### PHYSICAL REVIEW C 83, 065208 (2011)

# $J/\Psi$ -nuclear bound states

K. Tsushima, 1,\* D. H. Lu, 2,† G. Krein, 3,‡ and A. W. Thomas 1,§

<sup>1</sup>CSSM, School of Chemistry and Physics, University of Adelaide, Adelaide, South Australia 5005, Australia
<sup>2</sup>Department of Physics, Zhejiang University, Hangzhou 310027, People's Republic of China
<sup>3</sup>Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, São Paulo, SP, Brazil
(Received 28 March 2011; published 29 June 2011)

 $J/\Psi$ -nuclear bound state energies are calculated for a range of nuclei by solving the Proca (Klein-Gordon) equation. Critical input for the calculations, namely the medium-modified D and  $D^*$  meson masses, as well as the nucleon density distributions in nuclei, are obtained from the quark-meson coupling model. The attractive potential originates from the D and  $D^*$  meson loops in the  $J/\Psi$  self-energy in the nuclear medium. It appears that  $J/\Psi$ -nuclear bound states should produce a clear experimental signature provided that the  $J/\Psi$  meson is produced in recoilless kinematics.

DOI: 10.1103/PhysRevC.83.065208 PACS number(s): 21.85.+d, 13.75.Gx, 14.40.Lb, 24.85.+p

### I. INTRODUCTION

With the 12-GeV upgrade of the CEBAF accelerator at the Jefferson Lab (JLab) in the USA and with the construction of the FAIR facility in Germany, we expect tremendous progress in understanding the properties of charmonia and charmed mesons in the nuclear medium. These new facilities will be able to produce low-momenta charmonia and charmed mesons such as  $J/\Psi$ ,  $\psi(2S)$ , D, and  $D^*$  in atomic nuclei. Because the targets are nuclei and the nuclear Fermi momentum is available, it may also be possible to produce these mesons at subthreshold energies. While at JLab charmed hadrons will be produced by scattering electrons off nuclei, at FAIR they will be produced by the annihilation of antiprotons on nuclei. One of the major challenges is to find appropriate kinematical conditions to produce these mesons essentially at rest, or with small momentum relative to the nucleus. One of the most exciting experimental efforts may be to search for the  $J/\Psi$ -nuclear bound states, among many other possible interesting experiments in these facilities. The discovery of such bound states would provide evidence for a negative mass shift of the  $J/\Psi$  meson and a possible role of the QCD color van der Waals forces [1] in nuclei. Voloshin [2] presents a recent review of the properties of charmonium states and compiles a complete list of references for theoretical studies concerning a great variety of physics issues related to these

There is a relatively long history for studies of charmonium binding in nuclei. The original suggestion [1] that multiple gluon QCD van der Waals forces would be capable of binding a charmonium state led to an estimate of a binding energy as large as 400 MeV in an A=9 nucleus. However, using the same approach but taking into account the nucleon density distributions in the nucleus, Wasson [3] found a maximum of 30-MeV binding energy in a large nucleus. Along the same line, including the nuclear density distributions in nuclei

 $\eta_c$ -nuclear bound state energies were estimated in Ref. [4]. Based on Ref. [5], in which it was shown that the mass shift of charmonium in nuclear matter can be expressed in terms of the usual second-order Stark effect arising from the chromo-electric polarizability of the nucleon, the authors of Ref. [6] obtained a 10-MeV binding for  $J/\Psi$  in nuclear matter, in the limit of infinitely heavy charm quark mass. On the other hand, for the excited charmonium states, a much larger binding energy was obtained, e.g., 700 MeV for the  $\psi'(2S)$ state, an admittedly untrustworthy number. Following the same procedure, but keeping the charm quark mass finite and using realistic charmonium bound-state wave functions, Lee and Ko [7] found 8-MeV binding energy for  $J/\Psi$  in nuclear matter, but still over 100-MeV binding for the charmonium excited states. While a larger value for the QCD Stark effect is expected for excited states (because of the larger sizes of these states), the large values for the binding energies found in the literature for these states may be an overestimate. A possible source of this overestimate is the breakdown of the multipole expansion for the larger-sized charmonium

There are some other studies of charmonium interactions with nuclear matter, in particular involving the  $J/\Psi$  meson. QCD sum rules studies estimated a  $J/\Psi$  mass decrease in nuclear matter ranging from 4 to 7 MeV [8–10], while an estimate based on color polarizability [11] gave a value larger than 21 MeV. Since the  $J/\Psi$  and nucleons have no quarks in common, the quark interchange or the effective meson exchange potential should be negligible to first order in elastic scattering [1], and multigluon exchange should be dominant. Furthermore, there is no Pauli blocking even at the quark level. Thus, if the  $J/\Psi$ -nuclear bound states are formed, the signal for these states will be sharp and show a clear, narrow peak in the energy dependence of the cross section, as will be explained later. This situation has a tremendous advantage compared to the cases of the lighter vector mesons [12].

Previously, we have studied [13] the  $J/\Psi$  mass shift (scalar potential) in nuclear matter based on an effective Lagrangian approach, including the D and  $D^*$  meson loops in the  $J/\Psi$  self-energy. This is a color-singlet mechanism at the hadronic level, and it may be compared to the color-octet

<sup>\*</sup>kazuo.tsushima@gmail.com

<sup>†</sup>dhlu@zju.edu.cn,

<sup>‡</sup>gkrein@ift.unesp.br

<sup>§</sup>anthony.thomas@adelaide.edu.au

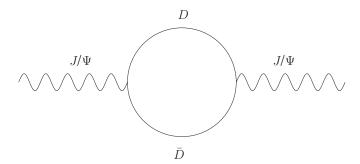


FIG. 1. DD-loop contribution to the  $J/\Psi$  self-energy. We include also  $DD^*$  and  $D^*D^*$  contributions.

mechanism of multigluon exchange or QCD van der Waals forces. In this work, we extend our previous study made in nuclear matter [13] to finite nuclei, and we compute the  $J/\Psi$ -nuclear bound state energies by explicitly solving the Proca (Klein-Gordon) equation for the  $J/\Psi$  meson produced nearly at rest. Furthermore, the structure of heavy nuclei, as well as the medium modification of the D and  $D^*$  masses, is explicitly included based on the quark-meson coupling (QMC) model [14].

A first estimate for the mass shifts (scalar potentials) for the  $J/\Psi$  meson [and  $\Psi(3686)$  and  $\Psi(3770)$ ] in the nuclear medium including the effects of the D meson loop in the  $J/\Psi$ self-energy (see Fig. 1) was made in Ref. [7]. By employing a gauged effective Lagrangian for the coupling of D mesons to the charmonia, the mass shifts were found to be positive for  $J/\Psi$  and  $\psi(3770)$  and negative for  $\psi(3660)$  at normal nuclear matter density  $\rho_0$ . These results were obtained for densitydependent D and  $\bar{D}$  masses that decrease linearly with density, such that at  $\rho_0$  they are shifted by 50 MeV. The loop integral in the self-energy (Fig. 1) is divergent and was regularized using form factors derived from the  ${}^{3}P_{0}$  decay model with quarkmodel wave functions for  $\psi$  and D. The positive mass shift is at first sight puzzling, since even with a 50-MeV reduction of the D masses, the intermediate state is still above threshold for the decay of  $J/\Psi$  into a  $D\bar{D}$  pair and so a second-order contribution should be negative. However, as we have shown in Ref. [13], this is a result of the interplay of the form factor used and the gauged nature of the interaction used in Ref. [7]. We have estimated [13] the mass shift (scalar potential) of the  $J/\Psi$ meson including the  $D\bar{D}$ ,  $D\bar{D}^*$ ,  $D^*\bar{D}$ , and  $D^*\bar{D}^*$  meson loops in the self-energy using nongauged effective Lagrangians. The density dependence of the D and  $D^*$  masses is included by an explicit calculation [15] using the QMC model [16]. The QMC model is a quark-based model for nuclear structure which has been very successful in explaining the origin of many-body or density-dependent effective forces [17] and hence describing nuclear matter saturation properties. It has also been used to predict a great variety of changes of hadron properties in the nuclear medium, including the properties of hypernuclei [18]. A review of the basic ingredients of the model and a summary of results and predictions can be found in Ref. [19].

## II. $J/\Psi$ POTENTIAL IN INFINITE NUCLEAR MATTER

We briefly review how the  $J/\Psi$  scalar potential is calculated in nuclear matter [13]. We use the effective Lagrangian densities at the hadronic level for the vertices,  $J/\Psi DD$ ,  $J/\Psi DD^*$ , and  $J/\Psi D^*D^*$  (where in the following we denote by  $\psi$  the field representing  $J/\Psi$ ):

$$\mathcal{L}_{\psi DD} = i g_{\psi DD} \psi^{\mu} [\bar{D}(\partial_{\mu} D) - (\partial_{\mu} \bar{D}) D], \tag{1}$$

$$\mathcal{L}_{\psi DD^{*}} = \frac{g_{\psi DD^{*}}}{m_{\psi}} \varepsilon_{\alpha\beta\mu\nu} (\partial^{\alpha} \psi^{\beta}) [(\partial^{\mu} \bar{D}^{*\nu}) D + \bar{D}(\partial^{\mu} D^{*\nu})], \tag{2}$$

$$\mathcal{L}_{\psi D^* D^*} = i g_{\psi D^* D^*} \{ \psi^{\mu} [(\partial_{\mu} \bar{D}^{*\nu}) D_{\nu}^* - \bar{D}^{*\nu} (\partial_{\mu} D_{\nu}^*)] + [(\partial_{\mu} \psi^{\nu}) \bar{D}_{\nu}^* - \psi^{\nu} (\partial_{\mu} \bar{D}_{\nu}^*)] D^{*\mu} + \bar{D}^{*\mu} [\psi^{\nu} (\partial_{\mu} D_{\nu}^*) - (\partial_{\mu} \psi^{\nu}) D_{\nu}^*] \}.$$
(3)

The Lagrangian densities above are obtained on the basis of SU(4) invariance for the couplings among the pseudoscalar and vector mesons [20,21]. We use the values for the coupling constants,  $g_{\Psi DD} = g_{\Psi D^*D^*} = g_{\Psi DD^*} = 7.64$ , where the former two values were obtained using the vector meson dominance model [20]. The form of the  $J/\Psi D^*\bar{D}^*$  coupling respects Yang's theorem [22,23] and so for spin one  $J/\Psi$  the virtual  $D^*$  and  $\bar{D}^*$  must not be simultaneously transverse. The difference with the gauged Lagrangian of Ref. [7] for the  $\Psi DD$  vertex amounts to adding to Eq. (1) a term  $2g_{\Psi DD}^2 \psi^\mu \psi_\mu \bar{D}D$ . It may be of interest to investigate in future whether the meson loops investigated here might play some role in the understanding of the well-known  $J/\psi \to \rho \pi$  puzzle [24].

Only the scalar part contributes to the  $J/\Psi$  self-energy, and the scalar potential for the  $J/\Psi$  meson is the difference of the in-medium,  $m_{\psi}^*$ , and vacuum,  $m_{\psi}$ , masses of  $J/\Psi$ ,

$$\Delta m = m_{\psi}^* - m_{\psi}, \tag{4}$$

with the masses obtained from

$$m_{\psi}^2 = (m_{\psi}^0)^2 + \Sigma (k^2 = m_{\psi}^2).$$
 (5)

Here  $m_{\psi}^0$  is the bare mass and  $\Sigma(k^2)$  is the total  $J/\Psi$  self-energy obtained from the sum of the contributions from the DD,  $DD^*$ , and  $D^*D^*$  loops. The in-medium mass,  $m_{\psi}^*$ , is obtained likewise, with the self-energy calculated with medium-modified D and  $D^*$  meson masses.

In the calculation, we use phenomenological form factors to regularize the self-energy loop integrals following a similar calculation for the  $\rho$  self-energy [25]:

$$u_{D,D^*}(\mathbf{q}^2) = \left(\frac{\Lambda_{D,D^*}^2 + m_{\psi}^2}{\Lambda_{D,D^*}^2 + 4\omega_{D,D^*}^2(\mathbf{q})}\right)^2.$$
 (6)

For the vertices  $\Psi DD$ ,  $\Psi DD^*$ , and  $\Psi D^*D^*$ , the form factors used are, respectively,  $F_{DD}(\mathbf{q}^2) = u_D^2(\mathbf{q}^2)$ ,  $F_{DD^*}(\mathbf{q}^2) = u_D(\mathbf{q}^2) u_{D^*}(\mathbf{q}^2)$ , and  $F_{D^*D^*}(\mathbf{q}^2) = u_{D^*}^2(\mathbf{q}^2)$ , with  $\Lambda_D$  and  $\Lambda_{D^*}$  being the cutoff masses, and the common values  $\Lambda_D = \Lambda_{D^*}$  are used. For the calculation in finite nuclei, we chose two values within the range expected to be reasonable,  $\Lambda_D = \Lambda_{D^*} = 1500$  and 2000 MeV, to estimate the model dependence. The scalar potential,  $(m_{\Psi}^* - m_{\Psi})$ , calculated via Eq. (5) in nuclear matter [13], is shown in Fig. 2 as a function of nuclear matter density.

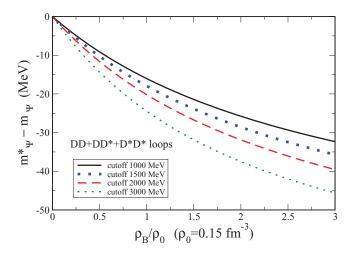


FIG. 2. (Color online) The scalar potential arising from DD,  $DD^*$ , and  $D^*D^*$  loops as a function of nuclear matter density for different values of the cutoff mass  $\Lambda_D = \Lambda_{D^*}$  [13].

## III. $J/\Psi$ BOUND STATES IN FINITE NUCLEI

In this section we discuss the case when  $J/\psi$  is placed in a finite nucleus. The nucleon density distributions for  $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ , and  $^{208}\text{Pb}$  are obtained using the QMC results for finite nuclei [14]. For  $^4\text{He}$ , we use the parametrization for the density distribution obtained in Ref. [26]. Then, using a local density approximation we calculate the  $J/\Psi$  potentials in nuclei. Note that also the in-medium masses of D and  $D^*$  mesons, which are necessary to estimate the  $J/\Psi$  self-energy consistently, are calculated in the QMC model. We emphasize that the same quark-meson coupling constants are used between the applied meson mean fields and the light quarks in the nucleon and those in the D and  $D^*$  mesons [15], where these coupling constants are calibrated by the nuclear matter saturation properties [19].

As examples, we show the  $J/\Psi$  potentials calculated for  ${}^4{\rm He}$  and  ${}^{208}{\rm Pb}$  nuclei in Fig. 3 for the two values of the cutoff

mass, 1500 and 2000 MeV. The maximum depth of the  $J/\Psi$  potential ranges roughly from 18 to 24 MeV.

Using the  $J/\Psi$  potentials in nuclei obtained in this manner, we next calculate the  $J/\Psi$ -nuclear bound state energies. We follow the procedure applied in the earlier work [27] for the  $\eta$ - and  $\omega$ -nuclear bound states. In this study, we focus entirely on the situation where the  $J/\Psi$  meson is produced nearly at rest (recoilless kinematics in experiments). Then, it should be a very good approximation to neglect the possible energy difference between the longitudinal and transverse components [28] of the  $J/\Psi$  wave function  $(\phi_{\Psi}^{\mu})$  as was assumed for the  $\omega$  meson case [28]. After imposing the Lorentz condition,  $\partial_{\mu}\phi_{\Psi}^{\mu}=0$ , to solve the Proca equation, aside from a possible width, becomes equivalent to solving the Klein-Gordon equation with the reduced mass of the system  $\mu$  in vacuum [29,30]:

$$\{\nabla^2 + E_{\Psi}^2 - \mu^2 - 2\mu [m_{\Psi}^*(r) - m_{\Psi}]\} \phi_{\Psi}(\mathbf{r}) = 0, \tag{7}$$

where  $E_{\Psi}$  is the total energy of the  $J/\Psi$  meson, and  $\mu = m_{\Psi}M/(m_{\Psi}+M)$  with  $m_{\Psi}(M)$  being the vacuum mass of the  $J/\Psi$  meson (nucleus). Since in free space the width of the  $J/\Psi$  meson is  $\sim 93$  keV [31], we can ignore this tiny natural width in the following. When the  $J/\Psi$  meson is produced nearly at rest, its dissociation process via  $J/\Psi+N\to\Lambda_c^++\bar{D}$  is forbidden in the nucleus (since the threshold energy in free space is about 115 MeV above). This is because the same number of light quarks there participates in the initial and final states and hence the effects of the partial restoration of chiral symmetry, which reduces mostly the amount of the light quark condensates, would effect a similar total mass reduction for the initial and final states [15,32]. Thus, the relative energy ( $\sim 115$  MeV) to the threshold would not be modified significantly.

We also note that once the  $J/\Psi$  meson is bound in the nucleus, the total energy of the system is below threshold for nucleon knockout and the whole system is stable. The exception to this is the process  $J/\Psi + N \rightarrow \eta_c + N$ , which is exothermic. An estimate of this cross section from which we deduce a width for the bound  $J/\Psi$  of order 0.8 MeV

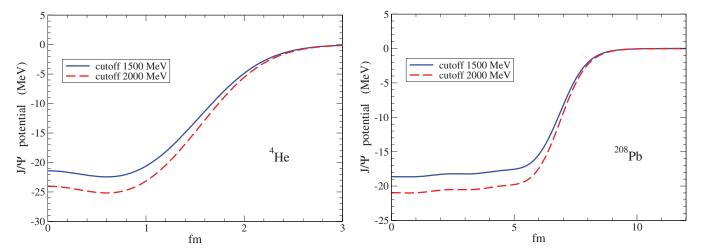


FIG. 3. (Color online)  $J/\Psi$  potential in a <sup>4</sup>He nucleus (left panel) and in a <sup>208</sup>Pb nucleus (right panel) for two values of the cutoff mass in the form factors, 1500 and 2000 MeV.

TABLE I. Bound state energies calculated for the  $J/\Psi$  meson, solving the Klein-Gordon equation with the reduced mass, Eq. (7). Under the situation that the  $J/\Psi$  meson is produced in recoilless kinematics, the width due to the strong interactions is set to zero, as well as its tiny natural width of  $\sim$  93 keV [31] in free space. (See also the text.)

		$\Lambda_{D,D^*} = 1500 \text{ MeV}$	$\Lambda_{D,D^*} = 2000 \text{ MeV}$
		E (MeV)	E (MeV)
<sup>4</sup> <sub>Ψ</sub> He	1 <i>s</i>	-4.19	-5.74
$_{\Psi}^{\dot{1}\dot{2}}\mathrm{C}$	1 <i>s</i>	-9.33	-11.21
	1 p	-2.58	-3.94
$_{\Psi}^{16}\mathrm{O}$	1 <i>s</i>	-11.23	-13.26
	1 p	-5.11	-6.81
$^{40}_{\Psi}\mathrm{Ca}$	1 <i>s</i>	-14.96	-17.24
	1 p	-10.81	-12.92
	1 <i>d</i>	-6.29	-8.21
	2s	-5.63	-7.48
$_{\Psi}^{90}\mathrm{Zr}$	1 <i>s</i>	-16.38	-18.69
	1 p	-13.84	-16.07
	1d	-10.92	-13.06
	2s	-10.11	-12.22
<sup>208</sup> Pb	1 <i>s</i>	-16.83	-19.10
	1 p	-15.36	-17.59
	1d	-13.61	-15.81
	2 <i>s</i>	-13.07	-15.26

(nuclear matter at  $\rho_B = \rho_0/2$ ) has been provided in Ref. [33]. We therefore expect that the bound  $J/\Psi$ , while not being completely stable under the strong interaction, should be narrow enough to be clearly observed. It would be worthwhile to investigate this further. Then, provided that experiments can be performed to produce the  $J/\Psi$  meson in recoilless kinematics, the  $J/\Psi$  meson is expected to be captured by the nucleus into one of the bound states, which has no strong interaction originated width. This is a completely different and advantageous situation compared to that of the  $\eta$  and  $\omega$ meson cases [27]. Thus, in the end, we may simply solve the Klein-Gordon equation with the reduced mass, Eq. (7), without worrying about the width, under the situation we consider now. The calculation is carried out in momentum space by the method developed in Refs. [29,30]. Bound state energies for the  $J/\Psi$  meson, obtained solving the Klein-Gordon equation [Eq. (7)], are listed in Table I.

The results in Table I show that the  $J/\Psi$  are expected to form nuclear bound states in all the nuclei considered. This is insensitive to the values of cutoff mass used in the form

factors. It is very interesting to note that one can expect to find a  $J/\Psi^{-4}$ He bound state. It will be possible to search for this state as well as the states in a  $^{208}$ Pb nucleus at JLab after the 12-GeV upgrade. In addition, one can expect quite rich spectra for medium- and heavy-mass nuclei. Of course, the main issue is to produce the  $J/\Psi$  meson with nearly stopped kinematics, or nearly zero momentum relative to the nucleus. Since the present results imply that many nuclei should form  $J/\Psi$ -nuclear bound states, it may be possible to find such kinematics by careful selection of the beam and target nuclei.

### IV. SUMMARY AND DISCUSSION

In this study, we have calculated the  $J/\Psi$ -nuclear potentials using a local density approximation, with the inclusion of the DD,  $DD^*$ , and  $D^*D^*$  meson loops in the  $J/\Psi$  self-energy. The nuclear density distributions, as well as the in-medium Dand  $D^*$  meson masses, are consistently obtained by employing the quark-meson coupling model. In the model, all the coupling constants between the applied meson mean fields and the light quarks in the nucleon, as well as those in the D and  $D^*$  mesons, are all equal and calibrated by the nuclear matter saturation properties. Using the  $J/\Psi$ -nuclear potentials obtained in this manner, we have solved the Klein-Gordon equation, which is reduced from the Proca equation, and obtained the  $J/\Psi$ -nuclear bound state energies. For this, we have been able to set the strong interaction width of the  $J/\Psi$  meson to be zero (and neglect its tiny natural width of  $\sim 93$  keV in free space). Combined with the generally advocated color-octet gluon-based attraction, or QCD color van der Waals forces, we expect that the  $J/\Psi$  meson will form nuclear bound states and that the signal for the formation should be experimentally very clear, provided that the  $J/\Psi$  meson is produced in recoilless kinematics.

## ACKNOWLEDGMENTS

The authors would like to thank S. Brodsky and A. Gal for helpful communications. KT would like to acknowledge IFT UNESP (Brazil), where a part of this work was carried out. The work of DHL was supported by the Science Foundation of Chinese University, while the work of GK was partially financed by CNPq and FAPESP (Brazilian agencies). This work was also supported by the University of Adelaide and by the Australian Research Council through the Australian Laureate (AWT) program.

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