d^2b \left|f_{sd}^{\pi p}(b)\right|^2 \left[\frac{e^{-\mathrm{Im}\, f_{\rm el}^{\pi p}(b)} - e^{-\mathrm{Im}\, f_{\rm el}^{ap}(b)}}{\mathrm{Im}\, f_{\rm el}^{ap}(b) - \mathrm{Im}\, f_{\rm el}^{\pi p}(b)}\right]^2 \approx \int d^2b \left[\mathrm{Im}\, f_{\rm el}^{\pi p}(b)\right]^2 e^{-2\mathrm{Im}\, f_{\rm el}^{\pi p}(b)}. \tag{15}$$

In the last step here we used the relation (7).

We see that the unitarity corrections to the elastic cross section, Eq. (14), and the absorptive corrections to diffraction, Eq. (15), act in opposite directions: they enhance the diagonal but suppress the off-diagonal diffractive processes.

To estimate the magnitude of the difference between the cross sections (14) and (15) we employ the conventional Gaussian form of the impact parameter dependence for the input single-Pomeron elastic partial amplitude,

$$\mathrm{Im}\, f_{\rm el}^{\pi p}(b) = \frac{\sigma_{\rm tot}^{\pi p}}{4\pi\, B_{\rm el}^{\pi p}} \exp\left(-\frac{b^2}{2B_{\rm el}^{\pi p}}\right). \tag{16}$$

Then we can evaluate the correction factor $K_{\rm AR} = \sigma_{sd}^{\pi p}/\sigma_{\rm el}^{\pi p}$, which should be applied to the right-hand side of Eq. (2). We used (14), (15), and (16) with $\sigma_{\rm tot}^{\pi p} = 13.6\ \mathrm{mb} \times s^{0.08} + 19.24\ \mathrm{mb} \times s^{-0.458}$, $B_{\rm el}^{\pi p} = B_0 + 2\alpha'_{\rm IP} \ln s$, where s is in GeV^2, $B_0 = 6$ GeV^{-2}, $\alpha'_{\rm IP} = 0.25$ GeV^{-2}. The results are depicted in Fig. 1 as function of ν. We see that the absorptive corrections cause a deviation from the AR of about 30%, which is not a dramatic effect. This is because the π-p elastic amplitude is still far from the unitarity bound. However, at much higher energies (still unreachable in neutrino experiments) the correction factor is expected to drop significantly.

V. COHERENT NEUTRINO PRODUCTION OF PIONS ON NUCLEI

According to the conventional terminology, coherent production on nuclei is a process which leaves the nucleus intact. Correspondingly, in an incoherent process the nucleus is supposed to break up to fragments.

In what follows we assume the validity of the AR for a nucleon target (unless specified), in order to identify the net nuclear effects for the AR. This section is devoted to coherent diffractive pion production. The production amplitudes on different nucleons interfere, and the interference is enhanced by the condition that the nucleus remains in the ground state. Such effects of coherence can lead to substantial deviations from the AR and from simplified expectations, as is demonstrated below.

A. Important time scales

There are several length scales characterizing the coherence effects in diffractive neutrino scattering on nuclei. The first length scale is controlled by the longitudinal momentum transfer q_L^π in diffractive production of a pion by an axial current of energy ν and virtuality Q^2,

$$l_c^\pi = \frac{1}{q_L^\pi} = \frac{2\,\nu}{Q^2 + m_\pi^2}. \tag{17}$$

Within this distance the pion production amplitudes $\nu N \to l\pi N$ on different nucleons interfere and shadow each other. If the axial current virtuality is small, $Q^2 \sim m_\pi^2$, the coherence length l_c^π is rather long even at low energies. It reaches the size of a heavy nucleus at energies as low as several hundreds of MeV. Such an early onset of coherence is a peculiar feature of the axial current. It would be impossible for the vector current; even for real photoproduction of ρ mesons the onset of shadowing is delayed up to energies of several GeV. Notice that, in addition to diffractive neutrino production of pions, the early onset of shadowing also occurs for the total neutrino-nucleus cross section [20–22] at low Q^2.

Another, much shorter length scale corresponds to diffractive transitions between the axial current and heavy states, which are represented by the effective axial-vector meson a in our model,

$$l_c^a = \frac{1}{q_L^a} = \frac{2\,\nu}{Q^2 + m_a^2}. \tag{18}$$

This coherence length controls neutrino diffractive dissociation to heavy hadronic states, and also the energy dependence of absorptive corrections to the cross section of neutrino production of pions on nuclei (see below). At small $Q^2 \lesssim m_\pi^2$ it is two orders of magnitude shorter than l_c^π.

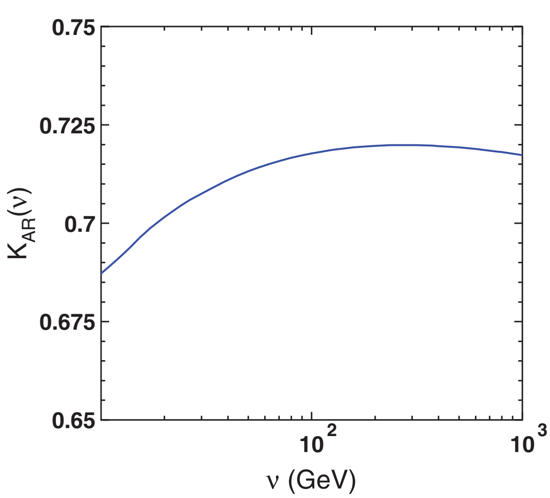

FIG. 1. (Color online) The absorptive correction factor for the Adler relation for diffractive neutrino production of pions on protons.

B. The amplitude

The process of coherent neutrino production of pions on nuclei, $\nu A \to l\pi A$, is possible only if $l_c^\pi \gtrsim R_A$, otherwise the nuclear form factor suppresses the cross section [11]. Even if l_c^π is long, the second length scale l_c^a might still be short. Correspondingly there are few energy regimes where the coherent length scales vary from very short up to much longer values than the nuclear radius. Thus the imaginary part of the partial amplitude of coherent production of a pion contains two terms,

$$M_{\nu A\to l\pi A}(\nu, Q^2, b) = M_1(\nu, Q^2, b) - M_2(\nu, Q^2, b), \tag{19}$$

where

$$\begin{aligned} &M_1(\nu, Q^2, b)\\ &\quad = M_{\nu N\to l\pi N}(\nu, Q^2)\int_{-\infty}^{\infty} dz\, e^{iq_L^\pi z}\rho_A(z,b)\, e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b,z)}, \end{aligned} \tag{20}$$

$$\begin{aligned} M_2(\nu, Q^2, b) = {}& M_{\nu N\to laN}(\nu, Q^2) M_{aN\to\pi N}(\nu)\int_{-\infty}^{\infty} dz\, e^{i(q_L^\pi - q_L^a)z}\\ &\times \rho_A(z,b)\, e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b,z)}\int_{-\infty}^{z} dz_1 e^{iq_L^a z_1}\\ &\times \rho_A(z_1,b)\, e^{-(1/2)\sigma_{\rm tot}^{aN}[T_A(b,z_1)-T_A(b,z)]}. \end{aligned} \tag{21}$$

Here the first term M_1 is the amplitude of pion production at the point with longitudinal coordinate z and impact parameter b, integrated over z weighted by the nuclear density ρ_A [11]. The z-dependent nuclear thickness function is defined as

$$T_A(b,z) = \int_z^{\infty} dz'\, \rho_A(b,z'), \tag{22}$$

and we denote $T_A(b) \equiv T_A(b, z \to -\infty)$.

The second term M_2 corresponds to diffractive production of the heavy state a preceding the pion production. This is the first-order Gribov inelastic shadowing correction [23] to the coherent pion production amplitude.

As long as the amplitude Eq. (19) is known, we can calculate the cross section of coherent pion production,

$$\frac{d\sigma(\nu A\to l\pi A)}{dQ^2\, d\nu\, d^2p_T} = \left|\int \frac{d^2b}{2\pi}\, e^{i\vec{p}_T\cdot\vec{b}} M_{\nu A\to l\pi A}(\nu, Q^2, b)\right|^2, \tag{23}$$

where $\vec{p}_T$ is the transverse momentum transfer to the target, and we neglect the real part of the amplitude.

The cross section on a nucleon target according to (16) has the form

$$\frac{d\sigma(\nu N\to l\pi N)}{dQ^2\, d\nu\, d^2p_T} = \frac{e^{-B_{\rm el}^{\pi p}p_T^2}}{(2\pi)^2}\left|M_{\nu N\to l\pi N}(\nu, Q^2)\right|^2. \tag{24}$$

C. Characteristic regimes in the energy dependence

In the general expressions Eq. (19)–(21) one can identify several characteristic regimes, which are controlled by the interplaying coherence scales l_c^π and l_c^a.

1. $l_c^\pi \lesssim R_A$

In this regime the AR on a nucleus trivially breaks down, as one can see from Eq. (20). The cross section is falling with decreasing l_c^π and vanishes at $l_c^\pi \ll R_A$, as calculated in [11]. The reason is obvious: the AR relation is supposed to hold at the pion pole at $Q^2 = -m_\pi^2$, and the extrapolation to $Q^2 = 0$ leads to a strong variation of the amplitude, if the longitudinal momentum transfer l_c^π is comparable with R_A, or shorter.

Notice that one should be cautious applying Eq. (20) at low energies where the neglected contribution of s-channel resonances and/or reggeons is important [24,25]. Also the neglected real part of the amplitude becomes large. Therefore, our calculations for this energy regime of $l_c^\pi \lesssim R_A$ only present an estimate of the effects related to the nuclear form factor [11].

2. $l_c^\pi \gg R_A$, $l_c^a \ll R_A$

Equations (20) and (21) are significantly simplified in this regime of very long l_c^π and very short lifetime l_c^a of the $\pi \to a$ fluctuations compared to the nuclear radius R_A. In this case the amplitude $M_2(\nu, Q^2, b)$, Eq. (21), is strongly suppressed by the oscillating exponential and can be neglected. At the same time, $l_c^\pi \gg R_A$; therefore the nonvanishing term in (19), the amplitude $M_1(\nu, Q^2, b)$, can also be simplified by integrating over z in Eq. (20) analytically,

$$M_1(\nu, Q^2, b) = \frac{2M_{\nu N\to l\pi N}(\nu, Q^2)}{\sigma_{\rm tot}^{\pi N}}\left[1 - e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b)}\right]. \tag{25}$$

At this point we concentrate on nuclear effects leading to breakdown of the AR; therefore hereafter we assume that the amplitude of pion neutrino production on a nucleon satisfies the AR Eq. (2),

$$M_{\nu N\to l\pi N}(\nu, Q^2 = 0) = \xi(E, \nu)\,\tfrac{1}{2}\sigma_{\rm tot}^{\pi N}(\nu), \tag{26}$$

where we employ the optical theorem, neglecting the real part of the diffractive amplitude. The amplitudes are normalized as $d\sigma/dQ^2 d\nu = |M_{\nu N\to l\pi N}(\nu, Q^2)|^2$. Applying (22) to (20) we get the relation

$$M_1(\nu, Q^2 = 0, b) = \xi(E, \nu)\left[1 - e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b)}\right], \tag{27}$$

which is exactly the AR for the partial amplitude of neutrino production of pions. We conclude that if the AR is correct for a pion production on a nucleon, it should also be correct for a nuclear target, provided that $l_c^\pi \gg R_A$, but $l_c^a \ll R_A$.

3. $l_c^a \gg R_A$

In this regime all the phase shifts in Eq. (19) can be neglected and the integration over z and z_1 can be performed analytically,

$$\begin{aligned} M_2(\nu, Q^2, b)\big|_{l_c^a\gg 1} &= M_{\nu N\to aN}(\nu, Q^2)\, M_{aN\to\pi N}(\nu)\,\frac{4}{\sigma_{\rm tot}^{aN}}\left\{\frac{1}{\sigma_{\rm tot}^{\pi N}}\left[1 - e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b)}\right] - \frac{e^{-(1/2)\sigma_{\rm tot}^{aN}T_A(b)} - e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b)}}{\sigma_{\rm tot}^{\pi N} - \sigma_{\rm tot}^{aN}}\right\} \\ &= M_{\nu N\to aN}(\nu, Q^2)\, M_{aN\to\pi N}(\nu)\,\frac{4}{(\sigma_{\rm tot}^{aN})^2}\left\{1 - \left[1 + \frac{1}{2}\sigma_{\rm tot}^{\pi N}T_A(b)\right]e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b)}\right\}. \end{aligned} \tag{28}$$

The first term in (19), the amplitude M_1, was already calculated in this limit ($l_c^a \gg R_A$) in (25). In both expressions the neutrino-production amplitudes $M_{\nu N\to hN}(\nu, Q^2)$ are related to the hadronic ones, $a + N \to h + N$, by the AR. In addition, based on the assumed dominance in the axial current of the effective pole a, we can extrapolate these relations to nonzero Q^2 with the pole propagator $(Q^2 + m_a^2)^{-1}$. Thus, we get new relations,

$$M_{\nu N\to laN}(\nu, Q^2) = \frac{\xi(\nu)\, m_a^2}{Q^2 + m_a^2}\, M_{aN\to aN}, \tag{29}$$

$$M_{\nu N\to l\pi N}(\nu, Q^2) = \frac{\xi(\nu)\, m_a^2}{Q^2 + m_a^2}\, M_{aN\to\pi N}. \tag{30}$$

We rely on these relations in what follows.

Eventually, summing the amplitudes Eqs. (25) and (28), we arrive at

$$\left.\frac{M_{\nu A\to l\pi A}(\nu, Q^2, b)}{M_{\nu N\to l\pi N}(\nu, Q^2)}\right|_{l_c^a\gg 1} = T_A(b)\, e^{-(1/2)\sigma_{\rm tot}^{\pi N}T_A(b)}. \tag{31}$$

One can observe the striking difference in the A dependence of the cross sections corresponding to the regimes of short and long coherence length l_c^a, Eqs. (25) and (31), respectively. The AR holds in the former case and the nuclear cross section behaves as $A^{2/3}$ (for very heavy nuclei). However, in the latter case the cross section is proprotional to $A^{1/3}$, so the AR breaks down. Notice that in this regime of $l_c^a \gg R_A$ the calculations [11] based on AR are not correct. Also, the results of [26] are not correct at any energy, since they rely on a wrong model for the pion-nucleus cross section (see discussion in [11]).

D. Numerical results

Such a nontrivial behavior of nuclear effects as a function of energy is confirmed by the results of numerical calculations of the p_T-integrated cross sections with Eqs. (19)–(21). The ratio

$$R^{\rm coh}_{A/N}(\nu, Q^2) = \frac{d\sigma(\nu A \to l\pi A)/dQ^2\, d\nu}{A\, d\sigma(\nu N \to l\pi N)/dQ^2\, d\nu} = \frac{4\pi B^{\pi N}_{\rm el}}{A} \int d^2b |M_{\nu A \to l\pi A}(\nu, Q^2, b)|^2 \quad (32)$$

is plotted in Fig. 2 for lead, aluminum, and carbon targets as a function of ν at $Q^2 = 0$.

For the sake of simplicity the calculations were performed with a constant cross section $\sigma^{\pi N}_{\rm tot} = 25$ mb.

We see that at low energies $\nu \lesssim 1$ GeV the nuclear ratio $R^{\rm coh}_{A/N}$, plotted by solid curves, rises with ν and saturates at the level corresponding to the AR applied to nuclei, depicted by dashed horizontal lines. Because the survival probability of a pion propagating along a long path length in the nuclear medium is low, the pion production points are pushed to the back surface of the nucleus [compare with Eq. (25)]. Therefore, the cross section depends on nuclear atomic number as $\sim A^{2/3}$ and coincides with the prediction of AR in the saturated regime of $q^\pi_c \ll 1/R_A$.

The observed strong deviation from the AR prediction at very low energies is a simple consequence of the suppression of the coherent cross section caused by the nuclear form factor due to finiteness of the momentum transfer, $q^\pi_c \sim 1/R_A$.

The energy dependence of the nuclear ratio forms a plateau from several hundreds of MeV up to several GeV in the regime described in Sec. V C2. It also agrees well with the prediction of the AR shown by dashed lines in Fig. 2.

At energies $\nu \gtrsim 10$ GeV the nuclear cross section considerably drops and saturates at a new level exposing a significant deviation from the expectations based on the AR, depicted by dashed curves. This happens due to the transition to the new regime of full coherence explained in Sec. V C3.

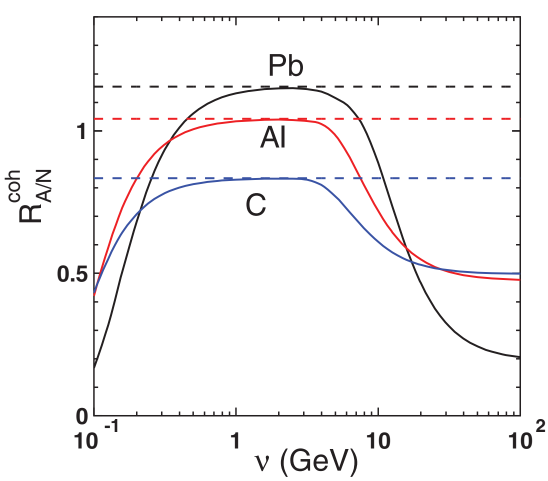

FIG. 2. (Color online) Solid curves: nuclear ratio $R^{\rm coh}_{A/N}(\nu, Q^2)$ of p_T-integrated cross sections of coherent neutrino production of pions, $\nu + A \to l + \pi + A$, calculated with Eq. (32) at $Q^2 = 0$. Dashed lines: the results of the Adler relation applied to nuclear targets, lead, aluminum, and carbon, from top to bottom.

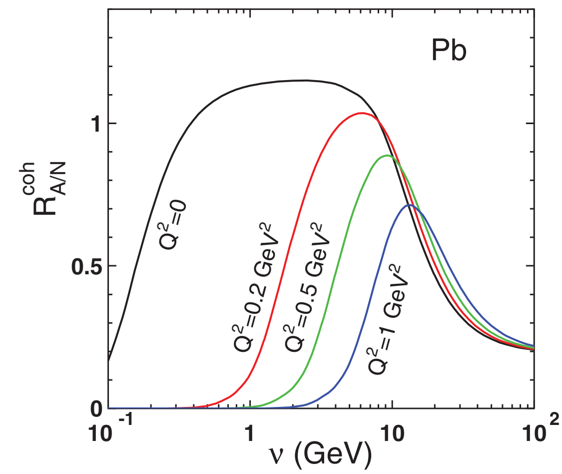

FIG. 3. (Color online) As Fig. 2 for lead at $Q^2 = 0,\ 0.2,\ 0.5$, and 1 GeV^2.

It worth commenting that the height of the plateaus for different nuclei shows that the A dependence of the cross section in this regime is slightly steeper than linear. This is different from the simple expectation of $R^{\rm coh}_{A/N} \propto A^{-1/3}$ corresponding to the black disk limit. This happens because the cross section $\sigma^{\pi N}_{\rm tot}$ is rather small and the pion-nucleus partial amplitude is still far from the unitarity bound. So the pion-nucleus elastic cross section is considerably smaller than πR^2_A. This is why it rises as A^α with $\alpha > 1$.

Thus, the cross section of diffractive coherent neutrino production of pions on nuclei exposes a peculiar energy dependence. It starts from zero at very small energies, then rises and saturates at a large magnitude, and eventually drops down to a value proportional to $A^{1/3}$ at higher energies. The AR relation is severely broken at the regimes of short $l^\pi_c \ll R_A$ and long $l^a_c \gg R_A$, but is rather accurate within the intermediate regime.

The specific energy dependence of nuclear effects presented in Fig. 2 at $Q^2 = 0$ drastically changes with rising Q^2. Indeed, the plateau in the energy dependence, which spans a wide energy range, is related to the significant difference between the length scales Eqs. (17) and (18), $l^\pi_c \gg l^a_c$. This holds, however, only for tiny values of $Q^2 \lesssim m^2_\pi$. With rising Q^2 both scales contract down to the same order of magnitude, and the plateau in the energy dependence of $R^{\rm coh}_{A/N}$ shrinks and becomes a peak. This is illustrated in Fig. 3 for neutrino production of pions on lead for few values of $Q^2 = 0,\ 0.2,\ 0.5$, and 1 GeV^2.

We do not extend our predictions to larger values of Q^2 for several reasons. First of all, at large Q^2 the effects of color transparency make the nuclear medium more transparent than we evaluated. These effects cannot be reproduced within the two-channel model employed. In hadronic representation, color transparency results from superposition of many singularities in the dispersion relation with masses up to $M^2 \sim Q^2$ [27,28]. In addition, one should take care of the correct (negative) signs of the off-diagonal diffractive amplitudes and provide a fine tuning between different amplitudes, which must essentially cancel each other at high Q^2, in order to end up with color transparency. This is a difficult task, which can be solved much more effectively within the dipole representation [27].

Another reason for not extending our calculation to larger values of Q^2 is the missed contributions of the transverse component of the axial current and of the vector current. Both vanish at $Q^2 \to 0$, but should be added and have growing importance with rising Q^2.

We presented numerical results for nuclear effects only for p_T-integrated cross sections, since their p_T dependence is rather simple and well known. The p_T distribution of coherent pion production forms a narrow peak at small p_T, with a slope of the order of $\frac{1}{3}R_A^2$, caused by the nuclear form factor. More accurately, the p_T dependence of the cross section is given by Eq. (23). The large p_T slope of the cross section is the signature of the coherent process; this is usually used to disentangle it from the incoherent background which has a much smaller slope, as in production on a free nucleon.

VI. INCOHERENT PION PRODUCTION

As a result of momentum transfer in diffractive neutrino production on a bound nucleon, the nucleus can be excited or break up to fragments, $\nu + A \to l + \pi + A^*$. Although pions diffractively produced at different impact parameters do not interfere in this process, characterized by rather large transverse momentum transfer, the amplitudes on bound nucleons with the same impact parameter do interfere. Evaluation of the cross section is more involved than in the case of coherent production, but can be simplified by summing up all nuclear final states and employing completeness. We perform calculations within the two-channel model for the axial current introduced earlier. The results are presented in the form of a nuclear ratio defined as in the case of coherent production, Eq. (32),

$$R^{\rm inc}_{A/N}(\nu, Q^2) = \frac{d\sigma(\nu A \to l\pi A^*)/dQ^2\, d\nu}{A\, d\sigma(\nu N \to l\pi N)/dQ^2\, d\nu}. \tag{33}$$

A. Effects of coherence for incoherent production

As in the case of coherent production, one can identify several contributions in the nuclear factor $R^{\rm inc}_{A/N}$, characterized via different mechanisms [29]:

$$R^{\rm inc}_{A/N} = R^{\rm inc}_1 + R^{\rm inc}_2 - R^{\rm inc}_3. \tag{34}$$

The three terms in the right-hand side of this equation correspond to the following mechanisms of incoherent pion production.

(i) The incoming neutrino does not interact in the nucleus up to the point with coordinates (b, z), where it diffractively produces the pion, $\nu + N \to l + \pi + N$, which survives propagating through the nucleus. The corresponding amplitude squared, summed over the final state of the nucleus, and integrated over coordinates of the bound nucleon has the form

$$R^{\rm inc}_1 = \frac{1}{A}\int d^2b \int_{-\infty}^{\infty} dz\, \rho_A(b,z)\, e^{-\sigma^{\pi N}_{\rm in} T_A(b,z)} = \frac{1}{A\sigma^{\pi N}_{\rm in}}\int d^2b \left[1 - e^{-\sigma^{\pi N}_{\rm in} T_A(b)}\right]. \tag{35}$$

(ii) Prior to the pion production the neutrino interacts with another bound nucleon at the point (b, z_1), and produces diffractively an a meson, $\nu + N \to l + a + N$, which is the effective state representing different products of diffractive excitations of a pion, as introduced in Sec. II. Then the a meson propagates further and produces a pion diffractively, $a + N \to \pi + N$ $(z > z_1)$. The corresponding term in the nuclear factor derived in [29] has the form

$$\begin{aligned} R^{\rm inc}_2 = {} & \frac{\sigma^{\pi N}_{\rm tot}}{2A\,\sigma^{\pi N}_{\rm el}}\left(\sigma^{\pi N}_{\rm in} - \sigma^{\pi N}_{\rm el}\right)\int d^2b \int_{-\infty}^{\infty} dz_1\, \rho_A(b, z_1) \\ & \times \int_{z_1}^{\infty} dz_2\, \rho_A(b, z_2)\cos\left[q^\pi_c(z_2 - z_1)\right] \\ & \times \exp\left[-\frac{1}{2}\left(\sigma^{\pi N}_{\rm in} - \sigma^{\pi N}_{\rm el}\right)T_A(b, z_2) - \frac{1}{2}\sigma^{\pi N}_{\rm tot}T_A(b, z_1)\right]. \end{aligned} \tag{36}$$

Here we fixed $\sigma^{aN}_{\rm tot} = \sigma^{\pi N}_{\rm tot}$, as follows from the AR in the two-channel model employed; z_1 and z_2 (36) are the longitudinal coordinates of diffractive neutrino production of the intermediate a meson in the two interfering amplitudes. The final pion is produced diffractively, $a + N \to \pi + N$, but incoherently, i.e., on the same nucleon, with coordinates (b, z) in both amplitudes.

(iii) In the first two terms of (34) we summed up all final states of the nucleus including the ground state. The latter corresponds to coherent pion production evaluated in the previous section, and should be subtracted. Thus,

$$R^{\rm inc}_3 = \frac{\left(\sigma^{\pi N}_{\rm tot}\right)^2}{4A\,\sigma^{\pi N}_{\rm el}}\int d^2b \left|\int_{-\infty}^{\infty} dz\, \rho_A(b,z)\, e^{iq^\pi_c z} e^{-(1/2)\sigma^{\pi N}_{\rm tot}\, T_A(b,z)}\right|^2. \tag{37}$$

As in the case of coherent production, one can identify three regimes of energy dependence of the incoherent cross section.

1. $l^\pi_c \lesssim R_A$, $l^a_c \ll R_A$

In the low-energy limit of $q^\pi_c \gg R_A$ only the first term in (34) survives and $R^{\rm inc}_{A/N}|_{q^\pi_c \gg R_A} = R^{\rm inc}_1$ given by Eq. (35).

At higher energies, when $q^\pi_c \to 0$, all integrations on longitudinal coordinates in (35)–(37) can be performed analytically,

$$R^{\rm inc}_{A/N}\big|_{q^\pi_c \to 0} = \int d^2b\, T_A(b) e^{-\sigma^{\pi N}_{\rm in} T_A(b)}. \tag{38}$$

This shows a considerable drop of the nuclear ratio from the low-energy limit given by Eq. (35) toward the high-energy limit. The interpolation between the two regimes is performed with the full expression Eqs. (32).

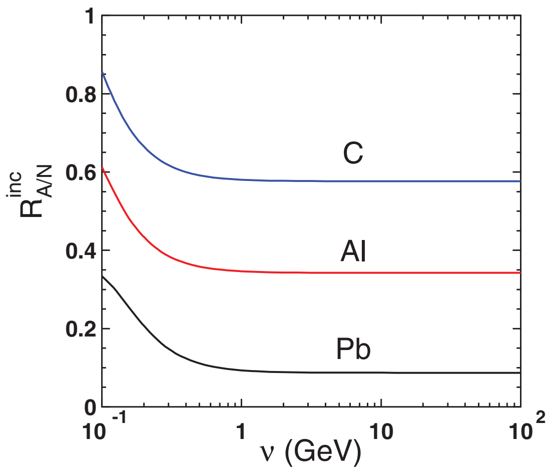

FIG. 4. (Color online) As Fig. 2 for incoherent pion production $\nu + A \to l + \pi + A^*$.

The numerical results at $Q^2 = 0$ for several nuclei depicted in Fig. 4 indeed demonstrate a considerable drop with energy of the nuclear ratio.

Notice that a similar behavior predicted in [29] for electroproduction of vector mesons was nicely confirmed later by the HERMES experiment [30] (see also [31]).

At large values of Q^2 the regime of short l_c^π propagates to higher energies, as is demonstrated in Fig. 5.

So far we have assumed that l_c^π may be short or long, but the second length scale l_c^a is always short. In this case, as for the coherent process in this regime, the AR is valid. Indeed, Eq. (38) is equivalent to the Glauber formula for the nuclear ratio in quasielastic pion scattering on a nucleus, i.e., is exactly what follows from the AR.

2. $l_c^a \gg R_A$

At higher energies l_c^a also becomes long, which leads to breakdown of the AR in the coherent process (see the previous section and Fig. 2). What happens in this case with incoherent

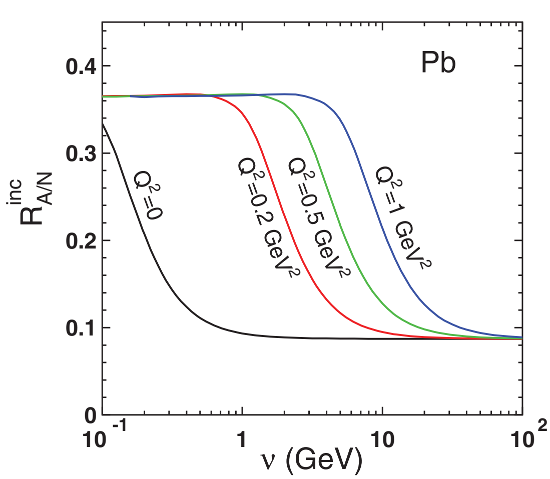

FIG. 5. (Color online) As Fig. 5 for lead at $Q^2 = 0,\ 0.2,\ 0.5,$ and 1 GeV^2.

pion production? In the asymptotic regime of $l_c^a \gg R_A$ the answer is easy,

$$R^{\rm inc}_{A/N}\Big|_{q_c^a \gg R_A} = \int d^2b\, \frac{e^{-\sigma_{\rm in}^{aN} T_A(b)} - e^{-\sigma_{\rm in}^{\pi N} T_A(b)}}{\sigma_{\rm in}^{\pi N} - \sigma_{\rm in}^{aN}}. \tag{39}$$

We have shown above, Eq. (7), that in the two-channel model under consideration the AR leads to the equality $\sigma_{\rm in}^{aN} = \sigma_{\rm in}^{\pi N}$. In this case Eq. (39) is equivalent to (38). Thus, we have arrived at the remarkable conclusion that in the case of incoherent neutrino production of pions on nuclear targets the AR is always correct.

VII. SUMMARY

At high energies neutrinos show hadronic properties similar to those of photons, since they also interact with a target via hadronic fluctuations. Although it is tempting to interpret the AR as pion dominance, the pion pole is excluded due to conservation of the leptonic current (for neutral current; otherwise it is suppressed by the lepton mass). In fact, the AR imposes a mysterious relation between the pion interaction with the target and the contribution of heavy axial states to the neutrino interaction. The former corresponds to elastic pion scattering in the process of diffractive neutrino production of pions, while the latter is related to off-diagonal diffraction of a pion, excluding elastic scattering. It is known that these two processes are subject to absorptive corrections which affect them quite differently, namely, they enhance diagonal diffraction (elastic scattering) but suppress inelastic diffraction. Therefore, the AR cannot be universal and target independent.

We checked the role of absorptive corrections for diffractive neutrino production of pions on protons and nuclei. Assuming that the AR holds on a proton target without absorptive corrections, we estimated the magnitude of deviation from AR at about 30% (see Fig. 1).

Much stronger effects were found on heavy nuclei. In coherent production of pions, $\nu + A \to l + \pi + A$, the AR holds with a good accuracy at energies $\nu \approx$ 1–10 GeV. However, it is severely broken at lower and higher energies (see Fig. 2). Our numerical results at low energies in the regime of $l_c^\pi \lesssim R_A$ are rather schematic, since we do not include the contribution of resonances and the large real part of the diffractive amplitudes.

For incoherent pion production, $\nu + A \to l + \pi + A^*$, when the nucleus decays into fragments, we found a considerable variation of nuclear effects with energy (see Fig. 4), similar to photoproduction of vector mesons. Remarkably, however, no deviations from the AR were detected, and it holds at all energies.

While the two-channel model employed may be numerically not very accurate, it allows a simplification of the calculation of the absorptive corrections and estimatation of the magnitude of deviations from the AR. In addition, explicit involvement of heavier singularities in the dispersion relation would lead to the appearance of many unknown parameters. An alternative description, which allows us to include all of them would be the light-cone color dipole representation [27]. The corresponding results will be presented elsewhere [32].

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