Examination of quantized multiskyrmions as possible nuclear bound states of antikaons

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The spectrum of strange multibaryons (baryon number $B = 2$ and 3) is considered within the chiral soliton model using one of several possible SU(3) quantization models (the bound state rigid oscillator version). The states with energy below that of an antikaon and its corresponding nucleus can be interpreted as antikaon-nucleus bound states. In the formal limit of small kaon mass the number of such states becomes large, and for a real value of this mass there are at least several states with positive and negative parity in the energy gap of one kaon mass. For large values of binding energies the interpretation of such states as just antikaon-nuclear bound states becomes more ambiguous.

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I. INTRODUCTION

The studies of multibaryon states with different values of flavor quantum numbers are always of interest. They are closely related to the problem of the existence of strange quark matter and its fragments, strange stars (analogs of neutron stars). In addition to traditional approaches to this problem, based usually on the potential and/or quark models, the chiral SU(3) dynamics, mean-field theories, etc., the chiral soliton approach (CSA) proposed by Skyrme [1] is effective and has certain advantages over the conventional methods (some early descriptions of this model can be found in Ref. [2]). The quantization of the model performed first in the SU(2) configuration space for the baryon number one states [3], and somewhat later for configurations with an axial symmetry [4,5] and for multiskyrmions [6–8], allowed, in particular, to describe the properties of nucleons and the Δ isobar [3] and, more recently, some properties of light nuclei, including the so-called "symmetry energy" [9] (recently the neutron-rich isotope ^{18}B has been found to be unstable relative to the decay $^{18}\mathrm{B} \to {}^{17}\mathrm{B} + n$ [10], in agreement with prediction of the CSA [9]; this can be considered as an illustration of the fact that the CSA provides quite realistic predictions for the case of nonstrange nuclei) and many other properties [11].

The SU(3) quantization of the model has been performed within the rigid [12] or soft [13] rotator approach and also within the bound state model [14]. The binding energies of the ground states of light hypernuclei have been described within a version of the bound state chiral soliton model [15], in qualitative, and even semiquantitative, agreement with empirical data [16]. The collective motion contributions have been taken into account here (single-particle excitations should be added), and a special subtraction scheme has been used to remove uncertainties in absolute values of masses intrinsic to the CSA. It makes sense therefore to extend such an investigation to the higher in energy (excited) states, because some of them may be interpreted as antikaon-nuclei bound states.

Recently the antikaon-nuclei interactions and possible bound states of antikaons and nuclei have attracted much attention [17–27]. Theoretically, deeply bound states of antikaons in nuclei have been obtained as a solution of a many-body problem by Akaishi and Yamazaki [17,18]. Most recent reviews of this topic within the framework of conventional approaches can be found in Refs. [26] and [27]. Here we investigate the possibility of interpreting such states as quantized multiskyrmions (a configuration with baryon number 1 is usually called a skyrmion). The spectrum of quantized multiskyrmions is very rich, and some of these states are appropriate for interpretation as bound antikaon-nuclei states.

Within the CSA there is a simple argument that at small value of the kaon mass m_K there should be quantized states of multiskyrmions with the mass below the sum of masses of the kaon and the corresponding number of nucleons. Indeed, the strangeness (flavor) excitation energies at small m_K are proportional to m_K^2, both in the rotator [12,13] and in the bound state models of skyrmion quantization [14]. Therefore, the mass of any state with baryon number B, strangeness S, isospin I, and spin J can be presented as sum of two terms (see Sec. V below)

$$M(B, S, I, J, \dots) \simeq M(B, S = 0, \dots) + m_K^2 \Gamma_B\, C(B, S, I, J, \dots), \quad (1)$$

where Γ_B is the Σ term (see Table I), and $C(B, S, I, J, \dots)$ is some quantity of the order $\sim$1, depending on quantum numbers of the system. Evidently, at small enough m_K the contribution given by (1) is smaller than the sum $M(B, S = 0, \dots) + |S| m_K$, and the number of states with the mass given by (1) in the gap between $M(B, S = 0, \dots)$ and $M(B, S = 0, \dots) + |S| m_K$ becomes large. This argument is quite rigorous, however, for realistic value of m_K it is a question of calculating numerically to find out which states have the energy below that of the multibaryon plus the antikaon system (here we consider the case of strangeness $S = -1$).

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TABLE I. Characteristics of classical skyrmion configurations which enter the mass formulas for multibaryons. The numbers are taken from Refs. [32] and [33]: moments of inertia Θ and Σ-term Γ are in units GeV^{-1}, ω_S is in MeV, and μ_S is dimensionless (see the next sections for explanations). (In some formulas we add a lower index B for all quantities to emphasize the dependence on the baryon number, e.g., $\mu_{S,B}$.) Parameters of the model $F_\pi = 186$ MeV; $e = 4.12$ [16,32,33].

B	Θ_I	Θ_J	Θ_3	Θ_S	Γ	ω_S	μ_S
1	5.55	5.55	5.55	2.04	4.80	306	3.165
2	11.47	19.74	7.38	4.18	9.35	293	3.081
3	14.4	49.0	14.4	6.34	14.0	289	3.066
4	16.8	78.0	20.3	8.27	18.0	283	2.972

The interpretation of these states with fixed external quantum numbers in terms of hadronic constituents is not straightforward and not unique. Each state is the whole Fock column of hadronic components with different weights. We could only, in some particular situations, make statements about the dominance of some components of this Fock column.

It should be especially pointed out that here we are using one of the possible SU(3) quantization models—the rigid oscillator version of the bound state model [15] which seems to be the simplest one. This quantization scheme can provide quantized states with definite restrictions on allowed quantum numbers of the states, including their spatial parity, e.g., only positive parity baryons appear when the basic baryon number 1 hedgehog-type SU(2) configuration is quantized in this way. To get the states with negative parity, for example, the low-mass $\Lambda(1405)$ state, an actual candidate to be the antikaon-nucleon bound state, one should provide at least a second-order expansion in the mesonic fluctuations around the basic classical configuration (hedgehog). Considerable success in describing the properties of $\Lambda(1405)$ has been reached in this way in Ref. [28].

For the case of multiskyrmions a similar approach is technically very complicated and has not been performed, except for a few attempts [29,30]. In the SU(2) case the qualitative description of some dibaryon states was obtained in Ref. [30]. Therefore, the expected spectrum of the negative strangeness states may be considerably richer than that obtained in present paper.

In the next section the isotopical properties of the $\bar{K}NN$ and $\bar{K}NNN$ systems are briefly discussed. Section III contains a description of the starting positions of the CSA; in Sec. IV we recollect the spectrum of SU(2) quantized dibaryons. Section V contains the formulas summarizing the CSA results for strange (flavored) multiskyrmions, and our main results for the spectrum of strange baryonic states with $B = 2$ and 3 are presented in Secs. VI and VII. Our former results for strange dibaryons are recollected in Sec. VI. Excitations of the ground states of the $B = 2$ and 3 systems in some cases could be interpreted as antikaon-nuclei bound states.

II. PHENOMENOLOGY

The K^-pp cluster has been proposed in Ref. [18] as a fundamental unit which plays an important role in the formation of similar strangeness $S = -1$ clusters in heavier nuclei.

Here we discuss first some consequences of isotopic invariance of strong interactions involving strange particles. The state K^-pp, which has the third component of isospin $I_3 = 1/2$, is in fact a coherent combination of states with isospins $I = 3/2$ and $I = 1/2$:

$$|K^-pp\rangle = \sqrt{\frac{1}{3}}|\bar{K}NN;\, 3/2, +1/2\rangle + \sqrt{\frac{2}{3}}|\bar{K}NN;\, 1/2, +1/2\rangle. \tag{2}$$

Another physical state with the same quantum numbers is

$$|\bar{K}^0(pn)_{I=1}\rangle = \sqrt{\frac{2}{3}}|\bar{K}NN;\, 3/2, +1/2\rangle - \sqrt{\frac{1}{3}}|\bar{K}NN;\, 1/2, +1/2\rangle, \tag{3}$$

where the $(pn)_{I=1}$ system has isospin $I = 1$. So, the same cluster which can be seen in the K^-pp system should be seen also in the $\bar{K}^0(pn)$ system, but with approximately four times smaller probability (we take into account that the pn system has isospin $I = 1$ with probability 1/2). In the $\bar{K}NN$ state with isospin $I = 3/2$ includes the state with charge +2, it is $\bar{K}^0pp$, and the state with charge −1, it is K^-nn.

Another possibility to have the state with isospin $I = 1/2$ is to combine the antikaon state with the isospin zero $2N$ state:

$$|\bar{K}NN;\, 1/2, +1/2\rangle = |\bar{K}\rangle|(pn)_{I=0}\rangle. \tag{4}$$

In total, for the $\bar{K}NN$ system we have eight different components which can be split into a quartet (isospin $I = 3/2$) and two doublets. Within the CSA we shall obtain the states with baryon number 2 and the quantum numbers—strangeness, isospin, spin—as indicated above, and estimate their masses.

This is similar for the $B = 3$ systems. In the case of a $\bar{K}NNN$ system we have in total 16 components which can be separated into one quintet with a maximal isospin $I = 2$, three triplets with $I = 1$, and two singlets. The maximal value of the third component of isospin is $I_3 = +2$ ($\bar{K}^0ppp$ system), and the minimal value is $I_3 = -2$ (K^-nnn system). For an arbitrary nucleus with atomic number $A = B$ the isospin of any antikaon-nucleus state should evidently satisfy the inequality $I_{\bar{K}A} \leqslant (A+1)/2$. As it was shown previously and as we shall see here, within the CSA there is a specific dependence of the mass of baryonic system on its isospin—usually states with a higher isospin have greater energy.

III. BASIC INGREDIENTS AND FEATURES OF THE CSA

The CSA is based on a few principles and ingredients incorporated into the *truncated* effective chiral Lagrangian

[1–3],

$$L^{\rm eff} = -\frac{F_\pi^2}{16}{\rm Tr}\, l_\mu l_\mu + \frac{1}{32e^2}{\rm Tr}[l_\mu l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{8}{\rm Tr}(U + U^\dagger - 2) + \cdots, \quad (5)$$

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $U \in SU(3)$ is the unitary matrix depending on chiral fields, m_π is the pion mass, F_π is the pion decay constant known experimentally, and e is the only parameter of the model in its minimal variant proposed by Skyrme [1].

The mass term $\sim F_\pi^2 m_\pi^2$ changes the asymptotics of the profile f and the structure of the multiskyrmions at large B, in comparison with the massless case. For the SU(2) case,

$$U = \cos f + i(\vec{n}\vec{\tau}) \sin f, \quad (6)$$

the unit vector $\vec{n}$ depends on two functions, α, β. Three profiles $\{f, \alpha, \beta\}(x, y, z)$ parametrize the four-component unit vector on the three-sphere S^3.

The topological soliton (skyrmion) is a configuration of chiral fields, possessing topological charge identified with the baryon number B [1],

$$B = \frac{1}{2\pi^2}\int s_f^2 s_\alpha I\, [(f, \alpha, \beta)/(x, y, z)]\, d^3r, \quad (7)$$

where I is the Jacobian of the coordinate transformation, $s_f = \sin f$. So, the quantity B shows how many times the unit sphere S^3 is covered when integration over 3D space R^3 is made.

The chiral and flavor symmetry breaking term in the Lagrangian density depends on the kaon mass and decay constant m_K and F_K ($F_K/F_\pi \simeq 1.23$ from experimental data):

$$L^{\rm FSB} = \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{24} Tr(U + U^\dagger - 2)(1 - \sqrt{3}\lambda_8) - \frac{F_K^2 - F_\pi^2}{48} Tr(U l_\mu l_\mu + l_\mu l_\mu U^\dagger)(1 - \sqrt{3}\lambda_8). \quad (8)$$

This term defines the mass splittings between strange and nonstrange baryons (multibaryons), modifies some properties of the skyrmions, and is crucially important in our consideration.

As we have stressed in previous publications, the great advantage of the CSA is that multibaryon states—nuclei, hypernuclei, etc.—can be considered on equal footing with the $B = 1$ case. Masses, binding energies of classical configurations, the moments of inertia Θ_I, Θ_J, . . . the Σ term (we call it Γ), and some other characteristics of chiral solitons contain implicit information about the interaction between baryons. Minimization of the mass functional $M_{\rm cl}$ provides three profiles $\{f, \alpha, \beta\}(x, y, z)$ and allows to calculate the moments of inertia, etc.

IV. MASS FORMULA FOR MULTIBARYONS QUANTIZED IN SU(2)

In the SU(2) case, the rigid rotator model (RRM) used at first in Ref. [3] for the $B = 1$ case is most effective and successful. It allowed to describe successfully the properties of nucleons, the $\Delta(1232)$ isobar, as well as many properties of light nuclei [11], and also mass splittings of nuclear isotopes, including neutron-rich nuclides with atomic numbers up to $\sim$30 [9].

When the basic classical configuration possesses definite symmetry properties, the interference between iso- and usual space rotations becomes important. We consider here first an example of the axially symmetrical configuration which is believed to provide the absolute minimum of the classical static energy (mass) for the $B = 2$ case. The mass formula for the axially symmetric configuration has been obtained first for the nonstrange states in Ref. [4] and, in greater detail somewhat later, in Ref. [5]:

$$M(B, I, J, \kappa) = M_{\rm cl} + \frac{I(I+1)}{2\Theta_I} + \frac{J(J+1)}{2\Theta_J} + \frac{\kappa^2}{2}\left(\frac{1}{\Theta_{I,3}} - \frac{1}{\Theta_I} - \frac{4}{\Theta_J}\right), \quad (9)$$

where $\kappa = I_3^{\rm bf}$, $I^{\rm bf}$ and $J^{\rm bf}$ are the body-fixed isospin and spin of the system, and the relation takes place $J_3^{\rm bf} = -2I_3^{\rm bf}$ as a consequence of the generalized axial symmetry of the $B = 2$ classical configuration [see Eq. (10)].

This formula is in agreement with the known quantum mechanical formulas for the energy of the axially symmetrical rotator [31]. The classical characteristics of the lowest baryon numbers states, moments of inertia $\Theta_I, \Theta_J, \Theta_{I,3}$, which enter formula (9), as well as other quantities, necessary for calculating the spectrum of SU(3) quantized states, are given in Table I.

The rational map approximation [34] considerably simplifies the calculations of the various characteristics of classical multiskyrmions presented in Table I (explicit expressions for the quantities shown in this table can be found in Refs. [11] and [33]). To get the numbers presented in Table I we used the SU(2) configurations given by the rational map ansatz [34] as the starting configurations in the full 3D minimization algorithm [35], where the configuration is described by eight functions of three variables [the complete SU(3) case]. The final numbers differ from the initial numbers given by rational maps by a only a few percent, thus showing the good quality of the rational map approximation. The value of Θ_J in Table I for the baryon numbers $B = 3$ and 4 is the average one of the diagonal elements of the orbital inertia tensor.

Here in Tables II and III we present for completeness the result of the calculation of the dibaryon spectrum according to the above formula. Many of the states shown in Table II have been presented in Ref. [5], where parametrization of the model by Adkins *et al.* [3] has been used, which allowed to describe the absolute values of the masses of the nucleon and the $\Delta(1232)$ isobar. Unlike Ref. [5] and following the approach proposed later, we calculate the differences of the masses of the quantized states, since the absolute values of the masses depend on the poorly known Casimir energy of the states calculated approximately for the $B = 1$ case; see Refs. [36] and [37]. For our choice of the model parameters the mass differences presented in Tables II and III are somewhat smaller (by few tens of MeV) than the mass differences which can be extracted from the results of Ref. [5] obtained with the parameters of Ref. [3].

TABLE II. The quantum numbers, possible hadronic content, and the energy (in MeV) of positive parity dibaryon states above the singlet NN scattering state with $I = 1,\ J = 0$.

I	J	κ	Content	ΔE	ΔE (MeV)
1	0	0	$NN(^1S_0)$	0	0
0	1	0	$NN(^3S_1)$	$1/\Theta_J - 1/\Theta_I$	−36
*1	1	0	$NN\pi$?	$1/\Theta_J$	51
1	2	0	$NN(^1D_2); \Delta N(^5S_2)$	$3/\Theta_J$	153
0	3	0	$NN(^3D_3); \Delta\Delta(^7S_3)$	$6/\Theta_J - 1/\Theta_I$	219
2	1	0	$\Delta N(^3S_1); \Delta N(^3D_1)$	$1/\Theta_J + 2/\Theta_I$	225
3	0	0	$\Delta\Delta(^1S_0); NN\pi\pi$	$5/\Theta_I$	435
2	4	2	$\Delta N(^5D_4); NN\pi$	$2/\Theta_J + 1/\Theta_I + 2/\Theta_{I,3}$	462

As it was shown first in Ref. [5], the parity of such dibaryon states is

$$P_{\mathrm{ax},B=2} = (-1)^\kappa. \tag{10}$$

The deuteronlike state ($I = 0,\ J = 1$) has an energy that is 36 MeV lower than the NN scattering state ($I = 1,\ J = 0$) [4,5] (the measured value of the deuteron binding energy is $\epsilon_d \simeq 2.2$ MeV; within the CSA the deuteronlike state is lower in energy than the singlet NN scattering state because the orbital inertia Θ_J is considerably—by a factor 1.5—greater than the isotopic moment of inertia Θ_I; see Table I. This remarkable property takes place in all known variants of the CSA), so the value ∼40 MeV can be considered as the uncertainty of our predictions of the masses in the SU(2) case.

The coefficient after κ^2 in Eq. (9) is negative, therefore, the states with a maximal possible value of $|\kappa|$ at fixed I, J have the lowest energy, linearly dependent on I and J after cancellation of the quadratic terms,

$$E_{\mathrm{kin}} = \frac{I}{2\Theta_I} + \frac{J}{2\Theta_J} + \frac{I}{2\Theta_{I,3}}. \tag{11}$$

This formula is valid for negative parity states with $I = 1$, $J = 2$, $\kappa = \pm 1$, or, generally, $J = 2I$, $\kappa = \pm I$.

Some comment is necessary concerning the state with $I = J = 1$, $\kappa = 0$, which is forbidden by the Finkelstein-Rubinstein (FR) constraint and cannot decay into the NN pair due to the Pauli principle. Such a state, if it exists, is an example of an elementary particle with $B = 2$, which is different from an ordinary deuteron or a singlet scattering state consisting mainly of two nucleons [38]. Such states have been considered earlier in Refs. [39] and [40], where their masses were found to be higher—greater than 2120 MeV. An experimental situation with a possible observation of such states has been described in Ref. [41].

The energy of such a state, shown in Table II, does not include the possible difference of Casimir energies (or loop corrections) between the FR-allowed and FR-forbidden states. If this energy is large, this state, as well as the $I = J = 0$ state, should have an energy larger than shown in Table II.

Generally, for multiskyrmions the internal constituents—nucleons, first of all—are not identifiable immediately. Some guesses and analyses of quantum numbers are necessary for this purpose. A possible hadronic content of the dibaryon states is shown in Tables II and III. Evidently, states with a value of isospin $I \geqslant 2$ cannot be made of two nucleons only; additional pions are needed, or Δ isobars instead of some nucleons. For the same reason, the states with $I \geqslant 2$ cannot be observed in nucleon-nucleon interactions. The states with isospin 0 or 1 could appear as some enhancements in the corresponding partial wave of the NN scattering amplitude.

For a configuration with baryon number $B = 3$ the symmetry properties of the classical configuration important for quantization have been established first by Carson [6] and exploited recently in Ref. [11]. As a consequence of the symmetry properties of the classical $B = 3$ configuration, which has a characteristic tetrahedral shape, the equality between the body fixed spin and isospin takes place, $K = L$. The parity of the quantized states equals [6]

$$P = (-1)^{(K_3+L_3)/2} = (-1)^{M_3/2}. \tag{12}$$

The analysis and interpretation of the $B = 3$ states are more complicated than the $B = 2$ states, and only a few of them were considered in Refs. [6] and [11]. Only a negative parity state with $M_3 = \pm 2$ was found to be allowed for the case

TABLE III. The quantum numbers, possible hadronic content, and the energy of negative parity dibaryon states above the NN scattering state with $I = 1,\ J = 0$.

I	J	κ	Content	ΔE	ΔE (MeV)
1	2	±1	$NN(^3P_2)$	$1/\Theta_J - 1/2\Theta_I + 1/2\Theta_{I,3}$	75
1	3	±1	$NN(^3P_3, ^3F_3)$	$4/\Theta_J - 1/2\Theta_I + 1/2\Theta_{I,3}$	229
2	2	±1	$\Delta N(^3P_2); NN\pi$	$1/\Theta_J + 3/2\Theta_I + 1/2\Theta_{I,3}$	249
2	3	±1	$\Delta N(^5P_3); NN\pi$	$4/\Theta_J + 3/2\Theta_I + 1/2\Theta_{I,3}$	402
2	4	±1	$\Delta N(^3F_4); NN\pi$	$8/\Theta_J + 3/2\Theta_I + 1/2\Theta_{I,3}$	606

of grandspin $M = 3$ ($\vec{M} = \vec{K} + \vec{L}$) due to the symmetry properties of the basic classical configuration with $B = 3$ [6].

V. SPECTRUM OF MULTIBARYONS WITH STRANGENESS IN THE RIGID OSCILLATOR MODEL

The observed spectrum of the strange multibaryon states (hypernuclei) is obtained by means of the SU(3) quantization procedure and depends on the quantum numbers and characteristics of skyrmions presented in Table I.

Within the bound state model (BSM) [14–16], the antikaon is bound by the SU(2) skyrmion. The mass formula takes place,

$$M = M_{\rm cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{\rm HFS}, \qquad (13)$$

where strangeness and antistrangeness excitation energies are

$$\omega_S = N_c(\mu_S - 1)/8\Theta_S, \quad \omega_{\bar{S}} = N_c(\mu_S + 1)/8\Theta_S, \qquad (14)$$

$$\mu_S = \sqrt{1 + \bar{m}_K^2/M_0^2} \simeq 1 + \frac{\bar{m}_K^2}{2M_0^2},$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_S) \sim N_c^0, \qquad (15)$$

$$\bar{m}_K^2 = m_K^2 F_K^2/F_\pi^2, \quad \mu_S \sim N_c^0,$$

and N_c is the number of colors of the underlying Quantum Chromodynamics (QCD).

The hyperfine splitting correction depending on the hyperfine splitting constants c_S, $\bar{c}_S$, isospin, "strange isospin" I_S, and angular momentum J equals, in the case when interference between usual space and isospace rotations is negligible or not important,

$$\Delta M_{\rm HFS} = \frac{J(J+1)}{2\Theta_J} + \frac{c_S I_r(I_r+1) - (c_S - 1)I(I+1) + (\bar{c}_S - c_S)I_S(I_S+1)}{2\Theta_I}. \qquad (16)$$

The hyperfine splitting constants are equal:

$$c_S = 1 - \frac{\Theta_I}{2\Theta_S\mu_S}(\mu_S - 1), \quad \bar{c}_S = 1 - \frac{\Theta_I}{\Theta_S\mu_S^2}(\mu_S - 1), \qquad (17)$$

The strange isospin equals $I_S = 1/2$ for $S = \pm 1$, and for negative strangeness in most cases of interest $I_S = |S|/2$, which minimizes this correction (but generally it may be not so). We recall that the body-fixed isospin $\vec{I}^{\rm bf} = \vec{I}_r + \vec{I}_S$, where $\vec{I}_r$ is the isospin of the skyrmion without added antikaons. It is quite analogous to the so-called "right" isospin within the rotator quantization scheme. When $I_S = 0$, i.e., for nonstrange states, $I = I_r$, and this formula goes over into the SU(2) formula for multiskyrmions,

$$E_{\rm kin} = \frac{J(J+1)}{2\Theta_J} + \frac{I(I+1)}{2\Theta_I}, \qquad (18)$$

where we neglect the interference terms [11,33]. Correction $\Delta M_{\rm HFS} \sim 1/N_c$ is small at large N_c, and also for heavy flavors [14,33].

For the case of a classical state with a generalized axial symmetry ($B = 2$), an additional term appears,

$$\Delta E^{\rm axial} = \frac{\kappa^2}{2}\left[\frac{1}{\Theta_3} - \frac{1}{\Theta_I} - \frac{4}{\Theta_3}\right] = \kappa^2\delta(\Theta), \qquad (19)$$

which differs for states with different parities (different κ, see Tables II and II).

The mass splitting within SU(3) multiplets is important and convenient for us here since the unknown for the $B > 1$ soliton Casimir energy cancels in the mass splittings. For the difference of energies of states with strangeness S and with $S = 0$ which belong to multiplets with equal values of (p, q) numbers we obtain, using the above expressions for the constants c_S and $\bar{c}_S$,

$$\Delta E(p,q;I,J,S;I_r,J_0,0) = |S|\omega_S + \frac{\mu_S - 1}{4\mu_S\Theta_S}[I(I+1) - I_r(I_r+1)] + \frac{(\mu_S - 1)(\mu_S - 2)}{4\mu_S^2\Theta_S}I_S(I_S+1) + \frac{1}{2\Theta_J}[J(J+1) - J_0(J_0+1)] + \left(\kappa^2 - \kappa_0^2\right)\delta(\Theta), \qquad (20)$$

if the underlying classical configuration possesses an axial symmetry. For arbitrary strangeness $I_S \leqslant |S|/2$, and $J_0 = J$ if these states belong to the same SU(3) multiplet. The values of the quantities which enter the above formulas are shown in Table I. As mentioned in the Introduction, $\Delta E(p,q;I,J,S;I_r,J,0) \sim m_K^2$ at small m_K, since in this case $\mu_S - 1 \sim m_K^2$ and $\omega_S \sim m_K^2$.

VI. DIBARYONS WITH STRANGENESS

Strange dibaryons have attracted much attention beginning with the pioneering papers [39,40,42–44]. Recent discussion of this topic and important references can be found in Ref. [26]. Here we do not discuss the $S = -2$ H dibaryon [42], which is the SU(3) singlet and appears as the SO(3) soliton within the chiral soliton approach [45–47].

For completeness we present here the former results by Schwesinger *et al.* [48] for energies of different strange dibaryons within the soft rotator model with an SU(3) configuration mixing. As can be seen from Table IV, we did not predict in Ref. [48] the bound states of dibaryons; all states of the lowest energy shown in this table are above the corresponding two-baryon thresholds (for consistency we took the theoretical values of the baryon masses which do not coincide with the empirical values). These lowest states can be and should be interpreted as virtual states, or scattering states

TABLE IV. The energy above threshold ΔE in MeV for dibaryons with $J^P = 0^+$, and different values of strangeness S and isospin I. The SU(3) multiplet, which the main component of the dibaryon configuration belongs to, is indicated in the upper line (see Fig. 1). Calculations are made according to the soft rotator model [48].

Multiplet	$\{\overline{10}\}$	{27}	$\{\overline{10}\}$	{27}	{27}	{27}	{27}	{35}	{28}
S, I	$-1,\ 1/2$	$-1,\ 1/2$	$-2,\ 1$	$-2,\ 0$	$-3,\ 1/2$	$-3,\ 3/2$	$-4,\ 0$	$-5,\ 1/2$	$-6,\ 0$
State	ΛN	ΛN	ΞN	$\Lambda\Lambda$	$\Lambda\Xi$	$\Sigma\Xi$	$\Xi\Xi$	$\Xi\Omega$	$\Omega\Omega$
$\Delta E^{\rm SRM}$	30	70	100	110	140	90	150	40	30

similar to the (NN) $^1S_0$ scattering state, the so-called singlet deuteron. The presence of such states leads to the enhancement of the scattering cross section in the corresponding channel, as seen in the ΛN or $\Lambda\Lambda$ data; see, e.g., Ref. [26]. In view of considerable numerical uncertainty about these results, there still remains a chance that the dibaryons, nearest to the threshold, can be bound.

In a previous publication on this subject [49] we obtained bound dibaryons, but the poorly known Casimir energies of the order of N_c^0 [36,37] (discussed already in this paper in connection with nonstrange dibaryons) have not been taken into account in Ref. [49]. In fact, we should write for the baryons

$$M_1(p,q;Y,I) = M_1^{\rm class} + \Delta M_1(p,q;Y,I) + M_1^{\rm Cas} \tag{21}$$

and for the dibaryons (multibaryons in general case)

$$M_2(p,q;Y,I) = M_2^{\rm class} + \Delta M_2(p,q;Y,I) + M_2^{\rm Cas}. \tag{22}$$

$\Delta M(p,q;Y,I)$ is the quantum-number-dependent quantum correction, hypercharge $Y = B + S$, and $M^{\rm Cas} \sim N_c^0$ is the Casimir energy or loop correction. When we calculated the energy (mass) difference,

$$\begin{aligned}\Delta M =\ & M_1(p_1,q_1;Y_1,I_1) + M_1(p_2,q_2;Y_2,I_2)\\ & - M_2(p,q;Y,I) = 2M_1^{\rm class} - M_2^{\rm class}\\ & + \Delta M_1(p_1,q_1;Y_1,I_1) + \Delta M_1(p_2,q_2;Y_2,I_2)\\ & - \Delta M(p,q;Y,I) + 2M_1^{\rm Cas} - M_2^{\rm Cas},\end{aligned} \tag{23}$$

in Ref. [49] we ignored the term $2M_1^{\rm Cas} - M_2^{\rm Cas}$ and obtained strong binding due to the large contribution of $\Delta M_1(p_1,q_1;Y_1,I_1)$ and $\Delta M_1(p_2,q_2;Y_2,I_2)$. This very large binding seemed apparently unrealistic, and a reasonable way out of this situation appeared when it was recognized that the contributions of the order of N_c^0 due to poorly known loop corrections, or to the Casimir energy, make a large negative contribution both to M_1 [36,37] and, probably, to M_2. To obtain the NN singlet scattering state on the right place, we should have [48]

$$2M_1^{\rm Cas} - M_2^{\rm Cas} \simeq -820 \text{ MeV} \tag{24}$$

for the choice of parameters made in Ref. [48], and results shown in Table IV follow immediately. Up to now these contributions to the classical masses of the skyrmions were calculated very approximately only for the unit $(B = 1)$ skyrmion [36,37]. These contributions are negative $M_1^{\rm Cas} \sim -1$ GeV, i.e., they act in the right direction. For larger baryon numbers the Casimir energy has not been calculated yet, because it is a very nontrivial computational problem.

The prediction of the $S = -3$ dibaryons with $(J^P;\ I) = (1^+, 2^+; 1/2)$ below the $\Lambda\Xi$ threshold was made long ago by Goldman *et al.* [43] within a variant of the Massachusetts Institute of Technology (MIT) bag model. Recently a strong attraction was found in some two-baryon channels with strangeness $S = -3$ and -4, in the leading order of the chiral effective field theory, suggesting the possible existence of bound states [50]. In the $S = -1$ channel quasibound $\pi\Lambda N$ state with $J^P = 2^+$, $I = 3/2$ has been obtained in Ref. [25] by solving the nonrelativistic Faddeev equation. The latest studies of strange dibaryons within quark models are presented in Ref. [51] and references therein.

To get the spectrum of the strange dibaryons in our chiral soliton approach we should transform basic formula (16) for the quantum correction to the energy of multiskyrmions to

$$\begin{aligned}\Delta M =\ & |S|\omega_S + \frac{1}{2\Theta_I}[cI_r(I_r+1) + (1-c)I(I+1)\\ & + (\bar{c}-c)I_S(I_S+1)] + \frac{J(J+1)}{2\Theta_J}\\ & + \frac{\kappa^2}{2}\left(\frac{1}{\Theta_{I,3}} - \frac{1}{\Theta_I} - \frac{4}{\Theta_J}\right),\end{aligned} \tag{25}$$

where $B = 2$ in all quantities Θ, ω_S to be taken from Table I. I_r (the right isospin within the rigid rotator quantization scheme) is the isospin of the nonstrange state, $I_S \leqslant |S|/2$ is the isospin carried by the strange mesons, and the observed isospin $\vec{I} = \vec{I}_r + \vec{I}_S$. For $S = 0$ and $I = I_r$ we recover the

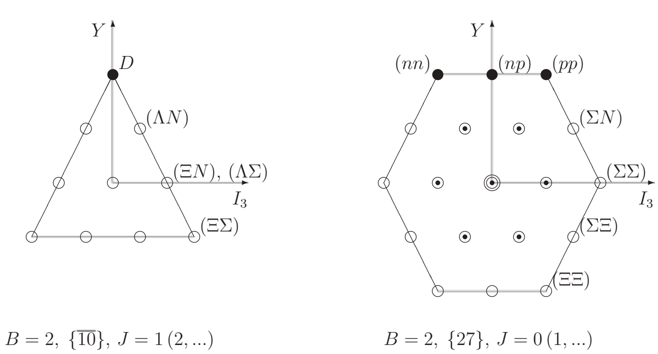

FIG. 1. I_3-Y diagrams of the lowest multiplets of dibaryons, $B = 2$. Virtual levels (scattering states) are shown in brackets, e.g., (ΛN) scattering state which appears as near threshold enhancement.

TABLE V. $B = 2$ states: Set of quantum numbers and the energy ΔE above the NN singlet scattering state for the $S = -1$ states with $I_r = 0$ and different values of spin, to be ascribed to the antidecuplet, $(p, q) = (0, 3)$, shown in Fig. 1(a).

I_r	J	I	S	κ	ΔE (MeV)
0	1	1/2	−1	0	289
0	2	1/2	−1	0	392
0	3	1/2	−1	0	546

above formula (9) for the quantum correction to the SU(2) quantized dibaryons.

For the difference of energies of states which belong to antidecuplet and singlet (NN) $^1S_0$ state we obtain

$$\begin{aligned} &E(0,3;I,J,S) - E(2N,{}^1S_0) \\ &= |S|\omega_S + \frac{\mu_{S,B}-1}{4\mu_{S,B}\Theta_{S,B}} \times I(I+1) \\ &+ \frac{(\mu_{S,B}-1)(\mu_{S,B}-2)}{4\mu_{S,B}^2\Theta_{S,B}} I_S(I_S+1) \\ &+ \frac{1}{2\Theta_J}J(J+1) - \frac{1}{\Theta_I} + \kappa^2\delta(\Theta), \end{aligned} \tag{26}$$

and in our case of $S = -1$ we should take $I_S = 1/2$. The only allowed possibility for κ is $\kappa = 0$, because $I_r = 0$. The numerical values of the dibaryon energies are given for several lowest states in Table V and Fig. 2.

The state with $J = I_r = 0,\ S = -1,\ I = 1/2$, not shown in Table V, has energy $\Delta E(0, 0, 1/2, -1) \simeq 238$ MeV, but this state cannot belong to the antidecuplet containing a deuteron with $J = 1$.

For dibaryon states which belong to {27}-plet we can use Eq. (20) with $I_r = 1,\ I_S = 1/2,\ J_0 = 0,\ \kappa_0 = 0$. Numerical results are shown in Table VI and Fig. 2. We would like to stress again that we are not fitting—here and previously—the absolute values of masses of nucleons, hyperons, and nuclei (opposite to what was done in Refs. [3] and [11]) because they are controlled by poorly known loop corrections or Casimir energy [see the discussion of Eqs. (21)–(24)].

TABLE VI. $B = 2$ states: Set of quantum numbers and the energy above the NN threshold for the $S = -1$ states with $I_r = 1$, which can be ascribed to the 27-plet, $(p, q) = (2, 2)$—see Fig. 1(b).

I_r	J	I	S	κ	ΔE (MeV)
1	0	1/2	−1	0	262
1	1	1/2	−1	0	313
1	1	3/2	−1	0	357
1	2	1/2	−1	0	416
1	2	1/2	−1	1	339
1	2	3/2	−1	1	434
1	3	1/2	−1	1	493

As we can see from Table VI, the state with isospin $I = 3/2$ has a greater mass than the state with $I = 1/2$ and the same other quantum numbers $(J = 1,\ S = -1,\ P = +1)$. At $J = 2$ the state with negative parity has a smaller mass than the state with positive parity. For $J^P = 2^+,\ I = 3/2$ the energy of the state is greater than the threshold energy of the $\pi\Lambda N$ system and close to the $\bar{K}NN$ threshold, so we do not obtain a quasibound $\pi\Lambda N$ state. However, this cannot be considered as a contradiction with Ref. [25], where such a quasibound state was obtained, because we used only one of the possible ways of the SU(3) quantization of the $B = 2$ skyrmion.

The following comment is necessary here. The CSA with our choice of the quantization scheme and model parameters $F_\pi,\ F_K,\ e$ overestimates systematically the strangeness excitation energies ω_S for all baryon numbers (Table I). For this

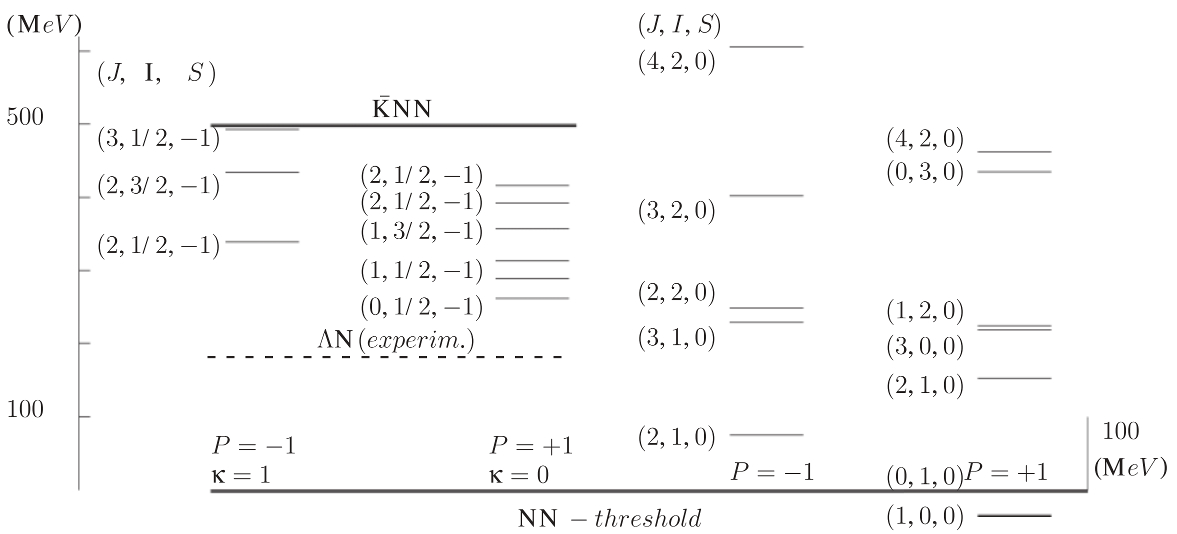

FIG. 2. Position of the $B = 2$ states above the NN threshold with negative strangeness, negative and positive parities (first two columns), and with zero strangeness, negative and positive parities (columns 3 and 4). The $\bar{K}NN$ threshold is shown by a black line, as well as the NN threshold. The dashed line indicates the ΛN threshold with an empirical value of M_Λ. The accuracy of the calculation is not better than ~40 MeV.

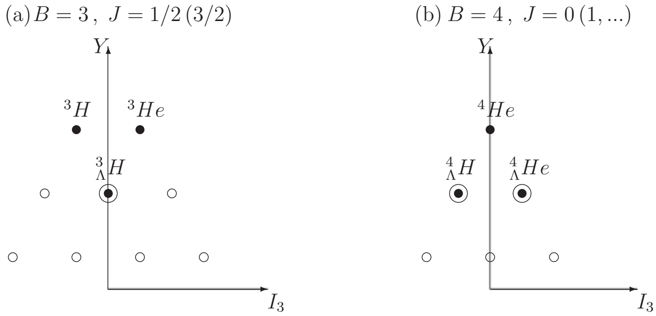

FIG. 3. (a) Location of the isoscalar ground state (shown by double circle) and isovector states with B number $B = 3$ and $S = -1$ in the upper part of the (I_3-Y) diagram. (b) The same for the isodoublet states with $B = 4$, $S = -1$ $^4_\Lambda$H, and $^4_\Lambda$He. Excited states with greater values of the right isospin I_r should belong to larger SU(3) multiplets. The lower parts of the diagrams with $Y \leqslant B - 3$ are not shown here.

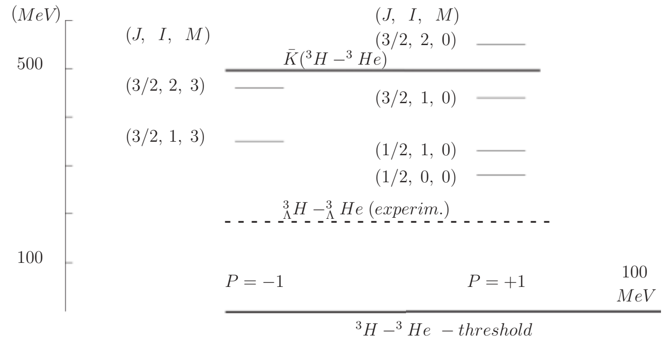

FIG. 4. Some of the $B = 3$ rotationally excited states (above the ground states of the ^{3}H-^{3}He doublet), strangeness $S = -1$, different isospin and spin, negative and positive parities, $I_r = J$.

reason, to estimate the binding energies of light hypernuclei in Ref. [16] we used the double subtraction scheme, where differences of the strangeness excitation energies enter into the final result for the differences of the binding energies of hypernuclei and ordinary nuclei—see Fig. 3, where the location of the lightest hypernuclei within the SU(3) multiplets is shown. More realistic (lower) values of ω_S will lead to an increase of the number of excited states in the gap of the one m_K presented in Fig. 2 as well as in Fig. 4 for the $B = 3$ state.

VII. SOME OF THE $B = 3$, $S = -1$ STATES

For the $B = 3$ system the expression for the difference of energies (masses) of state with strangeness S, isospin I, spin J, and the ground state with zero strangeness, isospin I_r is similar to (26)

$$\begin{aligned}
&\Delta E(p, q; I, J, S; I_r, J_0, 0)\\
&\simeq |S|\omega_S + \frac{\mu_{S,B} - 1}{4\mu_{S,B}\Theta_{S,B}}[I(I+1) - I_r(I_r+1)]\\
&\quad + \frac{(\mu_{S,B}-1)(\mu_{S,B}-2)}{4\mu_{S,B}^2\Theta_{S,B}} I_S(I_S+1) + \frac{1}{2\Theta'_J}\\
&\quad \times\left[J(J+1) - J_0(J_0+1) + M(M+1)\frac{\Theta_{\rm int}}{\Theta_I - \Theta_{\rm int}}\right],
\end{aligned} \tag{27}$$

TABLE VII. Some of the possible $B = 3$ states: Set of quantum numbers and the energy ΔE above the ^{3}H-^{3}He nuclei for states with $I_r = 1/2$ and 3/2, strangeness $S = -1$, and different values of spin, isospin, grandspin M, and parity $P = (-1)^{M_{3/2}}$ [6].

I_r	J	I	$M,\ P$	ΔE (MeV)
1/2	1/2	0	0, +	280
1/2	1/2	1	0, +	330
3/2	3/2	1	0, +	440
3/2	3/2	2	0, +	550
3/2	3/2	1	3, −	350
3/2	3/2	2	3, −	460

with $\Theta'_J = (\Theta_J\Theta_I - \Theta_{\rm int}^2)/(\Theta_I - \Theta_{\rm int})$, all quantities should be taken from Table I for $B = 3$, $\Theta_{\rm int} \simeq -9.4$ GeV^{-1}. $\vec{M} = \vec{K} + \vec{L}$ is the sum of the body-fixed spin and isospin of the system [6]. Therefore, the relation $0 \leqslant M \leqslant 2I_r$ takes place.

For the ground $B = 3$ state the SU(3) multiplet with $(p, q) = (1, 4)$, $I_r = J_0 = 1/2$ ($\overline{\{35\}}$-plet) is shown in Fig. 3(a). Figure 3(b) for even B numbers is included for illustration. The equality $J_0 = I_r$ follows from the symmetry properties of the $B = 3$ classical configuration, which has a tetrahedral form—see Ref. [6].

Our results for $S = -1$ excited tribaryons are presented in Table VII and Fig. 4. The lowest in energy state with $J = I_r = 1/2$, $I = M = 0$ can be naturally interpreted as a $^3_\Lambda$H hypernucleus (see the discussion in Sec. IV and at the end of the previous section). States with $J = I_r = 3/2$ should belong to other SU(3) multiplets.

These results should be considered as preliminary; further studies of this issue are desirable, also for greater baryon numbers $B \geqslant 4$.

The restriction on the allowed isospin of nonexotic states (i.e., the states without additional quark-antiquark pairs) takes place: $I \leqslant (3B + S)/2$, and for antikaon-nuclei bound states, evidently, $I \leqslant (B + 1)/2$. The second restriction becomes stronger for $B \geqslant 2$, so only states with not too large an isospin can be interpreted as antikaon-nuclei bound states. Generally, rotational excitations have an additional energy $\Delta E = J(J + 1)/2\Theta_J$. The orbital inertia grows fast with increasing baryon (atomic) number, $\Theta_J \sim B^p$, and p is between 1 and 2 [11,33]. For this reason the number of rotational states becomes greater with increasing baryon number.

VIII. SUMMARY AND CONCLUSIONS

To summarize, we have considered here rotational-type excitations of $S = -1$ baryonic systems (nuclei) with baryon number $B = 2$ and, partly, $B = 3$ using the chiral soliton approach. It was assumed that during the collective motion the shape of the basic classical configuration is not changed. We did not consider the vibration-breathing excitations which are possible as well. For the baryon number 1 it was possible to describe in this way some properties of the negative parity $\Lambda(1405)$ state [28]. For the case of dibaryons, some nonstrange states have been considered in Ref. [30], although numerical results have not been presented. Similar states should exist for strange multibaryons, but numerical computations are extremely complicated.

We investigated only one of several possible variants of multiskyrmion quantization in the SU(3) extension of the chiral soliton model, the rigid oscillator variant. The rich spectrum of strange multibaryons is predicted within this approach, with positive as well as with negative parities. There is a rigorous theoretical statement that at a small value of the kaon mass there should be quantized states with strangeness -1, with energy below the $NN\cdots + \bar{K}$ threshold.

The existence of strange excited nuclear states, which could be interpreted as bound antikaon-nuclear states within the CSA, seems to be quite natural and not unexpected. For a realistic value of the kaon mass some states are definitely predicted, but with considerable numerical uncertainty in their position (∼40 MeV). The deepest states are the most probable to be bound relative to the decay into the antikaon and the corresponding nuclear state. For $B = 2$ these are the states with $J^P = 2^-,\ I = 1/2$; $J^P = 0^+,\ I = 1/2$; $J^P = 1^+,\ I = 1/2$ and $I = 3/2$—see Fig. 2. For $B = 3$ these states have quantum numbers $J^P = 3/2^-,\ I = 1$ and $J^P = 1/2^+,\ I = 1$ (Fig. 4).

When the energy below the threshold becomes large, the interpretation of such states as the bound state of an antikaon and its corresponding nonstrange nucleus becomes less straightforward, due to the increase of the weight of the other components, first of all, containing hyperons. In view of theoretical uncertainties, experimental investigations could play an important role in determining the position of these states. Since several such states are expected in the energy gap equal to one kaon mass, a good enough experimental resolution in the energy (mass) of the observed states is of great importance. Another option can be that there are several wide overlapping states, and in this case better resolution in energy will not help much. The partial wave analysis of the decay product angular distributions would be useful in this situation.

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