Orbital electron capture of hydrogen- and helium-like ions

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Corrections to the ratio of electron capture (EC) rates in hydrogen- and helium-like ions are calculated. We find that the most significant contribution is the electron screening effect. The correction has a simple form $(1 - 5/16Z)^3(1 - \delta_3)$ which ranges from almost 50% in helium to 1% in heavier nuclei. We discuss also EC in helium-like ions accompanied by an emission of the remaining electron into the continuum, a new decay channel, for which we calculate the decay probability. It is a very exotic type of Auger electron emission.

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I. INTRODUCTION

Progress in experimental techniques based on projectile fragmentation, in-flight separation, and heavy-ion storage rings has enabled investigations of β decay of highly-charged ions [1–6]. Presently only two facilities worldwide are capable to produce isotopic beams of radioactive nuclides in high atomic charge states and to store them for a given time in the ultrahigh vacuum of a storage ring [7]. These facilities are the FRS-ESR at GSI in Darmstadt and the HIRFL-CSR at IMP in Lanzhou, where the corresponding in-flight fragment separators FRS [8] and RIBLL2 [9] are coupled to the cooler-storage rings ESR [10] and CSRe [11], respectively. However, while the experimental program on β-decay studies at GSI is running since about two decades, the experiments at IMP are still in their planning phase.

Recently, first experiments on orbital electron capture (EC) decay of few-electron ions have been performed at the ESR employing the so-called time-resolved Schottky mass spectrometry (SMS) [12–15]. Time-resolved SMS is sensitive to single-stored ions [15]. This property has been employed to measure individual EC decays of single H-like ^{140}Pr and ^{142}Pm ions stored in the ESR, yielding a still unexplained [16] observation of a non-exponential decay behavior [17,18].

Continuum β^+ decay and EC decay branches of bare, hydrogen-like (H-like) and helium-like (He-like) ^{140}Pr, ^{142}Pm, and ^{122}I ions have been investigated [19–22]. A striking result has been obtained, that in spite of the fact that the number of bound electrons is reduced from two to one in H-like ions compared to He-like ions, the EC-decay rate has increased by about 50%. The H-like ^{140}Pr and H-like ^{142}Pm ions decay by EC even faster than the corresponding neutral atoms. First theoretical decay studies have shown that the experimental results can be explained by taking into account the conservation of the total angular momentum (and its projection) of the nucleus plus leptons system [23,24].

In this work, we extend the former theoretical investigations and discuss in detail the corrections connected to different electron densities and energy transitions in H- and He-like ions. In addition, we calculate the probability that the remaining electron in the decay of a He-like ion is simultaneously excited to the continuum. We note, that both experimental and theoretical [25–27] studies of the EC process are important for the understanding of stellar evolution [28–30]. It concerns, in particular, highly ionized atoms which are abundant in stellar plasmas [31].

In the EC decay, an orbital electron (e^-) is captured by a nucleus thereby transmuting one of its protons into a neutron accompanied by an emission of an electron neutrino (ν_e)

$$(Z, N) + e^- \rightarrow (Z - 1, N + 1) + \nu_e, \tag{1}$$

where Z and N denote the number of protons and neutrons in the parent nucleus, respectively. The EC-decay probability per time unit P is given by Fermi's golden rule [32]

$$P = \frac{2\pi}{\hbar} |\langle f|\hat{O}|i\rangle|^2 \rho_f, \tag{2}$$

where ρ_f is the density of the neutrino final states per energy unit, which is proportional to the square of the decay energy Q_{EC}, i and f represent the initial and final states, and $\hat{O}$ is the weak interaction operator.

II. EC DECAY OF H-LIKE IONS

A H-like ion is the system consisting of an atomic nucleus with spin I and a single bound electron with spin $s = 1/2$. For a nucleus having a positive (negative) magnetic moment, the ground state of the ion has the total spin F_i equal to $F_i = I - 1/2$ $(F_i = I + 1/2)$.

EC decay transforms a parent nucleus with spin I into a daughter nucleus with spin $I \pm \Delta I$. We assume that the neutrino is emitted with the most probable orbital angular momentum $\Delta I - 1$. The daughter nucleus with the spin $I \pm \Delta I$ and the neutrino with the orbital angular momentum $\Delta I - 1$ can couple to total spins ranging from $I \pm 1/2$ to $|I \pm 2\Delta I \mp 1| \pm 1/2$. However, only transitions $F_i = I \pm 1/2 \rightarrow F_f = I \pm 1/2$ conserve the total angular momentum and are, thus, allowed. It is easy to check that for the neutrino angular momentum equals to $\Delta I - 2$ there exist no final state corresponding to the spin F_i.

There exist $2(I \pm 1/2) + 1$ initial states with different angular momentum projections and equal occupation probabilities [34]

$$P_H^{\pm} = \frac{2\pi \varrho_f}{\hbar[2(I \pm 1/2) + 1]} \times \sum_M |_{N\nu}\langle I \pm 1/2, M|\hat{O}|I \pm 1/2, M\rangle_H|^2, \quad (3)$$

where $P_H^{\pm}$ is the EC probability of the H-like ion with spin $I \pm 1/2$, $|I \pm 1/2, M\rangle_H$ and $|I \pm 1/2, M\rangle_{N\nu}$ denote the initial and final states, respectively, and ϱ_f is the density of neutrino final states.

III. EC DECAY OF HE-LIKE IONS

The initial state of He-like ion can be constructed as a product of the nuclear part having the spin I (with its projection M) and the singlet wave function of two electrons

$$|I, M\rangle_{\rm He} = |I, M\rangle_N \otimes \frac{|+\rangle_{e_1}|-\rangle_{e_2} - |-\rangle_{e_1}|+\rangle_{e_2}}{\sqrt{2}}, \quad (4)$$

where $|+\rangle$ and $|-\rangle$ denote Dirac spinors for the $1s$ state in a H-like ion [33] with spin projections $+1/2$ and $-1/2$, respectively, and with the screened charge $Z' = Z - q$, where q denotes the screening correction. The energy $E(Z, Z')$ for two self-interacting electrons in the potential of a point-like nucleus with charge Z it can be written in electron-mass units as [33]

$$E(Z, Z') = 2\sqrt{1-\alpha^2 Z'^2} + \frac{2\alpha^2 Z'(Z'-Z)}{\sqrt{1-\alpha^2 Z'^2}} + \frac{\alpha^2 Z'\left(\sqrt{1-\alpha^2 Z'^2} - \frac{4^{-2\sqrt{1-\alpha^2 Z'^2}}\Gamma(4\sqrt{1-\alpha^2 Z'^2})}{\Gamma(2\sqrt{1-\alpha^2 Z'^2})^2}\right)}{1-\alpha^2 Z'^2}, \quad (5)$$

where α is the fine-structure constant. The screened charge Z' minimizes the expectation value $E(Z, Z')$ of the relativistic Dirac Hamiltonian with the fixed nuclear charge Z. The screening correction q as a function of the nuclear charge Z has been calculated numerically and is plotted in Fig. 1.

In the final state f, the nuclear spin changes by ΔI units $I_f = I \pm \Delta I$. The remaining electron is described by the relativistic spinor with spin 1/2 [33]. In the most probable case, the neutrino carries out the orbital angular momentum $\Delta I - 1$ and its total angular momentum equals to $\Delta I - 1/2$ or $\Delta I - 3/2$. However, only the neutrino with the total angular momentum $\Delta I - 1/2$, the remaining electron with spin 1/2, and the nucleus with spin $I \pm \Delta I$ can couple to the angular momentum I. The basis of final states has the form

$$|M', k, l\rangle_{N,\nu,1e}^{ns} = |I \pm \Delta I, M'\rangle_N \otimes |\Delta I - 1/2, k\rangle_\nu \otimes |1/2, l\rangle_{1e}^{ns}, \quad (6)$$

where M', k, l denote the projection of the angular momentum for the daughter nucleus, neutrino and the remaining electron in the bound state ns, respectively. The EC probability for He-like ions, similar to the H-like case, can be expressed as

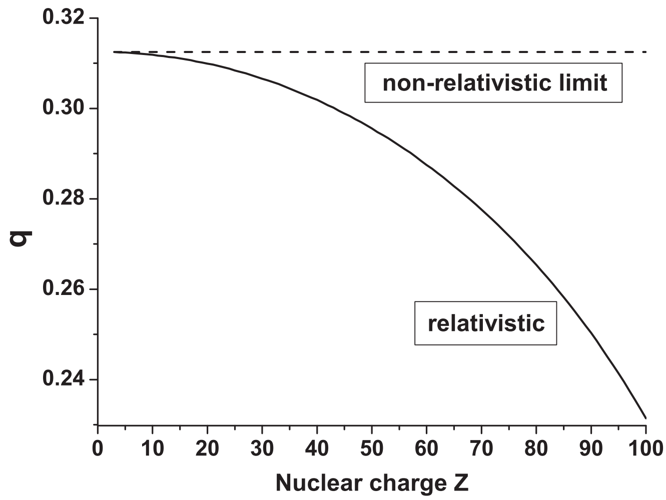

FIG. 1. The calculated relativistic screening correction q plotted as a function of the nuclear charge Z. The nonrelativistic limit, independent from Z, is equal to $q = 5/16$ [34].

$$P_{He} = 2\frac{2\pi \varrho'_f}{\hbar(2I+1)} \sum_{M,M',k,l,n} \left|{}_{N,\nu,1e}^{\;\;ns}\langle M', k, l|\hat{O}|I, M\rangle_{\rm He}\right|^2 = \frac{2\pi \varrho_f}{\hbar(2I+1)}\left(1 - \delta_1 + \frac{\delta_2}{Q_{EC}}\right) \times \sum_{M,M',k,l} |_N\langle I \pm \Delta I, M'|_\nu\langle \Delta I - 1/2, k|\hat{O}|I, M\rangle_N |1/2, l\rangle_{2e}|^2, \quad (7)$$

where ϱ'_f and ϱ_f are densities of the neutrino final states in the He- and H-like cases, respectively. The factor 2 in Eq. (7) accounts for two possibilities, namely, that the electron indexed as $1e$ or as $2e$ can be captured by the nucleus.

The ratio of the two neutrino states densities equals approximately to

$$\varrho'_f/\varrho_f = \frac{(\delta Q_n + Q_{EC})^2}{Q_{EC}^2} \approx 1 + 2\delta Q_n/Q_{EC},$$

where Q_{EC} is the mass difference between parent H-like ion and bare daughter nucleus in the EC decay, and δQ_n is defined by Eq. (11).

TABLE I. Calculated corrections δ_1, δ_2 (in keV) and δ_3 listed for a few selected nuclei. Z denotes the atomic number. q is the relativistic screening correction.

Z	q	$(1 - 5/16Z)^3$	$\log_{10}(\delta_1)$	δ_2 (keV)	$10^2\delta_3$
2	0.312	0.601	−0.88	0.0	−0.05
12	0.312	0.924	−2.87	−0.2	−0.25
22	0.309	0.958	−3.43	−0.4	−0.40
32	0.306	0.971	−3.74	−0.6	−0.53
42	0.301	0.978	−3.96	−0.8	−0.64
52	0.294	0.982	−4.11	−1.1	−0.75
62	0.286	0.985	−4.21	−1.3	−0.84
72	0.275	0.987	−4.28	−1.5	−0.92
82	0.263	0.989	−4.31	−1.8	−1.00
92	0.247	0.990	−4.31	−2.1	−1.06

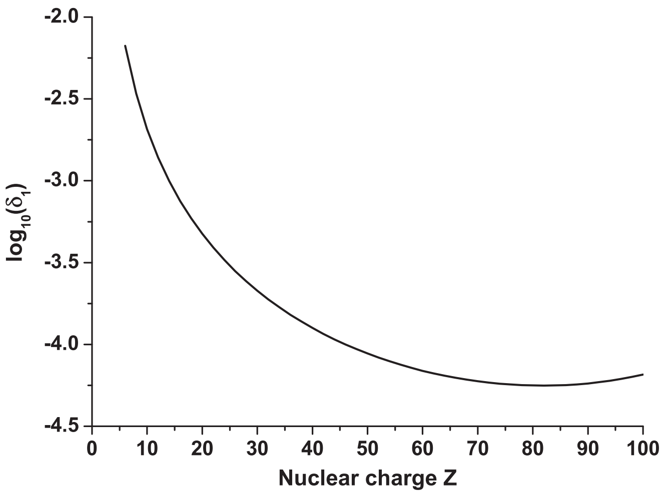

FIG. 2. The calculated probability δ_1 (logarithmic scale) that the He-like ion decays via EC to an atomic nucleus with the remaining electron excited into the continuum. It is about 0.1 for light nuclei and monotonically decreases down to 5.6×10^{-5} for Pb isotopes.

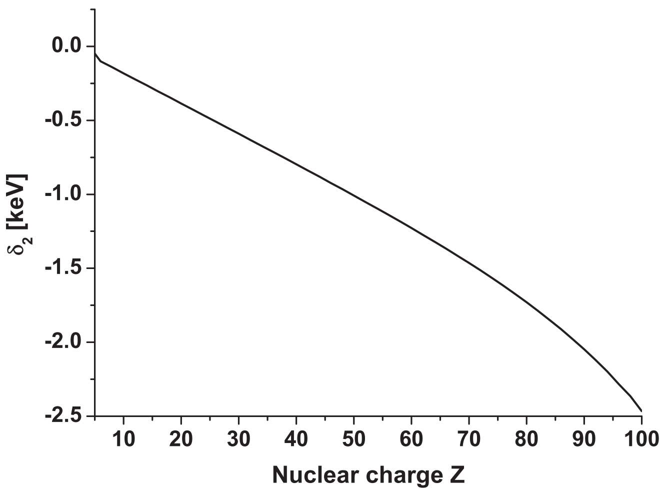

FIG. 3. The calculated correction term δ_2 expressed in keV as a function of the nuclear charge Z.

The quantity δ_1 in Eq. (7) (see Fig. 2 and Table I) is

$$\delta_1 = 1 - \sum_n |\langle ns, Z-1|1s, Z-q\rangle|^2, \tag{8}$$

and is the estimate of the probability that the remaining electron is unbound. The matrix elements

$$\langle ns, Z-1|1s, Z-q\rangle \equiv \int_0^\infty [f_{ns,Z-1}(r) f_{1s,Z-q}(r) + g_{ns,Z-1}(r) g_{1s,Z-q}(r)] r^2 dr, \tag{9}$$

are calculated for two spinors [33]. The correction term δ_2 (see Fig. 3) is introduced to account for the decays to the excited states ns in the H-like ion

$$\delta_2 = 2\sum_n |\langle ns, Z-1|1s, Z-q\rangle|^2 \delta Q_n, \tag{10}$$

where we used

$$\delta Q_n = B_1^{1s}(Z) + B_1^{ns}(Z-1) - B_2(Z). \tag{11}$$

Here, $B_1^{ns}(Z)$ denotes the binding energy of an electron in the bound state ns of the H-like ion with the nuclear charge Z. The total ionization energy $B_2(Z)$ for the He-like ions was taken from [35].

The quantity δQ_1 increases as a function of Z. It is equal to 0.1 keV for $Z = 10$ and reaches 0.8 keV for $Z = 73$. The value of δQ_1 is typically smaller than the experimental uncertainties of Q_{EC}.

In Eq. (7), nuclear states and captured electron states form the basis $|I, M\rangle_N |1/2, l\rangle_{2e}$ with $2(2I+1)$ independent vectors. The latter can be expanded into H-like ion states with fixed angular momenta: $|I+1/2, m\rangle_H$ and $|I-1/2, m\rangle_H$. These two bases have together again $2(2I+1)$ vectors. In a similar way a basis of the final states $|I \pm \Delta I, M\rangle_N |\Delta I - 1/2, k\rangle_\nu$ can be expanded into $2\Delta I - 1$ separate bases with fixed angular momenta: $|I \pm 1/2, m\rangle_{N\nu}$, $|I \pm 3/2, m\rangle_{N\nu}, \ldots$ The weak interaction operator $\hat{O}$, responsible for the EC decay [36,37] has nonzero matrix elements only between states with identical total angular momentum and its projection. Therefore, holds the following equality:

$$\sum_{M,M',k,l} |_N\langle I \pm \Delta I, M'|_\nu\langle \Delta I - 1/2, k|\hat{O}|I, M\rangle_N |1/2, l\rangle_{2e}|^2 = \sum_m |_{N\nu}\langle I \pm 1/2, m|\hat{O}|I \pm 1/2, m\rangle_H|^2, \tag{12}$$

where the electron spinors were taken with the effective charge Z'.

By combining Eqs. (3), (7), and (12), and by expressing the electron density at the nucleus in terms of the relativistic spinors with charge Z [as in Eq. (3) for H-like ions] we finally get

$$P_{\rm He} = \frac{[2(I \pm 1/2)+1]}{2I+1} P_H \left(1 - \delta_1 + \frac{\delta_2}{Q_{EC}}\right) \times \left(1 - \frac{5}{16Z}\right)^3 (1+\delta_3), \tag{13}$$

where $(1 - \frac{5}{16Z})^3$ is the ratio of nonrelativistic electron densities in He-like and H-like ions [34]. The relativistic correction δ_3 can be expressed as [33,34]

$$\delta_3 = \frac{Z^3 \tilde{\rho}_e(Z-q)}{(Z-5/16)^3 \tilde{\rho}_e(Z)} - 1. \tag{14}$$

The quantity $\tilde{\rho}_e(Z)$ describes the relativistic $1s$ electron density $f_{1s}(r,Z)^2 + g_{1s}(r,Z)^2$ calculated for uniformly distributed nuclear charge and it is averaged over the nuclear volume with radius $R_a = 1.24\ A^{1/3}$ fm. It can be written as

$$\tilde{\rho}_e(Z) \equiv \frac{3}{R_a^3} \int_0^{R_a} [f_{1s}(r,Z)^2 + g_{1s}(r,Z)^2] r^2 dr. \tag{15}$$

The calculated absolute value of the relativistic correction δ_3 is smaller than 0.012 and is presented in Fig. 4.

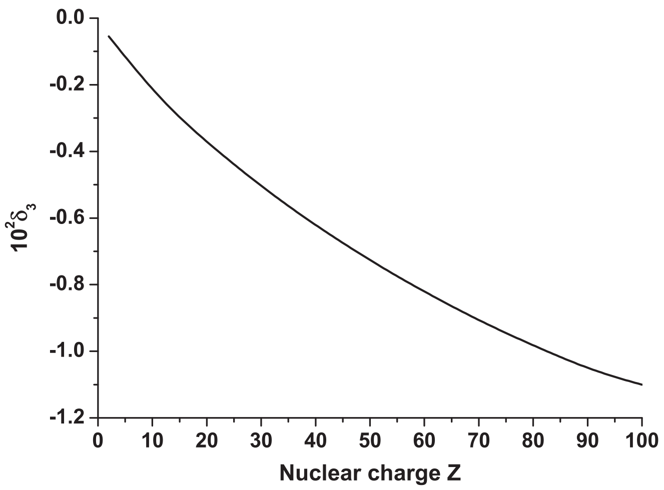

FIG. 4. The relativistic correction δ_3 calculated for uniformly distributed charge as a function of the nuclear charge Z.

IV. ARBITRARY NEUTRINO ORBITAL MOMENTUM

In the preceding subsections we have analyzed the EC processes assuming that the neutrino is emitted with an orbital angular momentum $\Delta I - 1$. Here we extend our discussion to the case, that $\Delta I \geqslant 0$ and that the neutrino takes the orbital momentum $\leqslant \Delta I$. Thus, both hyperfine states $I \pm 1/2$ of the H-like ion can decay with the probabilities denoted as $P_H^{\pm}$, respectively. It can be demonstrated (in a similar way as in the two previous sections), that the following relation holds combining the EC probabilities for He-like and H-like ions:

$$P_{\mathrm{He}} = \frac{P_H^-[2(I-1/2)+1] + P_H^+[2(I+1/2)+1]}{2I+1} \times \left(1 - \delta_1 + \frac{\delta_2}{Q_{EC}}\right)\left(1 - \frac{5}{16Z}\right)^3 (1+\delta_3). \quad (16)$$

For the transition $I \to I-1$ both probabilities $P_H^{\pm}$ satisfy the following approximate relation:

$$\frac{P_H^+}{P_H^-} \approx (kR)^2, \quad (17)$$

where k denotes the neutrino momentum (divided by $\hbar$) and R is the average nuclear radius. For the transition $I \to I+1$ Eq. (17) should be inverted. Assuming $Q_{EC} = 5$ MeV and $R = 5$ fm, we get $(kR)^2 \approx 0.016$.

V. SUMMARY

Various corrections for the EC-decay rates in He-like ions have been estimated, which include the effect of a new decay channel, (δ_1), the decays to the excited states ns, (δ_2), as well as relativistic and screening effects, $[(1-5/16Z)^3(1+\delta_3)]$. From all discussed corrections, the most significant one is the screening term $(1-5/16Z)^3$. It has a value of 0.91 for Ne ions. However, for Fm ions it reaches nearly unity and equals to 0.99.

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