Shear viscosity to entropy density ratio in the Boltzmann-Uehling-Uhlenbeck model

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The ratio of shear viscosity (η) to entropy density (s) for an equilibrated system is investigated in intermediate-energy heavy-ion collisions below $100A$ MeV within the framework of the Boltzmann-Uehling-Uhlenbeck model. After the collision system almost reaches a local equilibration, the temperature, pressure and energy density are obtained from the phase-space information and η/s is calculated using the Green-Kubo formulas. The results show that η/s decreases with incident energy and tends toward a smaller value around 0.5, which is not so drastically different from the BNL Relativistic Heavy Ion Collider results in the present model.

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I. INTRODUCTION

Studying the behavior of nuclear matter under extreme conditions is one of the most important problems in heavy-ion collisions. Due to the van der Waals nature of the nucleon-nucleon interaction, it is expected that multifragmentation may exhibit features of liquid-gas phase transition (LGPT) in intermediate-energy heavy-ion collisions [1,2]. Evidence of this has been provided from various observables, such as the nuclear caloric curve, fluctuation, fragment mass distribution, moment analysis, etc. [3–10]. Recent progress on nuclear liquid-gas phase transition has been reviewed, especially for the signals of LGPT in theory and experiment [11,12].

Empirical observation of the temperature or incident energy dependence of the shear viscosity to entropy density ratio (η/s) for H_2O, He, and Ne_2 reveals a minimum in the vicinity of the critical point for phase transition [13]. Furthermore, a lower bound of $\eta/s > 1/4\pi$ obtained by Kovtun-Son-Starinets (KSS) in certain gauge theories is speculated to be valid for several substances in nature [14,15]. In ultrarelativistic heavy-ion collisions [16–21], people have used the shear viscosity to entropy density ratio to study the quark-gluon plasma phase and get the minimum value of η/s, so it is very interesting to study shear viscosity or η/s in intermediate-energy heavy-ion collisions [22–26]. Unlike the studies on η/s at relativistic energies, there have been very limited investigations in intermediate-energy heavy-ion collisions.

In this work, we study the thermodynamic and transport properties of nuclear reactions and try to see how the η/s evolves with the beam energy or temperature in a transport model. We study the equilibration of a nuclear system within a finite volume using the Boltzmann-Uehling-Uhlenbeck (BUU) model. To make the system contain enough number of nucleons in the fixed spherical volume, we choose the Au + Au system in a head-on collision ($b = 0$ fm). The system evolves with time, sufficiently long enough that it is in the freeze-out stage.

In the final stage of the central collisions, the system can be viewed as locally equilibrated. The equilibrium in intermediate-energy heavy-ion collisions can be judged by using the temperature and other dynamical variables [27]. After the system is in equilibrium, we calculate the thermodynamic parameters (pressure, energy density, and entropy density) from phase-space information of the system. The shear viscosity coefficient is calculated from stress tensor fluctuations around the equilibrium state using the Green-Kubo formula [25,28]. Finally, we compare η/s at different incident energies with different nuclear equations of state and discuss the results.

The rest of the paper is organized as follows: In Sec. II, we describe the situation of system equilibrium. In Sec. III, we calculate the viscosity coefficient and entropy density. Finally, a brief summary and outlook is made in Sec. IV.

II. EQUILIBRATION OF FINITE NUCLEON SYSTEM

We calculate the shear viscosity to entropy density ratio η/s of an equilibrated nuclear system in intermediate-energy heavy-ion collisions using the BUU model, which is a one-body microscopic transport model based upon the Boltzmann equation [29,30].

The BUU equation reads [31]

$$\frac{\partial f}{\partial t} + v\cdot\nabla_r f - \nabla_r U\cdot\nabla_p f = \frac{4}{(2\pi)^3}\int d^3p_2 d^3p_3 d\Omega \frac{d\sigma_{NN}}{d\Omega} V_{12}[f_3 f_4(1-f)(1-f_2) - f f_2(1-f_3)(1-f_4)]\delta^3(p+p_2-p_3-p_4). \quad (1)$$

It is solved with the method of Bertsch and Das Gupta [32]. In Eq. (1), $\frac{d\sigma_{NN}}{d\Omega}$ and V_{12} are the in-medium nucleon-nucleon cross section and the relative velocity for the colliding nucleons, respectively, and U is the mean-field potential including the isospin-dependent term

$$U(\rho,\tau_z) = a\left(\frac{\rho}{\rho_0}\right) + b\left(\frac{\rho}{\rho_0}\right)^{\sigma} + C_{\rm sym}\frac{(\rho_n-\rho_p)}{\rho_0}\tau_z, \quad (2)$$

where ρ_0 is the normal nuclear matter density; ρ, ρ_n, and ρ_p are the nucleon, neutron, and proton densities, respectively; and τ_z equals 1 or −1 for neutrons and protons, respectively. The coefficients a, b, and σ are parameters for the nuclear equation of state (EOS). Two sets of mean-field parameters are used in

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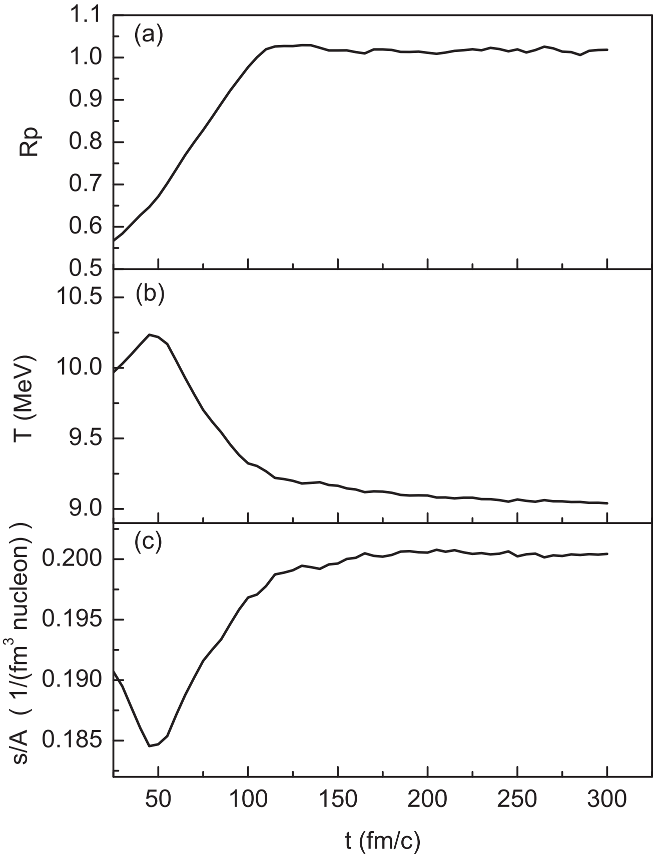

FIG. 1. R_p (a), temperature (b), and entropy density per nucleon (c) as a function of time (after 24 fm/c) for the head-on Au + Au collision within a 5-fm-radius sphere at 50A MeV.

this work, namely, the soft EOS with the compressibility K of 200 MeV ($a = -356$ MeV, $b = 303$ MeV, $\sigma = 7/6$) and the hard EOS with K of 380 MeV ($a = -124$ MeV, $b = 70.5$ MeV, $\sigma = 2$). $C_{\rm sym}$ is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium; here $C_{\rm sym} = 32$ MeV is used.

In this work, we focus on the thermodynamic and transport properties of a nuclear system. For this purpose, we investigate the process of the head-on Au + Au collision in a spherical volume with the radius of 5 fm. In the following calculation, we show the results with the hard EOS except in the case we mentioned.

First we check the evolution of the equilibration situation and temperature. The anisotropy ratio, which is a measure of the degree of equilibration reached in a heavy-ion reaction, is defined as

$$R_p = \frac{2}{\pi}\frac{R_{\parallel}}{R_{\perp}}, \tag{3}$$

where $R_{\parallel} = \langle\sqrt{p_x^2 + p_y^2}\rangle$ and $R_{\perp} = \langle\sqrt{p_z^2}\rangle$ are calculated by the momentum of nucleons in the given sphere. As an example, the time evolutions of R_p for Au + Au systems within a 5-fm-radius sphere at 50A MeV are shown in Fig. 1(a). When R_p approaches 1 at around 100 fm/c, the nuclear system is under equilibrium.

The time evolution of the temperature is also used to judge the state of equilibration. The temperature of the system can be derived from the momentum fluctuations of particles in the center-of-mass frame of the fragmenting source [33]. The variance σ^2 is obtained from the Q_z distribution through

$$\sigma^2 = \langle Q_z^2\rangle - \langle Q_z\rangle^2, \tag{4}$$

where Q_z is the quadruple moment, which is defined by $Q_z = 2p_z^2 - p_x^2 - p_y^2$, and p_x, p_y, and p_z are three components of momentum vector extracted from the phase space of the BUU model. If the mean equals zero, the second term vanishes. Q_z^2 is described by

$$\langle Q_z^2\rangle = \int d^3p\left(2p_z^2 - p_x^2 - p_y^2\right)^2 f(p). \tag{5}$$

Assuming a Maxwellian distribution for the momentum distribution, i.e.,

$$f(p) = \frac{1}{(2\pi mT)^{3/2}} e^{-\frac{p_x^2+p_y^2+p_z^2}{2mT}}, \tag{6}$$

we can obtain

$$\langle Q_z^2\rangle = 4m^2A^2T^2 \tag{7}$$

after the Gaussian integral, where m is the mass of a nucleon and A is the mass number of the fragment. For a nucleonic system, we have $A = 1$ and can calculate the evolution of the temperature using this equation. Figure 1(b) shows the temperature's evolution after 25 fm/c. It is seen that temperature reaches a maximum around 50 fm/c when the system is in the most compressible stage and then it starts to cool down when the system expands; later on the system tends toward thermodynamic equilibrium. For an equilibrated system, the kinetic energy distributions approach the Boltzmann distribution as time increases [34]. After the expansion process, the system will approach an equilibrate state, so we can then investigate the viscosity coefficient and entropy density in the system.

Except for temperature, other thermodynamic variables can be calculated during heavy-ion collisions. Energy density inside a volume with the 5-fm radius can be defined as

$$\varepsilon = \frac{1}{V}\sum_{r_i<r_0} E_i, \tag{8}$$

where E_i is $\sqrt{p_i^2 + m_i^2}$, r_i is the position of the ith nucleon in the center of mass, and r_0 is the selected radius (here we set $r_0 = 5$ fm), and pressure can be defined as

$$P = \frac{1}{3V}\sum_{r_i<r_0}\frac{p_i^2}{E_i}. \tag{9}$$

After we get the energy density, pressure, and temperature, entropy density can be calculated by the Gibbs formula

$$s = \frac{\varepsilon + P - \mu_n\rho}{T}, \tag{10}$$

where μ_n is the nucleon chemical potential and ρ is nucleon density of system within the given sphere. In principal, once we have the temperature T and $f(p)$, we can fit to a Fermi-Dirac function to extract the chemical potential. However, we can

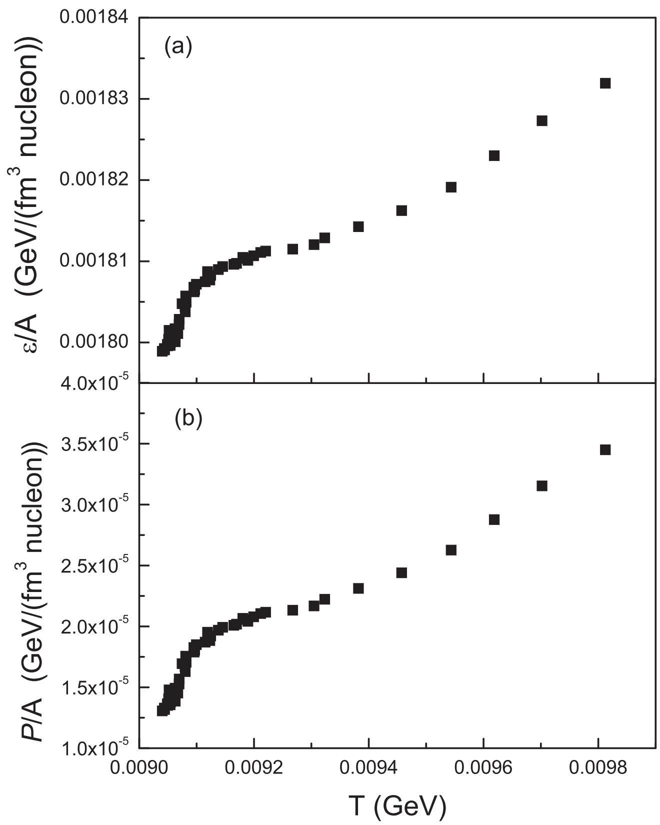

FIG. 2. Energy density per nucleon (a) and pressure per nucleon (b) as a function of temperature for the head-on Au + Au collision within a 5-fm-radius sphere at 50A MeV.

assume, for simplicity, zero nucleon chemical potential, or μ_n can be taken around 20 MeV in the present calculation [35]. In the following calculations of entropy and η/s, we only show the results with $\mu_n = 20$ MeV. But we have also checked the results with zero chemical potential; this will increase the entropy about 8% and then lead to the decreasing of η/s about 8%. However, this does not change our conclusions of this work. Figure 1(c) shows the entropy density per nucleon (s/A) evolves with time after 25 fm/c. It seems the entropy density per nucleon reaches a minimum when the system is in the most compressed stage and rises up in the expansion phase; it finally reaches an asymptotic value. From the viewpoint of phase space, the number of occupied states is most limited in the high density phase. In the high density phase, Fermi blocking forces some of the nucleons into higher momentum states but still in a spatially confined region, which makes the entropy density the minimum. When the system expands, the Fermi blocking is reduced, the system cools, but more coordinate volume is occupied. Therefore it is possible that this leads to a non-isentropic expansion until the system is in the freeze-out stage.

Energy density and temperature can be calculated at different times when the reaction is going on. Figure 2(a) shows energy density per nucleon versus temperature for the studied system after 25 fm/c. Similarly, pressure per nucleon increases with temperature are shown in Fig. 2(b). From the figure, we can see that both energy density and pressure increase with temperature. In a given volume, the increasing of temperature reflects a stronger thermal motion of nucleons; therefore the kinetic energy will contribute more to energy density and pressure.

III. VISCOSITY COEFFICIENT AND ENTROPY DENSITY

Now let's move on to the investigation of transport coefficients of the Au + Au system in a given volume by the BUU model. Viscosity is one of the transport coefficients that characterize the dynamical fluctuation of dissipative fluxes in a medium. Transport coefficients can be measured, as in the case of condensed matter applications. Also, they should be, in principle, calculable from the first principle. Monte-Carlo simulation for transport coefficients is a powerful tool when studying transport coefficients using Green-Kubo relations [36,37]. In high-energy heavy-ion collisions, the calculation of transport coefficients of shear viscosity for a binary mixture [38] and the calculation of coefficients of a hadrons gas have been studied [34,39]. The situation of nuclear gas in intermediate-energy heavy-ion collisions is similar to that of hadron gas. To study the extended irreversible dynamic processes, we use the Kubo fluctuation theory to extract transport coefficients [28]. The formula relates linear transport coefficients to near-equilibrium correlations of dissipative fluxes and treats dissipative fluxes as perturbations to local thermal equilibrium. The Green-Kubo formula for shear viscosity is defined by

$$\eta = \frac{1}{T}\int d^3r \int_0^{\infty} dt \langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t)\rangle, \tag{11}$$

where T is the equilibrium temperature of the system, t is the postequilibration time (the above formula defines $t = 0$ as the time the system equilibrates and is determined by equilibrium time), and $\langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t)\rangle$ is the shear component of the energy momentum tensor. The expression for the energy momentum tensor is defined by $\pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij}T_i^i$, and the momentum tensor reads [34]

$$T_{ij}(r,t) = \int d^3p \frac{p^i p^j}{p^0} f(x,p,t), \tag{12}$$

where $f(x,p,t)$ is the phase-space density of the particles in the system. To compute an integral, we assume that nucleons are uniformly distributed in the space. Meanwhile, the isolated spherical volume with the radius of 5 fm is fixed, so the viscosity becomes

$$\eta = \frac{V}{T}\langle \pi_{ij}(0)^2\rangle \tau_\pi, \tag{13}$$

where τ_π is calculated by

$$\langle \pi_{ij}(0)\pi_{ij}(t)\rangle \propto \exp\left(-\frac{1}{\tau_\pi}\right). \tag{14}$$

As shown in Fig. 3(a), $\langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t)\rangle$ is plotted as a function of time for Au + Au at 50A MeV. The correlation function is damped exponentially with time and can be fitted by Eq. (14) to extract the inverse slope correspondence as the relaxation time. Figure 3(b) summarizes the relaxation time decrease as

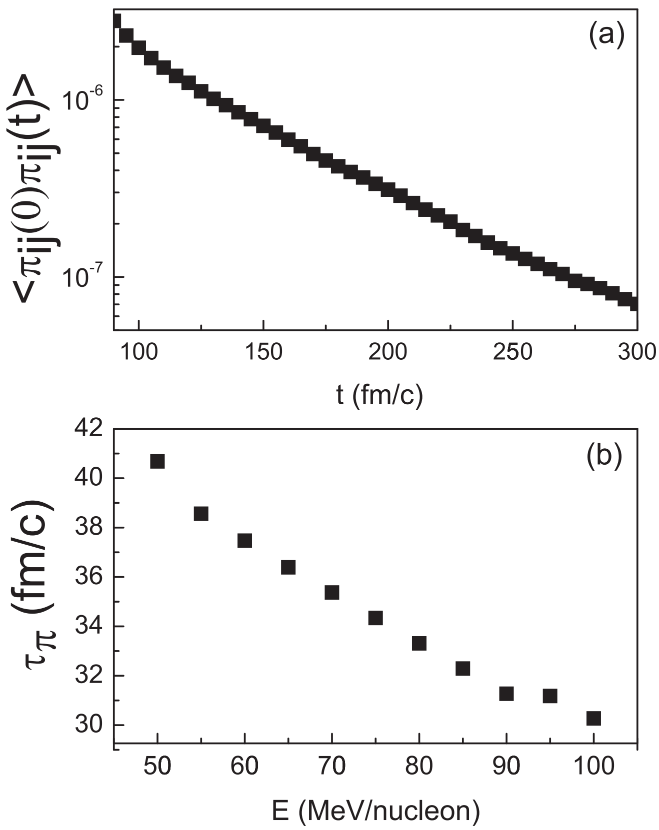

FIG. 3. (a) $\langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t)\rangle$ evolves with time for the head-on Au + Au collision in a given 5-fm-radius volume at 50A MeV; (b) Relaxation time as a function of incident energy for the head-on Au + Au collision in a given 5-fm-radius volume.

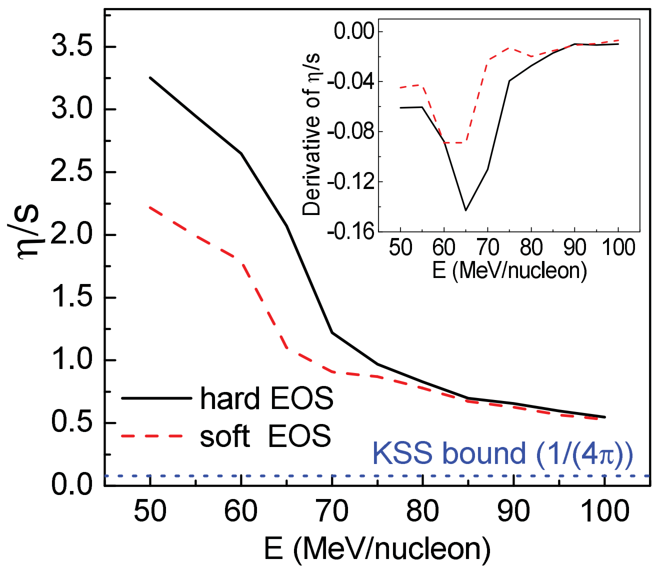

FIG. 4. (Color online) η/s as a function of beam energy for the head-on Au + Au collision in a spherical volume with radius of 5 fm. The inset shows the derivative of η/s versus beam energy.

the incident energy increases, indicating that the system can approach equilibration faster at higher incident energy.

Using the above method, in Fig. 4 we present the value of η/s as a function of incident energy after the studied system has been in equilibrium. The two sets of nuclear equations of state are used. The η/s value shows a rapid fall with the increasing of incident energy up to $E < 70A$ MeV and then drops slowly to a value close 0.5 when $E > 70A$ MeV. Because the BUU equation is one-body theory, fragmentation that originates from the fluctuation and correlation cannot be treated in the present model. In this case, the phase transition behavior cannot be predicted in the BUU model. The continuous drop of the ratio of shear viscosity to entropy density does not show a minimum at a certain beam energy, which indicates no obvious phase change or critical behavior in the present model. This is a shortcoming of the BUU model itself, especially when it is applied to higher beam energy. Actually, when we calculate the differential values of η/s versus the beam energy (see the inset of Fig. 4), there seems to be a turning point around $E \sim 65A$ MeV. This turning point could indicate a change in the dynamical behavior of the system; in other words, other mechanisms may be needed to be taken into account in the model especially at higher beam energies, e.g., multifragmentation [40]. Alternatively, a lack of minimum η/s perhaps shows that behavior with a local minimum of η/s at phase transition temperature might not be universal [41]. In the present BUU calculation, all calculated values of η/s are well above the conjectured KSS lower bound of $1/4\pi$ [14,15]. Comparing these values of η/s for our finite nuclei in the BUU model, we see they are not drastically different either from the BNL Relativistic Heavy Ion Collider (RHIC) results [17,42] or from the results of the usual finite nuclei at low temperature from the widths of giant vibrational states in nuclei [26]. As pointed out in Ref. [26], it is possible that the strong fluidity is a characteristic feature of the strong interaction of the many-body nuclear systems in general and not just of the state created in the relativistic collisions. Another interesting point from Fig. 4 is that η/s shows EOS sensitivity at lower beam energy: the hard EOS displays larger η/s than the soft one; i.e., larger compressibility of nuclear matter can lead to higher η/s values.

IV. SUMMARY

In summary, we studied thermodynamic variables as well as viscosity and entropy density for heavy-ion collision after the system tends toward equilibrium in intermediate-energy heavy-ion collisions in the framework of the BUU model. The Green-Kubo relation has been applied for nucleonic matter in a central region with a moderate volume when the system has been in the equilibrium stage for central heavy-ion collisions of Au + Au. It is found that the ratio of shear viscosity to entropy density η/s decreases very quickly before 70A MeV and then drops slowly toward a smaller value of η/s around 0.5 at higher beam energy in the Au + Au system. The η/s values are not drastically different either from the RHIC results or from the results of the usual finite nuclei at low temperature. However, no obvious minimum η/s value occurs within the investigated energy range. This may reflect that no liquid-gas phase transition behavior is displayed in the present model due to the shortcoming of the model itself, which lacks dynamical fluctuation and correlation. Relating the shortcoming, the equilibrium temperature could be a little higher than other models (QMD, SMM) which consider fragment formation [43] and where the shear viscosity and entropy density might be influenced by the cluster formation. Therefore, other models that can incorporate a liquid-gas phase transition should be checked for shear viscosity and entropy density. For instance, it will be very interesting to use a quantum-molecular-dynamics-type model to check if a minimum of η/s will occur around the liquid-gas phase transition. Work along this direction is in progress. Of course, experimental studies on shear viscosity are more important to demonstrate the relation of η/s and the liquid-gas transition point.

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