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Relativistic quark-diquark model of baryons

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A relativistic quark-diquark mass operator with direct and exchange interaction has been constructed in the framework of point form dynamics. The nonstrange baryon spectrum has been calculated and compared with experimental data.

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I. INTRODUCTION

The notion of diquark is as old as the quark model itself. Gell-Mann [1] mentioned the possibility of diquarks in his original paper on quarks, and then soon afterward, Ida and Kobayashi [2] and Lichtenberg and Tassie [3] introduced effective degrees of freedom of diquarks in order to describe baryons as composed of a constituent diquark and a constituent quark. Since its introduction, many articles have been written on this subject (for a review see Ref. [4]) and, more recently, the diquark concept has been applied to various calculations [5–13]. It has long been known that any interaction that is strong enough to bind π and ρ mesons in the so-called rainbow-ladder approximation of QCD's Dyson-Schwinger equation (DSE) will also produce (non-point-like) diquarks [14]. Furthermore, indications for diquark confinement have also been provided [15]. This makes it sufficiently plausible to include diquarks as part of the baryon's wave function. Moreover, very important phenomenological indications for diquark-like correlations have been collected over the years, like some regularities in hadron spectroscopy, the Regge behavior of hadrons, the $\Delta I = \frac{1}{2}$ rule in weak nonleptonic decays [16], some regularities in parton distribution functions [17] and in spin-dependent structure functions [17], and in $\Lambda(1116)$ and $\Lambda(1520)$ fragmentation functions [5–7]. Recently, diquark effective degrees of freedom have proven to be useful also in the study of transversity problems and fragmentation functions [18].

The idea of this article is to construct a relativistic quark-diquark model "inspired" by the good properties of the recently constructed nonrelativistic interacting quark-diquark model [19], which has been used to correlate different phenomenological data. We will reformulate the nonrelativistic interacting quark-diquark model using the point form formalism [20]. This formalism has allowed the development of point-form three-quark CQM's for baryons, such as the chiral constituent quark model [21], which is the point form reformulation of Glozman and Riska's nonrelativistic chiral quark model [22], and the point-form relativistic hypercentral model [23], i.e., the point-form reformulation of the nonrelativistic hypercentral model [24], but up to now no one has tried to construct a point-form relativistic quark-diquark model. This is the subject of the present article. We will show that the nonstrange baryon spectrum is reproduced with a quality comparable to that of three-quark CQM's and, most important, that no missing state is present under 2 GeV.

II. NONRELATIVISTIC QUARK-DIQUARK STATES

We assume that baryons are composed of a constituent quark, q, and a constituent diquark, $q^2 = Q^2$. We consider the diquark as two correlated quarks with no internal spatial excitations (thus in S wave) or at least we hypothesize that their internal spatial excitations are higher in energy than the scale of masses of the resonances we are going to study (2 GeV). Their color-spin-flavor wave functions then must be antisymmetric.

Moreover, as we take only light nonstrange baryons into account, composed of (u, d) quarks, the internal group is restricted to the Wigner spin-isopin $SU_{si}(4)$. Using the conventional notation of denoting spin and isospin by their values and color by the dimension of the representation, the quark has spin $s_2 = \frac{1}{2}$, isospin $t_2 = \frac{1}{2}$, and $C_2 = \mathbf{3}$. Since the hadron must be colorless, the diquark must transform as $\bar{\mathbf{3}}$ under $SU_c(3)$ and therefore one can have only the symmetric $SU_{si}(4)$ representation $\mathbf{10}_{si}(S)$, containing $s_1 = 0, t_1 = 0$, and $s_1 = 1, t_1 = 1$, i.e., the scalar and axial-vector diquarks, respectively. Thus, if only nonstrange baryons are considered, one can use the isospin of the diquark, t_1, and quark, 1/2, and the total isospin, T, to get the following basis states in the spin-isospin space:

$$\left|s_1 = 0, t_1 = 0; \tfrac{1}{2}, \tfrac{1}{2}; S = \tfrac{1}{2}, T = \tfrac{1}{2}\right\rangle, \quad (1a)$$

$$\left|s_1 = 1, t_1 = 1; \tfrac{1}{2}, \tfrac{1}{2}; S = \tfrac{1}{2}, T = \tfrac{1}{2}\right\rangle, \quad (1b)$$

$$\left|s_1 = 1, t_1 = 1; \tfrac{1}{2}, \tfrac{1}{2}; S = \tfrac{1}{2}, T = \tfrac{3}{2}\right\rangle, \quad (1c)$$

$$\left|s_1 = 1, t_1 = 1; \tfrac{1}{2}, \tfrac{1}{2}; S = \tfrac{3}{2}, T = \tfrac{1}{2}\right\rangle, \quad (1d)$$

$$\left|s_1 = 1, t_1 = 1; \tfrac{1}{2}, \tfrac{1}{2}; S = \tfrac{3}{2}, T = \tfrac{3}{2}\right\rangle. \quad (1e)$$

III. RELATIVISTIC QUARK-DIQUARK MODEL

A. Two-particles velocity states

The general quark-diquark state, defined on the product space $H_1 \otimes H_2$ of the one-particle spin s_1 [here 0 (or 1)] and spin s_2 (here 1/2) positive energy representations $H_1 = L^2(R^3) \otimes S_1^0$ or $H_1 = L^2(R^3) \otimes S_1^1$ and $H_2 = L^2(R^3) \otimes S_2^{1/2}$ of the Poincaré Group, can be written as

$$|p_1, p_2, \sigma_1, \sigma_2\rangle, \quad (2)$$

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where p_1 and p_2 are the four-momenta of the diquark and the quark, respectively, while σ_1 and σ_2 are the z projections of their spins. The spin s_1 of the diquark is chosen to be only 0 or 1, since we have made the hypothesis that the internal excitation of the non-point-like diquark will not be important for the spectrum under 2 GeV, but in principle all the excited states of the diquark can be considered within the same formalism. The only change would be in an increased number of basis states.

We introduce the velocity states (see Ref. [20]) by applying a Lorentz boost $U_B(v)$ to the states of the previous equation in the quark-diquark rest frame

$$|v, \vec{k}_1, \vec{k}_2, \sigma_1, \sigma_2\rangle = U_{B(v)}|k_1, k_2, \sigma_1, \sigma_2\rangle_0, \tag{3}$$

where the suffix 0 means that the diquark and the quark three-momenta $\vec{k}_1$ and $\vec{k}_2$ satisfy the condition $\vec{k}_1 + \vec{k}_2 = 0$: $\vec{k}_1$ and $\vec{k}_2$ are called internal momenta. We choose $U_B(v)$ to be a canonical boost, obtaining that the transformed tetramomenta are $p_1 = B(v)k_1$ and $p_2 = B(v)k_2$. Instead of the internal momenta $\vec{k}_1$ and $\vec{k}_2$ we use the relative momentum $\vec{q}$, conjugate to the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$, thus considering the following velocity basis states:

$$|v, \vec{q}, \sigma_1, \sigma_2\rangle = U_{B(v)}|k_1, k_2, \sigma_1, \sigma_2\rangle_0. \tag{4}$$

The important point is that a Lorentz transformation applied to a velocity state acts as [20]

$$U_\Lambda|v, \vec{q}, \sigma_1, \sigma_2\rangle = \sum_{\sigma_1', \sigma_2'} |\Lambda v, R_w\vec{q}, \sigma'_1, \sigma'_2\rangle_0 D^{s_1}_{\sigma_1', \sigma_1}(R_w) D^{s_2}_{\sigma_2', \sigma_2}(R_w), \tag{5}$$

that means that all the Wigner rotations are the same and thus one can couple angular momenta and/or spins as in the nonrelativistic case [20]. In particular since $\vec{q}$ is defined as a linear combination of the internal momenta $\vec{k}_1$ and $\vec{k}_2$, it undergoes the same Wigner rotation.

Moreover, one can define states

$$|v, L, M_L, q, \sigma_1, \sigma_2\rangle = \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}|\vec{q}, \sigma_1, \sigma_2\rangle_0, \tag{6}$$

where L is the total orbital angular momentum. We couple the spins s_1 and s_2 to obtain the total spin S and their z components to $\sigma_1 + \sigma_2 = S_z$:

$$\left|v, L, M_L, q, S = \frac{1}{2}, M_S\right\rangle = \left\langle 0, \sigma_1, \frac{1}{2}, \sigma_2 \,\middle|\, \frac{1}{2}, S_z\right\rangle \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}|\vec{q}, \sigma_1, \sigma_2\rangle_0, \tag{7a}$$

$$\left|v, L, M_L, q, S = \frac{1}{2}, M_S\right\rangle = \left\langle 1, \sigma_1, \frac{1}{2}, \sigma_2 \,\middle|\, \frac{1}{2}, S_z\right\rangle \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}|\vec{q}, \sigma_1, \sigma_2\rangle_0, \tag{7b}$$

and

$$\left|v, L, M_L, q, S = \frac{3}{2}, M_S\right\rangle = \left\langle 1, \sigma_1, \frac{1}{2}, \sigma_2 \,\middle|\, \frac{3}{2}, S_z\right\rangle \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}|\vec{q}, \sigma_1, \sigma_2\rangle_0. \tag{7c}$$

Finally, we couple L and the total spin S together to the total angular momentum J and its z component, obtaining the following basis

$$\left|v, J, M, q, L, S = \frac{1}{2}, s_1 = 0, s_2 = \frac{1}{2}\right\rangle = \left\langle L, M_L, S = \frac{1}{2}, S_z \middle| J, M\right\rangle \left\langle s_1 = 0, \sigma_1, s_2 = \frac{1}{2}, \sigma_2 \middle| S = \frac{1}{2}, S_z\right\rangle \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}\,|\vec{q}, \sigma_1, \sigma_2\rangle_0\,, \tag{8a}$$

$$\left|v, J, M, q, L, S = \frac{1}{2}, s_1 = 1, s_2 = \frac{1}{2}\right\rangle = \left\langle L, M_L, S = \frac{1}{2}, S_z \middle| J, M\right\rangle \left\langle s_1 = 1, \sigma_1, s_2 = \frac{1}{2}, \sigma_2 \middle| S = \frac{1}{2}, S_z\right\rangle \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}|\vec{q}, \sigma_1, \sigma_2\rangle_0, \tag{8b}$$

$$\left|v, J, M, q, L, S = \frac{3}{2}, s_1 = 1, s_2 = \frac{1}{2}\right\rangle = \left\langle L, M_L, S = \frac{3}{2}, S_z \middle| J, M\right\rangle \left\langle s_1 = 1, \sigma_1, s_2 = \frac{1}{2}, \sigma_2 \middle| S = \frac{3}{2}, S_z\right\rangle \int d^2\hat{q}\; Y_{LM_L}(\hat{q}) U_{B(v)}|\vec{q}, \sigma_1, \sigma_2\rangle_0, \tag{8c}$$

where the repeated indices mean as usual a sum and the Clebsch-Gordan coefficients are indicated as usual with the brackets. The isospin part of the states is the same as in the description of Eq. (1).

B. The mass operator

We consider a quark-diquark system, where $\vec{r}$ is the relative coordinate between the two constituents and $\vec{q}$ is the conjugate momentum of $\vec{r}$. We have already said that we use the relative momentum $\vec{q}$ instead of the internal momenta $\vec{k}_1$ and $\vec{k}_2$, which undergo the same Wigner rotation as $\vec{q}$.

We propose a relativistic quark-diquark model based on the following baryon rest frame mass operator:

$$M = E_0 + \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + M_{\rm dir}(r) + M_{\rm cont}(r) + M_{\rm ex}(r), \tag{9}$$

where E_0 is a constant, $M_{\rm dir}(r)$ and $M_{\rm ex}(r)$, respectively, are the direct and the exchange diquark-quark interaction, m_1 and m_2 stand for the diquark and quark masses, where m_1 is either m_S or $m_{\rm AV}$ according if the mass operator acts on a scalar or an axial vector diquark [5,6,25–37], and $M_{\rm cont}(r)$ is a contact interaction.

The direct term is a Coulomb-like interaction with a cutoff plus a linear confinement term

$$M_{\rm dir}(r) = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r. \tag{10}$$

The importance of the Coulomb-like interaction was emphasized long ago by Lipkin [38]. A simple mechanism that generates a Coulomb-like interaction is one-gluon exchange.

One needs also an exchange interaction, as emphasized by Lichtenberg [39]. This is indeed the crucial ingredient of a quark-diquark description of baryons. We have

$$M_{\rm ex}(r) = (-1)^{l+1} e^{-\sigma r}[A_S(\vec{s}_1 \cdot \vec{s}_2) + A_I(\vec{t}_1 \cdot \vec{t}_2) + A_{SI}(\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)], \tag{11}$$

where $\vec{s}$ and $\vec{t}$ are the spin and the isospin operators.

Moreover, we consider a contact interaction similar to that introduced by Godfrey and Isgur [40]

$$M_{\rm cont} = \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon} \frac{\eta^3 D}{\pi^{3/2}} e^{-\eta^2 r^2}\, \delta_{L,0}\delta_{s_1,1} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon}, \tag{12}$$

where $E_i = \sqrt{q^2 + m_i^2}$ $(i = 1, 2)$, ϵ, η, and D are parameters of the model.

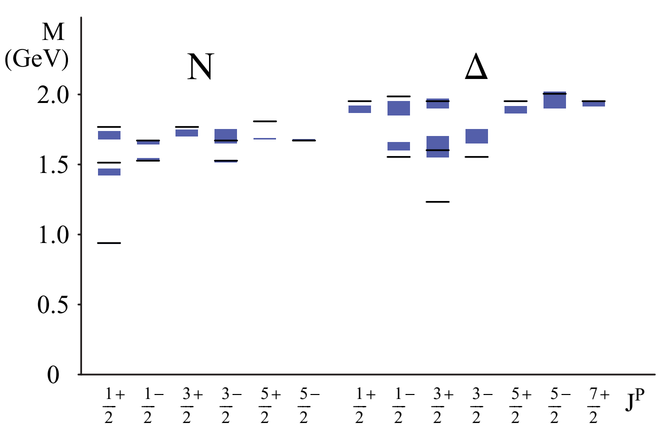

FIG. 1. (Color online) Comparison between the calculated masses (black lines) of the 3* and 4* nonstrange baryon resonances (up to 2 GeV) and the experimental masses from PDG [41] (boxes).

TABLE I. Resulting values for the model parameters.

$m_q = 200$ MeV	$m_S = 600$ MeV	$m_{\rm AV} = 950$ MeV
$\tau = 1.25$	$\mu = 75.0\ {\rm fm}^{-1}$	$\beta = 2.15\ {\rm fm}^{-2}$
$A_S = 375$ MeV	$A_I = 260$ MeV	$A_{SI} = 375$ MeV
$\sigma = 1.71\ {\rm fm}^{-1}$	$E_0 = 154$ MeV	$D = 4.66\ {\rm fm}^2$
$\eta = 10.0\ {\rm fm}^{-1}$	$\epsilon = 0.200$	

The Hamiltonian of the nonrelativistic model of Ref. [19] is

$$H = E_0 + \frac{q^2}{2\mu} - \frac{\tau}{r} + \beta r + [B + C\delta_0]\delta_{s_1,1} + (-1)^{l+1} 2A e^{-\alpha r}[(\vec{s}_1 \cdot \vec{s}_2) + (\vec{t}_1 \cdot \vec{t}_2) + (\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)], \tag{13}$$

TABLE II. Comparison between the experimental values [41] of the masses of the nonstrange baryon resonances (up to 2 GeV) and the numerical ones (all values are expressed in MeV). In the second column the "status" of each resonance is reported according to the classification given by PDG [41]. Tentative assignments of 2* and 1* resonances are shown in the second part of the table. J^P and L^P are respectively the total angular momentum and the orbital angular momentum of the baryon, including the parity P; S is the total spin, obtained coupling the spin of the diquark s_1 to the one of the quark; finally n_r is the number of nodes in the radial wave function.

Resonance	Status	$M^{\rm expt}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{\rm calc}$ (MeV)
$N(939)\ P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	0	939
$N(1440)\ P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	1	1513
$N(1520)\ D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1535)\ S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1650)\ S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1^-	$\frac{1}{2},\frac{3}{2}$	1	0	1671
$N(1675)\ D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1671
$N(1680)\ F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(1700)\ D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2},\frac{3}{2}$	1	0	1671
$N(1710)\ P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1768
$N(1720)\ P_{13}$	****	1700–1750	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1768
$\Delta(1232)\ P_{33}$	****	1231–1233	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1233
$\Delta(1600)\ P_{33}$	***	1550–1700	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1602
$\Delta(1620)\ S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1700)\ D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1900)\ S_{31}$	**	1850–1950	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986
$\Delta(1905)\ F_{35}$	****	1865–1915	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1910)\ P_{31}$	****	1870–1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1920)\ P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1930)\ D_{35}$	***	1900–2020	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	2005
$\Delta(1950)\ F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$N(2100)\ P_{11}$	*	1855–1915	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	2	1893
$N(2090)\ S_{11}$	*	1869–1987	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$N(1900)\ P_{13}$	**	1820–1974	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(2080)\ D_{13}$	**	1740–1940	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$\Delta(1750)\ P_{31}$	*	1708–1780	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1858
$\Delta(1940)\ D_{33}$	*	1947–2167	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986

TABLE III. Mass difference (in MeV) between scalar and axial-vector diquarks according to some previous studies.

m_S (MeV)	$m_{\rm AV} - m_S$ (MeV)	Source
–	210	Jaffe [5]
–	290	Wilczek [6]
–	360	Orginos [25]
688	202	Maris [26]
595	205	Lichtenberg *et al.* [27]
–	200÷300	Lichtenberg, Johnson [28]
420	520	Schäfer *et al.* [29]
770	140	de Castro *et al.* [30]
692	330	Cahill, Gunner [31]
750	100	Flambaum *et al.* [32]
590	210	
—	162	Babich *et al.* [33]
—	270	Eichmann *et al.* [34]
640	220	Hecht *et al.* [45]
730	210	Bloch *et al.* [46]
737	212	Burden *et al.* [37]

where δ_0, a short notation for $\delta_{n,0}\delta_{L,0}$, means that the contact term acts only on the ground state.

Comparing the mass operator of Eq. (9) to the nonrelativistic one of Eq. (13), one can note the disappearance of the terms $B\delta_{s_1,1}$ and $C\delta_0\delta_{s_1,1}$, where B and C are constants. The term $B\delta_{s_1,1}$, introduced in Ref. [19] to take the scalar-axial vector diquark mass difference into account, is here neglected because this mass difference results already automatically in the two different diquark masses m_S and $m_{\rm AV}$ in the kinetic energy operator, while the term $C\delta_0\delta_{s_1,1}$ is here replaced with the explicit contact interaction (12). The presence of the split between the two diquark configurations is a key ingredient of a quark-diquark model [19].

Moreover, we have introduced a cutoff in the Coulomb-like term to regularize it for short values of r, in order to avoid numerical problems arising in the relativistic case. Finally, it has to be noted that in the present work all the calculations are performed without any perturbative approximation.

We can thus state that the mass operator of Eq. (9) is not simply the relativistic extension of the interacting quark diquark model of Eq. (6) and Ref. [19], since at the end it results to be only inspired by the nonrelativistic previous model.

In point form dynamics Eq. (9) corresponds to a good mass operator since it commutes with the Lorentz generators and with the four velocity. We diagonalize (9) in the Hilbert space spanned by the velocity states. Since the mass operator and the velocity commute, the eigenstates of the mass operator are eigenstates also of the velocity operator.

IV. RESULTS AND DISCUSSION

Figure 1 and Table II show the comparison between the experimental data [41] and the results of our quark-diquark model calculation, obtained with the set of parameters of Table I. The overall quality in the reproduction of the experimental data (considering only 3* and 4* resonances) is comparable to that of other three quarks CQM's [22,24,42–44], but our model is not plagued with the problem of missing resonances (see Table II).

It has to be noted that this particular model does not predict missing states below the energy of 2 GeV, while the three quarks CQM's give rise to several missing states [41]. For example, Capstick and Isgur's model [42] has five missing states up to 2 GeV, the hypercentral CQM [24] has 8, the Glozman and Riska's model has 4 [22], and the U(7) model has 17 [44].

While the absolute values of the diquark masses are model dependent, their difference is not. Comparing our result for the mass difference $m_{\rm AV} - m_S$ between the axial vector and the scalar diquark to those of Table III, it is interesting to note that our estimation is comparable with the other ones. Such evaluations come from phenomenological observations [5,6, 27]; lattice QCD calculations [25,33]; instanton liquid model calculations [29]; applications of Dyson-Schwinger, Bethe-Salpeter, and Fadde'ev equations [26,31,32,34,37,45,46]; and constituent quark-diquark model calculations [28,30], even if in the latter it is not calculated the entire spectrum.

The whole mass operator of Eq. (9) has been diagonalized by means of a numerical variational procedure, based on harmonic oscillator trial wave functions. With a variational basis made of $N = 256$ harmonic oscillator shells the results converge very well, even if we have noticed that convergence is already satisfying for $N \approx 150$.

The present work can be expanded to include strange baryons or baryons with one or two heavy quarks and also to include an interaction term able to mix the various configurations with the same J^P but different spin-flavor content.

The application of our model to the description of strange baryons is straightforward and does not require a complete modification of the mass operator but just a change in the potential terms of Eq. (11), which should contain a flavor dependence instead of the isospin one.

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