$(sd)^2$ states or superclusters in ^{10}Be

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(Received 19 April 2011; published 9 August 2011)

A set of states in ^{10}Be have very large α widths and very small neutron strengths. We review the data and investigate whether they are $(sd)^2$ states and/or α clusters.

DOI: 10.1103/PhysRevC.84.024304 PACS number(s): 21.10.Jx, 21.10.Tg, 21.60.Gx, 27.20.+n

I. INTRODUCTION

A state at 10.15 MeV in ^{10}Be has been observed [1] as a resonance in ^{6}He + α elastic scattering and assigned $J^{\pi} = 4^{+}$. This assignment appears firm, as it comes from an angular distribution of two spin-zero objects—about as model independent as it gets. An earlier assignment [2] of 3^{-} to a state at nearly the same energy was based on a fit to an angular correlation in the reaction ^{7}Li(^{7}Li,α^{6}He)α, but that assignment was model dependent—involving assumptions about the reaction mechanism and *m*-state population. Given the tendency for ^{10}Be to exhibit close-lying doublets of opposite parity at lower excitation energy, it would not be surprising if both 3^{-} and 4^{+} states are present. In any event, we take the 4^{+}assignment as definitive.

The 0^{+} and 2^{+} states [3] at 6.179 and 7.542 MeV, respectively, in ^{10}Be are excellent candidates for the two lowest $(sd)^2$ states coupled to ^{8}Be(g.s.). Such states are now clearly known in ^{14,16}C [4] and ^{12}Be [5], where they were strongly populated in the (*t*,*p*) reaction. Of course, because ^{8}Be is unstable, that reaction is not possible for these states of ^{10}Be.

These 0^{+} and 2^{+} states, together with the 4^{+} at 10.15(2) MeV, have been frequently suggested as members of a rotational band, possessing very large α-particle strengths. Several theoretical [6–12] and experimental [1,2], [13–24] papers have dealt with this supposed band. It is not at all obvious that the two descriptions are incompatible. Because ^{6}He is well described as two proton holes in ^{8}Be(g.s.), states of ^{10}Be with large $2n$ strengths could easily have large overlaps with ^{6}He + α. In the present paper, we examine these three states (and others) in detail and compare their properties with those expected in the two descriptions. We also suggest additional experiments. But first we summarize the experimental situation for the relevant states.

II. A BRIEF HISTORY

Hamada *et al*. [13] used the ^{7}Li(α,*p*) reaction at 65 MeV to investigate cluster strengths in ^{10}Be. They analyzed angular distributions with zero-range distorted-wave Born-approximation calculations (ZRDWBA) and extracted relative cluster spectroscopic factors $R = S/S_{\rm g.s.}$. They suggested a state at 11.76 MeV as the 4^{+} member of the ground-state (g.s.) band. They noted *R*'s of 0.067 and 0.049 for 2^{+} and 4^{+} members of this band, but stated that the small values of *R* might be misleading. They stated that they were unable to locate the 2^{+} and 4^{+} members of the 0_2^{+} band. However, the 7.54-MeV 2^{+} state was stronger that the lowest 2^{+} state, and the 0^{+} state at 6.18 MeV had almost the same cross section as the g.s. Use of the same number of oscillator quanta for the two 0^{+} states gave $R = 0.86$ for 6.18. They suggested 2^{+} for a state at 9.64(10) MeV, which they identified with the compiled state at 9.4 MeV. A state at 10.2 MeV is not in their table, but comparing its angular distribution (their Fig. 6) with that of the proposed 4^{+} at 11.76 MeV (their Fig. 4), they appear nearly identical in magnitude and shape. However, in their spectrum (their Fig. 2), the 10.2-MeV state is much weaker than 11.76. Furthermore, they show an $L = 2$ curve for the 10.2-MeV state, whereas 4^{+} would correspond to $L = 3$.

Soic *et al*. [14] used the reaction ^{7}Li(^{7}Li,α^{6}He), at a bombarding energy of 8 MeV, to observe $\alpha + {}^{6}$He decays of ^{10}Be* states at 9.6 and 10.2 MeV. They suggested the latter might be in the 0_2^{+} band. No neutron decay was observed. They gave a width of <400 keV. They noted the close similarity in angular distributions for 10.2 and 11.8 MeV energy in the (α,*p*) reaction. [13]

Milin *et al*. [15] studied the reaction ^{6}He(^{6}Li,*d*)^{10}Be*, using a ^{6}He beam of energy 17.0 MeV, and analyzed their data with the finite-range DWBA. The 2^{+} state at 7.54 MeV was not resolved from the nearby 3^{-}, but they extracted a ratio of $S(2_3^{+} + 3_1^{-})/S(2_1^{+})$ of about 3, suggesting that at least one member of the doublet was strong.

Freer *et al*. [16] used the reaction ^{12}C(^{12}Be,^{10}Be*)^{14}C at 378 MeV to populate states of ^{10}Be. They did not observe the 10.2-MeV state, and suggested the state at 10.57 MeV as the 4^{+} member of the g.s. band.

Curtis *et al*. [2] populated states of ^{10}Be with the reactions Li(^{7}Li,^{10}Be*) at 34.5 MeV. Their results were consistent with 2^{+} for the 9.6-MeV state and they assigned 3^{-} for the 10.15 MeV state. They stated that no 4^{+} state was found. They gave a width of 296(15) keV for the 10.15 MeV state.

Liendo *et al*. [17], with ^{7}Li + ^{7}Li at 34 MeV, reported the α decay of the 2^{+} state at 7.54 MeV with a branching ratio (BR) of $\Gamma_{\alpha}/\Gamma = 3.5(12) \times 10^{-3}$, implying a very large α-cluster strength. They speculated that the 10.57-MeV state was the 4^{+} member of the 0_2^{+} band, and the 11.2-MeV state was the 4^{+} member of the g.s. band.

Ashwood *et al.* [18] investigated ^{10}Be with the one-neutron transfer reaction ^{9}Be(^{9}Be,^{8}Be)^{10}Be at 48 MeV. They stated that no 4^+ member of the g.s. band was seen, and concluded that the g.s. band terminates at 2^+. They suggested that the 9.4-MeV state in the literature had been misidentified. Their state at 9.58 MeV was in agreement with previous work.

Miljanic [19] concluded, on the basis of accumulated evidence, that the 9.5-MeV state was not 3^+, but rather a 2^+ *p*-shell state.

In the ^{12}C(^{10}Be,^{10}Be*) inelastic scattering, at 302 MeV, Ahmed, *et al.* [20] saw the states at 9.6(1) and 10.2(1) MeV, but found no evidence for the 4^+ member of the g.s. band, causing them to conclude that the ground and 2_1^+ states are not cluster states.

Curtis *et al.* [21], with ^{7}Li(^{7}Li,^{10}Be*) at 58 MeV, identified the 10.15-MeV state as 3^- and preferred 6^+ over (4^+) for 11.76. They stated they did not see the 4^+ member of the g.s. band.

Milin *et al.* [22] used an 18-MeV ^{6}He beam to investigate the ^{6}He(^{6}Li,d)^{10}Be* reaction. They suggested that the strong 10.2-MeV state is the 4^+ member of the band beginning at 6.18 MeV. They gave a limit for the BR of the 7.54-MeV state as $>2.0(6) \times 10^{-3}$, consistent with the value of Ref. [17].

Freer *et al.* [1], with $^{6}\mathrm{He} + {}^{4}\mathrm{He}$ resonance elastic scattering, obtained a firm assignment of 4^+ for the state at 10.15 MeV. Their Breit-Wigner fit gave a g.s. α width of $\Gamma_{\alpha 0} = 130(10)$ keV, and $\Gamma_{\alpha 0}/\Gamma = 0.46(3)$, leading to the conclusion that it has one of the largest α cluster spectroscopic factors known. No other decays were observed, but they surmised the missing width might correspond to α decay to the 2^+ state of ^{6}He.

Curtis *et al.* [23], with the reaction ^{10}Be(^{14}C,^{10}Be*), observed states at 9.56, 2^+; 10.15, 3^-; 11.23; and 11.76, 4^+ MeV. They stated that they did not observe states at 6.18, 7.54, and 10.15 MeV, as expected.

Bohlen, *et al.* [24] used the ^{12}C(^{12}C,^{14}O)^{10}Be* reaction at 211 MeV. They concluded that the 11.8-MeV state was the 4^+ member of the g.s. band, and 10.55 was 3^-.

III. THE 0_2^+ BAND

The positive-parity states of interest in ^{10}Be are displayed in Fig. 1, along with those of ^{14}C. Omitted from this plot are the two ground states and the 2^+ state at 3.368 MeV in ^{10}Be. This low-lying 2^+ state is absent in ^{14}C because the $p_{1/2}$ space does not support a 2^+ state, whereas $p_{3/2}$ does. The similarity of the two sets of energy levels is striking. We show as dashed lines the two predominantly *p*-shell 2^+ states. In ^{14}C, this 2^+ state and the $(sd)^2$ 2^+ state are thoroughly mixed. The 0^+ mixing is also stronger in ^{14}C than in ^{10}Be (see below). Energies of the g.s. and 0_2^+ "bands" are plotted in Fig. 2.

The possible *n* and α decays of the $(sd)^2$ (or cluster) 2^+ and 4^+ states are displayed in Fig. 3. Decay widths are listed in Table I. We have calculated *n* and α single-particle (sp) widths in a Woods-Saxon potential model, using $r_0 = 1.25$ and $a = 0.65$ fm for the neutron well, and 1.40 and 0.60 fm for the α particle. (Here, $R = r_0 A_{\mathrm{core}}^{1/3}$.) For the α widths, if the states have two nucleons in the *sd* shell, the number of oscillator quanta $q = 2N + L$ (where N is the number of radial nodes) is 6. Thus, for $L = 2$, the radial wave function has two nodes, and one for $L = 4$.

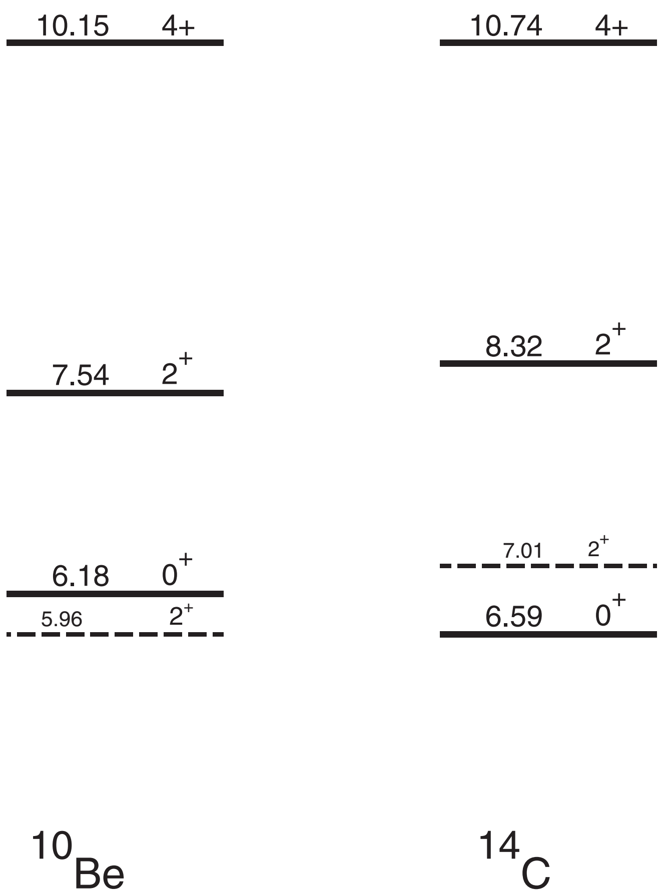

FIG. 1. Diagram of the relevant levels in ^{10}Be and ^{14}C.

We can perhaps gain some insight by examining the analogs in ^{10}B. If isospin is not mixed in ^{10}B, the properties of $T = 1$ states, including the values of S for both nucleon and α decay, should be the same in ^{10}Be and ^{10}B. Some isospin mixing is

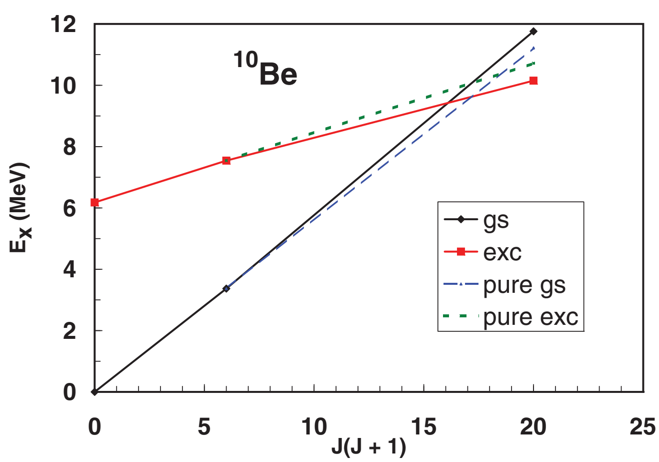

FIG. 2. (Color online) E_x vs $J(J+1)$ for g.s. band and 0_2^+ band. Dashed lines extend 0^+-2^+ slope; solid lines connect to known 4^+ states.

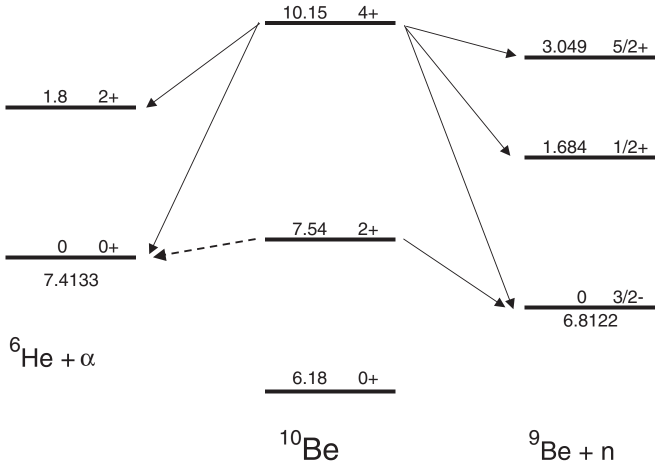

FIG. 3. Cluster states in ^{10}Be and their possible decays to ^{6}He and ^{9}Be.

definitely present in the 1^- and 2^- states in ^{10}B, but there is no evidence of such mixing in the positive-parity states.

A. 0^+

The 6.179-MeV 0^+ state has long been known as a core-excited state and frequently interpreted as the start of a rotational band. The purity of the state can be ascertained from its lack of sp character. From the ^{12}C(t,p) reaction, an estimate [25] of 12% was obtained for the 0^+ mixing in ^{14}C, whereas in ^{10}Be (for the mirror state in ^{10}B) the mixing is only about 1% [26].

B. 2^+

The 7.542-MeV 2^+ state of ^{10}Be is the likely next member of this band. We will investigate the ^{10}Be 2^+ mixing before we proceed. The neutron width of the 2^+ state at 7.542 MeV in ^{10}Be is 6.3(8) keV for decay to ^{9}Be(g.s.). The sp n width for this decay is about 700 keV. Thus, the value of S_n is less than 1%. In ^{10}B the analog at 8.894 MeV has $\Gamma =$ 40(1) keV, with $\Gamma_p/\Gamma = 0.35$. The proton width is given as 12.6(13) keV, and our calculated sp width is about 1.5 MeV. Thus, we have $C^2S \sim 0.84(9) \times 10^{-2}$ (where C is a Clebsch-Gordan coefficient), but with $C^2 = 1/2$, the value of S is $\sim 0.017(2)$. Therefore, in both nuclei, the nucleon spectroscopic factor is very small. The largely p-shell 2^+ state at 5.96 MeV has $S = 0.43$ – 1.0 experimentally and 0.69 theoretically. So we conclude that the 2^+ mixing is small.

A large α-particle reduced width has been one of the properties assigned to the 2^+ member of the supposed band. The 2^+ state at 7.542 MeV has a total width of 6.3(8) keV, and an α branching ratio [17] of $3.5(12) \times 10^{-3}$, resulting in an α width of 22(8) eV. The researchers in another experiment [22] stated that they could place a limit on this BR $> 2.0(6) \times 10^{-3}$. Our sp α widths are listed in Table I for the geometrical parameters given above. We see that the 2^+ state has an enormous α spectroscopic factor, $S = \Gamma_\alpha/\Gamma_{\rm sp} = 51(19)$. The sp α width is very sensitive to the precise energy of this state [$d\Gamma/(\Gamma dE)$ is about 6%/keV]. But the energy is well enough known (uncertainty of 0.7 keV), so this could not be the source of the unreasonably large spectroscopic factor. As the experimental α width for the ^{10}Be 2^+ state is not quite 3σ from zero, we also looked at the α width of the analog 2^+ state at 8.894 MeV in ^{10}B. Here, the α width is also known, and has a much smaller uncertainty. The nucleon and α widths and spectroscopic factors for these two states are listed in Table II. In ^{10}B, the nucleon spectroscopic factor is multiplied by the square of a Clebsch-Gordan coefficient $C^2 = 1/2$: $\Gamma_{\rm expt} = C^2 S \Gamma_{\rm sp}$. The α width in ^{10}B is for the isospin-allowed decay to the 0^+, $T = 1$ state of ^{6}Li. Any appreciable $T = 0$ component in the 2^+ state of ^{10}B would allow α decay to the $T = 0$ states of ^{6}Li (strongly favored on phase-space considerations), and none is observed. With an α width of 23(3) keV and a sp α width of 13.5 keV, we obtain an α spectroscopic factor of 1.70(22), drastically different from the value of 51(19) in ^{10}Be, though not statistically different. Various workers have suggested that these states might have very large radii, because of their supposed $\alpha nn\alpha$ extended structure. We find that doubling the radius of the well increases the sp width in ^{10}Be by about a factor of 5 (to 2.2 eV), greatly reducing the value of S, but still leaving it at 10(4) for the ^{10}Be 2^+ state. Milin *et al.* [15] used an 18-MeV ^{6}He beam on a ^{6}Li target to investigate the ^{6}He(^{6}Li,d)^{10}Be reaction. The 2^+ and nearby 3^- states were not resolved, but using the finite-range DWBA, they found that $S_\alpha(3^- + 2^+)/S_\alpha(2^+{}_1) \sim 3$.

TABLE I. Excitation energies and decays of 2^+ and 4^+ cluster states in ^{10}Be. (Energies in MeV, widths in keV).

Initial state		Decay	Final state		E_n or E_α	$\Gamma_{\rm expt}$	ℓ or L,N	$\Gamma_{\rm sp}$	S
E_x	J^π		E_x	J^π					
7.54	2^+	^{9}Be + n	0	$3/2^-$	0.730	6.3(8)	1	$\sim$700	$\leqslant$0.01
		^{6}He + α	0	0^+	0.129	$22(8) \times 10^{-3}$	2,2	0.43×10^{-3} [a]	51(19) [a]
10.15	4^+	^{9}Be + n	0	$3/2^-$	3.34	Not seen	3	220	—
			1.68	$1/2^+$	1.69	Not seen	4	0.47	—
			3.05	$5/2^+$	0.29	Not seen	2	5.1	—
		^{6}He + α	0	0^+	2.74	130(10)	4,1	42	3.1(2)
			1.8	2^+	0.94	153(22)	2,2	65	2.4(3)

[a]Doubling the potential-well radius increases $\Gamma_{\rm sp}$ to 2.2 eV, and hence $S = 10(4)$.

TABLE II. Decays of ^{10}Be(7.542) and ^{10}B(8.894) 2^+ states. (Widths in keV, unless stated otherwise.)

Nucleus	Nucleon decay			α decay		
	Γ_{expt}	Γ_{sp}	S	Γ_{expt}	Γ_{sp}	S
^{10}Be	6.3(8)	~700	<0.01	22(8) eV	0.43 eV	51(19)
^{10}B	12.6(13)	~1500	<0.02	23(3)[a]	13.5	1.70(22)

[a] α decay to ^{6}Li*(0^+,$T=1$).

C. 4^+

The history of the search for the 4^+ member of this band has been outlined above. It now appears firmly established as the state at 10.15 MeV, with $\Gamma_0 = 130(10)$ keV, $\Gamma_0/\Gamma = 0.46(3)$, with no neutron decays observed. (The width for g.s. n decay is said to be less than 10^{-3} of the α width.) With an α sp width of 42 keV for g.s. decay, we get $S_\alpha = 3.1(2)$. If the remaining width of 153(22) keV is α decay to the 2^+ state, our sp α width of 65 keV leads to $S_\alpha = 2.4(3)$ for this decay. Again, increasing the radius would reduce the S's somewhat. The analog 4^+ state in ^{10}B has not been identified. The neutron and α decays of these 2^+ and 4^+ states are listed in Table I. No neutron decays have yet been observed for either of the two 4^+ states, but the only quantitative limit is for the lower 4^+ state to ^{9}Be(g.s.): $\Gamma[10.15 \rightarrow {}^9\mathrm{Be(g.s.)}]$ is $<10^{-3}$ of Γ_{tot} [1]. Alpha decays of both 4^+ states to the first excited state of ^{6}He have been reported [27], but no branching ratios or widths are given. For possible mixing of 4^+ states, see Sec. IV.

D. OTHER STATES

If these 0^+, 2^+, and 4^+ states have $(sd)^2$ character, then in the range of about 9–10 MeV excitation there should exist three other $(sd)^2$ states with $J^\pi = 0^+$, 2^+, and 3^+. Nothing is known about them at this point. These three states will be difficult to populate. One has unnatural parity, and the other two have very little $2n$ cluster strength. The lower 0^+ and 2^+ have most of this $(sd)^2$ cluster strength. The published wave functions for $(sd)^2$ 0^+ states in ^{14}C in Ref. [4] provide $2n$ cluster spectroscopic factors of $S_{2n}(0_1^+) = 0.807$ and $S_{2n}(0_2^+) = 0.015$. These calculations ignored the $d_{3/2}$ orbital. Including it would increase both numbers slightly. Similar remarks hold for the 2^+ states. All three of these states in ^{14,16}C are very weak in the ^{12,14}C(t,p) reactions, and they are still unknown in ^{12}Be. If ^{8}Be were stable, it might have been possible to populate them in ^{10}Be via the (t,p) reaction. Coupled with all this is the general difficulty of populating non-yrast states at high excitation. If 4^+ mixing is indeed present, and the first 4^+ has been pushed down by this mixing, these states might be above the 4^+.

To make $(sd)^2$ states with $J > 4$ requires excitation of the ^{8}Be core. For example, coupling the $(sd)^2$ 4^+ state to the 2^+ first excited state of ^{8}Be would produce a 6^+ level at about 13.2 MeV, along with several lower-J ($J = 2 - 5$) states. Exciting ^{8}Be to 4^+ would produce an 8^+ state near 21.5 MeV. The configuration $^8\mathrm{Be}(2^+) \times (sd)_2^2$ would produce another 4^+ state near 10.57 MeV. The highest-J members of these core-excited multiplets are plotted in Fig. 4.

For a rotational band, with the moment-of-inertia parameter obtained from the 0^+-2^+ splitting, the 6^+state would be at 15.7 MeV, and the 8^+ at 22.5 MeV. If, instead, these states are of $(sd)^4$ character, as suggested in at least one paper [28], the band will (without ^{6}He core excitation) extend to 8^+. That paper predicts the 6^+ state near 14 MeV. None of these 6^+ and 8^+ states have been identified.

The energies of the supposed $(sd)^2$ states (lowest states of $J^\pi = 0^+, 2^+$, and 4^+) in ^{10}Be are compared in Fig. 5 with those in ^{12}Be and ^{14,16}C. The similarity is apparent.

IV. OTHER p-SHELL STATES

The p-shell calculations [29] produce other states: 0^+, 11.1 MeV; 1^+, 8.1 and 10.2 (plus two higher); 2^+, 9.2 and 10.3 MeV, 3^+, 9.8 and 13.3 MeV (plus two higher); and 4^+, 11.6, 15.7, and 16.7 MeV. The first 3^+ and one 2^+ state might correspond to the states near 9.6 and 9.4 MeV, respectively. The 4^+ is probably at 11.76 MeV. In fact, there is some indication of mixing between this 4^+ and the $(sd)^2$ state. The 10.15-MeV

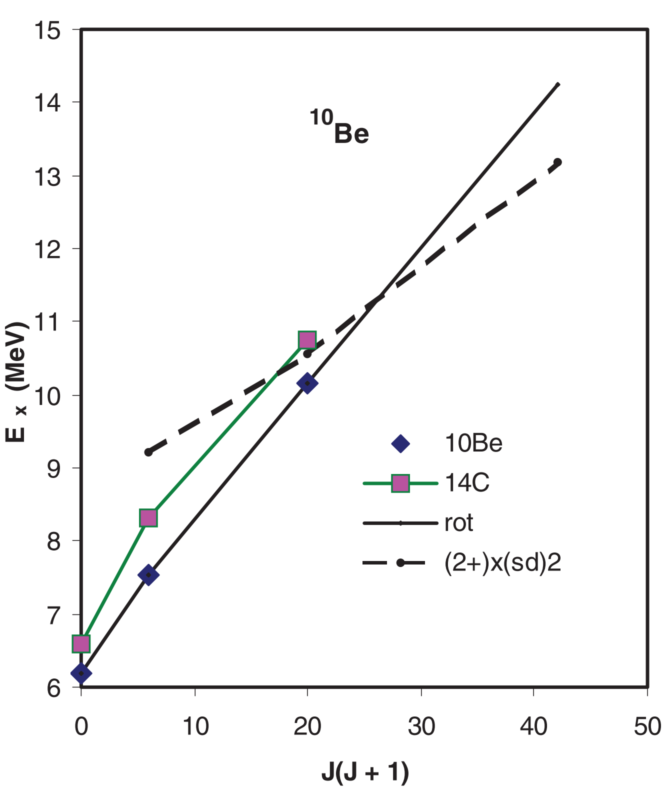

FIG. 4. (Color online) E_x vs $J(J+1)$ for the 0_2^+ band (diamonds), for yrast $(sd)^2$ states in ^{14}C (squares), for the extended rotational band (solid line), and for the hypothetical band built on ^{8}Be(2^+) (dashed line). The last would have the lowest-energy 6^+ state.

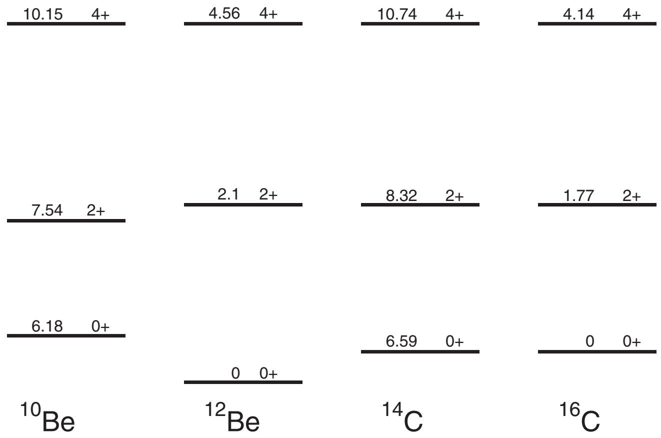

FIG. 5. Energy levels of $(sd)^2$ yrast states in ^{10,12}Be and ^{14,16}C, with 4^+ states aligned.

state is lower in energy than would be expected from the 0^+-2^+ splitting, and the 11.76 -MeV state is higher than the 4^+ member of a "g.s. band" should be. (See Fig 2.) If this mixing is present, the lower state could have acquired additional α strength at the expense of the upper [which has a total (all α) width of 120(10) keV] [3]. A p-shell 4^+ state at 11.76 MeV has a sp α width of 165 keV for decay to ^{6}He(g.s.) and 1.1 MeV for $L=2$ decay to the 2^+ state. If the 11.76-MeV state has $J^\pi = 6^+$, as suggested in Ref. [21], its sp α width for g.s. decay is 0.73 keV, and for decay to ^{6}He(2^+) it is 29 keV. Given that the experimental width of this state is 120(10) keV [3], it is unlikely to be 6^+.

V. SUMMARY

We have examined the properties of the supposed 0^+, 2^+, and 4^+ members of the 0_2^+ band and their analogs in ^{10}B. Mixing with the underlying p-shell states is essentially nonexistent for the 0^+ and 2^+ members, but could be important for 4^+. We have computed α-particle and nucleon sp widths for 2^+ and 4^+ states, and compared them to measured widths to produce spectroscopic factors. The 2^+ state in ^{10}Be has a spectroscopic factor that is much too large (but with a large uncertainty). Its analog in ^{10}B has a reasonable value (though still large). Even with a radius increased by a factor of 2, S_α in ^{10}Be is still too large—both on an absolute scale and by comparison with the analog in ^{10}B. We expect that a remeasurement of the BR will result in a much smaller value, in the vicinity of 1.4×10^{-4}, or less.

The 4^+ and 2^+ states have large S_α. It is likely that the 0^+ state also does. There is nothing wrong with having $S_\alpha > 1$. The three rotational-band 0^+, 2^+, and 4^+ states in ^{8}Be all have $S_\alpha \sim 1.5$ [30]. Coupling two sd-shell neutrons to this intrinsic structure and then recoupling to make $^6\text{He} \times \alpha$ should still provide large S_α, even though here the α will consist of two nucleons in the p shell and two in sd, rather than four in the p shell. This is another example of the principle that each mass partition is separately complete [31]. States can simultaneously have large S_n and large S_α.

Finally, the purity of the 4^+ states (and possible mixing with the p-shell 4^+) could perhaps be probed by looking for n decays from them. The most likely neutron decay of the $(sd)^2$ 4^+ state is to the $5/2^+$ state of ^{9}Be, for which the spectroscopic factor is about 1.9 and the sp width is 5 keV. One signature of 4^+ mixing would be the observation of neutron decay of the 11.76-MeV 4^+ state to ^{9}Be($5/2^+$) (allowed only through mixing), for which the sp width is about 450 keV. We expect the $(sd)^2$ 4^+ analog state to be at about 11.6 MeV in ^{10}B. Observation of its α decay to 0^+ and 2^+ $T=1$ states of ^{6}Li could shed additional light on the structure of these states.

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