# Search for systematic behavior of incomplete-fusion probability and complete-fusion suppression induced by <sup>9</sup>Be on different targets

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We present a trial to obtain a systematic behavior of the results available in the literature on the complete and incomplete fusion induced by the weakly bound projectile <sup>9</sup>Be on targets with different masses and/or charges. We stress that although the incomplete-fusion probability and the complete-fusion suppression are very closely related quantities, the first is an experimental value whereas the later is model dependent. A trend of systematic behavior for the incomplete-fusion probability as a function of the target charge is achieved, but not for the complete-fusion suppression.

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## I. INTRODUCTION

In recent years, efforts have been made to understand the effects of the breakup of weakly bound nuclei on the fusion cross section [1]. Following the projectile breakup, several processes may occur: one of the fragments may fuse with the target in a process named incomplete fusion (ICF) or all the fragments may fly away from the target in a process called noncapture breakup. Fusion of all the fragments with the target would lead to sequential complete fusion (SCF). The sum of ICF + SCF + the usual complete fusion is called total fusion (TF). Complete fusion (CF) is the sum of SCF + DCF. One approach to investigate this subject is to compare fusion data with predictions from some theory. The difference between them,  $\Delta \sigma_{\rm F}$ , is attributed to the ingredients missing in the theory. There are two kinds of effect to be investigated. The first is static effects, caused by the longer tail of the optical potential, owing to the low binding energies of the weakly bound and especially the halo nuclei. This effect gives rise to lower and thicker barriers when compared with tightly bound systems, and enhances fusion cross section at sub-barrier energies not too much below the barrier [2,3]. The second kind of effect is dynamic, caused by the strong coupling between the elastic channel and the continuum states representing the breakup channels. When one compares data with theory, the comparison may be (i) with a single channel with standard densities of the nuclei ( $\Delta \sigma_{\rm F}$  is due to static + dynamic effects); (ii) with a single channel with realistic densities ( $\Delta \sigma_{\rm F}$  comes from all channels but the static effect is already taken into account); (iii) with coupled-channel calculations taking into account all bound channels ( $\Delta \sigma_{\rm F}$  comes from the couplings to the continuum); (iv) Continuum Discretized Coupled Channels (CDCC) calculations ( $\Delta \sigma_{\rm F}$  should vanish). The comparison is, for sure, strongly model dependent.

Recently it has been shown [2,3], by using a comparison of fusion data with theoretical predictions which do not take into account the dynamic breakup plus transfer channel effects, that for energies not too much above the barrier (from around  $1.1V_{\rm B}$  to  $1.5V_{\rm B}$ ), CF involving the stable weakly bound projectiles <sup>6.7</sup>Li and <sup>9</sup>Be on heavy targets (<sup>208</sup>Pb and <sup>209</sup>Bi) [5–7] are suppressed by around 30%, whereas TF for

the same projectiles on targets of any mass do not seem to be affected by dynamic breakup + transfer effects [2–4]. In the theoretical model, a double-folding potential is used. As one observes suppression of CF above the barrier and no effect on TF in the same energy range, one attributes the suppression of CF to the presence of ICF. For lighter targets, as the Coulomb breakup becomes weaker, one expects that the suppression of CF becomes smaller than for heavy targets.

So far, there is no systematic behavior of the CF suppression as a function of the charge or mass of the targets. This is due to the difficulty in separating events from complete fusion and incomplete fusion for light systems, since most of their evaporation residues coincide. It is an experimental challenge to separate CF and ICF for light systems. As the charge of the target decreases, one expects that the Coulomb breakup becomes weaker, and consequently the CF suppression and ICF probability decrease.

For fusion induced by <sup>9</sup>Be, there are at present four systems for which CF was measured separated from ICF with the targets <sup>208</sup>Pb [5–7], <sup>144</sup>Sm [8,9], <sup>89</sup>Y [10], and <sup>124</sup>Sn [11]. For lighter targets, there are TF measurements for <sup>64</sup>Zn [12,13] and <sup>27</sup>Al [14]. So, in the present work we try to obtain a systematic behavior for the suppression of CF induced by <sup>9</sup>Be as a function of the target charge, and we also analyze the behavior of the probability of ICF as a function of the target charge for systems for which the data are available.

In Sec. II we present the conceptual difference between ICF probability and CF suppression. In Sec. III we describe the data available in the literature, whereas in Sec. IV we show the theoretical prediction for the variation of these quantities with the target charge. In Secs. V and VI we analyze those data and compare with the predictions. In Sec. VII we compare the results with the universal fusion function, and finally in Sec. VIII we present some conclusions.

# II. THE CONCEPTUAL DIFFERENCE BETWEEN ICF PROBABILITY AND CF SUPPRESSION

Although the concept that the ICF probability, defined as  $P_{\rm ICF} = \sigma_{\rm ICF} / \sigma_{\rm TF}$ , and the CF suppression are similar quantities

is widely used and accepted [5-11,15-17], we would like to point out important differences between these concepts, which will be used in the present work.  $P_{ICF}$  is, essentially, an experimental quantity. It does not depend on theoretical models for the interacting potential nor on coupled-channel calculations. On the other hand, CF suppression is a quantity very sensitive to the theoretical model used. When one talks about enhancement or hindrance of fusion, one must be very clear as to what one is talking about.

As we mentioned in the Introduction of this paper, if the theoretical calculations consider correctly all static and dynamical effects due to the characteristics of weakly bound nuclei, including breakup, there should be no difference between data and theory. That is what we expect if complete CDCC calculations are performed, including possible transfer channel effects. Nevertheless, it is still a challenge for theory to separate ICF from CF in a full quantum mechanical calculation. This has been achieved only for a few systems with very specific characteristics. If one wants to investigate the effect of the breakup process on the fusion, the breakup effects should not be included in the calculations. This means that resonances in the continuum of the weakly bound nuclei should not be included in the coupling scheme, nor should the bare potential be obtained by taking into account experimental fusion excitation functions and barrier distributions, because the data are certainly already affected by the breakup process to be investigated. In Refs. [5-11], resonances of the weakly bound nuclei and/or fit of the data were used in the theoretical calculations.

So, in this paper we present results of the comparison of CF cross-section data with predictions from a doublefolding potential based on realistic densities without any data fit search, and coupled-channel calculations involving only the target inelastic excitations. So the differences between experimental and theoretical cross sections will be attributed to full dynamical breakup effects plus any relevant transfer effect. The potential that we use is the Sao Paulo potential (SPP) [18,19], which has been proved to be a reliable potential for hundreds of systems, including the stable weakly bound projectile [20] under study in the present work. However, other reliable potentials may be used within the present approach, such as the Akyüs-Winther potential [21].

It is important to mention that the effects we investigate correspond to variations between 5% and 30% of the CF cross sections. Therefore, conclusions about a possible systematic behavior cannot be drawn from data with large uncertainties, as one finds sometimes in the literature.

## III. THE DATA AVAILABLE FOR THE PRESENT WORK

In the present work we analyze four systems for which CF cross sections induced by <sup>9</sup>Be are available. The targets are <sup>208</sup>Pb, <sup>144</sup>Sm, <sup>124</sup>Sn, and <sup>89</sup>Y. Brief comments on those data will be made before we show our results. The first of those targets to be investigated was <sup>208</sup>Pb [5,7]. The experiments were performed at ANU, Canberra, by measuring  $\alpha$  particles emitted by the evaporation residues, online and offline. All evaporation channels of CF and ICF were clearly identified,

and the error bars of the cross sections are small. By ICF one means the fusion of one  $\alpha$  particle with the target. Then, the  $^{144}$ Sm target was investigated [8,9]. The measurements were performed at the Tandar laboratory of Buenos Aires, using the offline  $K_{\alpha}$  x-ray technique. All evaporation channels of the CF were measured and the most important channel of the ICF was also measured. In Refs. [8,9] the authors report the lower limit of the ICF cross section. In the present work we corrected those results by including the predictions from the PACE code [22], in order to take into account the <sup>148</sup>Gd and <sup>146</sup>Gd channels of the ICF. The <sup>89</sup>Y target experiments [10] were performed at BARC, Mumbai, using the offline  $\gamma$ -ray technique. Most of the CF cross section was measured, and corrections were made, based on the PACE code, to obtain the full CF cross section. ICF could not be measured, since the compound nucleus decays by the  ${}^{92}Nb_m$  and  ${}^{92}Nb_{gs}$  channels, and only the first one could be measured. The  $^{\tilde{12}4}\mathrm{Sn}$  target experiments were also performed at the same laboratory in Mumbai, but using the online  $\gamma$ -ray technique [11]. Only a very small correction had to be considered in the CF cross section to take into account the unmeasured channels. The ICF could not be measured. Finally, it should be mentioned that in Ref. [12], only TF for <sup>64</sup>Zn is reported, although an estimation that ICF should be smaller than 10% is presented, based on PACE predictions and the large error bars obtained. In the following we use this value as an upper limit for this system. We do not use the results of  ${}^{9}\text{Be} + {}^{27}\text{Al}$  [14] because only TF was measured for that system.

## IV. AN EMPIRICAL PREDICTION FOR THE INCOMPLETE FUSION PROBABILITY

Although there is no theory on the dependence of the target charge  $Z_{\text{target}}$  of  $P_{\text{ICF}}$ , there is an empirical formula obtained by Hinde et al. [23], in a well-performed experiment, when the sub-barrier prompt breakup of <sup>9</sup>Be was measured and <sup>208</sup>Pb was the target. Following a careful analysis of the angular distributions of single and coincident  $\alpha$  particles, prompt <sup>9</sup>Be breakup was identified and separated from other events producing  $\alpha$  particles. Their results suggest that the prompt breakup is due largely to a process close to the nuclear surface. So the breakup probability is taken to be proportional to the gradient of the nuclear potential  $V'_{\rm N}$ , multiplied by an exponential factor  $f(R_s)$ , which is dependent on the surface-to-surface separation  $R_s$  and independent of nuclear structure. The fit to their data gives  $f(R_s)$  proportional to  $\exp(-0.924R_{\rm s})$ . The  $P_{\rm ICF}$  for <sup>208</sup>Pb was then scaled to predict  $P_{\rm ICF}$  for any target as

$$P_{\rm ICF} = P_{\rm ICF}(^{208}\rm{Pb}) \frac{V'_{\rm N}}{V'_{\rm N}^{\rm Pb}} \exp\{-0.924[R_{\rm s} - R_{\rm s}(^{208}\rm{Pb})]\}$$
(1)

when all quantities are evaluated at the fusion barrier radius  $R_{\rm B}$ . In the present work we use the parameters obtained from Refs. [15,24]. The nuclear potential was evaluated using the empirical potential of Christensen and Winther [25],

$$V'_{\rm N} = -50 \frac{R_{\rm P} R_{\rm T}}{R_{\rm P} + R_{\rm T}} \exp\left(-\frac{R_{\rm s}}{0.63}\right),$$
 (2)

where  $R_{\rm s} = R_{\rm B} - R({}^{9}{\rm Be}) - R_{\rm T}$  fm,  $R({}^{9}{\rm Be}) = 3.92$  fm [26], and  $R_{\rm i} = 1.2333 A_{\rm i}^{1/3} - 0.978 A_{\rm i}^{-1/3}$  fm. In expression (2)  $R_{\rm P} = R({}^{9}{\rm Be})$  and  $R_{\rm T}$  is the target radius. The barrier radius was calculated using the Sao Paulo potential.

Although this empirical prediction, based on geometrical assumptions, is not a theory that has to be necessarily in agreement with the data, it is a good reference curve, and it has the expected behavior that  $P_{ICF}$  should decrease with the target charge, due to the relatively smaller importance of the Coulomb breakup.

## V. THE PROBABILITY OF ICF AS A FUNCTION OF THE TARGET CHARGE

At present it is difficult to describe a systematic behavior for the  $P_{\rm ICF} = \sigma_{\rm ICF} / \sigma_{\rm TF}$  quantity as a function of  $Z_{\rm target}$ , since there are only two systems for which this quantity could be measured. In Fig. 1 we show this quantity for both systems, as a function of the center-of-mass energy in the range  $1.1V_{\rm B} < E < 1.5V_{\rm B}$ . In Fig. 2 we show the result of the average value for each system. In this figure we also show as a dashed point the estimate of the upper limit of the ICF for the <sup>64</sup>Zn target [12]. The shaded band in Fig. 2 is the empirical prediction described in the previous section. Uncertainties of the prediction are associated with the error bar of the radius of the Coulomb barrier obtained from the SPP. The results show a reasonable agreement between data and the prediction, although the  $P_{\rm ICF}$  for the <sup>144</sup>Sm target is below the shaded band. Although there are no available data for several systems,  $P_{\rm ICF}$  for the three systems follows the trend of the empirical prediction. As expected, PICF decreases when Ztarget decreases.



FIG. 1. (Color online) Experimental probability of incomplete fusion at energies slightly above the Coulomb barrier, defined as  $P_{\text{ICF}} = \sigma_{\text{ICF}} / \sigma_{\text{TF}}$  for the two systems for which it is available.



FIG. 2. (Color online) Mean experimental probability of incomplete fusion at energies slightly above the Coulomb barrier, for the two systems for which it is available. The dashed point is an estimation of  $P_{\rm ICF}$  for the <sup>64</sup>Zn target [12]. The shaded band is an empirical prediction by Hinde [23] for this value.

# VI. THE CF SUPPRESSION IN RELATION TO CALCULATIONS THAT DO NOT TAKE INTO ACCOUNT THE BREAKUP AND TRANSFER PROCESSES

In this section we compare the measured CF cross sections for the four mentioned systems with the theoretical predictions of calculations that do not take into account the breakup process. As there are no transfer cross-section data for these systems, required to derive reliable spectroscopic factors, no transfer couplings were included in the calculations. So the differences between CF data and the theoretical predictions for TF will come from breakup + transfer effects. The bare potential used is the SPP. In the coupled-channel calculations, we include only the inelastic excitations of the targets. No resonance in the continuum states of the <sup>9</sup>Be was included in the calculations, since this is already part of the breakup process. The <sup>9</sup>Be ground state deformation was not included. If we include this deformation, all results will change a little toward a larger theoretical value and therefore a larger value of the suppression factor. All coupled-channel calculations were performed with the FRESCO code [27]. In the coupled-channel calculations for the  ${}^{9}\text{Be} + {}^{124}\text{Sn}$  system the one-phonon quadrupole  $2_1^+$  (1.1317 MeV) and octupole  $3_1^-$ (2.6025 MeV) states were included. Also, the two-phonon quadrupole states were included in the coupling scheme [the  $2_2^+$  (2.1296 MeV),  $4_1^+$  (2.1017 MeV), and  $0_2^+$  (2.1921 MeV) states]. The quadrupole and octupole deformation parameters were taken from the systematics of Refs. [28] and [29], respectively. The same deformation parameter was assumed for the Coulomb and nuclear excitations. Only first-order transitions were included. In the case of  ${}^{9}\text{Be} + {}^{89}\text{Y}$ , as the target is odd and assuming that the weak-coupling approximation of the odd particle is valid, the coupled-channel calculations were

TABLE I. TF cross section and CF suppression for the  ${}^{9}\text{Be} + {}^{89}\text{Y}$  system.  $V_{\text{B}} = 21.1$  MeV.

$E_{\rm c.m.}$ (MeV)	$E_{\rm c.m.}/V_{\rm B}$	$\sigma_{\rm CF}$ (mb) [10]	$\sigma_{\mathrm{TF}} (\mathrm{mb})$	CF supp. (%)
24.1	1.14	$265 \pm 20$	375.9	$29.5 \pm 5.3$
24.9	1.18	$348\pm20$	464.6	$25.9\pm4.3$
25.9	1.23	$361 \pm 35$	555.4	$35.0\pm6.3$
27.7	1.31	$537 \pm 31$	696.5	$22.9\pm4.4$
28.6	1.36	$541 \pm 33$	796.1	$29.2\pm4.1$
29.8	1.41	$559\pm32$	833.1	$32.9\pm3.8$

performed for two targets: 90Zr and 88Sr, considering that 89Y is a proton hole of <sup>90</sup>Zr and one extra proton of the <sup>88</sup>Sr nuclei. For the <sup>88</sup>Sr the low-lying one-phonon states of multipolarity 2  $(2_1^+ \text{ state with energy } 1.8361 \text{ MeV})$  and  $3(3_1^- \text{ state with energy } 1.8361 \text{ MeV})$ 2.7341 MeV) were included in the coupling scheme. The deformation parameters were taken from Refs. [28] and [29], respectively. For the <sup>88</sup>Sr target the first  $2_1^+$  (2.1863 MeV) and  $5_1^-$  (2.3190 MeV) excited states were included. For the quadrupole excitation the deformation parameters were taken from Ref. [28]. As for  $\lambda = 5$  there is no systematic, as far as we know, we used the deformation parameter obtained from the B(E5) reported in the database of Ref. [30]. The results of the fusion cross-section calculations for both systems were very similar and the mean values were used. For the  ${}^{9}\text{Be} + {}^{144}\text{Sm}$ system the one-phonon quadrupole  $(2^+_1)$  state with excitation energy of 1.660 MeV) and octupole  $(3^{-}_{1})$  state with excitation energy of 1.8101 MeV) excitations of the target were included in the coupling scheme. The deformation parameters were taken from the same systematic as in the previous systems. The coupling scheme of the  ${}^{9}\text{Be} + {}^{208}\text{Pb}$  system was formed by the low-lying excited states of the target:  $3_1^-$  (2.61 MeV) and  $5_1^-$  (3.2 MeV). The deformation parameters were taken from Ref. [7].

Tables I–IV show the TF cross sections calculated with the above-mentioned coupling scheme and the CF data for the four systems. The quantity  $1 - (\sigma_{CF})_{exp}/(\sigma_{TF})_{calculated}$ , called CF suppression, is also shown.  $V_{\rm B}$  is calculated from the systematics of the SPP.

Figure 3 shows the suppression factor for the four systems as a function of the center of mass energy in the range  $1.1 < V_{\rm B} < 1.5V_{\rm B}$ . Figure 4 shows the average suppression factor for each system, as a function of  $Z_{\rm target}$ . The curve is

TABLE II. TF cross section and CF suppression for the  ${}^{9}\text{Be}$ + ${}^{124}\text{Sn}$  system.  $V_{\text{B}} = 25.6$  MeV.

$E_{\rm c.m.}$ (MeV)	$E_{\rm c.m.}/V_{\rm B}$	$\sigma_{\rm CF}$ (mb) [11]	$\sigma_{\mathrm{TF}} (\mathrm{mb})$	CF supp. (%)
28.08	1.10	$200 \pm 3$	300.8	$33.5 \pm 1.0$
29.03	1.13	$244 \pm 7$	395.5	$38.3 \pm 1.8$
29.98	1.17	$322 \pm 15$	485.7	$33.7 \pm 3.1$
30.93	1.21	$369 \pm 21$	549.4	$35.2\pm3.8$
31.88	1.25	$437 \pm 23$	649.3	$32.7\pm3.5$
32.82	1.28	$517 \pm 11$	723.1	$28.5\pm1.5$
33.77	1.32	$584 \pm 34$	793.5	$26.4 \pm 4.3$
34.71	1.36	$656\pm43$	857.5	$23.5\pm5.0$

TABLE III. TF cross section and CF suppression for the  ${}^{9}\text{Be}+{}^{144}\text{Sm}$  system.  $V_{\text{B}} = 31.1$  MeV.

$\overline{E_{\text{c.m.}} (\text{MeV})}$	$E_{\rm c.m.}/V_{\rm B}$	$\sigma_{\rm CF} ({\rm mb})  [8, 9]$	$\sigma_{\mathrm{TF}}$ (mb)	CF supp. (%)
34.8	1.12	$295\pm21$	362.0	$18.5\pm5.8$
35.8	1.15	$395\pm27$	468.0	$15.6\pm5.8$
37.5	1.21	$496\pm42$	610.1	$18.7\pm6.9$
38.6	1.24	$577\pm69$	695.2	$17.0\pm9.9$
39.5	1.27	$669 \pm 56$	762.0	$12.2 \pm 7.4$
41.4	1.33	$770\pm66$	891.2	$13.6\pm7.4$

the same empirical prediction shown in Fig. 2, using the  $P_{ICF}$  for the <sup>208</sup>Pb target as reference, with the corresponding error bars. The points depend on the ratio between CF data and TF calculations. A first comment on this figure is that the CF suppression factor is indeed very similar to  $P_{ICF}$  for Pb, but not exactly equal. The results for the other systems do not follow the predictions within the error bars. For the <sup>144</sup>Sm, the CF is below the prediction, whereas for the two systems investigated in Mumbai the CF suppression is well above the prediction and similar to the suppression obtained for Pb. If that is correct, the CF suppression should be almost independent of the target value, contrary to the expectations which consider that the Coulomb breakup probability decreases when the target charge decreases.

It is important to compare the CF suppression obtained by the method described in the present work with those from the original works where the data were presented. For the  ${}^{9}\text{Be} + {}^{208}\text{Pb}$  system, the result is in excellent agreement with the present ones:  $(30 \pm 8)\%$  in [5,7] and  $(31.5 \pm 0.5)\%$  in this work. For  ${}^{9}\text{Be} + {}^{144}\text{Sm}$ , the results are  $(10 \pm 3)\%$  in [8,9] and  $(16.1 \pm 2.8)\%$  in this work. For  ${}^{9}\text{Be} + {}^{124}\text{Sn}$  the results are  $(28 \pm 5)\%$  in [11] and  $(32.9 \pm 0.7)\%$  in the present work. However, for the  ${}^{9}\text{Be} + {}^{89}\text{Y}$  system the results are quite different:  $(20 \pm 5)\%$  in [10] for <sup>89</sup>Y and  $(29.7 \pm 1.6)\%$  in the present work. These discrepancies show very clearly how sensitive are the CF suppression factors to the coupled-channel calculations and bare potentials used for comparison with the data. Once more we would like to emphasize that when one talks about enhancement or suppression of the fusion cross section, it is very important to be clear about the reference to which one is comparing the data.

TABLE IV. TF cross section and CF suppression for the  ${}^{9}\text{Be}$ +  ${}^{208}\text{Pb}$  system.  $V_{\text{B}} = 38.5$  MeV.

$\overline{E_{\text{c.m.}}}$ (MeV)	$E_{\rm c.m.}/\rm V_B$	$\sigma_{\rm CF}$ (mb) [5,7]	$\sigma_{\mathrm{TF}} (\mathrm{mb})$	CF supp. (%)
42.23	1.10	$243.7 \pm 4.9$	365.4	$33.3 \pm 1.3$
43.13	1.12	$303.1\pm6.5$	438.6	$30.9\pm1.5$
44.09	1.15	$351.4\pm6.6$	514.5	$31.7\pm1.3$
45.05	1.17	$403.6\pm7.7$	588.6	$31.4 \pm 1.3$
46.01	1.20	$453.8\pm8.6$	657.7	$31.0\pm1.3$
46.97	1.22	$507 \pm 10$	774.0	$34.5\pm1.3$
47.93	1.24	$570 \pm 12$	790.6	$27.9 \pm 1.5$
48.88	1.27	$594 \pm 12$	854.7	$30.5\pm1.4$



FIG. 3. (Color online) Complete-fusion suppression at energies slightly above the Coulomb barrier. See text for details.

One can observe that, for the two systems for which  $P_{ICF}$  has been measured,  ${}^{9}\text{Be} + {}^{144}\text{Sm}$  and  ${}^{208}\text{Pb}$ , the values obtained for  $P_{ICF}$  and the CF suppression factor are in excellent agreement.

From Fig. 4 one cannot draw conclusions about systematic behavior for the CF suppression as a function of the target charge (or mass). We see two possibilities; in both of them we take the results for the <sup>9</sup>Be + <sup>208</sup>Pb system as reference. The first possibility, using the Mumbai data for <sup>89</sup>Y and <sup>124</sup>Sn targets, shows a CF suppression factor of the order of 30% and independent of  $Z_{target}$ . The <sup>144</sup>Sm data, in this hypothesis, is out of the systematics, showing a much smaller CF suppression. The other possibility is that the <sup>144</sup>Sm data



FIG. 4. (Color online) Mean complete fusion suppression at energies slightly above the Coulomb barrier. The shaded band is an empirical prediction by Hinde [23] for this value. See text for details.

are in qualitative agreement with the empirical prediction, in quantitative agreement with  $P_{ICF}$  and CF suppression, and these quantities decrease when the  $Z_{target}$  decreases. In this hypothesis, the data from Mumbai are out of the systematics.

We believe that there are two possible explanations for the lack of a systematic behavior of CF suppression for these four systems: (i) something special occurs with one or more systems, such as different roles of transfer channels; (ii) there are some problems with data. In the following we explore both hypotheses.

An analysis of the ground state (g.s.) transfer Q values for the main channels for the four systems shows that the stripping of one neutron has a large positive value for the four systems (+2.271 MeV for <sup>208</sup>Pb, +5.09 MeV for <sup>144</sup>Sm, +4.067 MeV for <sup>124</sup>Sn, and +5.192 MeV for <sup>89</sup>Y). So the one-neutron transfer channel may play an important role in the influence of coupled-channel calculations on the CF and TF of these systems. Furthermore, there are other transfer channels with small positive or negative g.s. Q values, which also might play some role in the fusion cross-section calculation. As mentioned before, since there are no transfer data available for those systems, it is not possible yet to disentangle the effects of breakup and transfer channels on the CF suppression for <sup>9</sup>Be-induced fusion.

Concerning the data, we do not see reasons to believe that there is anything wrong with them. However, we would like to point out that in Ref. [11], the measured  $\sigma_{ICF}$  for the  ${}^9\text{Be} + {}^{124}\text{Sn}$  system is shown, considering the only evaporation channel that could be measured. This lower limit for  $P_{ICF}$ corresponds to up to 7% of the measured CF. If one corrects this result by the PACE prediction, the actual ICF will be of the same order as the CF cross section, a very unexpected result. The reason for that might be the misidentification of some  $\gamma$ -ray lines, which is one of the main difficulties associated with the method of derivation of fusion cross sections by the online  $\gamma$ -ray spectroscopy method [31]. This problem might affect either the ICF or CF cross section or both.



FIG. 5. (Color online) Renormalized experimental fusion function for the four systems analyzed. The full curve is the universal fusion function (UFF) and the dashed line corresponds to 70% of the UFF.

## VII. COMPARISON OF THE CF FUSION FUNCTION WITH THE UNIVERSAL FUSION FUNCTION FOR THE FOUR SYSTEMS

If one wants to investigate a systematic behavior of the fusion cross section for several systems in the same plot, with different mass, charge, and barrier parameters, the data should be reduced in order to eliminate the static effects. It has been shown [2,3] that the traditional reduction procedures of dividing the cross section by  $\pi R_{\rm B}^2$  and the center-of-mass energy by  $V_{\rm B}$  do not work properly ( $R_{\rm B}$  and  $V_{\rm B}$  stand for the barrier radius and height, respectively). Instead, a fusion function  $F_{\text{expt}} = 2E\sigma_{\text{F}}/(\hbar\omega R_{\text{B}}^2)$  should be plotted against the quantity  $x = (E - V_{\rm B})/\hbar\omega$ , where  $\hbar\omega$  is the barrier curvature. As one wants to eliminate the effect of all target inelastic channels in the present investigation, it is necessary to renormalize  $F_{\text{expt}}$  to  $\bar{F}_{\text{expt}} = F_{\text{expt}}(\sigma_{\text{FW}} / \sigma_{\text{FCC}})$ , where  $\sigma_{\text{FW}}$  is the fusion cross section calculated by the Wong approximation [32] and  $\sigma_{FCC}$  is the cross section obtained with a reliable bare potential and coupled-channel calculations including all bound excited channels.  $\bar{F}_{expt}$  is then compared with a universal fusion function (UFF)  $F_0(x) = \ln[1 + \exp(2\pi x)]$ . It has been shown [2,3] that for tightly bound systems for which only inelastic channels are important in the coupling scheme,  $\bar{F}_{exp}$ coincides with the UFF at energies not too much above the barrier. For the weakly bound systems, the difference between  $\bar{F}_{expt}$  and the UFF is attributed to the coupling of breakup and transfer channels not included in the CC calculations. We show in Fig. 5 the results for CF of the four systems investigated. The barrier parameters used in the reduction procedure were obtained from the SPP systematic. One can observe that the behaviors of CF for <sup>208</sup>Pb, <sup>124</sup>Sn, and <sup>89</sup>Y are very similar, with a suppression factor of the order of 30%, whereas <sup>144</sup>Sm show a much smaller suppression, of the order of 15%. If one plots the  $\bar{F}_{expt}$  for TF of stable weakly bound projectiles (<sup>6</sup>Li, <sup>7</sup>Li, and <sup>9</sup>Be) with any target [2–4], no suppression of the TF is observed, including for <sup>9</sup>Be + <sup>27</sup>Al, <sup>64</sup>Zn [4].

## VIII. CONCLUSIONS

From the results shown in the present paper we observe that there is a trend of systematic behavior of  $P_{\rm ICF}$  as a function of the target charge.  $P_{ICF}$  decreases almost linearly when  $Z_{target}$  decreases. However, there are two possibilities for the CF suppression in relation to calculations that do not take into account the breakup and transfer channels of the weakly bound projectile. In both possibilities, the results for  ${}^{9}\text{Be} + {}^{208}\text{Pb}$  are taken as the reference, for both  $P_{\text{ICF}}$  and CF suppression. One possibility, using the Mumbai data for <sup>89</sup>Y and <sup>124</sup>Sn targets, shows a CF suppression factor of the order of 30% and independent of  $Z_{\text{target}}$ . The <sup>144</sup>Sm data, in this hypothesis, is out of the systematics, showing a much smaller CF suppression. The other possibility is that the <sup>144</sup>Sm data are in qualitative agreement with the empirical prediction, in quantitative agreement with the  $P_{ICF}$  and CF suppression, and these quantities decrease when  $Z_{\text{target}}$  decreases. In this hypothesis, the data from Mumbai are out of the systematics. The lack of a clear systematic behavior for the CF suppression, as a function of the target charge, may be explained by different effects of transfer channels, specially one-neutron stripping, on the CF or TF.

Also, we have obtained CF suppression quite different from that originally reported for the  ${}^{9}\text{Be} + {}^{89}\text{Y}$  system, which shows how important is to have reliable theoretical calculations to be compared with the data. The values obtained for  $P_{ICF}$  and CF suppression for the  ${}^{9}\text{Be} + {}^{144}\text{Sm}, {}^{208}\text{Pb}$  systems are in excellent agreement.

In conclusion, one needs to investigate more systems, especially light ones, to understand the role of the Coulomb breakup in the CF process and in the breakup itself. The measurement of transfer cross sections is also very important in such investigations. Also, the comparison between data and calculations has to be carefully made, in order to avoid misleading conclusions.

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