Statistical significance of the gallium anomaly

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We calculate the statistical significance of the anomalous deficit of electron neutrinos measured in the radioactive source experiments of the GALLEX and SAGE solar neutrino detectors, taking into account the uncertainty of the detection cross section. We found that the statistical significance of the anomaly is $\sim 3.0\sigma$. A fit of the data in terms of neutrino oscillations favors at $\sim 2.7\sigma$ short-baseline electron neutrino disappearance with respect to the null hypothesis of no oscillations.

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The GALLEX [1–3] and SAGE [4–7] gallium solar neutrino experiments have been tested with intense artificial ^{51}Cr and ^{37}Ar radioactive sources placed inside the detectors. The results of these gallium radioactive source experiments' indicate a ratio R of measured and predicted ^{71}Ge event rates which is smaller than unity:

$$R_B^{G1} = 0.953 \pm 0.11, \quad (1)$$

$$R_B^{G2} = 0.812^{+0.10}_{-0.11}, \quad (2)$$

$$R_B^{S1} = 0.95 \pm 0.12, \quad (3)$$

$$R_B^{S2} = 0.791^{+0.084}_{-0.078}, \quad (4)$$

where G1 and G2 denote the two GALLEX experiments with ^{51}Cr sources, S1 denotes the SAGE experiment with a ^{51}Cr source, and S2 denotes the SAGE experiment with a ^{37}Ar source.

Assuming Gaussian probability distributions and taking into account the asymmetric uncertainties of R_B^{G2} and R_B^{S2}, we have the probability distributions shown by the dashed, dotted, dashed-dotted, and dashed-dotted-dotted lines in Fig. 1. The combined probability distribution $p_{R_B^{Ga}}(r)$ shown in Fig. 1 gives the average ratio

$$R_B^{Ga} = 0.86^{+0.05+0.10+0.15}_{-0.05-0.10-0.15}, \quad (5)$$

where the uncertainties are at 68.27% C.L. (1σ), 95.45% C.L. (2σ), and 99.73% C.L. (3σ). Thus, the number of measured events is $\sim 2.8\sigma$ smaller than the prediction. This is the gallium anomaly.

As indicated by the "B" subscript, the ratios in Eqs. (1)–(5) have been calculated with respect to the rate estimated using the best-fit values of the cross section of the detection process

$$\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^- \quad (6)$$

calculated by Bahcall [8],

$$\sigma_B^{bf}(^{51}\mathrm{Cr}) = \left(58.1^{+2.1}_{-1.6}\right) \times 10^{-46}\ \mathrm{cm}^2, \quad (7)$$

$$\sigma_B^{bf}(^{37}\mathrm{Ar}) = \left(70.0^{+4.9}_{-2.1}\right) \times 10^{-46}\ \mathrm{cm}^2. \quad (8)$$

The uncertainties of these cross sections are not taken into account in the experimental ratios in Eqs. (1)–(4). These uncertainties are large [8–10], because only the cross section of the transition from the ground state of ^{71}Ga to the ground state of ^{71}Ge is known with precision from the measured rate of electron capture decay of ^{71}Ge to ^{71}Ga. Electron neutrinos produced by ^{51}Cr and ^{37}Ar radioactive sources can be absorbed also through transitions from the ground state of ^{71}Ga to two excited states of ^{71}Ge at 175 and 500 keV, with cross sections which are inferred using a nuclear model from $p + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + n$ measurements [11].

Hence, at least part of the deficit of measured events with respect to the prediction could be explained by an overestimation of the transitions to the two excited states of ^{71}Ge [6,7,12]. However, since the contribution of the transitions to the two excited states of ^{71}Ge is only 5% [8], even the complete absence of such transitions would reduce the ratio of measured and predicted ^{71}Ge event rates to approximately 0.91 ± 0.05, leaving an anomaly of $\sim 1.7\sigma$ [13].

We think that, for a correct assessment of the statistical significance of the gallium anomaly, simple approaches based on either accepting the Bahcall cross section in Eq. (7) without taking into account its uncertainty or suppressing without theoretical motivations the transitions to the two excited states of ^{71}Ge are insufficient. A correct assessment of the statistical significance of the gallium anomaly can be done by taking into account the large uncertainties of the transitions to the two excited states of ^{71}Ge [8–10]. The most reliable estimate of these transitions and their uncertainties have been done by Haxton in Ref. [10], leading to the total cross section for a ^{51}Cr source

$$\sigma_H(^{51}\mathrm{Cr}) = (63.9 \pm 6.8) \times 10^{-46}\ \mathrm{cm}^2. \quad (9)$$

Notice that the average value of this cross section is even larger than the Bahcall cross section in Eq. (7). This leads to an enhancement of the gallium anomaly. However, the uncertainty

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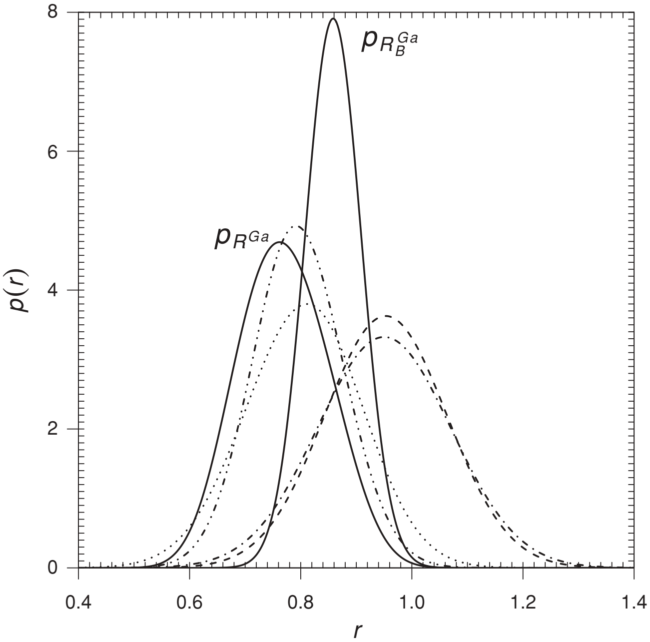

FIG. 1. Solid lines: Probability distributions $p_{R_B^{\rm Ga}}(r)$ and $p_{R^{\rm Ga}}(r)$, as indicated by the labels. Dashed, dotted, dashed-dotted, and dashed-dotted-dotted lines: Probability distributions $p_{R_B^{\rm G1}}(r)$, $p_{R_B^{\rm G2}}(r)$, $p_{R_B^{\rm S1}}(r)$, and $p_{R_B^{\rm S2}}(r)$, respectively.

of $\sigma_{\rm H}(^{51}{\rm Cr})$ is rather large. Hence, a correct assessment of the statistical significance of the gallium anomaly requires an accurate treatment of the cross-section uncertainty.

Since the ratios in Eqs. (1)–(3) have been calculated with respect to the best-fit value in Eq. (7) of the Bahcall cross section for a ^{51}Cr source, these ratios must be rescaled by

$$R_{\rm B}^{\rm H}(^{51}{\rm Cr}) = \frac{\sigma_{\rm H}(^{51}{\rm Cr})}{\sigma_{\rm B}^{\rm bf}(^{51}{\rm Cr})} = 1.10 \pm 0.12. \tag{10}$$

For the SAGE ^{37}Ar source experiment, we evaluate the detection cross section and its uncertainty as follows. The cross section is given by [8]

$$\sigma(^{37}{\rm Ar}) = \sigma_{\rm gs}\left(1 + 0.695\frac{{\rm BGT}_{175}}{{\rm BGT}_{\rm gs}} + 0.263\frac{{\rm BGT}_{500}}{{\rm BGT}_{\rm gs}}\right), \tag{11}$$

where $\sigma_{\rm gs} = 66.2 \times 10^{-46}$ cm^2 is the cross section from the ground state of ^{71}Ga to the ground state of ^{71}Ge, ${\rm BGT}_{\rm gs}$ is the corresponding Gamow-Teller strength, and ${\rm BGT}_{175}$ and ${\rm BGT}_{500}$ are the Gamow-Teller strengths of the transitions from the ground state of ^{71}Ga to the two excited states of ^{71}Ge at 175 and 500 keV. The coefficients of ${\rm BGT}_{175}/{\rm BGT}_{\rm gs}$ and ${\rm BGT}_{500}/{\rm BGT}_{\rm gs}$ are determined by phase space. In Ref. [10], Haxton estimated [in Ref. [10], the values of ${\rm BGT}_{175}/{\rm BGT}_{\rm gs}$ and ${\rm BGT}_{500}/{\rm BGT}_{\rm gs}$ can be extracted, respectively, from Eqs. (12) and (7), taking into account Eq. (1). As explained by Haxton, ${\rm BGT}_{175}$ requires a theoretical calculation, whereas for ${\rm BGT}_{500}$ it is reasonable to adopt the corresponding (p,n) value]

$${\rm BGT}_{175}/{\rm BGT}_{\rm gs} = 0.19 \pm 0.18, \tag{12}$$

$${\rm BGT}_{500}/{\rm BGT}_{\rm gs} = 0.13 \pm 0.02. \tag{13}$$

Thus, we obtain

$$\sigma_{\rm H}(^{37}{\rm Ar}) = (77.3 \pm 8.2) \times 10^{-46}\ {\rm cm}^2 \tag{14}$$

and

$$R_{\rm B}^{\rm H}(^{37}{\rm Ar}) = \frac{\sigma_{\rm H}(^{37}{\rm Ar})}{\sigma_{\rm B}^{\rm bf}(^{37}{\rm Ar})} = 1.10 \pm 0.12, \tag{15}$$

which has the same value of $R_{\rm B}^{\rm H}(^{51}{\rm Cr})$ in Eq. (10). Therefore, all the ratios in Eqs. (1)–(4) must be rescaled by $R_{\rm B}^{\rm H} = R_{\rm B}^{\rm H}(^{51}{\rm Cr}) = R_{\rm B}^{\rm H}(^{37}{\rm Ar})$.

One must also take into account that the value of the cross section is bounded from below by the cross section $\sigma_{\rm gs}$ of the transition from the ground state of ^{71}Ga to the ground state of ^{71}Ge [8]:

$$R_{\rm B}^{\rm H} \geqslant R_{\rm B}^{\rm gs} = \frac{\sigma_{\rm gs}}{\sigma_{\rm B}^{\rm bf}} = 0.95. \tag{16}$$

In the following we calculate the probability distribution of

$$R^{\rm Ga} = \frac{R_{\rm B}^{\rm Ga}}{R_{\rm B}^{\rm H}} \tag{17}$$

by taking into account the uncertainty of the denominator $R_{\rm B}^{\rm H}$ given in Eqs. (10) and (15). This is the theoretical uncertainty of the cross section that has not been taken into account in the ratios (1)–(5), which have been evaluated using the best-fit values of the Bahcall cross sections in Eqs. (7) and (8). We assume a Gaussian probability distribution truncated below $R_{\rm B}^{\rm gs}$:

$$p_{R_{\rm B}^{\rm H}}(r) \propto \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{r - \langle R_{\rm B}^{\rm H}\rangle}{\Delta R_{\rm B}^{\rm H}}\right)^2\right], & r \geqslant R_{\rm B}^{\rm gs}, \\ 0, & r < R_{\rm B}^{\rm gs}, \end{cases} \tag{18}$$

with $\langle R_{\rm B}^{\rm H}\rangle = 1.10$ and $\Delta R_{\rm B}^{\rm H} = 0.12$.

The probability distribution of the ratio $R^{\rm Ga}$ in Eq. (17) is given by (see Sec. 2.4.4 of Ref. [14])

$$p_{R^{\rm Ga}}(r) = \int_{R_{\rm B}^{\rm gs}}^{\infty} p_{R_{\rm B}^{\rm Ga}}(rs)\, p_{R_{\rm B}^{\rm H}}(s)\, s\, ds. \tag{19}$$

Figure 1 shows the probability distribution $p_{R_{\rm B}^{\rm Ga}}(r)$ of $R_{\rm B}^{\rm Ga}$ derived from the experimental data in Eqs. (1)–(4) and the result of the integral in Eq. (19). One can see that $p_{R^{\rm Ga}}(r)$ is peaked at a smaller value than $p_{R_{\rm B}^{\rm Ga}}(r)$, but the uncertainty is larger. We obtain

$$R^{\rm Ga} = 0.76^{+0.09+0.17+0.24}_{-0.08-0.15-0.21}, \tag{20}$$

where the uncertainties are at 68.27% C.L. (1σ), 95.45% C.L. (2σ), and 99.73% C.L. (3σ). From a comparison of these uncertainties and from Fig. 1, one can see that the probability distribution is approximately Gaussian, with slightly asymmetric uncertainties and tails which decrease slightly faster than Gaussian tails.

The probability of $R^{\rm Ga} < 1$ is 99.86% (3.0σ anomaly), slightly larger than the probability of $R_{\rm B}^{\rm Ga} < 1$, which is

99.75% (2.8σ anomaly). Therefore, the gallium anomaly remains statistically significant after taking properly into account the cross-section uncertainty.

For the four individual Gallium radioactive source experiments, using the same method as above, from the experimental values in Eqs. (1)–(4) we obtain

$$R^{\mathrm{G1}} = R_{\mathrm{B}}^{\mathrm{G1}} / R_{\mathrm{B}}^{\mathrm{H}} = 0.84^{+0.13+0.26+0.40}_{-0.12-0.23-0.33}, \quad (21)$$

$$R^{\mathrm{G2}} = R_{\mathrm{B}}^{\mathrm{G2}} / R_{\mathrm{B}}^{\mathrm{H}} = 0.71^{+0.12+0.24+0.36}_{-0.11-0.21-0.31}, \quad (22)$$

$$R^{\mathrm{S1}} = R_{\mathrm{B}}^{\mathrm{S1}} / R_{\mathrm{B}}^{\mathrm{H}} = 0.84^{+0.14+0.28+0.42}_{-0.13-0.24-0.35}, \quad (23)$$

$$R^{\mathrm{S2}} = R_{\mathrm{B}}^{\mathrm{S2}} / R_{\mathrm{B}}^{\mathrm{H}} = 0.70^{+0.10+0.21+0.31}_{-0.09-0.17-0.25}, \quad (24)$$

with 1σ, 2σ, 3σ uncertainties. A comparison of these uncertainties shows that the probability distributions are approximately Gaussian, with slightly asymmetric uncertainties.

Since the gallium anomaly is confirmed by the new statistical analysis which takes into account the uncertainty of the detection cross section, it is plausible that it is due to a physical mechanism. In the following, we consider the possibility of electron neutrino disappearance due to short-baseline oscillations [13,15–21] (another explanation based on quantum decoherence in neutrino oscillations has been proposed in Ref. [22]).

We consider the electron neutrino survival probability

$$P_{\nu_e \to \nu_e}^{\mathrm{SBL}}(L, E) = 1 - \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \quad (25)$$

where ϑ is the mixing angle, Δm^2 is the squared-mass difference, L is the neutrino path length, and E is the neutrino energy. This survival probability is effective in short-baseline (SBL) experiments in the framework of four-neutrino mixing schemes (see Refs. [23–26]), which are the simplest extensions of three-neutrino mixing schemes which can accommodate the two measured small solar and atmospheric squared-mass differences $\Delta m^2_{\mathrm{SOL}} \simeq 8 \times 10^{-5}\ \mathrm{eV}^2$ and $\Delta m^2_{\mathrm{ATM}} \simeq 2 \times 10^{-3}\ \mathrm{eV}^2$ and one larger squared-mass difference for SBL neutrino oscillations, $\Delta m^2 \gtrsim 0.1\ \mathrm{eV}^2$. The existence of a fourth massive neutrino corresponds, in the flavor basis, to the existence of a sterile neutrino ν_s.

We performed a maximum likelihood analysis (see Ref. [27]) of the gallium data as follows [a standard least-squares analysis would lead to misleading results, because it does not allow us to take into account the lower bound in Eq. (16) for $R_{\mathrm{B}}^{\mathrm{H}}$]. We started with the calculation, for each experiment, of the value of the ratio of the event rate as a function of $\sin^2 2\vartheta$ and Δm^2 and the event rate in absence of neutrino oscillations (see Ref. [19] for details):

$$R^k(\sin^2 2\vartheta, \Delta m^2) = \frac{\int_k dV\, L^{-2} \sum_i b_i^k \sigma_i^k P_{\nu_e \to \nu_e}^{\mathrm{SBL}}(L, E_i)}{\sum_i b_i^k \sigma_i^k \int_k dV\, L^{-2}}, \quad (26)$$

where the index k labels the experiments ($k = \mathrm{G1}, \mathrm{G2}, \mathrm{S1}, \mathrm{S2}$), the index i labels the ν_e lines emitted in $^{51}\mathrm{Cr}$ or $^{37}\mathrm{Ar}$ electron captures with energies E_i, b_i^k and σ_i^k are the corresponding branching ratios and cross sections (see Table I of Ref. [19]), L is the neutrino path length, and $\int_k dV$ is the integral over the volume of each detector (see Table II of Ref. [19]).

The uncertainty of $R_{\mathrm{B}}^{\mathrm{H}}$ is correlated in the calculation of the combined probability distribution of the four experimental ratios in Eqs. (21)–(24). Using a method similar to that utilized for the derivation of Eq. (19) (see Sec. 2.4.4 of Ref. [14]), we obtain the combined probability distribution

$$p_{\vec{R}}(\vec{r}) = \int_{R_{\mathrm{B}}^{\mathrm{gs}}}^{\infty} \left[\prod_k p_{R_{\mathrm{B}}^k}(r^k s)\right] p_{R_{\mathrm{B}}^{\mathrm{H}}}(s) s^4\, ds, \quad (27)$$

where $\vec{R} = (R^{\mathrm{G1}}, R^{\mathrm{G2}}, R^{\mathrm{S1}}, R^{\mathrm{S2}})$ and $\vec{r} = (r^{\mathrm{G1}}, r^{\mathrm{G2}}, r^{\mathrm{S1}}, r^{\mathrm{S2}})$. The authors of Ref. [28] considered a correlation of the systematic errors of the two GALLEX experiments and the two SAGE experiments. Since such a correlation is not documented in the experimental publications, where the combined ratio was calculated as a weighted average, without correlations, we adopt the conservative approach of considering the systematic experimental errors as independent [a correlation of the systematic experimental errors can be taken into account in Eq. (27) by replacing $\prod_k p_{R_{\mathrm{B}}^k}(r^k s)$ with a multivariate Gaussian distribution with the appropriate covariance matrix.]

The likelihood function of the oscillation parameters $\sin^2 2\vartheta$ and Δm^2 is given by

$$\mathcal{L}(\sin^2 2\vartheta, \Delta m^2) = p_{\vec{R}}[\vec{R}(\sin^2 2\vartheta, \Delta m^2)], \quad (28)$$

with the four components of $\vec{R}(\sin^2 2\vartheta, \Delta m^2)$ given by Eq. (26). Figure 2 shows the allowed regions in the

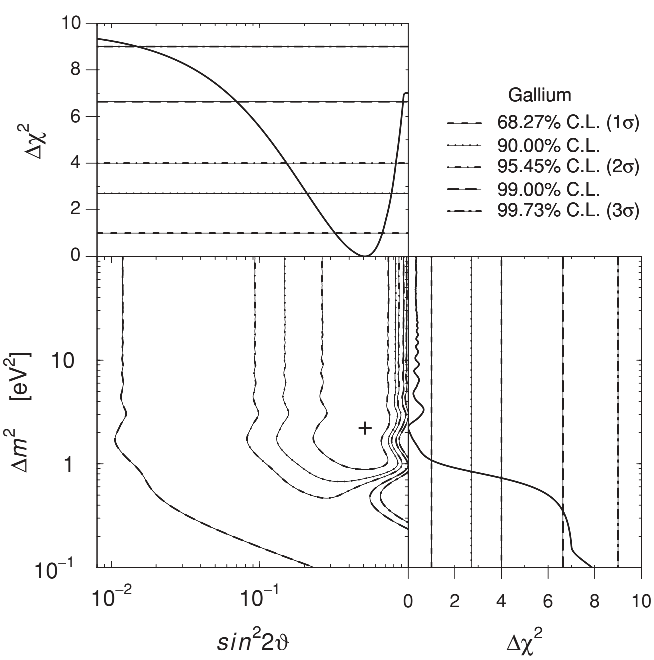

FIG. 2. Allowed regions in the $\sin^2 2\vartheta$–Δm^2 plane and marginal $\Delta\chi^2$ for $\sin^2 2\vartheta$ and Δm^2 obtained from the combined fit of the results of the two GALLEX $^{51}\mathrm{Cr}$ radioactive source experiments and the SAGE $^{51}\mathrm{Cr}$ and $^{37}\mathrm{Ar}$ radioactive source experiments. The best-fit point corresponding to χ^2_{min} is indicated by a cross.

$\sin^2 2\vartheta$–Δm^2 plane and the marginal $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ for $\sin^2 2\vartheta$ and Δm^2, from which one can infer the corresponding uncorrelated allowed intervals. In the maximum likelihood analysis $\chi^2(\sin^2 2\vartheta, \Delta m^2)$ is given by $-2\ln \mathcal{L}(\sin^2 2\vartheta, \Delta m^2) + \text{const.}$

The best-fit values of the oscillation parameters are

$$\sin^2 2\vartheta_{\text{bf}} = 0.50, \quad \Delta m^2_{\text{bf}} = 2.24 \text{ eV}^2. \qquad (29)$$

The value of the likelihood ratio between the null hypothesis of no oscillations and the oscillation hypothesis,

$$\frac{\mathcal{L}_0}{\mathcal{L}\left(\sin^2 2\vartheta_{\text{bf}}, \Delta m^2_{\text{bf}}\right)} = 8 \times 10^{-3}, \qquad (30)$$

is in favor of the oscillation hypothesis. It corresponds to $\Delta\chi^2 = 9.7$, which, with two degrees of freedom, disfavors the null hypothesis of no oscillations at 99.23% C.L. (2.7σ). The small difference between this statistical significance of the indication in favor of the gallium anomaly and that obtained from Eq. (20) (3.0σ) is due to the different analysis of the data. Although the neutrino oscillation analysis leads to a better fit of the four data in Eqs. (21)–(24) [the best-fit values of the oscillation parameters in Eq. (29) give $R^{\text{G1}} = 0.75$, $R^{\text{G2}} = 0.75$, $R^{\text{S1}} = 0.73$, and $R^{\text{S2}} = 0.72$], the correlation of the theoretical uncertainty of R^{H}_{B} slightly disfavors a fit in which the deviations of the data from the best-fit values do not have the same sign.

From Fig. 2 one can see that the marginal distributions of $\sin^2 2\vartheta$ and Δm^2 indicate that [these bounds are weaker than those presented in a previous version of this paper (arXiv:1006.3244v2) in which the correlation of the uncertainty of R^{H}_{B} in the calculation of the combined probability distribution of the four experimental ratios in Eqs. (21)–(24) was not taken into account]

$$\sin^2 2\vartheta > 0.07, \quad \Delta m^2 > 0.35 \text{ eV}^2, \qquad (31)$$

at 99% C.L. These bounds indicate that the short-baseline disappearance of electron neutrinos may be larger than that of electron antineutrinos, which is bounded by the results of reactor neutrino experiments [13,19,20]. This could be an indication of a violation of the CPT symmetry [29] (CPT implies that $P_{\nu_\alpha\to\nu_\alpha} = P_{\bar\nu_\alpha\to\bar\nu_\alpha}$ for any flavor $\alpha = e, \mu, \tau$; see Ref. [30]). However, according to a recent calculation [31] the $\bar\nu_e$ fluxes produced in nuclear reactors are ~3% larger than the standard ones used in the analysis of reactor antineutrino data (see Ref. [32]). A comparison of the new reactor antineutrino fluxes with the data of several reactor neutrino experiments suggests the existence of a reactor antineutrino anomaly [28], which is compatible with the gallium anomaly in a standard CPT-invariant neutrino oscillation framework. In this case, the indication in favor of CPT violation obtained by comparing the results of the neutrino oscillation analysis of gallium and reactor data is weakened, but the plausibility of the existence of a gallium anomaly is reinforced by its compatibility with the reactor antineutrino anomaly.

CPT violation in short-baseline electron neutrino disappearance can be tested with high accuracy in future experiments with pure and well-known ν_e and $\bar\nu_e$ beams, as beta-beam [33] and neutrino factory [34,35] experiments. Although the possibility of CPT violation is theoretically problematic [36], it cannot be dismissed in phenomenological analyses of experimental results. It is interesting to notice that recently another indication of a violation of the CPT symmetry has been found in the MINOS long-baseline ν_μ and $\bar\nu_\mu$ disappearance experiment [37,38].

There is also a growing experimental interest in favor of possible tests of the Gallium anomaly. In addition to the future experimental possibilities to test the SBL disappearance of electron neutrinos discussed in Ref. [13], the authors of Ref. [21] presented recently a plan to make an improved direct measurement of the gallium anomaly with the liquid gallium metal used in the SAGE experiment and a new vessel divided in two zones, which can measure a variation of the electron neutrino disappearance with distance. The Borexino collaboration is studying the possibility of a radioactive source experiment [39], which could provide a "smoking gun" signal by measuring the oscillation pattern inside the detector. Other possible measurements with radioactive sources and different detector types has been recently discussed in Ref. [40].

The existence of at least four massive neutrinos, one of which has a mass larger than ~0.6 eV in order to generate the squared-mass difference in Eq. (31), can have important implications for cosmology (see Refs. [41–43]). The current indications of cosmological data analyzed in the framework of the standard cosmological model are controversial. On one hand, there are indications that the effective number of neutrino species may be larger than three from Big Bang nucleosynthesis [44] and from the cosmic microwave background radiation [45]. This is consistent with a thermalization of sterile neutrinos due to active-sterile oscillations before Big Bang nucleosynthesis induced by the large values of the mixing parameters in Eq. (31) [46]. On the other hand, analyses of cosmic microwave background radiation data and large-scale structure data constrain the mass of a fourth thermalized neutrino to be smaller than ~0.7 eV [47–50]. Hence, either the heavy neutrino mass is close to the standard cosmological bound or the existence of SBL neutrino oscillations is connected with nonstandard cosmological effects, as those discussed in Refs. [51–53].

In conclusion, we have estimated the uncertainty of the deficit of electron neutrinos measured in the radioactive source experiments of the GALLEX [1–3] and SAGE [4–7] solar neutrino detectors, taking into account the uncertainty of the detection cross section estimated by Haxton in Ref. [10]. The result shows that the gallium anomaly is statistically significant, at a level of $\sim 3.0\sigma$. The analysis of the data in terms of neutrino oscillations indicates values of the oscillation amplitude $\sin^2 2\vartheta \gtrsim 0.07$ and squared-mass difference $\Delta m^2 \gtrsim 0.35 \text{ eV}^2$ at 99% C.L.

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