Strongly intensive quantities

M. I. Gorenstein[1,2] and M. Gaździcki[3,4]
[1]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[2]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
[3]*Institut für Kernphysik, University of Frankfurt, Frankfurt, Germany*
[4]*Jan Kochanowski University, Kielce, Poland*
(Received 8 February 2011; published 21 July 2011)

Analysis of fluctuations of hadron production properties in collisions of relativistic particles profits from use of measurable intensive quantities which are independent of system size variations. The first family of such quantities was proposed in 1992; another is introduced in this paper. Furthermore we present a proof of independence of volume fluctuations for quantities from both families within the framework of the grand canonical ensemble. These quantities are referred to as strongly intensive ones. Influence of conservation laws and resonance decays is also discussed.

DOI: 10.1103/PhysRevC.84.014904 PACS number(s): 25.75.Dw, 12.40.Ee, 24.10.Pa, 24.60.Ky

I. INTRODUCTION

An intensive quantity is a physical quantity which does not depend on the system volume. In contrast, an extensive quantity is proportional to the system volume. Clearly, the ratio of two extensive quantities is an intensive one. As an example, let us consider the number of particles, N, in the relativistic gas which fluctuates around its mean value, $\langle N\rangle$. Within the grand canonical ensemble, $\langle N\rangle$ is an extensive quantity, whereas the ratio of mean multiplicities of two different particle types is an intensive one. Particle number fluctuations are quantified by the variance $\langle N^2\rangle - \langle N\rangle^2$, which is an extensive quantity, while the scaled variance, $[\langle N^2\rangle - \langle N\rangle^2]/\langle N\rangle$, is an intensive one.

Statistical models are surprisingly successful in modeling multiparticle production in high-energy interactions [1]. They are used to describe properties of strongly interacting matter created in nucleus-nucleus collisions in terms of intensive quantities. In particular, an equation of state is usually given as a function relating pressure, temperature, and baryonic chemical potential. On the other hand, in high-energy collisions the volume of produced matter cannot be kept fixed. For instance, nucleus-nucleus collisions with different centralities may produce a statistical system with the same local properties (e.g., the same temperature and baryonic chemical potential) but with the system volume changing significantly from interaction to interaction. Thus, an important question is whether it is possible to measure the properties of the system without knowing its volume fluctuations, or equivalently, whether there are measurable quantities which are independent of volume fluctuations.

Within the grand canonical ensemble, the answer is yes, and the quantities with the required properties are referred to as strongly intensive ones. Ratios of mean particle multiplicities are strongly intensive quantities. In general, this is the case for all ratios of any two extensive quantities which correspond to the first moments of fluctuating variables. They are intensive and strongly intensive quantities. However, the situation is more complicated for the measures of fluctuations which include the second moments of fluctuating variables. For example, as is shown below, the scaled variance of a particle number distribution is an intensive quantity but not a strongly intensive one.

In this paper, we show that there are two families of strongly intensive quantities which characterize the second moments of random extensive variables used to study fluctuations and correlations in a physical system. The first family was introduced in 1992 [2], and another one is proposed in this paper.

The paper is organized as follows. In Sec. II, the two families of strongly intensive quantities are introduced. For simplicity, this is done within the model of independent particle sources. The relation of the strongly intensive quantities to previously used fluctuation measures is discussed in Sec. III. The proof that the quantities are in fact strongly intensive, that is, strictly independent of volume and volume fluctuations within the grand canonical ensemble, is given in Sec. IV. Finally, their properties within the canonical and microcanonical ensembles are discussed in Sec. V. The summary given in Sec. VI closes the paper.

II. TWO FAMILIES OF STRONGLY INTENSIVE QUANTITIES

Let us start from the model of independent sources for multiparticle production in which the number of sources, N_s, changes from event to event. The first and the most popular example of this approach is the wounded nucleon model [3]. In the model of independent sources, extensive quantities (e.g., mean number of particles and mean transverse energy) are considered as those which are proportional to N_s. Two fluctuating extensive variables A and B can be expressed as

$$A = a_1 + a_2 + \cdots + a_{N_s}, \quad B = b_1 + b_2 + \cdots + b_{N_s}, \qquad (1)$$

where a_k and b_k denote the contributions from the kth source. One finds for event averages

$$\langle A\rangle = \langle a\rangle\langle N_s\rangle, \quad \langle A^2\rangle = \langle a^2\rangle\langle N_s\rangle + \langle a\rangle^2\big[\langle N_s^2\rangle - \langle N_s\rangle\big], \qquad (2)$$

$$\langle B\rangle = \langle b\rangle\langle N_s\rangle, \quad \langle B^2\rangle = \langle b^2\rangle\langle N_s\rangle + \langle b\rangle^2\big[\langle N_s^2\rangle - \langle N_s\rangle\big], \qquad (3)$$

$$\langle AB\rangle = \langle ab\rangle\langle N_s\rangle + \langle a\rangle\langle b\rangle\big[\langle N_s^2\rangle - \langle N_s\rangle\big], \qquad (4)$$

where $\langle a\rangle$, $\langle b\rangle$ and $\langle a^2\rangle$, $\langle b^2\rangle$, $\langle ab\rangle$ are the first and second moments of the distribution $P^*(a,b)$ for a single source. These quantities are independent of N_s and play the role of intensive quantities in the model of independent sources. The distribution $P^*(a,b)$ is assumed to be the same for all sources; that is, they are statistically identical. The probability distribution $P_s(N_s)$ of the source number is needed to calculate $\langle N_s\rangle$ and $\langle N_s^2\rangle$ and, in general, it is unknown. Using Eq. (2), the scaled variance ω_A which describes the event-by-event fluctuations of the extensive variable A can be presented as

$$\omega_A \equiv \frac{\langle A^2\rangle - \langle A\rangle^2}{\langle A\rangle} = \frac{\langle a^2\rangle - \langle a\rangle^2}{\langle a\rangle} + \langle a\rangle\frac{\langle N_s^2\rangle - \langle N_s\rangle^2}{\langle N_s\rangle} \equiv \omega_a^* + \langle a\rangle\omega_s, \tag{5}$$

where ω_a^* is the scaled variance of the quantity A for each source. A similar expression follows from Eq. (3) for the scaled variance ω_B. The scaled variances ω_A and ω_B are independent of the average number of sources $\langle N_s\rangle$. Thus, ω_A and ω_B are intensive quantities. However, they depend on the fluctuations of the number of sources via ω_s and, therefore, they are not strongly intensive quantities.

From Eq. (2) follows that a knowledge of $\langle A\rangle$ and $\langle A^2\rangle$ is not sufficient to derive any strongly intensive quantity. However, this is possible when two extensive random variables A and B are considered. In order to characterize fluctuations, one may then construct special combinations of the second moments in which the terms proportional to $\langle N_s^2\rangle$ in the right-hand side of Eqs. (2)–(4) are not present. Clearly, only two linearly independent combinations of this type result from the three equations (2)–(4). Note that in order to remove the dependence on $\langle N_s\rangle$, strongly intensive quantities should be in a form of reducible fractions. The following combinations seem the most convenient:

$$\sum^{AB} = \langle C\rangle^{-1}[\langle B\rangle\omega_A + \langle A\rangle\omega_B - 2(\langle AB\rangle - \langle A\rangle\langle B\rangle)], \tag{6}$$

$$\Delta^{AB} = \langle C\rangle^{-1}[\langle B\rangle\omega_A - \langle A\rangle\omega_B], \tag{7}$$

where $\langle C\rangle$ is the average of any extensive quantity, for example, $\langle A\rangle$ or $\langle B\rangle$. Straightforward calculation of (6) and (7) using Eqs. (2)–(5) gives

$$\sum^{AB} = \langle c\rangle^{-1}[\langle b\rangle\omega_a^* + \langle a\rangle\omega_b^* - 2(\langle ab\rangle - \langle a\rangle\langle b\rangle)], \tag{8}$$

$$\Delta^{AB} = \langle c\rangle^{-1}[\langle b\rangle\omega_a^* - \langle a\rangle\omega_b^*]. \tag{9}$$

Thus, Σ^{AB} and Δ^{AB} defined by Eqs. (6) and (7) depend on ω_a^* and ω_b^*, but they are independent of the average number of sources $\langle N_s\rangle$ and its fluctuations ω_s. They are, in fact, strongly intensive measures which quantify fluctuations of any two extensive random variables A and B. This is proved in Sec. IV within the grand canonical ensemble for a case of particle multiplicities.

There is an important difference between the Σ^{AB} and Δ^{AB} quantities. Namely, in order to calculate Δ^{AB} one needs to measure only the first two moments: $\langle A\rangle$, $\langle B\rangle$ and $\langle A^2\rangle$, $\langle B^2\rangle$. This can be done by independent measurements of the distributions $P_A(A)$ and $P_B(B)$. Quantity Σ^{AB} includes the correlations term, $\langle AB\rangle - \langle A\rangle\langle B\rangle$, and thus it also requires simultaneous measurements of A and B in order to obtain the joint probability distribution $P_{AB}(A,B)$. The quantities Σ^{AB} and Δ^{AB} also have different symmetry properties: $\Sigma^{AB} = \Sigma^{BA}$ and $\Delta^{AB} = -\Delta^{BA}$. We call all strongly intensive quantities which include the correlation term the Σ family, and those which include only variances are the Δ family.

III. RELATION TO OTHER FLUCTUATION MEASURES

The well-known fluctuation measure Φ was introduced in 1992 [2] for a study of transverse momentum fluctuations, and it belongs to the Σ family. In the general case [4], when $A \equiv X = \sum_{i=1}^{N} x_i$ represents any motional extensive variable as a sum of single-particle variables and $B \equiv N$ is the particle multiplicity, one gets

$$\Phi_x = \left[\frac{\langle X\rangle}{\langle N\rangle}\Sigma^{XN}\right]^{1/2} - [\overline{x^2} - \overline{x}^2]^{1/2}, \tag{10}$$

where Σ^{XN} is given by Eq. (6) with $C \equiv N$, and $\overline{x^2}$, $\overline{x}^2$ correspond to single-particle inclusive averages. Note that these inclusive quantities can also be presented in terms of event averages, namely $\overline{x} = \langle X\rangle/\langle N\rangle$ and $\overline{x^2} = \langle X_2\rangle/\langle N\rangle$, where $X_2 \equiv \sum_{i=1}^{N} x_i^2$. The measure Φ was extended in 1999 [5,6] for multiplicity fluctuations. For two particle types, A and B, the Φ measure was constructed by setting $x_i = 1$ if the ith particle is of the A type and $x_i = 0$ otherwise. One then finds $\overline{x} = \overline{x^2} = \langle A\rangle/[\langle A\rangle + \langle B\rangle]$ and thus $\overline{x^2} - \overline{x}^2 = \langle A\rangle\langle B\rangle/[\langle A\rangle + \langle B\rangle]^2$ with A and B denoting particle numbers. Taking into account these relations, and using $X = A$ and $N = A + B$ in Eq. (10), the expression for the Φ reads

$$\Phi = \frac{\sqrt{\langle A\rangle\langle B\rangle}}{\langle A+B\rangle}[(\Sigma^{AB})^{1/2} - 1], \tag{11}$$

where Σ^{AB} is given by Eq. (6) with $C \equiv A + B$.

A possible extension of Φ for the case of two motional variables has not been discussed yet; however, it can be done naturally within the framework of the Σ^{AB} and Δ^{AB} families presented here. It is also important to note that the Φ measure extended to the study of the third moment preserves its strongly intensive properties within the model of independent sources [7]. Study of strongly intensive quantities which include third and higher moments of extensive quantities is beyond the scope of this paper.

Another quantity frequently used to characterize the fluctuations of particle numbers A and B was introduced in 2002 [8] as

$$\nu_{\rm dyn}^{AB} \equiv \frac{\langle A(A-1)\rangle}{\langle A\rangle^2} + \frac{\langle B(B-1)\rangle}{\langle B\rangle^2} - 2\frac{\langle AB\rangle}{\langle A\rangle\langle B\rangle}. \tag{12}$$

Using Eq. (6) with $C \equiv A + B$, one easily finds the relation

$$\nu_{\rm dyn}^{AB} = \frac{\langle A+B\rangle}{\langle A\rangle\langle B\rangle}[\Sigma^{AB} - 1]. \tag{13}$$

Equation (13) shows that $\nu_{\rm dyn}^{AB}$, similar to Σ^{AB}, is independent of fluctuations of the source number, but it decreases as $\nu_{\rm dyn}^{AB} \propto \langle N_s\rangle^{-1}$ and, thus, it is not an intensive quantity. Note that the quantity $\langle C\rangle\nu_{\rm dyn}^{AB}$, where C can be chosen as A, B, or $A + B$, is a strongly intensive quantity from the Σ family. Despite the fact that specific examples of the Σ^{AB} family were introduced and discussed a long time ago, the Δ^{AB} family is now proposed in this paper.

IV. PROOF WITHIN THE GRAND CANONICAL ENSEMBLE

Let us now prove within the grand canonical ensemble (GCE) that the two families of quantities, Σ^{AB} and Δ^{AB}, are strongly intensive. The proof is limited to a case of particle multiplicities; that is, A and B are the numbers of particles of type A and B, respectively. The GCE partition function Ξ of the quantum gas, which is a mixture of different types of particles, reads

$$\Xi = \exp\left\{V\sum_j \eta_j d_j \int \frac{d^3p}{(2\pi)^3}\ln[1+\eta_j\lambda_j\exp(-\epsilon_j/T)]\right\}, \tag{14}$$

where V and T denote, respectively, the system volume and temperature; λ_j is the fugacity, which is related to particle chemical potential μ_j as $\lambda_j \equiv \exp(\mu_j/T)$; d_j denotes the number of a particle's internal degrees of freedom; $\epsilon_j \equiv (m_j^2+\mathbf{p}^2)^{1/2}$ is the particle energy with m_j and $\mathbf{p}$ being its mass and momentum; $\eta_i=-1$ for bosons; $\eta_i=1$ for fermions; and $\eta_i=0$ corresponds to the classical Boltzmann approximation. The GCE averages are calculated as

$$\begin{aligned}\overline{A} &= \frac{1}{\Xi}\lambda_A\frac{\partial}{\partial\lambda_A}\Xi\\ &= V\int\frac{d^3p}{(2\pi)^3}\frac{d_A}{\lambda_A^{-1}\exp(\epsilon_A/T)+\eta_A}\equiv Vn_A,\end{aligned} \tag{15}$$

$$\begin{aligned}\overline{A^2} &= \frac{1}{\Xi}\left(\lambda_A\frac{\partial}{\partial\lambda_A}\right)^2\Xi\\ &= V^2n_A^2+V\int\frac{d^3p}{(2\pi)^3}\frac{d_A\lambda_A^{-1}\exp(\epsilon_A/T)}{\left[\lambda_A^{-1}\exp(\epsilon_A/T)+\eta_A\right]^2},\end{aligned} \tag{16}$$

$$\overline{AB} = \frac{1}{\Xi}\lambda_A\frac{\partial}{\partial\lambda_A}\lambda_B\frac{\partial}{\partial\lambda_B}\Xi = V^2n_An_B, \tag{17}$$

where $n_A=\overline{A}/V$ and $n_B=\overline{B}/V$ denote the particle number densities. The corresponding expressions for $\overline{B}$ and $\overline{B^2}$ are obtained by replacing A by B in Eqs. (15) and (16).

For the GCE scaled variance, one finds

$$\omega_A^* \equiv \frac{\overline{A^2}-\overline{A}^2}{\overline{A}} = n_A^{-1}\int\frac{d^3p}{(2\pi)^3}\frac{d_A\lambda_A^{-1}\exp(\epsilon_A/T)}{\left[\lambda_A^{-1}\exp(\epsilon_A/T)+\eta_A\right]^2}. \tag{18}$$

It corresponds to the particle number fluctuations at a fixed volume V. It is an intensive quantity and depends only on T and μ_A. Note that $\omega_A^*>1$ for bosons, $\omega_A^*<1$ for fermions, and $\omega_A^*=1$ for classical Boltzmann particles.

We introduce volume fluctuations, assuming that local properties of the system within the GCE (i.e., the temperature and chemical potentials) are volume independent. The volume fluctuations are described by the probability density function $F(V)$. Thus, the full averaging denoted as $\langle\cdots\rangle$ includes both the GCE averaging (15)–(17) at a fixed volume and an averaging over the volume fluctuations:

$$\begin{aligned}\langle A\rangle &= \langle V\rangle n_A,\\ \langle A^2\rangle &= \langle V^2\rangle n_A^2+\langle V\rangle n_A\omega_A^*,\quad \langle AB\rangle = \langle V^2\rangle n_An_B,\end{aligned} \tag{19}$$

where $\langle V^k\rangle \equiv \int dV\, V^kF(V)$ for $k=1,2$. One finds

$$\omega_A \equiv \frac{\langle A^2\rangle-\langle A\rangle^2}{\langle A\rangle} = \omega_A^*+n_A\frac{\langle V^2\rangle-\langle V\rangle^2}{\langle V\rangle}\equiv\omega_A^*+n_A\omega_V. \tag{20}$$

The corresponding expression for ω_B is obtained by replacing A by B in Eq. (20). Equations (20) and (5) have similar structures. Namely, the first terms ω_A^* or ω_a^* correspond to the particle number fluctuations at a fixed volume V or fixed number of sources N_s, respectively. The second terms correspond to the contribution from the volume fluctuations in Eq. (20) and the fluctuations of the number of sources in Eq. (5).

By calculating (6) and (7) according to Eq. (19) with $C=A+B$, one gets

$$\begin{aligned}\Sigma^{AB} &= \frac{1}{n_A+n_B}(n_B\omega_A^*+n_A\omega_B^*),\\ \Delta^{AB} &= \frac{1}{n_A+n_B}(n_B\omega_A^*-n_A\omega_B^*).\end{aligned} \tag{21}$$

Equation (21) proves that Σ^{AB} and Δ^{AB} are *strongly intensive* quantities as they are strictly independent of average volume $\langle V\rangle$ and its fluctuations ω_V.

Note that in the GCE there are no correlations between the number of different particle species; that is, $\langle AB\rangle-\langle A\rangle\langle B\rangle=0$. In modeling of hadron production in high-energy collisions, particles stable with respect to strong decays are usually considered, whereas the GCE system includes also short-lived resonances which finally decay into stable particles. These resonance decays increase multiplicities of stable particles and thus change numerical values of n_A, n_B, ω_A^*, and ω_B^*. If decay products of a resonance R decay include both A and B hadrons, a correlation between them appears, and it can be expressed as

$$\begin{aligned}\langle AB\rangle-\langle A\rangle\langle B\rangle &= \sum_R\langle R\rangle(\langle AB\rangle_R-\langle A\rangle_R\langle B\rangle_R)\\ &\equiv \sum_R\langle R\rangle\rho_{AB}^R,\end{aligned} \tag{22}$$

where $\langle R\rangle$ is a mean multiplicity of R and $\langle\cdots\rangle_R$ means the averaging over its decay channels. The measure Σ^{AB} then has the form

$$\Sigma^{AB} = \frac{1}{n_A+n_B}\left(n_B\omega_A^*+n_A\omega_B^*-2\sum_R n_R\rho_{AB}^R\right), \tag{23}$$

and thus it remains a strongly intensive quantity.

V. PROPERTIES OF Σ^{AB} AND Δ^{AB} WITHIN CANONICAL AND MICROCANONICAL ENSEMBLES

For a large-volume system in equilibrium, the particle number distribution $P(A, B; V)$ in the GCE, canonical ensemble (CE), and microcanonical ensemble (MCE) can be written in a general form of the bivariate normal distribution (see Refs. [9,10]):

$$P(A,B;V) = \frac{1}{2\pi}\left[\omega_A^*\omega_B^*\left(1-\rho_{AB}^{*2}\right)\overline{A}\,\overline{B}\right]^{-1/2} \times \exp\left\{-\frac{1}{2\left(1-\rho_{AB}^{*2}\right)}\left[\frac{(A-\overline{A})^2}{\omega_A^*\overline{A}} - 2\,\rho_{AB}^*\frac{(A-\overline{A})(B-\overline{B})}{(\omega_A^*\omega_B^*\overline{A}\,\overline{B})^{1/2}} + \frac{(B-\overline{B})^2}{\omega_B^*\overline{B}}\right]\right\}, \tag{24}$$

where $\overline{A} \equiv n_A V$ and $\overline{B} \equiv n_B V$ are mean particle numbers. Averaging at a fixed volume V is defined as $(k = 1, 2)$

$$\overline{A^k} \equiv \sum_{A,B} A^k P(A,B;V), \quad \overline{B^k} \equiv \sum_{A,B} B^k P(A,B;V), \quad \overline{AB} \equiv \sum_{A,B} AB\,P(A,B;V). \tag{25}$$

The straightforward calculations of (25) with the distribution function (24) give

$$\frac{\overline{A^2}-\overline{A}^2}{\overline{A}} = \omega_A^*, \quad \frac{\overline{B^2}-\overline{B}^2}{\overline{B}} = \omega_B^*, \quad \frac{\overline{A\,B}-\overline{A}\,\overline{B}}{(\omega_A^*\omega_B^*\overline{A}\,\overline{B})^{1/2}} = \rho_{AB}^*. \tag{26}$$

Equation (26) reveals the meaning of the parameters in the distribution (24): the scaled variances ω_A^* and ω_B^*, and the correlation coefficient ρ_{AB}^*. In a mixture of relativistic ideal gases, particle numbers are not conserved, and thus A and B fluctuate in all statistical ensembles. This leads to nonzero positive values of ω_A^* and ω_B^* which are approaching constant values with system volume, increasing to infinity.[1] Particle correlations, and thus nonzero ρ_{AB}^*, result from exact material and motional conservation laws [12]. Thus the correlation coefficient ρ_{AB}^* equals to zero in the GCE and is nonzero in the CE and MCE. The exact conservation laws also influence the values of ω_A^* and ω_B^*. Quantities like the particle number densities do not depend on the choice of the statistical ensemble for large systems. This means thermodynamical equivalence of the statistical ensembles. Below we present results for the CE and MCE in the large volume limit in which all three statistical ensembles become thermodynamically equivalent. However, let us stress that the thermodynamical limits of the quantities (26) are different (see Ref. [12] for details) in the GCE, CE, and MCE ensembles.

[1] The volume dependence may be different for systems at the phase transition. For example, in a case of the Bose-Einstein condensation, one gets $\omega \propto V^{1/3}$ at $T = T_C$ and $\omega \propto V$ at $T < T_C$, as shown in Ref. [11].

We introduce now the volume fluctuations assuming that local properties of the system (e.g., temperature and conserved charge densities in the CE, or energy density and conserved charge densities in the MCE) are volume independent for sufficiently large volumes. In this case, the distribution (24) depends on the system volume only through the average multiplicities. The full averaging reads

$$\langle\cdots\rangle \equiv \int dV\,F(V)\sum_{A,B}\cdots P(A,B;V). \tag{27}$$

By calculating (6) and (7) according to Eq. (27) with $C = A + B$, one gets

$$\Sigma^{AB} = \frac{1}{n_A+n_B}\left[n_B\omega_A^* + n_A\omega_B^* - 2\rho_{AB}^*(n_An_B\omega_A^*\omega_B^*)^{1/2}\right], \tag{28}$$

$$\Delta^{AB} = \frac{1}{n_A+n_B}(n_B\omega_A^* - n_A\omega_B^*). \tag{29}$$

Equations (28) and (29) show that Σ^{AB} and Δ^{AB} are independent of the average system volume and its fluctuations. For the CE and MCE, this is valid if the volume fluctuates in a range in which all three statistical ensembles are thermodynamically equivalent.

A unique determination of five intensive quantities, n_A, n_B, ω_A^*, ω_B^*, and ρ_{AB}^*, from measurements of $\langle A\rangle$, $\langle B\rangle$, $\langle A^2\rangle$, $\langle B^2\rangle$, and $\langle AB\rangle$, is impossible as $\langle V\rangle$ and ω_V are in general unknown. The average particle multiplicities are given by $\langle A\rangle = n_A\langle V\rangle$ and $\langle B\rangle = n_B\langle V\rangle$. Therefore, only the ratio of particle number densities, $r_{AB} \equiv \langle A\rangle/\langle B\rangle = n_A/n_B$, can be found from the measurements of $\langle A\rangle$ and $\langle B\rangle$. The strongly intensive quantities, r_{AB}, Σ^{AB}, and Δ^{AB}, allow a unique determination of ω_A^* and ω_B^* in the GCE. In this case, there are no correlations between A and B at fixed volume, and from Eq. (21) one finds

$$\omega_A^* = 2(1+r_{AB})(\Sigma^{AB}+\Delta^{AB}), \quad \omega_B^* = 2\left(1+r_{AB}^{-1}\right)(\Sigma^{AB}-\Delta^{AB}). \tag{30}$$

However, if $\rho_{AB}^* \neq 0$, as in the CE and MCE, or if including correlations due to resonance decays in the GCE, even the knowledge of all strongly intensive quantities is not sufficient to reconstruct ω_A^*, ω_B^*, and ρ_{AB}^* in a unique way.

VI. SUMMARY

In summary, in this paper we consider two families of strongly intensive quantities Σ^{AB} and Δ^{AB} which characterize fluctuations of system properties. While specific measures from the Σ^{AB} family were introduced in 1992 [2], the Δ^{AB} family is proposed in this paper. We prove within the grand canonical ensemble that both Σ^{AB} and Δ^{AB} quantities are strictly independent of volume and volume fluctuations. In the canonical and microcanonical ensembles, they are approximately independent of volume and volume fluctuations for sufficiently large systems. Furthermore, we show that they are also independent of fluctuations of the number of sources in models of independent particle sources, for example, the wounded nucleon model [3]. This suggests that the Σ^{AB} and Δ^{AB} quantities may be approximately independent of the system size fluctuations in approaches which are neither strictly statistical nor strictly independent source models. Studies within transport models should help to clarify this conjecture. It would be also important to identify physical processes which may lead to a significant violation of the strong intensive properties of the Σ^{AB} and Δ^{AB} quantities.

Note that the Φ and ν_{dyn}^{AB} measures, which can be expressed in terms of Σ^{AB}, have already been used successfully to study transverse momentum and particle ratio fluctuations; see, for example, Refs. [13,14] and references therein. We hope that the results presented in this paper will be useful in further analysis of fluctuations of hadron production properties in collisions of relativistic particles.

ACKNOWLEDGMENTS

We thank V. V. Begun, W. Greiner, M. Hauer, S. Mrówczyński, and P. Seyboth for fruitful discussions. This work was in part supported by the Program of Fundamental Research of the Department of Physics and Astronomy of NAS, Ukraine, and the German Research Foundation (Grant No. GA 1480/2-1).

[1] For a recent review, see W. Florkowski, *Phenomenology of Ultra-relativistic Heavy Ion Collisions* (World Scientific Publishing Company, Singapore, 2009).
[2] M. Gaździcki and S. Mrówczyński, Z. Phys. C **54**, 127 (1992).
[3] A. Bialas, M. Bleszynski, and W. Czyz, Nucl. Phys. B **111**, 461 (1976).
[4] F. Liu, A. Tai, M. Gázdzicki, and R. Stock, Eur. Phys. J. C **8**, 649 (1999).
[5] M. Gaździcki, Eur. Phys. J. C **8**, 131 (1999).
[6] S. Mrówczyński, Phys. Lett. B **459**, 13 (1999).
[7] S. Mrówczyński, Phys. Lett. B **465**, 8 (1999).
[8] C. Pruneau, S. Gavin, and S. Voloshin, Phys. Rev. C **66**, 044904 (2002).
[9] M. Hauer, V. V. Begun, and M. I. Gorenstein, Eur. Phys. J. C **58**, 83 (2008).
[10] V. V. Begun, M. Gaździcki, and M. I. Gorenstein, Phys. Rev. C **80**, 064903 (2009).
[11] V. V. Begun and M. I. Gorenstein, Phys. Lett. B **653**, 190 (2007); Phys. Rev. C **77**, 064903 (2008).
[12] V. V. Begun, M. Gaździcki, M. I. Gorenstein, and O. S. Zozulya, Phys. Rev. C **70**, 034901 (2004); V. V. Begun, M. I. Gorenstein, M. Hauer, V. P. Konchakovski, and O. S. Zozulya, *ibid.* **74**, 044903 (2006); V. V. Begun, M. Gaździcki, M. I. Gorenstein, M. Hauer, V. P. Konchakovski, and B. Lungwitz, *ibid.* **76**, 024902 (2007).
[13] T. Anticic *et al.* (NA49 Collaboration), Phys. Rev. C **79**, 044904 (2009); C. Alt *et al.* (NA49 Collaboration), *ibid.* **70**, 064903 (2004).
[14] V. Koch and T. Schuster, Phys. Rev. C **81**, 034910 (2010).