Region of a hadron-quark mixed phase in hybrid stars

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Hadron-quark mixed phase is expected in a wide region of the inner structure of hybrid stars. However, we show that the hadron-quark mixed phase should be restricted to a narrower region because of the charge screening effect. The narrow region of the mixed phase seems to explain physical phenomena of neutron stars such as the strong magnetic field and glitch phenomena, and it would give a new cooling curve for the neutron star.

It is widely believed that there should be quark matter in the inner core region of neutron stars because the central density of neutron stars is sufficiently high for nucleons to dissolve into more elementary particles, namely, quarks. Nowadays we consider that compact stars consist of not only nuclear matter but also other matter such as hyperons and quarks. We call such stars *hybrid stars*.

Although deconfinement phase transition is still not clearly understood, many authors have studied the transition through modeling and first-principle calculations such as those of lattice QCD. Properties of quark matter have been actively studied theoretically in terms of quark-gluon plasma, color superconductivity [1,2], and magnetism [3–5], and experimentally in terms of relativistic heavy-ion collisions [6] and early-universe and compact stars [7,8]. Such studies are continuing to provide exciting results [9].

Because many theoretical calculations have suggested that deconfinement transition is of first order at low temperature and high density [10,11], we assume it to be a first-order phase transition here. The Gibbs condition (GC) [12] then gives rise to various structured mixed phases (SMPs). The SMPs suggested by Heiselberg *et al.* [13] and Glenndening and Pei [14] suggest a crystalline structure of the mixed phase in the core region of hybrid stars. Such structures are called "droplets,' "rods,' "slabs," "tubes," and "bubbles." These SMPs exist within a wide density range if we take a moderate value of the surface tension. However, if the surface effect is strong, the SMPs are limited to a narrow density region. On the other hand, Voskresensky *et al.* [15] reported the charge screening effect for several cases of droplets and slabs. They showed that even if we take a moderate value of the surface tension, SMPs cannot exist because of the charge screening effect. They referred to this phenomenon as *mechanical instability*. However, they used a linear approximation to solve the Poisson equation.

We have presented the equation of state (EOS) for the mixed phase taking into account the charge screening effect [16] without recourse to any approximation. The EOS is similar to that obtained from the Maxwell construction (MC). The allowed region of the mixed phase should then be narrow due to the charge screening effect.

Recently "black stars" have been proposed by Barcelo *et al.* [17], who suggested that quantum effects prevent a star from collapsing into a "Black hall." Therefore, the EOS of hadron matter in the high-density states plays an important role in determining the quantum effects within compact stars.

In this paper, we apply our EOS to study the structure of hybrid stars and demonstrate how charge screening affects the physical properties of the hybrid stars. We use the EOS given in our previous paper [16], which also presented our framework. Therefore, our approach is only briefly explained here. Thermal equilibrium is implicitly achieved at $T = 0$. We consider that both hadron and quark matter and the mixed phase are β stable. We employ density functional theory (DFT) under the local density approximation [18,19].

We consider the geometrically structured mixed phase (SMP) in which one phase is embedded in the other phase with a certain geometrical form. We divide the whole space into equivalent Wigner-Seitz cells with radius R_W. Such cells include an embedded phase with size R. We impose total charge neutrality and chemical equilibrium to satisfy the GC. Therefore, adjacent cells do not interact with each other.

The quark phase consists of u, d, and s quarks and electrons in β equilibrium. We consider that u and d quarks are massless and s quarks are massive ($m_s = 150$ MeV), and consider one-gluon exchange for quark interaction. The hadron phase consists of nucleons and electrons in β equilibrium. We use an effective potential parametrized to reproduce the saturation property of nuclear matter. We use empirical values for the binding energy ($\epsilon_{\text{bind}} = -15.6$ MeV), saturation density ($\rho_0 = 0.16$ fm^{-3}), symmetry energy ($S_0 = 18$ MeV), and compression modulus ($K_0 = 285$ MeV).

To account for the confinement, we introduce a sharp boundary between the two phases employing the bag model with surface tension parameter σ. Determination of the surface tension between hadron and quark matter is a difficult problem; thus, many authors [13,14,20] have treated the surface tension as a free parameter and observed its effect. We take the same approach in this study. We use the bag constant of $B = 120$ MeV/fm^3, which is the same as that used in Refs. [13,16].

As we need to account for the Coulomb interaction, the Coulomb potential appears in the total thermodynamic potential. Differentiating the thermodynamic potential Ω_{total} with respect to each constituent density ρ_i ($i = u, d, s, n, p, e$) or the Coulomb potential $V(\vec{r})$ with r being the radial coordinate,

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we obtain the equations of motion (EOMs) for ρ_i and $V(\vec{r})$. We then numerically solve these EOMs under the conditions of chemical equilibrium in each phase and at the hadron-quark boundary. In particular, we fully solve the Poisson equation without any approximation. Note that the Poisson equation becomes highly nonlinear because charged particle densities are complicated functions of the Coulomb potential.

Here, we must determine eight variables; i.e., the six chemical potentials μ_u, μ_d, μ_s, μ_p, μ_n, and μ_e and the radii R and R_W. First, we fix R and R_W. We then have four conditions due to the β equilibrium, expressed by the chemical equilibrium. Therefore, once the two chemical potentials μ_B and μ_e are given, we can determine the other four chemical potentials μ_u, μ_d, μ_s, and μ_p. Next, we determine μ_e according to the global charge neutrality condition:

$$f_V \rho_{ch}^Q + (1 - f_V)\rho_{ch}^H = 0, \qquad (1)$$

where the superscripts "Q" and "H" denote the quark and hadron phases, respectively. The volume fraction is given by $f_V = (\frac{R}{R_W})^d$, and d denotes the dimensionality of each geometrical structure. At this point, f_V is still fixed.

The pressure obtained from the surface tension is given by

$$P_\sigma = \sigma \frac{dS}{dV_Q}, \qquad (2)$$

where S is the surface area of the interface and V_Q is the volume of the quark matter. We then find the optimal value of R (R_W is fixed and f_V thus changes with R) using one of the GC:

$$P^Q = P^H + P_\sigma. \qquad (3)$$

The pressure in each phase, P^Q or P^H, is given by the thermodynamic relation $P^{Q(H)} = -\Omega_{Q(H)}/V_{Q(H)}$, where $\Omega_{Q(H)}$ is the thermodynamic potential for each phase. Finally, we determine R_W by minimizing the thermodynamic potential. Therefore, once μ_B is given, all other μ_i ($i = u, d, s, p, e$) can be obtained along with R and R_W. Note that we keep the GC throughout the numerical procedure.

To elucidate the charge screening effect, we also make the calculation without the screening effect. We refer to this calculation as the "no-screening" calculation and the calculation with the screening effect as the "screening" calculation for convenience.

We then apply the EOS derived in our recent paper [16] to the Tolemann-Oppenheimer-Volkov (TOV) equation. We present the results for the screening and no-screening cases. Figure 1 shows the mass-radius relations of stars in the cases of screening and no screening using the MC, and in the case of pure hadron matter, there is no notable difference in the low-mass region because there is no phase transition. On the other hand, there is a slight difference in the region around the maximum mass. The difference in the maximum masses between the cases of screening and no screening is about $0.05 M_\odot$. This difference is small compared with the total mass of the star. Thus, charge screening does not greatly affect the bulk properties of the star.

On the contrary, we find that the inner structure is greatly affected by charge screening. Figure 2 shows the inner core region of a hybrid star in the case without screening. The
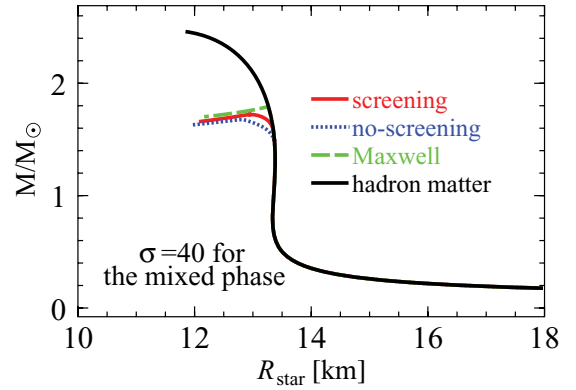

FIG. 1. (Color online) Mass-radius relation of stars. The maximum masses in the cases of pure hadron matter and screened and not screened mixed phases are 2.51, 1.73, and $1.68 M_\odot$, respectively. Note that we use a simple model for each type of matter. The difference between screened and not screened mixed phases is clearly small.

mixed phase appears in a wide region of the star. In this case, the mixed phase is spread over a radius of about 6 km, and we scarcely see the quark matter phase. In the case of screening shown in Fig. 3, on the other hand, there is mixed phase over only 3 km. The region of mixed phase clearly becomes more narrow and there is a quark matter phase in the central region because of the charge screening.

These results have possible implications for star phenomena. It has been stated that the strong magnetism of a neutron star is due to the spin polarization of quark matter [3], in which case pure quark matter should extend far into the core region. Our results suggest that quark matter could exist in the inner region of compact stars, and it is thus plausible to attribute the magnetism of compact stars to spin polarization.

Moreover, Bejger *et al.* [21] estimated the region of mixed phase in a hybrid star and suggested that the glitch phenomenon is caused by the presence of the mixed phase. Their results show that the mixed phase is constrained to
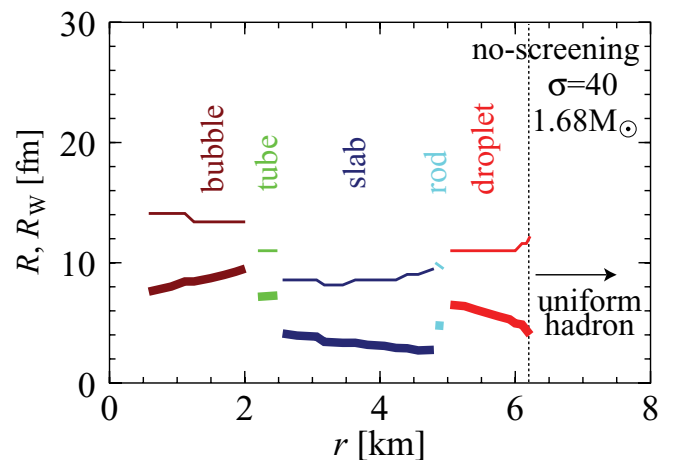

FIG. 2. (Color online) Structure size and cell size in the core region of hybrid stars in the case of no screening. There is about 6 km of mixed phase in this figure. The radius is 12.6 km. Thick lines denote R and thin lines denote R_W.
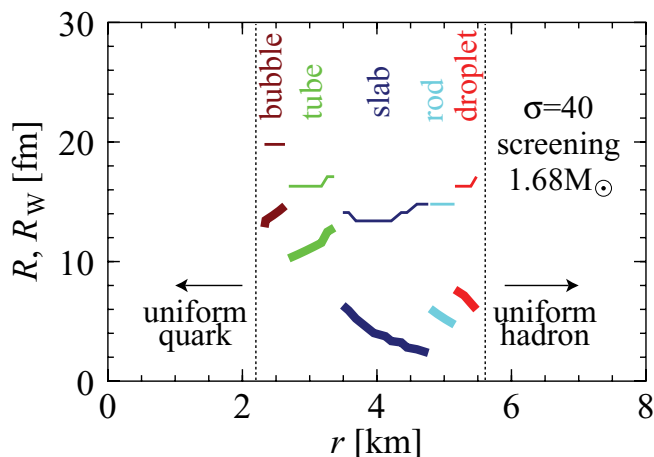
FIG. 3. (Color online) Structure size and cell size in the core region of hybrid stars in the case of screening. The mixed phase is restricted by the charge screening. The radius is 12.9 km.

a narrow region. In another study, gravitational waves have been commonly used to probe density discontinuities [22]. There should be a discontinuity in the hadron-quark phase transition if we use the MC. Although the MC is not valid strictly speaking, the EOS of the mixed phase is close to that obtained using the MC due to the finite size. In addition, many theoretical studies have used other models for the glitch phenomenon and gravitational waves. We cannot simply apply our EOS to studying these phenomena. However, it is interesting to compare our results with those of other studies [23].

It is, however, plausible to use our results in investigating the cooling problems of neutron stars. If there is nonuniformity in the core region of a hybrid star, it is interesting to investigate whether neutrinos would be scattered by the lumps. Cooling would then be prevented and the cooling curve should change. Although the results strongly depend on the region of the mixed phase, we suggest new cooling curves by taking into account the finite-size effects.

In this study, we demonstrated how charge screening affects the hadron-quark mixed phase in the core region of hybrid stars. We examined the effect by numerically solving the EOM for particle densities and the Poisson equation for the Coulomb potential. We demonstrated, by taking various SMPs as examples, that the charge screening effect and the rearrangement of charge densities play an important role in determining the inner structures of hybrid stars. We applied our EOS to the TOV equation and found that charge screening did not greatly affect the bulk properties of the star, such as the

radius and mass. However, we found that the inner structures are greatly affected. The region of the mixed phase in the star is highly restricted by the charge screening. If we ignore the charge screening effect, the mixed phase would appear in a wide region. On the basis of this result, some authors have suggested structured mixed phase with various geometrical structures, simply by including the finite-size effects and the surface and Coulomb energies [13,14,20,24]. However, once we take into account the charge screening effect, the mixed phase is restricted to a narrow region. In particular, the core could consist of quark matter and could appear due to the charge screening effect. As another case in which there is more than one chemical potential, kaon condensation has been studied [25] and the results are similar to those in our paper [16].

We considered surface tension, but its definite value is not yet clear and many authors treat it as a free parameter. If we use smaller (larger) values of the surface tension, the mixed phase is favored (disfavored). Here we use the value of $40\,\mathrm{MeV/fm^2}$, but the mixed phase is disfavored if we use $\sim 90\,\mathrm{MeV/fm^2}$ [13]. There are also many estimations of the surface tension at the hadron-quark interface in lattice QCD [26,27], in shell-model calculations [28–30], in model calculations based on the dual Ginzburg-Landau theory [31], and in a sigma model [32]. If we have a realistic value of the surface tension, we can reasonably deduce SMPs in the hadron-quark matter phase transition. Moreover, we can determine the region of mixed phase in hybrid stars. We are then able to provide important information for the EOS of the deconfinement phase transition, and clarify the phenomena of neutron stars and black stars.

We have assumed in relation to phenomenological implications that the temperature is zero. It would be interesting to include the finite-temperature effect. We could then study the structure of the protoneutron star and present new cooling curves of the neutron stars.

In this study, we used a simple model for quark matter to determine the finite-size effects acting on the SMP in a hybrid star. However, it has been suggested that color superconductivity is a ground state of quark matter [1,20]. To obtain a more realistic picture of the hadron-quark phase transition, we need to take into account color superconductivity. The description of nuclear matter needs to be improved; for example, we need to take into account the relativistic mean field theory [33]. We will then be able to provide more realistic results.

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