$\bar{K}NN$ system with chiral dynamics

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We have performed a calculation of the scattering amplitude for the three-body system $\bar{K}NN$ assuming $\bar{K}$ scattering against a NN cluster using the fixed center approximation to the Faddeev equations. The $\bar{K}N$ amplitudes, which we take from chiral unitary dynamics, govern the reaction and we find a $\bar{K}NN$ amplitude that peaks around 40 MeV below the $\bar{K}NN$ threshold, with a width in $|T|^2$ of the order of 50 MeV for spin 0 and has another peak around 27 MeV with similar width for spin 1. The results are in line with those obtained using different methods but implementing chiral dynamics. The simplicity of the approach allows one to see the important ingredients responsible for the results. In particular, we show the effects from the reduction of the size of the NN cluster due to the interaction with the $\bar{K}$ and those from the explicit consideration of the $\pi\Sigma N$ channel in the three-body equations.

I. INTRODUCTION

The study of the bound $\bar{K}NN$ system is rising much interest as the lightest nuclear system binding a $\bar{K}$ [1–6]. Earlier calculations on this system were done in the 1960s [7] and more recently in Ref. [8], yet the works of Refs. [1–6] improve considerably on those earlier works, and, according to Ref. [3], the results of Ref. [8] should be considered at best as a rough estimate. It is very instructive to recall the basics of these calculations. Those of Refs. [1–3,6] use Faddeev equations, in the formulation of Alt-Grassberger-Sandhas (AGS) [9], using a separable interaction for the potentials with energy-independent strength, with form factors depending on the three momenta. They also include as coupled channels $N\pi\Sigma$ and $NN\bar{K}$. On the other hand, the works of Refs. [4,5] use a variational method to obtain the binding energy and width by means of an effective potential [10] that leads to the strongly energy-dependent $\bar{K}N$ amplitudes of the chiral unitary approach [11–20]. The works of Refs. [4,5] make the variational calculation using the $\bar{K}N$ effective potential. This includes the $\pi\Sigma$ channel integrated into the effective $\bar{K}N$ potential but do not include the $N\pi\Sigma$ channel explicitly into the three-body system. The $\pi\Sigma$ two body channel is effectively considered through the $\bar{K}N$ potential, but the interaction of πN or ΣN which appears in the Faddeev equations would not be considered. Yet the πN interaction is relatively weak compared to the $\bar{K}N$ at these energies, and the effect of the ΣN is very small, as mentioned in Ref. [3], so one should look for other reasons to understand the differences between the two approaches. In Table IV of Ref. [3] one finds that there are 11-MeV differences considering just one channel $\bar{K}N$ or the two channels $\bar{K}N$ and $\pi\Sigma$, both in the evaluation of the $\bar{K}N$ amplitude and in the $\bar{K}NN - \pi\Sigma N$ system, with more binding in the case of two channels. One should note that this is not to be compared with the present approach or the one in Refs. [4,5], where the $\bar{K}N$ amplitude is always calculated with $\bar{K}N$, $\pi\Sigma$, and other coupled channels. As

argued here and in Ref. [10], this should take into account most of the effects of the $\pi\Sigma$ channel. The extra Faddeev diagrams with explicit $\pi\Sigma N$ intermediate states involve the πN or ΣN interactions which have small effects. Actually, this line of argumentation coincides with a hyphothesis of Schick and Gibson [21] which was substantiated numerically in the work of Ref. [22].

However, in Ref. [5] the authors make guesses on the reasons for the differences between the Faddeev approach and their approach and, among other reasons, they suggest that the three-body $\pi\Sigma N$ dynamics not accounted for in their work could be, in large part, responsible for the differences. In view of this persistent question we shall introduce here explicitly the $\pi\Sigma$ channel in the three-body system to find out an answer.

The two Faddeev approaches lead to binding energies higher than the variational approach, 50- to 70-MeV versus around 20-MeV binding, respectively. The details mentioned above regarding an energy-independent kernel used in the AGS equations is partly responsible for the extra binding of these approaches with respect to the chiral calculations. Indeed, as shown in Refs. [23,24], it is possible to obtain the same results as with the field theoretical chiral unitary approach using quantum mechanics with a separable potential but where the potential is energy-dependent and proportional to the sum of the two external meson energies in $\bar{K}N \to \bar{K}N$. As a consequence, a smaller $\bar{K}N \to \bar{K}N$ amplitude is obtained at lower $\bar{K}N$ energies, resulting in a smaller binding for the $\bar{K}NN$ system. This numerical result was mentioned in Ref. [5]. Actually, the same result is found within the approach of Refs. [1,6] when the energy dependence of the Weinberg-Tomozawa chiral potential is taken into account [25]. In return, this latter approach contained two poles in the scattering matrices of the three-body system, as a reminder of the two $\Lambda(1405)$ poles contained in Refs. [17–20]. We shall come back to this work later in the paper.

Yet, in most cases the widths are systematically larger than the binding energy, of the order of 70–100 MeV. This certainly makes the observation of these states problematic, as acknowledged in all these works. In view of this, the claim in Ref. [26] of a K^-pp bound state with 115 MeV binding was met with skepticism, and soon it was shown that the peak observed in [26] was easily interpreted in terms of conventional, unavoidable, reaction mechanisms which were well under control [27].

In between, the study of three-body systems involving mesons and baryons with the Faddeev equations has experienced a new qualitative jump. It is well known that different potentials leading to exactly the same on-shell two-body scattering amplitudes lead to different results in the Faddeev equations due to their different off-shell extrapolation. Yet, a novel result has emerged from the study of the two meson-one baryon systems with chiral dynamics, since it was shown in Ref. [28] that the off shell part of the two body scattering amplitudes cancels with genuine three-body terms that the same chiral Lagrangians provide at the same order (see also the Appendix of Ref. [29]). The same happens when systems of three mesons are studied within the same chiral unitary approach [30]. This finding is most welcome because the results for the three-body amplitudes do not depend on the unphysical off-shell two-body amplitudes. This cancellation obviously cannot be seen in phenomenological approaches like in Refs. [1–3,6] since one cannot systematically associate the corresponding three-body forces that would cancel the off-shell contribution of the two-body amplitudes. Thus, the results are linked to the off-shell extrapolation of the amplitudes, and though one should expect cancellations with some three-body forces, there is no way in those approaches to estimate the extent of these cancellations and the repercussion in the final binding energy. In view of it, awaiting hopefully a calculation in the line of Refs. [28–30], which is not technically easy, we want to present a simplified approach, yet one that is realistic enough and that explicitly relies only on the on-shell amplitudes of the interacting pairs and takes into account the energy dependence of the $\bar{K}N$ amplitude demanded by chiral dynamics. We find this possibility using the fixed center approximation to the Faddeev equations (FCA). As we shall discuss in the next section, the common findings of all the former approaches allow us to make the simplifying approximations that make the FCA (with some modifications) an easy and reliable tool to face this problem.

The presence of the $\Lambda(1405)$ in the $\bar{K}N$ amplitude is one complicating factor because it is well known that all present chiral theories [11–20] predict two poles close by in the 1405-MeV region. This means two states of the coupled channels $\pi\Sigma$, $\bar{K}N$ and other coupled channels. As found in Ref. [17] different amplitudes involving the $\Lambda(1405)$ have different shapes, and, as a consequence, the $\Lambda(1405)$ should appear with different shapes in different reactions. Since the pole with lower energy couples mostly to $\pi\Sigma$ and the one at higher mass mostly to $\bar{K}N$ [17], it was expected that in reactions where the $\Lambda(1405)$ production would be driven by the $\bar{K}N$ the peak would appear around 1420 MeV, where all different chiral approaches predict this second pole. It

did not take long to see this realized. Indeed, the study of the $K^-p \rightarrow \pi^0\pi^0\Sigma^0$ reaction at $p_K = 514$–750 MeV/c [31] revealed the peak of the $\Lambda(1405)$ clearly at 1420 MeV. This was interpreted in Ref. [32] as being due to a dominant mechanism in which a pion is emitted from the initial nucleon, forcing the $\Lambda(1405)$ to be created from the K^-p state. More recently, it was also found in Ref. [33] that a clear peak appeared for the $\Lambda(1405)$ in the $K^-d \rightarrow n\pi\Sigma$ reaction for kaons in flight. This is surprising considering that the K^-p threshold is about 30 MeV above the $\Lambda(1405)$ mass. However, it was explained in Ref. [34] that the peak was due to the rescattering of the kaon on a second nucleon, after it would lose some energy in the first collision against one nucleon and have the right kinematics to produce the $\Lambda(1405)$ in a second collision. The second surprise was that the peak of the resonance appears around 1420 MeV, not 1405 MeV, which is not surprising at all from the chiral dynamical perspective, since the resonance production is once again induced by the $\bar{K}N$ channel. A recent calculation [35] shows that the same features can already be observed for kaons produced at the DAFNE facility, and an experimental proposal is being prepared [36].

Coming back to the three-body works, the approach of Refs. [4,5] is fully consistent with these chiral dynamical features since it uses chiral theory as the underlying framework to construct the effective potential. This is not the case of the Faddeev approaches of [1–3,6], where to solve the AGS equations an energy independent kernel is used. In addition, providing just one $\Lambda(1405)$ resonance, the important dynamics of the two $\Lambda(1405)$ resonances is lost. With this in mind, the authors of these works try to evaluate uncertainties by fitting their parameters to different masses of the $\Lambda(1405)$ from 1405 to 1420 MeV, and differences in the binding of about 10–20 MeV appear. The fits with a mass of the $\Lambda(1405)$ of 1420 MeV provide less binding, closer to the results of the variational approach of Refs. [4,5].

An important step to conjugate the AGS equations with the $\bar{K}N$-$\pi\Sigma$ dynamics of the chiral Lagrangians has been done, as mentioned, in Ref. [25]. Indeed, two poles are found in qualitative agreement with other chiral approaches. One narrow pole around 1420 MeV, rather stable against changes of parameters, is found in agreement with all findings of the chiral unitary approach. The second, wider pole, is found at very low energies (1335–1341 MeV) and it is more unstable with respect to changes of parameters. This agrees qualitatively with the findings of the chiral unitary approach, but the energy is lower than in other approaches. To understand these differences, in Ref. [25] the $\pi\Sigma$ mass distribution of the $K^-p \rightarrow \pi\pi\pi\Sigma$ in the Hemingway experiment [37] is adjusted, assuming that it is proportional to $|T_{\pi\Sigma,\pi\Sigma}|^2$. Yet, as shown in Ref. [17], when one has two poles, the $T_{\pi\Sigma,\pi\Sigma}$ and $T_{\bar{K}N,\pi\Sigma}$ amplitudes differ and the $\Lambda(1405)$ production processes proceed via the combination of the two amplitudes $|T_{\pi\Sigma,\pi\Sigma} + \beta T_{\bar{K}N,\pi\Sigma}|^2$ [38]. The fact that a separable potential with form factors is used also induces off-shell effects with respect to the use of the on-shell scattering amplitudes of the chiral unitary approach. All these facts certainly introduce uncertainties that revert in the deduced $T_{\pi\Sigma,\pi\Sigma}$, $T_{\bar{K}N,\pi\Sigma}$ amplitudes and the position of the poles. These uncertainties should then translate into the

position of the poles of the $\bar{K}NN$-$\pi\Sigma N$ system reported in Ref. [25]. In addition, one would like to know the $T_{\bar{K}NN,\bar{K}NN}$ amplitude resulting from Ref. [25], which was not provided in the paper and which would get a contribution from the two poles of the three-body system. This information would be useful to eventually compare with experiments, and in the case of the $\bar{K}N$-$\pi\Sigma$ system, the physical amplitudes do not have the shapes of either of the poles alone.

In the approach that we use we follow the chiral unitary approach for the $\bar{K}N$ amplitudes, which provide the most important source of the binding of the three-body system, according to the former studies. But there is also another different technical aspect of the present calculation with respect to the former ones. All previous approaches have concentrated on looking for the binding, searching for poles in the complex plane or looking for the energy that minimizes the expectation value of the Hamiltonian. Here, inspired by the studies of Refs. [28–30], we shall look for bumps in the scattering matrices as a function of the energy of the three-body system. Furthermore, the amplitudes obtained here could, in principle, be used as input for final-state interaction when evaluating cross sections in reactions where eventually this $\bar{K}NN$ state is formed.

The discussion here shows that, in spite of the much and good work done so far on the subject, uncertainties remain and further studies like the one done here should be welcome. Although there are certainly approximations in the FCA with respect to the Faddeev approach, as there are also in the approximate three-body scheme of the AGS equations, one can rely on some findings of these more elaborate approaches to make the FCA results more solid, while it allows us to overcome the sources of uncertainties mentioned in the other approaches and provide a more intuitive and direct approach to the problem with the consequent transparency in the results obtained. Another of the novel things in the present work is that we show the existence of a less bound state in $S = 1$, $I = 0$ of the two nucleons. This state was not considered in Refs. [3–8,25], although hints of the existence could be guessed from the large negative real part of the K^-d scattering length deduced in the work discussed in next section.

II. FIXED CENTER FORMALISM FOR THE $\bar{K}(NN)$ INTERACTION

The findings of the former works simplify our task. In this sense we should mention that in Ref. [4] the nucleons are found to move slowly, having around 20 MeV of kinetic energy. It was also found there that the dependence on the type of NN interaction is rather weak. We rely on the results of Refs. [28–30] in the sense that only on-shell two-body amplitudes are needed. By this we mean the part of the analytical amplitudes obtained setting $q^2 = m^2$ for the external particles, even if the particles are below threshold.

We also assume, like in the other works, that the two-body interactions proceed in $L = 0$. The effect of the p-wave interaction in the main interacting pair, the $\bar{K}N$, was found to be small in Ref. [5], the main consequence being a moderate

increase in the width, so we should accept the width that we find as a lower bound of the actual width.

According to all the literature, the main component of the wave function corresponds to having a $\bar{K}N$ in $I = 0$ and hence the total isospin will be $I = 1/2$. The total spin can be $J = 0,1$, but the $J^P = 0^-$ state is the one found most attractive. We shall investigate both possibilities.

According to Ref. [3] the effect of the ΣN interaction is very small. We shall neglect it in our approach. Yet we shall incorporate the $\pi\Sigma N$ as an explicit channel in the three-body system, which in the $\bar{K}NN \to \bar{K}NN$ amplitude appears via the $\bar{K}N \to \pi\Sigma$ transition followed by $\pi N \to \pi N$ rescattering and $\pi\Sigma \to \bar{K}N$ recombination.

All this considered, the problem we shall solve is the interaction of a $\bar{K}$ with a NN cluster. We shall evaluate the scattering matrix for this system as a function of the total energy of the $\bar{K}NN$ system and look for peaks in $|T|^2$. In a second step we will allow explicitly the intermediate $\pi\Sigma$ state in the three-body system. We shall use in our study the FCA approximation to the $\bar{K}NN$ system. This approach has been used before in connection with the evaluation K^-d scattering length [22,39–42]; a followup of Ref. [42] is done in Ref. [43]. A discussion of these different approaches is done in Ref. [44], where it is shown that Refs. [40,41] do not take into account explicitly the charge exchange $K^-p \to \bar{K}^0n$ reactions and antisymmetry of the nucleons, while it is explicitly done in Ref. [42]. In the present approach we also consider the antisymmetry and charge exchange in our scheme although in a different technical way than in Ref. [42] and assuming isospin symmetry for the $\bar{K}N$ interaction.

The FCA for K^-d at threshold was found to be an acceptable approximation, within a few percentage points, to the more elaborate Faddeev equation in Ref. [22,44]. There it was also discussed that neglecting the $\pi\Sigma$ channel in the three-body system is a good approximation, as far as $\bar{K}N$ interaction is evaluated in terms of the $\bar{K}N$ and $\pi\Sigma$ coupled channels. We shall be able to corroborate this here by explicitly introducing the $\pi\Sigma$ channel in the three-body system. Technically, we follow closely the formalism of Ref. [45], where the FCA has been considered, using chiral amplitudes, in order to study theoretically the possibility of forming multi-ρ states with large spin.

An interesting shared result of all the K^-d calculation quoted above is a large and negative real part of the scattering length, of the order of -1.40 to 1.80 fm, which suggests the existence of a bound state in $J = 1$. Of course, the imaginary part of the scattering length is equally large, anticipating a broad state. We shall be able to make this more quantitative in the present approach. However, we shall also find that the most bound $\bar{K}NN$ system correspond to $J = 0$, in agreement with other approaches.

The NN interaction is of long range and very strong. It binds the deuteron in spin $S = 1$ and $I = 0$, with $L = 0$ to a very good approximation. It nearly binds the pp system also. The binding of the pp system is so close that a little help from an extra interaction, the strongly attractive $\bar{K}N$ interaction, is enough to also bind this system and we shall assume that this is the case here for pp or in general for two nucleons in $S = 0$, $I = 1$, $L = 0$.

Our strategy will be to assume as a starting point that the NN cluster has a wave function like the one of the deuteron (we omit the d wave). Later we will release this assumption and assume that the NN system is further compressed in the $\bar{K}NN$ system. Since in the FCA the input from the NN system is the NN form factor, taking into account an extra compression of the NN systems is very easy by smoothly modifying the form factor to have a smaller radius. As mentioned in the Introduction, we have taken information from previous studies, and in this sense it is interesting to recall that the calculations of Ref. [46] for $\bar{K}$ bound in nuclei point to a moderate compression of the nucleus due to the strong $\bar{K}N$ interaction. However, in the case of two isolated nucleons the decrease in the radius can be far bigger than that in nuclei, where nucleons are already close to saturation density. The information on the NN radius in the $\bar{K}NN$ molecule we get from Ref. [5], where the NN interaction is taken into account, including short-range repulsion that precludes unreasonable compression. Yet, the rms radius found is of the order of 2.2 fm, slightly above one-half the value of the deuteron rms radius of 3.98 fm (NN distance).

A. Calculation of the three-body amplitude

The formal derivation follows the steps of Ref. [45]. We assume a pure $L = 0$ interaction for all the $\bar{K}N$ and NN pairs, hence we have $S = J = 0$, $I_{NN} = 1$ or $S = J = 1$, $I_{NN} = 0$ for the two nucleons and we shall evaluate the interaction in both cases. Since $L = 0$ also for $\bar{K}N$, J will be the spin of the total $\bar{K}NN$ system. In the case of $I_{NN} = 0$ for NN, the total isospin will be $I = 1/2$. In the case of $I_{NN} = 1$ we shall also couple the total isospin I to $1/2$ to give room to the $\Lambda(1405)N$ configuration which is where the attraction concentrates, according to former studies.

The mechanism implicit in the FCA are depicted in Fig. 1. The first thing we need to do is to evaluate the interaction of a $\bar{K}$ with either of the two nucleons in each of the two configurations that we will consider: $S = 0$, $I_{NN} = 1$, $I = 1/2$ and $S = 1$, $I_{NN} = 0$, $I = 1/2$. The $\bar{K}N$ scattering matrix is obtained as follows. First, we write the wave function for the states.

$$|\bar{K}(NN)_{I_{NN}=0}, I = 1/2, I_z = 1/2\rangle = |1/2\rangle \frac{1}{\sqrt{2}}$$
$$\times (|1/2, -1/2\rangle - |-1/2, 1/2\rangle) \quad (1)$$

with the nomenclature $|I_z >$ for the states of $\bar{K}$ and $|I_{z_1}, I_{z_2} >$ for the NN system. We want to evaluate

$$\langle \bar{K}(NN)_{I_{NN}=0}, I = 1/2, I_z = 1/2|t_{31}|\bar{K}(NN)_{I_{NN}=0},$$
$$I = 1/2, I_z = 1/2\rangle \quad (2)$$

and the same one for t_{32}, where t_{31}, t_{32} stand for the scattering matrix of the $\bar{K}$ with nucleons 1 and 2, respectively.

In order to use the isospin-scattering matrices we write the state of Eqs. (1) in terms of good isospin states of $\bar{K}N$ as

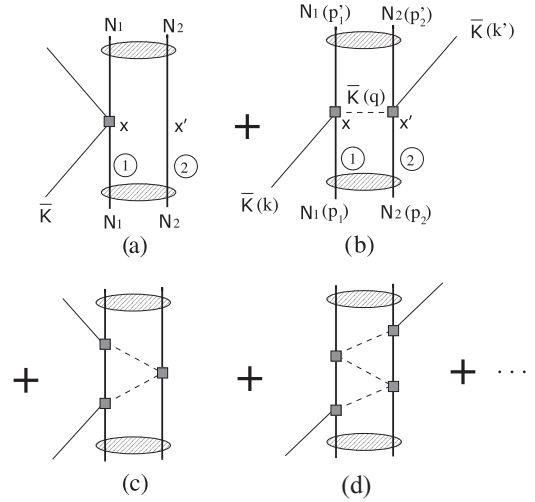$$\frac{1}{\sqrt{2}}|1/2\rangle(|1/2, -1/2\rangle - |-1/2, 1/2\rangle)$$

FIG. 1. Mechanism for the $\bar{K}NN$ interaction, including multiple scattering of the $\bar{K}$ with the nucleons. The equivalent diagrams where the $\bar{K}$ interacts first with the second nucleon should be added.

$$= \frac{1}{\sqrt{2}}(|1, 1\rangle|-1/2\rangle - \frac{1}{\sqrt{2}}|1, 0\rangle|1/2\rangle$$
$$- \frac{1}{\sqrt{2}}|0, 0\rangle|1/2\rangle), \quad (3)$$

where, in the second member of the equation, we use the nomenclature $|\bar{K}N, I, I_z\rangle|I_{N_2 z}\rangle$ and then we find immediately that

$$I_{NN} = 0 : t_{31} = \frac{3}{4}t^{I=1}_{\bar{K}N} + \frac{1}{4}t^{I=0}_{\bar{K}N}. \quad (4)$$

Similarly, combining the $\bar{K}$ isospin with that of the second nucleons we find

$$I_{NN} = 0 : t_{32} = \frac{3}{4}t^{I=1}_{\bar{K}N} + \frac{1}{4}t^{I=0}_{\bar{K}N}. \quad (5)$$

Similarly, we evaluate t_{31}, t_{32} for the case of $I_{NN} = 1$

$$|\bar{K}(NN)_{I_{NN}=1}, I = 1/2, I_z = 1/2\rangle = -\sqrt{\frac{2}{3}}|1/2,$$
$$-1/2\rangle|1, 1\rangle + \sqrt{\frac{1}{3}}|1/2, 1/2\rangle|1, 0\rangle \quad (6)$$

with nomenclature $|I_{\bar{K}}, I_{\bar{K},z}\rangle|I_{NN}, I_{NN,z}\rangle$

$$|\bar{K}(NN)_{I_{NN}=1}, I = 1/2, I_z = 1/2\rangle$$
$$= -\sqrt{\frac{2}{3}}|-1/2\rangle|1/2, 1/2\rangle$$
$$+ \frac{1}{\sqrt{3}}|1/2\rangle\left(\frac{1}{\sqrt{2}}|1/2, -1/2\rangle + \frac{1}{\sqrt{2}}|-1/2, 1/2\rangle\right), \quad (7)$$

where now we are using the nomenclature $|\bar{K}, I_z\rangle|I_{N_{1z}}, I_{N_{2z}}\rangle$. Combining $\bar{K}N_1$ to have good isospin states we obtain

$$|\bar{K}(NN)_{I_{NN}=1}, I = 1/2, I_z = 1/2\rangle$$

$$= -\sqrt{\frac{2}{3}}\frac{1}{\sqrt{2}}|1, 0\rangle|1/2\rangle + \sqrt{\frac{2}{3}}\frac{1}{\sqrt{2}}|0, 0\rangle|1/2\rangle$$

$$+ \frac{1}{\sqrt{6}}|1, 1\rangle| - 1/2\rangle + \frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}|1, 0\rangle|1/2\rangle$$

$$+ \frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}|0, 0\rangle|1/2\rangle \tag{8}$$

with nomenclature $|\bar{K}N, I, I_z\rangle|I_{N_2,z}\rangle$, and then we find immediately that

$$I_{NN} = 1 : t_{31} = \frac{1}{4}t_{\bar{K}N}^{I=1} + \frac{3}{4}t_{\bar{K}N}^{I=0} \tag{9}$$

and, similarly,

$$I_{NN} = 1 : t_{32} = \frac{1}{4}t_{\bar{K}N}^{I=1} + \frac{3}{4}t_{\bar{K}N}^{I=0}. \tag{10}$$

The S matrix for the single scattering of $\bar{K}N_{1,2}$ [Fig. 1(a)] is given by

$$S^{(1,1)} = -it_1\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_{\bar{K}}}}\frac{1}{\sqrt{2\omega'_{\bar{K}}}}\sqrt{\frac{2M}{2E_1}}\sqrt{\frac{2M}{2E'_1}}$$

$$\times (2\pi)^4\,\delta(k + k_{NN} - k' - k'_{NN}), \tag{11}$$

$$S^{(1,2)} = -it_2\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_{\bar{K}}}}\frac{1}{\sqrt{2\omega'_{\bar{K}}}}\sqrt{\frac{2M}{2E_2}}\sqrt{\frac{2M}{2E'_2}}$$

$$\times (2\pi)^4\,\delta(k + k_{NN} - k' - k'_{NN}). \tag{12}$$

while that for the double scattering, Fig. 1(b), is given by

$$S^{(2)} = -i(2\pi)^4\delta(k + k_{NN} - k' - k'_{NN})$$

$$\times \frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_{\bar{K}}}}\frac{1}{\sqrt{2\omega'_{\bar{K}}}}\sqrt{\frac{2M}{2E_1}}\sqrt{\frac{2M}{2E'_1}}\sqrt{\frac{2M}{2E_2}}\sqrt{\frac{2M}{2E'_2}}$$

$$\times \int\frac{d^3q}{(2\pi)^3}F_{NN}(q)\frac{1}{q^{02} - \vec{q}^{\,2} - m_{\bar{K}}^2 + i\epsilon}t_1t_2, \tag{13}$$

where $F_{NN}(q)$ is the form factor of the NN system.

On the other hand, if we consider the scattering of $\bar{K}$ with the NN system the S matrix is given by

$$S = -iT(2\pi)^4\delta(k + k_{NN} - k' - k'_{NN})\frac{1}{\mathcal{V}^2}$$

$$\times \frac{1}{\sqrt{2\omega_{\bar{K}}}}\frac{1}{\sqrt{2\omega'_{\bar{K}}}}\sqrt{\frac{2M_{NN}}{2E_{NN}}}\sqrt{\frac{2M_{NN}}{2E'_{NN}}}. \tag{14}$$

As we can see, the field normalization factors that appear in the amplitude of the different terms, for which we follow the Mandl and Shaw convention [47], differ. However, if we approximate $\omega_{\bar{K}} = m_{\bar{K}}$, $\sqrt{M_N/E_N} = \sqrt{2M_{NN}/2E_{NN}} = 1$, then all the normalization factors are the same. Then we can write T in terms of two partitions, T_1, T_2, summing all the

terms which begin with the $\bar{K}$ interaction with the nucleon 1 or nucleon 2, respectively. Then we find

$$T = T_1 + T_2, \tag{15}$$

$$T_1 = t_1 + t_1G_0T_2, \qquad T_2 = t_2 + t_2G_0T_1, \tag{16}$$

where

$$G_0 = \int\frac{d^3q}{(2\pi)^3}F_{NN}(q)\frac{1}{q^{02} - \vec{q}^{\,2} - m_{\bar{K}}^2 + i\epsilon}. \tag{17}$$

One can easily solve Eqs. (16) and we obtain

$$T = T_1 + T_2 = \frac{t_1 + t_2 + 2t_1t_2G_0}{1 - t_1t_2G_0^2}, \tag{18}$$

which, taking into account that $t_1 = t_2$, can be written as

$$T = \frac{2t_1}{1 - t_1G_0}. \tag{19}$$

In principle, one must also consider the form factor in the single scattering [45] but, as done in Ref. [45], taking into account that one has small momenta of $\bar{K}$ in the bound $\bar{K}N$ system, one can just take it equal to 1.

The variable q^0 in Eq. (17) is the energy carried by the $\bar{K}$, which is given by

$$q^0 = \frac{s + m_{\bar{K}}^2 - (2M_N)^2}{2\sqrt{s}}, \tag{20}$$

with $\sqrt{s}$ the rest energy of the $\bar{K}NN$ system. We also need the argument of the t_1 and t_2 function, s_1, given by

$$s_1 = m_{\bar{K}}^2 + m_N^2 + \frac{1}{2}(s - m_{\bar{K}}^2 - 4m_N^2). \tag{21}$$

For the form factor we take, as a starting point, the one of the deuteron which is given by

$$F(q) = \int_0^\infty d^3p \sum_{j=1}^{11}\frac{C_j}{\vec{p}^2 + m_j^2}\sum_{i=1}^{11}\frac{C_i}{(\vec{p} - \vec{q})^2 + m_i^2}, \tag{22}$$

where the C_i coefficients are given in Ref. [48]. The form factor is normalized to 1 at $\vec{q} = 0$ by dividing $F(q)$ by the expression of Eq. (22) at $\vec{q} = 0$. In further steps this form factor will be changed to accomodate the reduced size of the two N systems found in Ref. [5]. This is done by rescaling the masses m_i appearing in Eq. (22), demanding that the radius be the one of Ref. [5].

III. RESULTS

In Fig. 2 we plot the deuteron form factor $F(q)$ normalized to 1 at $q = 0$, as well as the one corresponding to the reduced NN radius, as described in the former section. In Fig. 3 we show the results for $|T|^2$ of Eq. (19), as a function of the total energy of the $\bar{K}NN$ system for the case of $S = 1$. The results obtained using $F(q)$ from the deuteron show a clear peak at $\sqrt{s} = 2350$ MeV, about 22 MeV below the threshold of $\bar{K}NN$. The width of the distribution is about 50 MeV. It is interesting to see what happens with the decreased NN radius. As one can see, the effects are clearly visible, the binding is increased by about 5 MeV, and the width is practically unchanged.
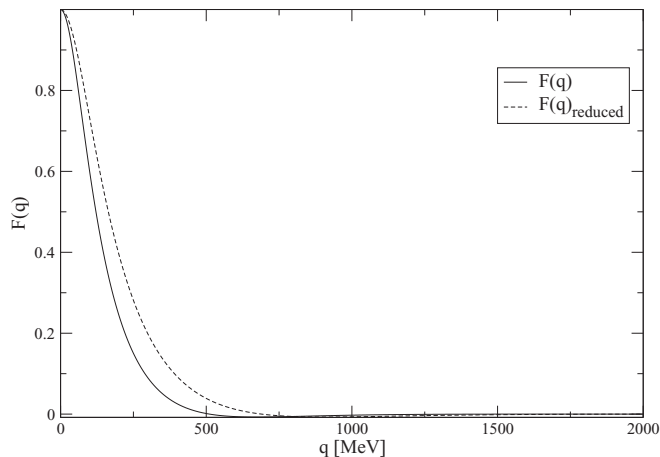
FIG. 2. Form factor of the deuteron, and the one corresponding to an NN system with a reduced radius from Ref. [5].

More interesting is to see what happens for $S = 0$. This is shown in Fig. 4. We find here that $|T|^2$ also shows a peak but is more bound than for $S = 1$. The position of the peak indicates a binding of the $\bar{K}NN$ system by 30 MeV. The width is of about 50 MeV. When we take the case of the reduced NN radius the peak is displaced to about 40 MeV binding and the width is still around 50 MeV.

The results obtained could be interpreted as giving support to two independent states since they have different total angular momentum, but some considerations are in order. The first important thing is to observe the different strengths of the two distributions. Indeed, $|T|^2$ is 4.5 times bigger in the $S = 0$ case than in $S = 1$. Second, in the case of $S = 0$ the system is more bound. The two independent states come in our approach because we have assumed an exact $L = 0$ contribution. Should we allow for some $L = 1$ contribution in the $\bar{K}N$ interaction, we can have a contribution to total spin $J = 0$ from a spin $S = 1$ of the two nucleons and then the different spins of
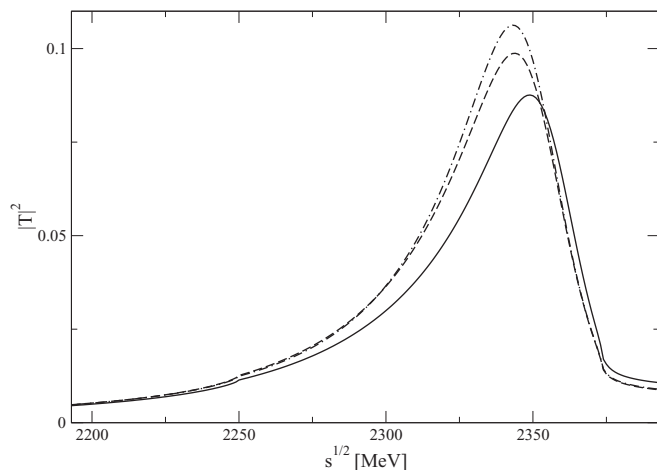

FIG. 3. Modulus squared of the three-body scattering amplitude for triplet ($S = 1$). The solid line indicates the $\bar{K}NN$ system in normal size and the dashed line indicates the same system with reduced NN radius Ref. [5]. The dot-dashed line indicates the result of the including the $\pi\Sigma$ channel with the reduced radius.
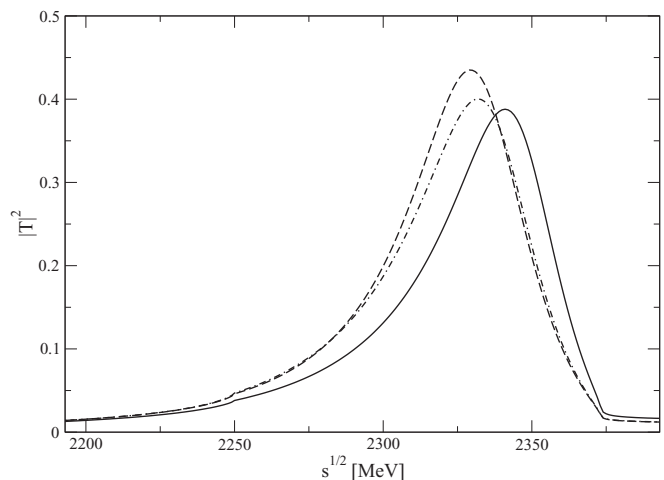

FIG. 4. Modulus squared of the three-body scattering amplitude for singlet ($S = 0$). The solid line indicates the $\bar{K}NN$ system in normal size and the dashed line indicates the same system with reduced NN radius Ref. [5]. The dot-dashed line indicates the result of the including the $\pi\Sigma$ channel with the reduced radius.

the nucleons would mix in the real state. But this mixing should be small in view of the weak effects of the $\bar{K}N$ p wave reported in Ref. [5]. It is clear from our results that the state with $J = 0$ will correspond to the most bound $\bar{K}NN$ system and that it will contain a small admixture of $S = 1(I = 0)$ of the two nucleons. This is the same conclusion reached in Ref. [4,5].

The approach followed here shows with clarity why the $S = 0, I_{NN} = 1$ configuration is preferred. Indeed, as one can see in Eqs. (4), (5), (9), and (10) in the case of $S = 1, I_{NN} = 0$, the $t_{1,2}$ amplitudes have a weight of 3/4 for $I_{\bar{K}N} = 1$ and 1/4 for $I_{\bar{K}N} = 0$, while in the case of $S = 0, I_{NN} = 1$ the weights are opposite, with 1/4 for $I_{\bar{K}N} = 1$ and 3/4 for $I_{\bar{K}N} = 0$. Since the strong attraction appears in $I_{\bar{K}N} = 0$, where the two $\Lambda(1405)$ states are generated, it becomes clear that one gets more attraction and more strength in $|T|^2$ for the case of $S = 0, I_{NN} = 1$ than for $S = 1, I_{NN} = 0$.

On the other hand, the exercise made here also tells that, while the $S = 1, I_{NN} = 0, L_{\bar{K}N} = 0$ configuration does not correspond to a pure state and does not lead to the most bound component of the $\bar{K}NN$ system, it is still a configuration with a sizable strength, although reasonably smaller than the one with $S = 0, I_{NN} = 1, L_{\bar{K}N} = 0$. In a reaction producing $\bar{K}NN$ (bound) in the final state, these configuration would all contribute to the cross section, resulting in a peak with larger width. Note that in such reactions one would have to look for decay products of $\bar{K}NN$ to identify the peak that we produce. A possible channel would be ΛN, but one should not expect to see such a clean peak as we have produced, since the production amplitude with ΛN in the final states will also have contribution from "uncorrelated" Λp production which might have a large strength compared to the contribution of Λp production mediated by the doorway mechanism of $\bar{K}NN$ (bound) production. An answer to this issue can be provided only by detailed calculations for each particular reaction. In
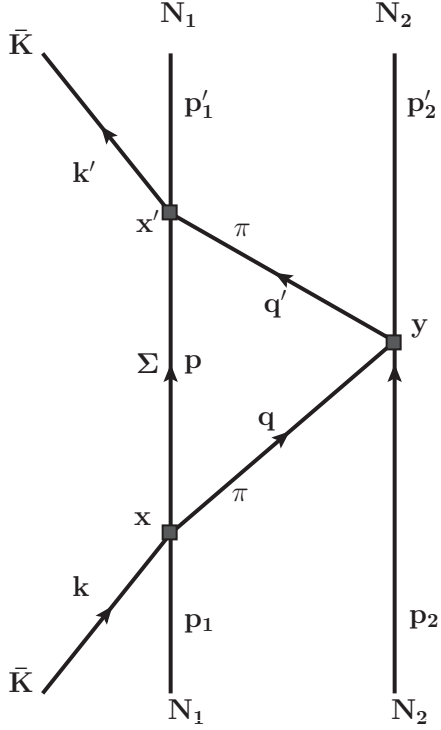
FIG. 5. Feynman diagram for the $\pi \Sigma N$ channel. The equivalent diagrams where the $\bar{K}$ interacts first with the second nucleon should be added.

such a case, the explicit evaluation of the $\bar{K}NN$ scattering amplitude that we have obtained would be of much use.

The width of the $\bar{K}NN$ state of about 50 MeV that we have obtained is in line with that obtained in most approaches. Some of them provide even larger widths [2–5]. Note that although our $\bar{K}N$ amplitudes contain the transition $\bar{K}N \to \pi \Sigma$ in intermediate states, and hence our approach contains $\pi \Sigma N$ intermediate states, we do not have diagrams in our approach, so far, in which the π or the Σ interact with the nucleon, although former works have indicated these effects to be small, as mentioned in the Introduction. We come back to this issue in the next section. Common to all the theoretical approaches is the lack of $\bar{K}$ absorption by two nucleons. In our approach, this is also not considered, and this would also make the width larger. Estimates done in Refs. [4,5] point to one increase of extra 25 MeV in the width from this source, leading to widths considerably larger than the binding, which added to other difficulties pointed out above, do not help when trying to observe experimentally this interesting state.

IV. EXPLICIT CONSIDIRATION OF THE $\pi \Sigma N$ CHANNNEL IN THE THREE-BODY SYSTEM

As we have explained, the $\pi \Sigma$ channel and other coupled channels are explicitly taken into account in the consideration of the $\bar{K}N$ amplitude which we have used in the FCA approach. This means that we account for the $\bar{K}N \to \pi \Sigma$

transition, and an intermediate $\pi \Sigma N$ channel, but this $\pi \Sigma$ state is again reconverted to $\bar{K}N$ leaving the other N as a spectator. This is accounted in the multiple scattering in the $\bar{K}N \to \bar{K}N$ scattering matrix on one nucleon. However, we do not consider the possibility that one has $\bar{K}N \to \pi \Sigma$ and the π rescatters with the second nucleon. If we want to have a final $\bar{K}NN$ system again, the π has to scatter later with the Σ to produce $\bar{K}N$. One may consider multiple scatterings of the π with the nucleons, but given the smallness of the πN amplitude compared to the $\bar{K}N$, any diagram beyond the one having one rescattering of the pion with the nucleon will be negligible. Then we must consider the diagram of Fig. 5 (for $\bar{K}N$ scattering on nucleon 1).

The S matrix for this process is given by

$$
\begin{aligned}
S^{(\pi \Sigma)} = &\int d^4x \int d^4x' \int d^4y \frac{1}{\sqrt{2\omega_{\bar{K}}\mathcal{V}}} \frac{1}{\sqrt{2\omega_{\bar{K}'}\mathcal{V}}} \\
&\times e^{-ikx} e^{ik'x'} \varphi_1(x)\varphi_2(y)\varphi_1^*(x')\varphi_2^*(y) \\
&\times \int \frac{d^4p}{(2\pi)^4} e^{-ip(x'-x)} \frac{2M_\Sigma}{2E_\Sigma(\vec{p})} \frac{1}{p^0 - E_\Sigma(\vec{p}) + i\epsilon} \\
&\times \int \frac{d^4q}{(2\pi)^4} e^{-iq(y-x)} \frac{1}{q^2 - m_\pi^2 + i\epsilon} \\
&\times \int \frac{d^4q'}{(2\pi)^4} e^{-iq'(x'-y)} \frac{1}{q'^2 - m_\pi^2 + i\epsilon} \\
&\times t_{\bar{K}N,\pi\Sigma} t_{\pi N,\pi N} t_{\pi\Sigma,\bar{K}N}.
\end{aligned}
\tag{23}
$$

We use the variables

$$
\begin{aligned}
&\vec{R} = \tfrac{1}{2}(\vec{x} + \vec{y}), \vec{R}' = \tfrac{1}{2}(\vec{x'} + \vec{y}), \vec{r} = (\vec{y} - \vec{x}), \\
&\vec{r}' = (\vec{x'} - \vec{y}), \vec{R}' = \vec{R} + \tfrac{1}{2}(\vec{r} + \vec{r}')
\end{aligned}
\tag{24}
$$

and perform explicitly the $x^0, x'^0, y^0, q^0, q'^0, p^0$ integrations following similar steps as in Sec. II A.

We also write

$$
\begin{aligned}
\varphi_1(\vec{x})\varphi_2(\vec{y}) &= \frac{1}{\sqrt{\mathcal{V}}} e^{i\vec{k}_{NN}\vec{R}} \varphi(\vec{r}), \\
\varphi_1^*(\vec{x'})\varphi_2^*((\vec{y}) &= \frac{1}{\sqrt{\mathcal{V}}} e^{-i\vec{k}'_{NN}\vec{R}'} \varphi^*(\vec{r}'),
\end{aligned}
\tag{25}
$$

where $\varphi(\vec{r})$ is the wave function of the NN system, for which we take the same expression as it was used to evaluate the form factor. In momentum space

$$
\tilde{\varphi}(\vec{q}) = \int d^3\vec{r} e^{i\vec{q}\vec{r}} \varphi(\vec{r}) = \frac{1}{N} \sum_j \frac{C_j}{\vec{q}^2 + m_j^2}
\tag{26}
$$

with

$$
N^2 = \frac{1}{(2\pi)^3} \int d^3\vec{p} \left(\sum_j \frac{C_j}{\vec{p}^2 + m_j^2} \right)^2.
\tag{27}
$$

Then we find

$$S^{(\pi\Sigma)} = i \frac{1}{\sqrt{2\omega_{\bar{K}}\mathcal{V}}} \frac{1}{\sqrt{2\omega_{\bar{K}'}\mathcal{V}}} \int \frac{d^3\vec{q}}{(2\pi)^3} \int \frac{d^3\vec{q}'}{(2\pi)^3} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{M_\Sigma}{E_\Sigma(\vec{p})} \tilde{\varphi}(\vec{p}+\vec{q})\tilde{\varphi}(\vec{p}+\vec{q}')(2\pi)^4$$

$$\times \delta(k+k_{NN}-k'-k'_{NN})t_{\bar{K}N,\pi\Sigma}t_{\pi N,\pi N}t_{\Sigma,\bar{K}N}\frac{1}{2}\frac{1}{\omega(\vec{q})+\omega(\vec{q}')}\frac{1}{\omega(\vec{q})\omega(\vec{q}')}\frac{1}{\sqrt{s_1}-\omega(\vec{q})-E(\vec{p})+i\epsilon}$$

$$\times \frac{1}{\sqrt{s_1}-\omega(\vec{q}')-E(\vec{p})+i\epsilon}[\sqrt{s_1}-E(\vec{p})-\omega(\vec{q})-\omega(\vec{q}')]. \tag{28}$$

One can also perform the angular integrations in $\vec{q}, \vec{q}'$ explicitly and considering the relationship of S to T from Eq. (14) we find

$$T^{(\pi\Sigma)} = -\frac{1}{N^2}\frac{1}{(2\pi)^3}\int p^2 dp \frac{M_\Sigma}{E_\Sigma(\vec{p})} \int q dq \int q' dq' \sum_j \frac{C_j}{2p}\ln\frac{p^2+q^2+2pq+m_j^2}{p^2+q^2-2pq+m_j^2} \sum_i \frac{C_i}{2p}\ln\frac{p^2+q'^2+2pq'+m_i^2}{p^2+q'^2-2pq'+m_i^2}$$

$$\times t_{\bar{K}N,\pi\Sigma}t_{\pi N,\pi N}t_{\Sigma,\bar{K}N}\frac{1}{\omega(\vec{q})+\omega(\vec{q}')}\frac{1}{\omega(\vec{q})\omega(\vec{q}')}\frac{1}{\sqrt{s_1}-\omega(\vec{q})-E(\vec{p})+i\epsilon}\frac{1}{\sqrt{s_1}-\omega(\vec{q}')-E(\vec{p})+i\epsilon}$$

$$\times [\sqrt{s_1}-E(\vec{p})-\omega(\vec{q})-\omega(\vec{q}')]. \tag{29}$$

Using the same arguments as in Sec. II A one can show that the $t_{\bar{K}N,\pi\Sigma}$ amplitude that appears in Eq. (29) is given by

$$t_{\bar{K}N,\pi\Sigma} = \frac{1}{4}t_{\bar{K}N,\pi\Sigma}^{I=1} + \frac{3}{4}t_{\bar{K}N,\pi\Sigma}^{I=0}; S=0 \tag{30}$$

$$t_{\bar{K}N,\pi\Sigma} = \frac{3}{4}t_{\bar{K}N,\pi\Sigma}^{I=1} + \frac{1}{4}t_{\bar{K}N,\pi\Sigma}^{I=0}; S=1 \tag{31}$$

$$t_{\pi N,\pi N} = \left(\frac{7}{12}-\frac{1}{\sqrt{6}}\right)t_{\pi N,\pi N}^{I=\frac{3}{2}} + \left(\frac{5}{12}+\frac{1}{\sqrt{6}}\right)t_{\pi N,\pi N}^{I=\frac{1}{2}}; S=0 \tag{32}$$

$$t_{\pi N,\pi N} = \left(\frac{11}{12}+\frac{2}{3}\frac{1}{\sqrt{3}}+\frac{2}{3}\frac{1}{\sqrt{6}}-\frac{1}{6}\sqrt{2}\right)t_{\pi N,\pi N}^{I=\frac{3}{2}}$$

$$+ \left(\frac{7}{12}-\frac{2}{3}\frac{1}{\sqrt{3}}-\frac{2}{3}\frac{1}{\sqrt{6}}+\frac{1}{6}\sqrt{2}\right)t_{\pi N,\pi N}^{I=\frac{1}{2}}; S=1. \tag{33}$$

We get $t_{\pi N,\pi N}$ from the work of Ref. [49].[1] We find that $T^{(\pi\Sigma)}$ is about a factor 20 smaller than $t_{\bar{K}N,\bar{K}N}$. Once we have $T^{(\pi\Sigma)}$ we add it coherently to $t_{\bar{K}N,\bar{K}N}(t_1)$ in Eq. (19) and recalculate the total T matrix. This will also contain rescattering effects.

The result of the including the $\pi\Sigma$ channel can be seen in Figs. 3 and 4 for $S=1$, $S=0$. We can see that the effects are moderate. For $S=0$ we find a small shift of the peak of about 2–3 MeV to smaller binding energies and a small broadening of the width by about 2–3 MeV. The small shift of the peak to lower energies does not contradict the findings of Ref. [3] where the comparison is made with Faddeev equations using $\bar{K}N$ and $\pi\Sigma$ channels or the same Faddeev equations using only $\bar{K}N$, both in the two body scattering matrices and in the three-body Faddeev equations. Here the $\pi\Sigma$, $\bar{K}N$ channels always have been considered in the two body $\bar{K}N$ amplitudes. This result answers a question raised in the Introduction, and

the approximately 11-MeV shift of the binding energy seen in Ref. [3] should be mostly attributed to the consideration of the $\pi\Sigma$ channel in the two-body scattering matrices rather than its explicit consideration in the extra diagrams of the Faddeev series that we have evaluated explicitly here.

For the case of $S=1$ we can see the result in Fig. 3. The effects of the $T^{(\pi\Sigma)}$ amplitude are also small and here we do not see any visible change in the peak position or the width, only a small increase in the $|T|^2$.

V. CONCLUSIONS

We performed calculations for the scattering amplitudes of the $\bar{K}NN$ system using the FCA of Faddeev equations and considering the scattering of the light $\bar{K}$ against the heavier NN cluster. We assume that an NN cluster is made, since in $S=1$, $I_{NN}=0$ the NN system is bound, and the $S=0$, $I_{NN}=1$, which is nearly bound in free space, gets the small push needed to bind from the strong attractive $\bar{K}N$ interaction. We reduced the "frozen" condition for the NN system by allowing its size to be changed, and for that we took the results for the NN radius obtained in the calculations of Ref. [5]. We found that the consideration of this reduced NN size reverted into a larger binding of the three-body system. For the $S=0$, $I_{NN}=1$ state with a "normal" NN size we found a peak around 30 MeV binding and a width of about 50 MeV, but when the reduced size was considered we found a shift of about 10 MeV to larger binding energies and practically no change in the width ($B=40$ MeV, $\Gamma=50$ MeV). We also found a peak with smaller strength for the $S=1$, $I_{NN}=0$ (with $L_{\bar{K}N}=0$) configuration around 27 MeV, when the reduced size was considered. We also saw that this configuration should weakly mix (with $L_{\bar{K}N}=1$) with the dominant configuration $S=0$, $I_{NN}=1$. The results that we obtain are in line with those obtained when a chiral amplitude is used for the $\bar{K}N$ interaction or when the energy dependence of the chiral approach is used [4–6]. Our results, which rely only on physical on-shell amplitudes, reinforce the findings of these latter works and they shed light on why other works could

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[1]We thank A. Ramos for providing us with the numerical amplitudes of the model of Ref. [49].

give different results, because they either force the $\Lambda(1405)$ with mass 1405 MeV to be a bound state of $\bar{K}N$ or because they use an energy independent kernel to describe the $\bar{K}N$ interaction. In any case, they introduce off-shell effects which cannot be controlled in those approaches.

The simplicity of the present approach also allows for a transparent interpretation of the results, which is not easy to see when one uses either a variational method or the full Faddeev equations. The dominance of the $S = 0$, $I_{NN} = 1$ channel could be anticipated once the $\bar{K}N$ amplitudes for a N belonging to a cluster with $S = 0$ or $S = 1$ is known. This conclusion is in agreement with results of other methods, which were found with more laborious ways. The results obtained here with our on shell approach with the FCA to Faddeev equations, should encourage more elaborate calculations with the on-shell approach to Faddeev equations developed in Refs. [28,29]. Comparison with the present results would tell us the degree of accuracy of the present method, and with this knowledge one could venture into similar problems with this technically much simpler approach.

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