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Final excitation energy of fission fragments

Karl-Heinz Schmidt* and Beatriz Jurado†

Centre d'Etudes Nucléaires de Bordeaux Gradignan (CENBG), CNRS/IN2P3, Chemin du Solarium, B.P. 120, F-33175 Gradignan, France

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We study how the excitation energy of the fully accelerated fission fragments is built up. It is stressed that only the intrinsic excitation energy available before scission can be exchanged between the fission fragments to achieve thermal equilibrium. This is in contradiction with most models used to calculate prompt neutron emission, where it is assumed that the total excitation energy of the final fragments is shared between the fragments by the condition of equal temperatures. We also study the intrinsic excitation-energy partition in statistical equilibrium for different level-density descriptions as a function of the total intrinsic excitation energy of the fissioning system. Excitation energies are found to be strongly enhanced in the heavy fragment, if the level density follows a constant-temperature behavior at low energies, e.g., in the composed Gilbert-Cameron description.

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Introduction. The final excitation energy found in the fission fragments, that is, the excitation energy of the fully accelerated fission fragments, and in particular its variation with the fragment mass, provides fundamental information on the fission process as it is influenced by the dynamical evolution of the fissioning system from saddle to scission and by the scission configuration, namely, the deformation of the nascent fragments. The final fission-fragment excitation energy determines the number of prompt neutrons and gamma rays emitted. Therefore, this quantity is also of great importance for applications in nuclear technology. To properly calculate the value of the final excitation energy and its partition between the fragments, one has to understand the mechanisms that lead to it. In particular, one has to take into account that the different contributions to the total excitation energy (TXE) of the fission fragments appear at different stages of the fission process.

The intrinsic excitation energy of the fissioning system acquired before scission $E^*_{\rm intr}$ may be exchanged between the nascent fragments. This process strongly influences the prompt neutron yield as a function of fragment mass. To investigate the sharing of $E^*_{\rm intr}$, one has to consider the level density of the emerging fission fragments before scission. For many years, there have been experimental indications that the nuclear level density at low excitation energies shows a constant-temperature behavior. This has already been realized by Gilbert and Cameron [1]. The constant-temperature behavior deviates from the Bethe formula [2], also named the Fermi-gas level density, which predicts a behavior of the level density as $e^{2\sqrt{aE^*}}$ in the independent-particle model (a is the level-density parameter). A suggestive solution of this apparent contradiction is the influence of residual interactions (e.g., pairing correlations), which explains the deviations from the independent-particle model at lower excitation energies. This idea led Gilbert and Cameron to propose their composed level-density formula, which describes the level density by a constant-temperature formula below and the back-shifted Fermi-gas formula (the Bethe formula with an energy shift) above a "matching energy" [3]. The value of the matching energy is determined by the assumption that there is a sudden transition from one description to the other and by the condition that the two descriptions coincide at the matching energy. Rather recently, new experimental results [4] suggest that the validity of the constant-temperature regime extends to appreciably higher excitation energies than the matching energy according to the composed Gilbert-Cameron description. Thus, the transition between the constant-temperature regime and the independent-particle regime is subject to current research. Still, the theoretical arguments for the validity of the independent-particle model at a higher excitation energy remain valid.

In a previous paper, we showed that thermodynamic processes lead to an energy sorting of the thermal excitation energy of the nascent fragments before scission in the constant-temperature regime [5]. The energy sorting is complete, i.e., all the intrinsic excitation energy is found in the heavy fragment, if the usual condition of equal temperatures of the nascent fragments is applied. In a more recent paper [6], we have discussed possible mechanisms of energy transfer between the fragments such as nucleon collisions at the neck or nucleon exchange through the neck. We showed that these transport phenomena are subject to substantial fluctuations. In this Rapid Communication, we investigate the sharing of excitation energy between the two nascent fragments in statistical equilibrium for different level-density descriptions, considering all available states of the nascent fragments. In particular, we study the energy partition at energies around the matching energy in the composed Gilbert-Cameron description. The relevance of this study is not only given by the fact that the Gilbert-Cameron description is used rather frequently for the modeling of nuclear properties and nuclear reactions for technical applications, but even more by the expectations that the transition from a constant-temperature-like behavior at low excitation energies to an independent-particle description at higher excitation energies is assumed to be a realistic feature of the nuclear level density. The essential conclusion of the present Rapid Communication is independent of the exact value of the transition energy, and the results may be adapted

*schmidt-erzhausen@t-online.de
†jurado@cenbg.in2p3.fr

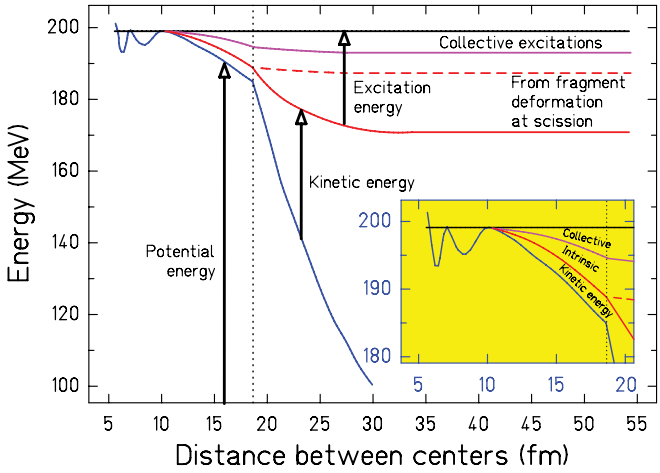

FIG. 1. (Color online) Schematic presentation of the different energies appearing in the fission process. The vertical dotted line indicates the scission point. The inset illustrates that the energy release due to the decreasing potential energy is partly dissipated into excitations of collective normal modes and intrinsic excitations. The remaining part feeds the pre-scission kinetic energy. The main figure demonstrates that the excitation energy of the fragments still increases right after scission, because the excess surface energy of the deformed fragments with respect to their ground states becomes available. Later also the collective excitations are damped into the intrinsic degrees of freedom. The figure represents the fission of ^{236}U with an initial excitation energy equal to the fission-barrier height.

easily to an extended validity range of the constant-temperature regime.

Contributions to the final fission-fragment excitation energy. The excitation energy of the fully accelerated fragments is composed of the following contributions: (i) the intrinsic excitation energy, (ii) the excitation energy stored in collective excitations, and (iii) the deformation energy (mostly due to the larger surface of the strongly deformed nascent fragments at scission) with respect to the ground state of the fragment. Figure 1 illustrates that these different contributions to the TXE appear at different stages of the fission process.

The intrinsic excitation energy $E^*_{\rm intr}$ at scission is given by

$$E^*_{\rm intr} = E^*_{\rm CN} - {\rm FB} + E_{\rm dis}, \qquad (1)$$

where $E^*_{\rm CN}$ is the initial energy of the fissioning nucleus. For example, in neutron-induced reactions it is given by the sum of the neutron binding energy and the incident neutron energy in the center-of-mass reference system. FB is the height of the fission barrier and $E_{\rm dis}$ is the amount of the energy released on the way from saddle to scission that is dissipated into intrinsic excitation energy. Let us present an example to give a quantitative idea of the amount of $E^*_{\rm intr}$. The measured prompt neutron yields shown in Fig. 1 of Ref. [5] [^{237}Np(n,f) with $E_n = 0.8$ and 5.5 MeV] correspond to an excitation energy of 0.1 MeV, respectively 4.8 MeV, above the fission barrier. According to Ref. [7], the energy release from saddle to scission in ^{238}Np is ∼17 MeV. Theoretical estimations given in Ref. [8] state that only a minor part of this energy is dissipated into intrinsic excitation energy at scission. Thus, the intrinsic excitation energies at scission, which are relevant for the discussion in Ref. [5], are expected to be ∼10 MeV, certainly well below 20 MeV. Of course, neutron energies above 5.5 MeV would lead to higher intrinsic excitation energies.

The intrinsic excitation energy $E^*_{\rm intr}$ that is present in the fissioning system before scission is divided between the fission fragments according to statistical equilibrium. The excitation energy stored in one of the collective normal modes (angular-momentum bearing and others) [9] is shared between the fragments according to the corresponding coordinated motions of the nascent fragments. It is dissipated into intrinsic excitations well after scission. The deformation of the fragments at scission is strongly favored by the mutual Coulomb repulsion of the nascent fragments that induces a considerable elongation of their shapes. An additional influence of shell effects is evidenced, e.g., by the low prompt neutron yields of the heavy fragment and the high total kinetic energies in partitions with heavy fragment masses close to $A = 132$. The energy stored in deformation is transformed into excitation energy of the fragments when they snap back to their ground-state deformation after separation. Therefore, the deformation and collective energies, which largely contribute to the TXE (see Fig. 1), are released well after scission and cannot be exchanged between the fragments. Indeed, there is no possibility to exchange nucleons after scission and practically no mechanism which allows for an exchange of energy between the fragments after scission (a small contribution due to Coulomb excitation in the field of the partner fragment may be neglected). Therefore, thermodynamical considerations on the fissioning system as a whole are not meaningful after scission.

Discussion on existing models. As has been shown above, only the intrinsic excitation energy available at scission can be shared between the nascent fragments, before the fragments separate. The total excitation energy of the separated fragments includes other contributions, e.g., from the larger deformations of the fragments in the scission configuration and from the damping of collective excitations after scission. Therefore, it is not appropriate to assume that the total excitation energy of the final fragments is shared between the fragments by the condition of statistical equilibrium. This is at variance with most of the existing models that have been developed for predicting prompt neutron emission from fission fragments [10–14]. Indeed, these models assume that the TXE is distributed among the fragments in a way such that the temperatures of the final fission fragments are equal. Since most often the Fermi-gas level density is used, where excitation energy and temperature are related via $E^* = aT^2$, this leads to

$$E^*_L/a_L = E^*_H/a_H, \qquad (2)$$

where a_L and a_H are the level-density parameters of the light (L) and heavy (H) fission fragments. E^*_L, E^*_H are the excitation energies of the fully accelerated fission fragments. That is,

$$E^*_L + E^*_H = Q - {\rm TKE} = {\rm TXE}, \qquad (3)$$

where Q is the Q value of the fission reaction and TKE is the total kinetic energy. Note that if the level-density parameter is assumed to be proportional to the nucleus mass, Eq. (2)

leads to an energy partition in proportion to the mass ratio of the fragments. Since neutrons evaporated from the fragments carry only small angular momentum on average [15], the rotational energy is mostly removed by $E2$ gamma emission. Therefore, for neutron emission the rotational energy must be subtracted from the TXE. This was done in Ref. [14], but it is often forgotten in other models. The temperature values often deduced from Eqs. (2) and (3) by the relation $T \propto \sqrt{\frac{E^*}{A}}$ are a measure of the excitation energies of the final fragments, just transformed into a parameter T with a relation valid for the Fermi gas with some level-density parameter a. These temperatures are certainly decisive for the energies of the evaporated neutrons. But neither the excitation energy of the separated fragments nor a somehow deduced temperature has any relevance for the exchange of intrinsic excitation energy between the nascent fragments before scission, discussed above. Thus, these temperature values should not be confounded with the temperature values, which govern the energy-sorting mechanism, discussed in Refs. [5] and [6].

In Refs. [11] and [14] it has been shown that a description following Eq. (2) does not reproduce experimental data on the average number of neutrons as a function of the fragment mass. To solve this problem, a fit parameter $R_T = T_L/T_H$ is introduced [13,14]; in Ref. [14] R_T is mass dependent. This parameter serves to somehow "simulate" the contribution of the deformation energy to the final excitation energy of the fission fragments. However, this is again not correct since the TXE contains other contributions that are not related to the fragment's deformation ($E^*_{\rm intr}$ and collective excitations). One does not need such a parameter, if the fission process is treated in the way described above: First the intrinsic excitation energy is statistically partitioned between the fragments, and then one calculates for each fragment the intrinsic excitation originating from the fragment's deformation and from the damping of collective modes. This is the procedure followed in the GEF code that calculates fission-fragment isotopic yields and neutron spectra from spontaneous fission to an excitation energy of ∼13 MeV for a wide range of heavy nuclei from polonium to fermium [16,17].

Partition of intrinsic excitation energy at scission: Dependence with the level-density description. The understanding of the nuclear level density is still incomplete and under debate. This is mainly because until very recently there has been a great lack of accurate experimental data. The data were mostly restricted to level counting at low excitation energies and the value deduced from neutron resonances at the neutron-separation energy. Therefore, the existing models use a variety of expressions for the level density. Also microscopic models seem to be unable to reproduce recent experimental data—see, e.g., Ref. [4]. Increasingly precise experiments on the level densities of nuclei with masses in the fission-fragment range (e.g., Ref. [18]) reveal a "constant-temperature" behavior where the logarithm of the level density increases in a good approximation as a linear function of excitation energy, at least in the energy range below the neutron-separation energy. In addition, the experimental work of Ref. [4] shows that the constant-temperature behavior persists up to excitation energies of 20 MeV. Assuming a constant nuclear temperature, independent of excitation energy, the configuration of the pre-scission dinuclear system with the largest entropy is characterized by a concentration of the intrinsic excitation energy in the fragment with the smallest logarithmic slope of the energy-dependent level density. Without regard to shell effects, this tends to be the larger of the two fragments. This is what we call the "energy-sorting process" [5]. However, the assumption of an energy-independent temperature is violated, since there are no states below the ground state. In a more realistic estimation, all states available in the two fragments (as given by realistic level densities) at a given total intrinsic excitation energy of the fissioning system should be considered. Thus, the mean excitation energy in, e.g., the light fragment $\langle E_L \rangle$, is obtained by the following condition of statistical equilibrium, which assumes that each available state of the fissioning system is populated with the same probability:

$$\langle E_L \rangle = \frac{\int_0^{E^*_{\rm intr}} E_L \rho_L(E_L) \rho_H(E^*_{\rm intr} - E_L) dE_L}{\int_0^{E^*_{\rm intr}} \rho_L(E_L) \rho_H(E^*_{\rm intr} - E_L) dE_L}, \qquad (4)$$

where ρ_L and ρ_H are the level densities of the light and heavy fragment, respectively. Of course, the sum of the average excitation energies in the light and heavy fragments must be equal to $E^*_{\rm intr}$. As we have shown in Ref. [6], the irregularities and structures in the measured level density are smoothed out by the averaging caused by the relatively large fluctuations involved in the process of energy transfer. In the example shown in Ref. [6], we demonstrate that the constant-temperature formula gives a very good description of the measured nuclear level density down to zero excitation energy. Therefore, we can apply Eq. (4) with the nuclear level densities following constant-temperature behavior. The result

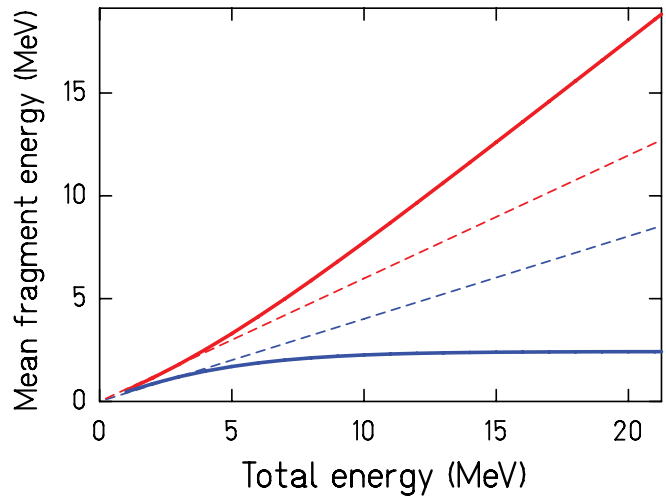

FIG. 2. (Color online) Mean excitation energies of the nuclei ^{94}Sr (upper lines) and ^{140}Xe (lower lines) in thermal contact as a function of the total intrinsic excitation energy. For the full lines, the level densities are described by the constant-temperature formula over the whole energy range above the ground state. The temperature parameters of the two level densities are obtained from the empirical parametrization of Ref. [19]. The dashed lines denote the partition according to the mass ratio.

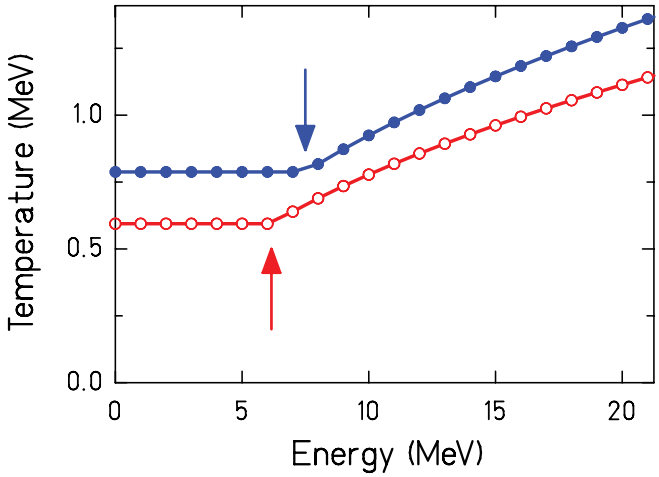

FIG. 3. (Color online) Nuclear-temperature values of ^{94}Sr (full circles) and ^{140}Xe (open circles) as a function of excitation energy according to the composite Gilbert-Cameron formula. The arrows denote the matching energies.

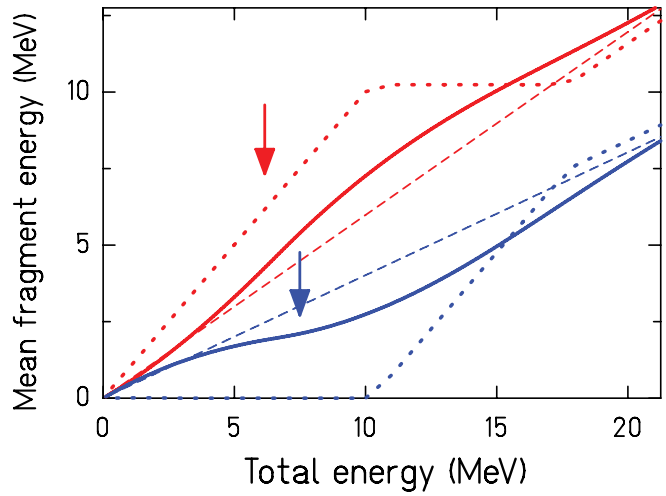

FIG. 4. (Color online) Mean excitation energies of the nuclei ^{94}Sr (upper lines) and ^{140}Xe (lower lines) in thermal contact as a function of the total intrinsic excitation energy. The dotted lines are derived from the condition of equal temperatures. The full lines result from the condition of statistical equilibrium, considering all states of the two nuclei. The level densities are described by the composite Gilbert-Cameron formula. The arrows denote the matching energies. The dashed lines denote the partition according to the mass ratio.

is shown in Fig. 2 for the two nuclei ^{94}Sr and ^{140}Xe, which are produced with high yields in the thermal-neutron-induced fission of ^{235}U. In contrast to the result based on the assumption of equilibration of temperatures, which leads to complete energy sorting [5], the excitation-energy division in statistical equilibrium obtained with Eq. (4) follows approximately the mass ratio up to a total intrinsic excitation energy of ∼4 MeV. This can be considered as a kind of border effect due to the absence of nuclear levels below the ground state. Thus, the integrals in Eq. (4) have to be truncated at zero excitation energy in either of the fragments. At higher total excitation energies, however, the excitation energy of the light fragment levels off and remains at ∼2 MeV. Practically all additional excitation energy ends up in the heavy fragment. Thus, excitation energies in excess of ∼4 MeV are subject to energy sorting. For the neutron yields of ^{237}Np discussed in Ref. [5], the intrinsic excitation energy to be partitioned is higher than 4 MeV for the two neutron energies. Therefore, energy sorting applies and explains in a transparent way why the increase of incident neutron energy leads to an increase of the number of neutrons emitted by the heavy fragment only.

The combined formula of Gilbert and Cameron [3] is very often used in nuclear-reaction calculations (see pp. 3148–3150 of Ref. [20]) due to its simplicity. Gilbert and Cameron established their composite formula on the basis of experimental indications for a constant-temperature behavior of the nuclear level density well below the neutron-separation energy. Above the particle-separation threshold, experimental data were practically absent, and the Fermi-gas model was assumed to be valid. This fact led to the creation of the composite Gilbert-Cameron formula consisting of the constant-temperature level density below the particle-separation threshold and the Fermi-gas level density above the particle-separation threshold. The normally used values of this transition energy are in contradiction with Ref. [4], where the constant-temperature behavior persists well above the neutron-separation energy. It is reasonable to expect a transition between the two regimes. However, it is still not clear up to which excitation energy the constant-temperature component of the Gilbert and Cameron formula is valid.

We will now investigate the partition of $E^*_{\rm intr}$ if the nuclear level density is described by the composite Gilbert-Cameron formula. The temperatures as a function of excitation energy are shown in Fig. 3. We used the global parameterization of the composed Gilbert-Cameron formula recommended in Ref. [20].

First, we apply the usual condition of equal temperatures. Figure 3 shows that the lowest temperature for which it is possible to have $T_1 = T_2$ is 0.8 MeV. For total excitation energies above 10 MeV, the excitation energies of the two fragments are given by the condition $T_1 = T_2$. For lower total excitation energies there is no solution with $T_1 = T_2$, and the process of thermal equilibration leads to complete energy sorting. The resulting excitation-energy division at thermal equilibrium is shown in Fig. 4. Complete energy sorting occurs up to ∼10 MeV. Even above the energy range of complete energy sorting, the excitation energy is not divided according to the ratio of the level-density parameters of the two nuclei [see Eq. (2)]. The heavier nucleus receives a much larger fraction. Only above 18 MeV, the division of excitation energy can be described without considering the constant-temperature behavior of the level density below the matching energy. Still, the division of excitation energy deviates from the ratio of the level-density parameters.

If we use expression (4) to determine the excitation-energy partition, the energy sorting is less pronounced. It appears appreciably washed out, but it is still clearly observed. The excitation energy of the heavy fragment is strongly enhanced with respect to the value corresponding to the ratio of the fragment masses. The enhancement is maximum at

approximately $E^*_{\text{intr}} = 10$ MeV, which exceeds the matching energies considerably.

Experimental results suggest that the validity range of the constant-temperature behavior of the level density extends to higher energies in the order of 20 MeV [4]. If the matching energies are shifted to higher values, the region of strong energy sorting will be enlarged. Thus, the excitation energies of the fragments would follow the behavior of the constant-temperature case shown in Fig. 2 up to higher energies.

Conclusions. We investigated how the final excitation energy in each fission fragment is built up from different contributions that arise at different steps of the fission process. Only the intrinsic excitation energy available at scission is shared between the fragments according to statistical equilibrium: The deformation and collective energies are dissipated into intrinsic excitation energy after scission, when the fragments are not in contact anymore. These two types of energies cannot be exchanged between the fragments. Therefore, the assumption of most fission models used for the prediction of prompt neutron emission that the total excitation energy TXE is shared between the fragments according to the condition of the equal final temperatures is not correct. We also investigated the intrinsic-excitation-energy partition for a specific mass split assuming two different level-density descriptions, disregarding shell effects: Energy sorting occurs for excitation energies in excess of ∼4 MeV if the level densities of the nascent fragments are characterized by a constant, energy-independent temperature. As a second case, we chose a combined description with a transition from a constant-temperature regime at low excitation energies to a Fermi-gas regime at higher excitation energies. In particular, we made a numerical calculation using the Gilbert-Cameron composite formula. This type of description leads to a strongly enhanced excitation energy in the heavy fragment at low energies, reaching well above the matching energies.

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