# $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states in a chiral quark model

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The *S*-wave  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states with isospin  $I = 1/2$  and spin  $S = 1/2$  are dynamically investigated within the framework of a chiral constituent quark model by solving a resonating group method equation. The results show that the interaction between  $\Sigma_c$  and  $\bar{D}$  is attractive, which consequently results in a  $\Sigma_c\bar{D}$  bound state with a binding energy of about 5–42 MeV, unlike the case of the  $\Lambda_c\bar{D}$  state, which has a repulsive interaction and thus is unbound. The channel-coupling effect of  $\Sigma_c\bar{D}$  and  $\Lambda_c\bar{D}$  is found to be negligible owing to the fact that the gap between the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  thresholds is relatively large and the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  transition interaction is weak.

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#### **I. INTRODUCTION**

Understanding the structure and dynamical origin of baryon resonances is one of the most important topics within the field of hadron physics. On the quark level, several constituent quark models have been developed to investigated the mass spectrum of excited baryon states. Isgur, Karl, and Capstick described the baryon resonances as excited states of three constituent quarks (*qqq*) which are confined by a phenomenological confinement potential and interact through a residual interaction inspired by one-gluon exchange (OGE) [\[1,2\]](#page-5-0). Glozman and Riska proposed a rather different interaction mechanism. In their model, two quarks interact via Goldstone boson exchanges (GBEs) in addition to a phenomenological confinement potential, and it is claimed that the flavor-dependent interaction is responsible for the low mass of the Roper resonance  $[N*(1440)]$  [\[3,4\]](#page-5-0). So far it is not clear whether the interactions among the three constituent quarks, which are assumed to form the baryon resonances, should be described by either OGE or GBE or a mixture of both  $[5,6]$ . In chiral constituent quark models, it is found that some nucleon resonances are able to be accommodated as baryon-meson dynamically generated resonances  $[7-9]$ . In Refs. [\[7,8\]](#page-5-0), the  $\Lambda K$  and  $\Sigma K$  states were dynamically investigated in a chiral SU(3) quark model, and it was shown that a resonance with the same quantum numbers as the *S*<sup>11</sup> nucleon resonances can be dynamically generated owing to the strong  $\Sigma K$  attraction. In Ref. [\[9\]](#page-5-0) also, the  $\overline{K}N$ and  $\pi \Sigma$  interactions were dynamically investigated within the extended chiral SU(3) quark model, and it was found that both  $\pi \Sigma$  and  $\bar{K}N$  are bound and the latter appears as a  $\pi \Sigma$ resonance in a coupled-channel calculation. This resonance is referred to as  $\Lambda(1405)$ .

At the hadron level, various sophisticated coupled-channel approaches have been formulated for the study of baryon resonances. In the *K*-matrix-approximation approach [\[10–12\]](#page-6-0), only on-shell intermediate states are taken into account when solving the scattering equation for two-body scattering, which prohibits virtual two-body intermediate states. There, all resonances are treated as genuine resonances and no dynamical poles are reported. The unitary isobar model was developed by Drechsel *et al.* [\[13\]](#page-6-0). It is a variation of the standard *K*-matrixapproximation approach, and all the resonances are included as genuine resonances described by Breit-Wigner forms. In the chiral unitary approach, which includes only the lowest-order interacting diagrams (i.e., the contact terms) in the scattering kernel, a completely different picture is delivered and resonances appear as dynamical effects through rescattering. In the baryon sector, the  $N^*(1535)$ ,  $N^*(1650)$ ,  $N^*(1700)$ ,  $\Delta^*(1700)$ , and  $\Lambda$ (1405) have been claimed to be dynamically generated from the interactions of the pseudoscalar meson octet or vector meson octet with the nucleon octet or  $\Delta$  decuplet [\[14–16\]](#page-6-0). The dynamical coupled-channel hadron-exchange models, capable of a quantitative description of the meson production processes, have been developed by the Jülich group and the Excited Baryon Analysis Center (EBAC) at Jefferson Lab to study the nucleon resonances  $[17–21]$ . In the Jülich model, the Roper *N*<sup>∗</sup>(1440) resonance appears as a dynamically generated resonance and the other resonances like *N*<sup>∗</sup>(1535), *N*<sup>∗</sup>(1650), and  $\Delta^*(1700)$  are included as genuine resonances [\[17–19\]](#page-6-0). In the EBAC model, all resonances needed for fitting the data are included explicitly and no dynamically generated resonance is reported  $[20,21]$ .

The situation we have presented so far clearly shows that the constituent quark models and the models on the hadron level do not give us a definite picture of the structures of the baryon resonances. Different models may give us different descriptions for the resonance structures even though they fitted the same set of data, since each model has its own uncertainties with tunable parameters. Thus it is still confusing to us whether the baryon resonances should be described by three-quark ( $qqq)$ ) or five-quark configurations ( $qqqqq\bar{q}$ ), or baryon-meson dynamically generated states, or a mixture of them.

The study of  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states is of particular interest. If there exists a  $\Sigma_c \bar{D}$  bound state or a  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  dynamically generated state, its energy will be around 4*.*3 GeV. Unlike the low-energy resonances, where the excitation energies, i.e., the energy differences of nucleon ground state and nucleon resonance states, are hundreds of MeV (usually comparable to the 3*q* configuration excitation energy), such a high-energy resonance, if it exists, will have more than 3*.*3 GeV excitation <span id="page-1-0"></span>energy. This definitely excludes its explanation as a three-lightquark configuration (*qqq*), and only its description as dominated by a hidden charm five-constituent-quark configuration  $(qqqc\bar{c})$ , or a  $\Sigma_c\bar{D}$  bound state, or a  $\Sigma_c\bar{D}$ - $\Lambda_c\bar{D}$  resonance state, or a mixture of them is possible.

In Refs. [\[22,23\]](#page-6-0), the interaction between  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  was studied within the framework of the coupled-channel unitary approach. There, a  $\Sigma_c \bar{D}$  bound state is obtained with energy of 4.269 GeV, which is about 52 MeV below the  $\Sigma_c \bar{D}$  threshold. This state is found not to couple to the  $\Lambda_c\bar{D}$  channel even though its energy is about 114 MeV above the  $\Lambda_c \bar{D}$  threshold. Since the unitary approach used in Refs. [\[22,23\]](#page-6-0) is restricted to the contact term interaction only, the momentum-dependent terms being neglected, a study of the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states in other approaches is imperative in order to check the model dependence and to confirm the possibility of the existence of such a  $\Sigma_c \bar{D}$  bound state.

In the past few years, the chiral  $SU(3)$  quark model and its extended version have been shown to be quite reasonable and useful models to describe the medium-range nonperturbative QCD effect in light-flavor systems. Some successes were achieved when these two models were applied to studies of the energies of the baryon ground states, the binding energy of the deuteron, the nucleon-nucleon (*NN*) and kaon-nucleon (*KN*) scattering phase shifts of different partial waves, and the hyperon-nucleon  $(YN)$  and antikaon-nucleon  $(\bar{K}N)$  cross sections [\[24–30\]](#page-6-0). In the chiral SU(3) quark model, the quarkquark interaction contains OGE, a confinement potential, and boson exchanges stemming from scalar and pseudoscalar nonets. In the extended chiral SU(3) quark model, the boson exchanges stemming from the vector nonets are also included, and as a consequence the OGE is greatly reduced by fitting to the energies of the octet and decuplet baryon ground states. Recently, these two models have also been applied to study the systems  $N\phi$ ,  $N\overline{\Omega}$ ,  $E\overline{K}$ ,  $\Omega\pi$ ,  $\Omega\omega$ ,  $\omega\phi$ , and  $D^0\overline{D}^{*0}$ , among others [\[31–37\]](#page-6-0).

In this work, we further extend the chiral SU(3) quark model and its extended version to perform a dynamical coupledchannel study of the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states in the framework of the resonating group method (RGM), a well-established method for studying the interactions among composite particles [\[38–40\]](#page-6-0). The quark configuration of the considered system is  $(qqc)-(q\bar{c})$  with *q* being the light-flavor quark *u* or *d*. We take the interaction between the light-flavor quark pair *qq* from our previous work, where the parameters were fixed by a fitting of the energies of octet and decuplet baryon ground states, the binding energy of the deuteron, the *NN* scattering phase shifts, and the *Y N* cross sections [\[24,25\]](#page-6-0). The light-heavy quark pair *qc* or  $q\bar{c}$  and the heavy-heavy quark pair  $c\bar{c}$  are considered here to interact via the OGE and confinement potential. The only adjustable parameter is the charm quark mass  $m<sub>c</sub>$ , while the parameters of the OGE and the confinement for  $qc$ ,  $q\bar{c}$ , and  $c\bar{c}$  interactions are fixed by the masses of the charmed baryons  $\Sigma_c$  and  $\Lambda_c$ , the charmed mesons *D* and *D*<sup>\*</sup>, and the charmonium  $J/\psi$  and  $\eta_c$ , and by the stability conditions of those hadrons. Our results show that the interaction between  $\Sigma_c$  and  $\bar{D}$  is attractive, which consequently results in a  $\Sigma_c \bar{D}$ bound state with binding energy of about 5–42 MeV, unlike

the  $\Lambda_c \bar{D}$  state, which has a repulsive interaction and thus is unbound. The channel-coupling effect of  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  is found to be negligible because the gap between the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  thresholds is relatively large and the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$ transition interaction is weak.

The paper is organized as follows. In the next section the framework is briefly introduced. The results for the  $\Sigma_c \bar{D}$  and  $\Lambda_c\bar{D}$  states are shown in Sec. [III,](#page-3-0) where some discussion is presented as well. Finally, the summary is given in Sec. [IV.](#page-5-0)

## **II. FORMULATION**

The chiral quark model used in the present work has been widely described in the literature [\[7,8,](#page-5-0)[27–30\]](#page-6-0), and we refer the reader to those references for details. Here we just present the salient features of this model. The total Hamiltonian is written as

$$
H = \sum_{i} T_i - T_G + \sum_{i,j} V_{ij},
$$
 (1)

where  $T_i$  is the kinetic energy operator for the *i*th quark, and  $T_G$ is the kinetic energy operator for the center-of-mass motion. *Vij* represents the interactions between quark-quark and quarkantiquark,

$$
V_{ij} = \begin{cases} V_{ij}^{\text{OGE}} + V_{ij}^{\text{conf}} + \sum_{M} V_{ij}^{M} & (ij = qq), \\ V_{ij}^{\text{OGE}} + V_{ij}^{\text{conf}} & (ij = qQ, q\bar{Q}, Q\bar{Q}), \end{cases}
$$
\n(2)

where *q* and *Q* represent the light quark *u* or *d* and heavy quark *c*, respectively;  $V_{ij}^{\text{OGE}}$  is the OGE potential

$$
V_{ij}^{\text{OGE}} = \frac{1}{4} g_i g_j \left( \lambda_i^c \cdot \lambda_j^c \right) \left[ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) \times \left( \frac{1}{m_{qi}^2} + \frac{1}{m_{qj}^2} + \frac{4}{3} \frac{\sigma_i \cdot \sigma_j}{m_{qi} m_{qj}} \right) \right],
$$
 (3)

and  $V_{ij}^{\text{conf}}$  is the confinement potential which provides the nonperturbative QCD effect at large distance,

$$
V_{ij}^{\text{conf}} = -(\lambda_i^c \cdot \lambda_j^c) (a_{ij}^c r_{ij} + a_{ij}^{c0}). \tag{4}
$$

 $V_{ij}^M$  represents the effective quark-quark potential induced by one-boson exchanges, and it is only considered for the light quark pairs. Generally,

$$
V_{ij}^{M} = V_{ij}^{\sigma_a} + V_{ij}^{\pi_a} + V_{ij}^{\rho_a}, \tag{5}
$$

with  $V_{ij}^{\sigma_a}$ ,  $V_{ij}^{\pi_a}$ , and  $V_{ij}^{\rho_a}$  stemming from scalar nonets, pseudoscalar nonets, and vector nonets, respectively. Their explicit forms are

$$
V^{\sigma_a}(\mathbf{r}_{ij}) = -C(g_{\text{ch}}, m_{\sigma_a}, \Lambda) X_1(m_{\sigma_a}, \Lambda, r_{ij}) \left(\lambda_i^a \lambda_j^a\right), \quad (6)
$$

$$
V^{\pi_a}(\mathbf{r}_{ij}) = C(g_{\text{ch}}, m_{\pi_a}, \Lambda) \frac{m_{\pi_a}^2}{12m_{q_i}m_{q_j}} X_2(m_{\pi_a}, \Lambda, r_{ij})
$$

$$
\times (\sigma_i \cdot \sigma_j) (\lambda_i^a \lambda_j^a), \qquad (7)
$$

$$
V^{\rho_a}(\mathbf{r}_{ij}) = C(g_{\text{chv}}, m_{\rho_a}, \Lambda) \left[ X_1(m_{\rho_a}, \Lambda, r_{ij}) + \frac{m_{\rho_a}^2}{6m_{q_i}m_{q_j}} \times \left( 1 + \frac{f_{\text{chv}}}{g_{\text{chv}}}\frac{m_{q_i} + m_{q_j}}{M_N} + \frac{f_{\text{chv}}^2}{g_{\text{chv}}^2}\frac{m_{q_i}m_{q_j}}{M_N^2} \right) \times X_2(m_{\rho_a}, \Lambda, r_{ij}) (\sigma_i \cdot \sigma_j) \left[ \left( \lambda_i^a \lambda_j^a \right), \right] \tag{8}
$$

where

$$
C(g_{\text{ch}}, m, \Lambda) = \frac{g_{\text{ch}}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} m,
$$
\n(9)

$$
X_1(m, \Lambda, r) = Y(mr) - \frac{\Lambda}{m} Y(\Lambda r), \qquad (10)
$$

$$
X_2(m, \Lambda, r) = Y(mr) - \left(\frac{\Lambda}{m}\right)^3 Y(\Lambda r), \quad (11)
$$

$$
Y(x) = \frac{1}{x}e^{-x},
$$
 (12)

with  $m_{\sigma_a}$  being the mass of the scalar meson,  $m_{\pi_a}$  the mass of the pseudoscalar meson, and  $m_{\rho_a}$  the mass of the vector meson.  $m_{q_i}$  is the constituent quark mass of the *i*th quark.  $g_{ch}$ is the coupling constant for the scalar and pseudoscalar nonets, and  $g_{\text{chv}}$  and  $f_{\text{chv}}$  the coupling constants for vector coupling and tensor coupling of vector nonets.

In this work, we take the parameters for the light-flavor quark system from our previous work [\[8,](#page-5-0)[33,34\]](#page-6-0), which gave a satisfactory description for the energies of the octet and decuplet baryon ground states, the binding energy of the deuteron, the *NN* scattering phase shifts, and the *NY* cross sections. The main procedure for determination of those parameters is the following. The initial input parameters, i.e., the harmonic-oscillator width parameter  $b<sub>u</sub>$  and the up (down) quark mass  $m_{u(d)}$ , are taken to have the usual values:  $b_u =$ 0*.*5 fm for the chiral SU(3) quark model and 0*.*45 fm for the extended chiral SU(3) quark model and  $m_{u(d)} = 313$  MeV. The coupling constant for scalar and pseudoscalar chiral field coupling, *g*ch, is fixed by the relation

$$
\frac{g_{\rm ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{NN\pi}^2}{4\pi} \frac{m_u^2}{M_N^2},\tag{13}
$$

with the empirical value  $g_{NN\pi}^2/4\pi = 13.67$ . For the vector meson field coupling, we consider three different cases. In model I, the coupling between vector meson field and quark field is not considered at all, which means  $g_{\text{chv}} = 0$ . Then, in models II and III, the coupling constant for vector coupling is taken to be  $g_{\text{chv}} = 2.351$  and 1.973, respectively, and the ratio for the tensor coupling and vector coupling is taken to be 0 and 2*/*3, respectively. The masses of the mesons are taken to be the experimental values, except for the  $\sigma$  meson.  $m_{\sigma}$  is obtained by fitting the binding energy of the deuteron. The cutoff radius  $\Lambda^{-1}$  is taken to have a value close to the chiral symmetry breaking scale [\[41–44\]](#page-6-0). The OGE coupling constants and the strengths of the confinement potential are fitted by the baryon masses and their stability conditions.

Note that, in light-flavor quark systems, the confinement potential is found to give negligible contributions between two color-singlet hadron clusters [\[7,](#page-5-0)[27–29\]](#page-6-0). Therefore different

TABLE I. Model parameters. Model I refers to the model where the coupling for vector nonets is not considered. Models II and III refer to the models where the coupling for vector nonets is included while the ratio for tensor coupling and vector coupling  $f_{\text{chv}}/g_{\text{chv}}$  is taken to be 0 and 2*/*3, respectively.

	$m_c$ (GeV)	$g_c$	$a_{uu}^c$	$a_{uc}^c$ $(fm^{-2})$ $(fm^{-2})$ $(fm^{-2})$ $(fm^{-1})$ $(fm^{-1})$ $(fm^{-1})$	$a_{cc}^c$	$a_{uu}^{c0}$	$a_{uc}^{c0}$	$a_{cc}^{c0}$
	1.43	0.35	0.44	1.07	1.74	$-0.38$	$-0.74$	$-0.73$
	1.55	0.37	0.44	1.08	1.77	$-0.38$	$-0.85$	$-0.93$
	1.87	0.43	0.44	1.10	1.81	$-0.38$	$-1.14$	$-1.44$
П	1.43	0.77	0.41	1.70	1.83	$-0.53$		$-1.15 -0.34$
	1.55	0.82	0.41	1.72	1.68	$-0.53$	$-1.27$	$-0.40$
	1.87	0.94	0.41	1.76	1.04	$-0.53$	$-1.57$	$-0.47$
Ш	1.43	0.57	0.37	1.68	2.19	$-0.46$	$-1.14$	$-0.71$
	1.55	0.60	0.37	1.69	2.16	$-0.46$	$-1.25$	$-0.85$
	1.87	0.69	0.37	1.74	1.94	$-0.46$	$-1.55$	$-1.17$

forms of confinement potential (linear or quadratic) do not have any visible influence on the theoretical results in lightflavor quark systems. In the present work we adopt a color linear confinement potential. The results from a calculation using a color quadratic confinement potential are discussed as well. Of course the *NN* scattering phase shifts and the *NY* cross sections are always well described irrespective of the confinement form because the contribution of any confinement to these systems is negligible.

The additional parameters needed in the present work are those associated with the charm quark. The only adjustable parameter is the charm quark mass  $m<sub>c</sub>$ . Here we take three typical values,  $m_c = 1.43$  GeV [\[45\]](#page-6-0), 1.55 GeV [\[46\]](#page-6-0), and 1.87 GeV  $[47]$ , to test the dependence of our results on  $m_c$ . The other parameters we need are the coupling constant of the OGE and confinement strengths for light and heavy quark pairs  $qc$  and  $q\bar{c}$  and for the heavy quark pair  $c\bar{c}$ . They are fixed by a fitting to the masses and stability conditions of the charmed baryons  $\Sigma_c$  and  $\Lambda_c$ , charmed mesons *D* and  $D^*$ , and charmonium  $J/\psi$  and  $\eta_c$ . The values of those parameters are listed in Table I. The corresponding masses of  $\Sigma_c$ ,  $\Lambda_c$ , *D*,  $D^*$ ,  $J/\psi$ , and  $\eta_c$  obtained with  $m_c = 1.55$  GeV are shown in Table II. There, model I refers to the model where the coupling for vector nonets is not considered. Models II and III refer to the models where the coupling for vector nonets is included while the ratio for tensor coupling and vector coupling, *f*chv*/g*chv, is taken to be 0 and 2*/*3, respectively.

TABLE II. The masses (in GeV) of  $\Sigma_c$ ,  $\Lambda_c$ , *D*,  $D^*$ ,  $J/\psi$ , and  $\eta_c$ obtained from models I, II, and III, respectively, with  $m_c$  being taken as 1*.*55 GeV. Experimental values are taken from the Particle Data Group (PDG) [\[48\]](#page-6-0).

	$\Sigma_c$	$\Lambda_c$	D	D*	$J/\psi$	$\eta_c$
Expt.	2.452	2.286	1.869	2.007	3.097	2.980
	2.436	2.269	1.883	1.947	3.052	3.024
Н	2.450	2.283	1.869	1.932	3.129	2.946
Ш	2.450	2.283	1.869	1.932	3.087	2.989

<span id="page-3-0"></span>With all parameters determined, the  $\Sigma_c\bar{D}$  and  $\Lambda_c\bar{D}$  systems can be dynamically studied in the framework of the RGM, where the wave function of the five-quark system is of the following form:

$$
\Psi = \sum_{\beta} \mathcal{A}\{[\hat{\phi}_A(\xi_1, \xi_2)\hat{\phi}_B(\xi_3)]_{\beta} \chi_{\beta}(R_{AB})\}.
$$
 (14)

Here  $\xi_1$  and  $\xi_2$  are the internal coordinates for the cluster *A* ( $\Lambda_c$  or  $\Sigma_c$ ), and  $\xi_3$  is the internal coordinate for the cluster *B* ( $\overline{D}$ ).  $R_{AB} \equiv R_A - R_B$  is the relative coordinate between the two clusters *A* and *B*, and  $\beta \equiv (A, B, I, S, L, J)$ specifies the hadron species (*A*,*B*) and quantum numbers of the baryon-meson channel.  $\hat{\phi}_A$  and  $\hat{\phi}_B$  are the internal cluster wave functions of *A* and *B*, and  $\chi_{\beta}(R_{AB})$  is the relative wave function of the two clusters. The symbol  $A$  represents the antisymmetrizing operator defined as

$$
\mathcal{A} \equiv 1 - \sum_{i \in A} P_{i4} \equiv 1 - 3P_{34}.
$$
 (15)

Substituting  $\Psi$  into the projection equation

$$
\langle \delta \Psi | (H - E) | \Psi \rangle = 0, \tag{16}
$$

we obtain the coupled integro-differential equation for the relative function *χβ* as

$$
\sum_{\beta'} \int [\mathcal{H}_{\beta\beta'}(R,R') - E\mathcal{N}_{\beta\beta'}(R,R')] \chi_{\beta'}(R') dR' = 0,
$$
\n(17)

where the Hamiltonian kernel  $H$  and normalization kernel  $\mathcal N$ can, respectively, be calculated from

$$
\begin{aligned}\n\left\{\frac{\mathcal{H}_{\beta\beta'}(R, R')}{\mathcal{N}_{\beta\beta'}(R, R')}\right\} &= \left\langle [\hat{\phi}_A(\xi_1, \xi_2)\hat{\phi}_B(\xi_3)]_{\beta}\delta(R - R_{AB}) \right. \\
&\times \left| \left\{ \begin{array}{c} H \\ 1 \end{array} \right\} \right| \mathcal{A}[[\hat{\phi}_A(\xi_1, \xi_2)\hat{\phi}_B(\xi_3)]_{\beta'}\delta \\
&\times (R' - R_{AB})]\right\rangle.\n\end{aligned} \tag{18}
$$

Equation (17) is the so-called coupled-channel RGM equation. Expanding the unknown  $\chi_{\beta}(R_{AB})$  by employing well-defined basis wave functions, such as Gaussian functions, one can solve the coupled-channel RGM equation for a bound-state or a scattering problem to obtain the binding energy or scattering *S* matrix elements for the two-cluster systems. The details of solving the RGM equation can be found in Refs. [\[38–40\]](#page-6-0).

## **III. RESULTS AND DISCUSSION**

As mentioned in the Introduction, the structures of the nucleon resonances below 2 GeV are not clear so far. Different models may give us different pictures even though they fit the same set of data, since each model has its own uncertainties, which are usually approximated by fitting parameters. It is still a challenging question for hadron physicists whether the low-energy baryon resonances should be described by a three- (*qqq*) or a five-constituent-quark configuration

 $(qqqq\bar{q})$ , or baryon-meson dynamically generated states, or a mixture of them. The  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states are of particular interest simply because if there exists a  $\Sigma_c \bar{D}$  bound state or a  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  dynamically generated resonance, its energy will be around 4*.*3 GeV, and the explanation of such a high-energy state as a three-constituent-quark configuration (*qqq*) will be definitely excluded; only the description that this state is dominated by a hidden charm five-constituent-quark configuration ( $qqqc\bar{c}$ ) or a  $\Sigma_c\bar{D}$ - $\Lambda_c\bar{D}$  baryon-meson state or a mixture of them will be possible. Thus the system of  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  is a good place to test whether we could have a nucleon resonance whose configuration is dominated by at least five quarks.

Here we perform a dynamical investigation of the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states with isospin  $I = 1/2$  and spin  $S = 1/2$  by solving the RGM equation [Eq.  $(17)$ ] in our chiral quark models as depicted in Sec. [II.](#page-1-0) Our purpose is to understand the interaction properties of the  $\Sigma_c \overline{D}$  and  $\Lambda_c \overline{D}$  states and to see whether there exists a  $\Sigma_c \bar{D}$  bound state or a  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  dynamically generated resonance within our chiral quark models.

Figure [1](#page-4-0) shows the diagonal matrix elements of the Hamiltonian for the  $\Sigma_c \bar{D}$  system in the generator coordinate method (GCM) [\[38\]](#page-6-0) calculation, which can be regarded qualitatively as the effective Hamiltonian of two color-singlet clusters  $\Sigma_c$  and  $\bar{D}$ . In Fig. [1,](#page-4-0)  $H_{\Sigma_c\bar{D}}$  includes the kinetic energy of  $\Sigma_c \bar{D}$  relative motion and the effective potential between  $\Sigma_c$  and  $\overline{D}$ , and *s* denotes the generator coordinate, which can qualitatively describe the distance between the two clusters  $\sum_c$  and  $\bar{D}$ . From Fig. [1,](#page-4-0) one sees that  $\Sigma_c$  and  $\bar{D}$  are attractive to each other in the medium range for all three values of the charm quark mass  $m_c = 1.43$ , 1.55, and 1.87 GeV and all three models I, II, and III (see Sec. [II](#page-1-0) for details of these three models). Our further analysis demonstrates that in model I the attraction between  $\Sigma_c$  and  $\bar{D}$  is dominated by  $\sigma$  exchange and the color magnetic force of the OGE; the latter exists between the two color-singlet clusters  $\Sigma_c$  and  $\overline{D}$  because of the antisymmetrizing  $[Eq. (14)]$  of the four constituent quarks in  $\Sigma_c \bar{D}$  required by the general Pauli principle. In models II and III, the OGE, among light-flavor quarks are greatly reduced by vector-meson exchanges and the  $\Sigma_c \bar{D}$  attraction is found to be dominated by  $\sigma$  and  $\rho$  exchanges.

Inspired by the moderately large  $\Sigma_c \bar{D}$  attraction, we solved the RGM equation for a bound-state problem to see whether or not there is a  $\Sigma_c \bar{D}$  bound state. Our results are listed in Table [III,](#page-4-0) where the first and second columns denote the model and the charm quark mass, respectively, and the third column shows the corresponding binding energy for each set of parameters. One sees that the  $\Sigma_c \bar{D}$  is really bound independent of the type of model and the value of the charm quark mass we use. The binding energy is around 9–42 MeV in various models, which corresponds to an energy of 4*.*279–4*.*312 GeV for the  $\Sigma_c \bar{D}$  bound state (the  $\Sigma_c \bar{D}$  threshold is 4.321 GeV).

Here we would like to discuss the dependence of our results on the phenomenological confinement potential. In light-flavor quark systems, the SU(3) flavor symmetry is approximately respected and thus the confinement potential between two color-singlet hadron clusters is found to give negligible contributions [\[7](#page-5-0)[,27–29\]](#page-6-0). If the charm quark is included, the SU(4) flavor symmetry is strongly violated since the charm

<span id="page-4-0"></span>

FIG. 1. (Color online) The GCM matrix elements of the Hamiltonian for the  $\Sigma_c \bar{D}$  system. The dotted, solid, and dash-dotted lines represent the results obtained in models I, II, and III, respectively.

TABLE III. The binding energy of  $\Sigma_c \bar{D}$  (in MeV) in models I, II, and III, respectively.

	$m_c$ (GeV)	$r$ confinement	$r^2$ confinement
I	1.43	9.3	4.5
	1.55	10.9	6.4
	1.87	15.3	11.0
П	1.43	28.3	9.3
	1.55	31.8	10.3
	1.87	41.6	10.0
Ш	1.43	19.7	7.3
	1.55	22.2	8.9
	1.87	28.6	11.3

quark mass is much bigger than that of the light-flavor quark. The consequence of this flavor symmetry violation is that the contribution of the confinement potential to the interaction between two hadron clusters may not be negligible. In the present work, we check the dependence of our results on the form of the confinement potential by replacing the linear confinement  $[Eq. (4)]$  $[Eq. (4)]$  $[Eq. (4)]$  with a quadratic one,

$$
V_{ij}^{\text{conf}} = -(\lambda_i^c \cdot \lambda_j^c) \left( a_{ij}^c r_{ij}^2 + a_{ij}^{c0} \right), \tag{19}
$$

with the parameters being fitted by using the same procedure as given in the previous section. With the quadratic confinement Eq. (19), we re-solve the RGM equation for the  $\Sigma_c \bar{D}$  boundstate problem, and the results are shown in the fourth column of Table III. One sees that the  $\Sigma_c \bar{D}$  is still bound in various models and the binding energy is around 5–11 MeV, which is a little smaller than that for linear confinement. The corresponding energy of the  $\Sigma_c \bar{D}$  bound state is 4.310–4.316 GeV.

We have also studied the  $\Lambda_c\bar{D}$  system. Figure [2](#page-5-0) shows the diagonal matrix elements of the Hamiltonian for the  $\Lambda_c \bar{D}$  system in the GCM calculation, which can be regarded qualitatively as the effective Hamiltonian of two color-singlet clusters  $\Lambda_c$  and  $\bar{D}$ . One sees that, unlike the  $\Sigma_c \bar{D}$  system, which is attractive in the medium range, the  $\Lambda_c \bar{D}$  system is strongly repulsive for all three models and all three values of charm quark mass. No  $\Lambda_c \bar{D}$  bound state will be found as a matter of course because of this repulsion.

Is there a  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  resonance in the coupled-channel study? In Refs. [\[7,8\]](#page-5-0), we dynamically investigated the  $\Sigma K$ and  $\Lambda K$  systems by using the RGM in our chiral quark model. There, it was found that the  $\Sigma K$  interaction is attractive and  $a \Sigma K$  bound state can be formed as a consequence, with a binding energy of about 17–44 MeV, while the  $\Lambda K$  state is repulsive and unbound. In the coupled-channel calculation, a  $\Sigma K$ - $\Lambda K$  dynamically generated resonance is obtained; it is located between the thresholds of  $\Sigma K$  and  $\Lambda K$  and has quantum numbers the same as those for nucleon  $S_{11}$  resonances. By analogy, one may expect a  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  dynamically generated resonance in the coupled-channel calculation since  $\Sigma_c \bar{D}$  is also attractive and bound just like  $\Sigma K$ . But actually, the coupled-channel effect of  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  is found to be negligible, and no  $\Sigma_c \bar{D}$ - $\Lambda_c \bar{D}$  resonance is found in our coupled-channel calculation. This is because the gap between the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  thresholds, 166 MeV, is comparatively big and the transition matrix elements between  $\Sigma_c \overline{D}$  and  $\Lambda_c \overline{D}$  are too weak, in contrast to the case of the  $\Sigma K$ - $\Lambda K$  system, where the gap between the two channel thresholds is only 78 MeV and the transition matrix elements between  $\Sigma K$  and  $\Lambda K$  are relatively large.

In brief, we obtain a  $\Sigma_c \bar{D}$  bound state in our model with energy of about 4*.*279–4*.*316 MeV, and the effect on this state of the  $\Lambda_c \bar{D}$  channel is negligible. In Refs. [\[22,23\]](#page-6-0), the  $\Sigma_c \bar{D}$ and  $\Lambda_c \bar{D}$  states were studied on the hadron level within the framework of the coupled-channel unitary approach. There, a  $\Sigma_c \bar{D}$  bound state was also found with energy of about 4.240–4.291 GeV, and this state does not couple to the  $\Lambda_c\bar{D}$ channel. Although the binding energy given in Refs. [\[22,23\]](#page-6-0) is bigger than that obtained in the present work, it makes sense that the results from different theoretical approaches

<span id="page-5-0"></span>

FIG. 2. (Color online) The GCM matrix elements of the Hamiltonian for the  $\Lambda_c \bar{D}$  system. The dotted, solid, and dash-dotted lines represent the results obtained in models I, II, and III, respectively.

are qualitatively similar. Note that the  $\Sigma_c \bar{D}$  bound state, if it exists, cannot be accommodated in the three light-flavor quark configuration (*qqq*), unlike the nucleon resonances below 2 GeV. Whether or not it can be explained as a hidden charm

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five-constituent-quark configuration  $(qqqc\bar{c})$  needs further detailed scrutiny. Investigations from other approaches and experiments are needed to further confirm the existence of this state and to pin down its structure and mass.

As discussed in Refs. [\[22,23\]](#page-6-0), the largest decay modes of the  $\Sigma_c \bar{D}$  bound state are expected to be  $\eta_c N$ ,  $K \Sigma$ ,  $\eta N$ ,  $\eta' N$ , and  $\pi N$  through *t*-channel  $D^*$  and  $D_s^*$  exchanges. A much smaller but much more easily detected decay mode is *J/ψN* through *t*-channel *D* exchange. Since  $D^*$ ,  $D_s^*$ , and *D* are very heavy, the total decay width of the state is expected to be small. The narrow width of the predicted state makes it easier to observe in experiments. Furthermore, compared with those baryons with hidden charms below the  $\eta_c$ *N* threshold proposed by other approaches [\[49\]](#page-6-0), the  $\Sigma_c \bar{D}$  bound state predicted by the present work is above the  $\eta_c N$  and  $J/\psi N$  thresholds, which also makes it much easier to detect experimentally.

### **IV. SUMMARY**

In this work, we perform a dynamical coupled-channel study of the  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$  states by solving the RGM equation in the framework of a chiral quark model. The model parameters for light-flavor quarks are taken from our previous work [8]; they gave a satisfactory description of the energies of the octet and decuplet baryon ground states, the binding energy of the deuteron, the *NN* scattering phase shifts, and the *NY* cross sections. The parameters associated with the charm quark are determined by fitting the energies and the stability conditions of  $\Sigma_c$ ,  $\Lambda_c$ ,  $D$ ,  $D^*$ ,  $J/\psi$ , and  $\eta_c$ . Our results show that the  $\Sigma_c$  and  $\overline{D}$  interaction is attractive and a  $\Sigma_c \overline{D}$ bound state can be formed as a consequence, with energy of about 4.279–4.316 GeV, while the  $\Lambda_c\bar{D}$  state is repulsive and unbound. The channel-coupling effect between  $\Sigma_c \overline{D}$  and  $\Lambda_c \overline{D}$ is negligible because of the large mass difference between the  $\Sigma_c \overline{D}$  and  $\Lambda_c \overline{D}$  thresholds and the small off-diagonal matrix elements of  $\Sigma_c \bar{D}$  and  $\Lambda_c \bar{D}$ . This  $\Sigma_c \bar{D}$  bound state, if it really exists, cannot be accommodated in a three-lightflavor-quark configuration (*qqq*). Further investigations using other approaches and experiments are needed to confirm the existence of this state and to pin down its structure and mass.

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