Two-phonon γ-vibrational states in rotating triaxial odd-A nuclei

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The distribution of the two-phonon γ-vibrational collectivity in the rotating triaxial odd-A nucleus ^{103}Nb, which is one of the three nuclides for which experimental data were reported recently, is calculated in the framework of the particle vibration coupling model based on the cranked shell model plus random phase approximation. This framework was previously utilized for analyses of the zero- and one-phonon bands in another mass region and is applied to the two-phonon band for the first time. In the present calculation, three sequences of two-phonon bands share collectivity almost equally at finite rotation whereas the $K = \Omega + 4$ state is the purest at zero rotation.

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I. INTRODUCTION

One of the properties most specific to the finite quantum many-body system, the atomic nucleus, is the exhibition of both single-particle and collective modes of excitation in similar energy scales.

Thanks to recent progress in computer power, the exact diagonalization of a given effective interaction in a huge model space is available for medium-mass nuclei including those far from stability. But in order to extract a physical picture of the dynamics from the huge amount of numerical data, it is necessary to have recourse to the concept of collectivity. Among the various kinds of collective modes of excitation, one that has been well studied and is common in the nuclear chart is the γ vibration. Vibrational excitations in many-fermion systems, that is, phonons, are made from the coherent superposition of particle-hole or two-quasiparticle excitations. Thus, on the one hand, multiple excitations are expected if they are really collective. On the other hand, their excitation spectra, in particular their anharmonicity, reflect the underlying nuclear structure.

On the nuclear chart, collective vibrations in rare earth nuclei with masses $A = 160$ to 170 are the best studied. The first experimental information of the two-phonon γ-vibration, denoted by 2γ hereafter, in ^{168}Er was reported by Davidson *et al.* [1] based on a high-resolution γ-ray study following neutron capture. The first theoretical analysis was carried out by Warner *et al.* [2] using the interacting boson model with *s* and *d* bosons; this work was critically assessed by Bohr and Mottelson [3]. They argued within the general framework applicable to deformed nuclei with rotational spectra [4]. Dumitrescu and Hamamoto [5] elucidated this problem by means of macroscopic and microscopic analyses. In contrast, Soloviev and Shirikova [6] argued that collective two-phonon excitations do not exist because of the strong Pauli principle among nucleons that constitute vibrational excitations based on the quasiparticle phonon model. Matsuo [7] and Matsuo and Matsuyanagi [8] applied the self-consistent collective coordinate method, which provides a quantized collective Hamiltonian, starting from the microscopic random phase approximation (RPA) and obtained the result that the $K = 4$ 2γ state in the triaxially deformed potential exists at an energy about 2.7 times that of the 1γ state. Piepenbring and Jammari [9] also obtained a similar result using the multiphonon method based on the Tamm-Dancoff approximation. The description using the interacting boson model was improved by Yoshinaga *et al.* [10] by introducing the *g* boson. The definitive experimental evidence, the absolute $B(E2)$ value between the $K = 4$ 2γ candidate and 1γ, which proves that the $K = 4$ state is really the 2γ, was given by means of γ-ray-induced Doppler broadening following neutron capture by Börner *et al.* [11] and using Coulomb excitation by Oshima *et al.* [12] and Härtlein *et al.* [13].

In Refs. [8,14], 2γ states of very similar character were predicted. These were observed in nearby nuclides: in ^{166}Er using Coulomb excitation by Fahlander *et al.* [15], using a $(n, n'\gamma)$ reaction by Garrett *et al.* [16], and in ^{164}Dy using thermal-neutron capture by Corminboeuf *et al.* [17]. In particular, the $K = 0$ 2γ state was also observed in ^{166}Er [16]. Sun *et al.* [18] studied the 2γ bands in ^{166,168}Er in terms of the triaxial projected shell model. This model gives the $K = 4$ 2γ states in a kinematical manner similar to Davydov-Filippov's asymmetric-rotor model [19]. In other mass regions, harmonic 2γ states were observed in ^{232}Th by Korten *et al.* [20,21] using Coulomb excitation and in ^{106,104}Mo by Guessous *et al.* [22,23] using spontaneous fission. The latter was discussed from the viewpoint of $X(5)$ symmetry [24]. The very limited number of observations indicates that whether or not the 2γ states exist depends strongly on the underlying single-particle level structure. This fact suggests that a microscopic description of the collective vibrational excitations is mandatory.

In odd-A nuclei, numerical predictions for rare earth nuclides were made by Durand and Piepenbring [25] in terms of the multiphonon method prior to experimental observation. The first experimental observation was made ten years later for ^{105}Mo by Ding *et al.* [26] using spontaneous fission. In these fission fragments, $^{104\text{–}106}$Mo, rotational band members built on the 2γ states were also populated. Soon after this, similar rotational bands were observed also for ^{103}Nb by Wang *et al.* [27] and for ^{107}Tc by Long *et al.* [28]. These nuclei exhibit anharmonicity: $E_{2\gamma}/E_{1\gamma} < 2$ in ^{105}Mo and ^{103}Nb, while $E_{2\gamma}/E_{1\gamma} \gtrsim 2$ in ^{107}Tc, where level energies are measured from the corresponding zero-phonon states. These

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are all the 2γ states that have been observed so far. The first theoretical calculation of an observed 2γ band in an odd-A nucleus, ^{103}Nb, was carried out by Sheikh *et al.* [29] using the triaxial projected shell model. It was discussed there that the observed anharmonicity is difficult to be reproduced with the triaxial parameter that gives a good description of the 1γ band.

In the present paper, we take a complementary approach to the 2γ band in ^{103}Nb; the particle vibration coupling (PVC) calculation based on the RPA phonon constructed in the rotating frame allowing static triaxial deformation.

II. THE MODEL

In the model adopted in this study, elementary modes are quasiparticles and γ-vibrational RPA phonons excited on top of the common vacuum as in traditional calculations; however, the vacuum is the yrast configuration of an even-even nucleus rotating with a frequency $\omega_{\rm rot}$. This vacuum configuration can be zero quasiparticle, two quasiparticle, and so on, as seen from the nonrotating ground state. Excitations are labeled by the signature quantum number $r = \exp(-i\pi\alpha)$, $I = \alpha + \text{even}$, appropriate for rotating reflection-symmetric objects. Two kinds of γ-vibrational excitations exist, denoted by $X^\dagger_{\gamma(\pm)}$ with $r = \pm 1$, respectively. The phonon space is limited to zero-, one-, and two-phonon states. The vacuum mean field is rotating and static triaxial deformation is also possible; this means that the Ω mixing in quasiparticles and the K mixing in RPA phonons are naturally taken into account. Here Ω is the projection of the single-particle angular momentum onto the third axis.

The formulation is summarized as follows. We begin with a one-body Hamiltonian in the rotating frame:

$$h' = h - \omega_{\rm rot} J_x, \tag{1}$$

$$h = h_{\rm Nil} - \Delta_\tau (P^\dagger_\tau + P_\tau) - \lambda_\tau N_\tau, \tag{2}$$

$$h_{\rm Nil} = \frac{\mathbf{p}^2}{2M} + \frac{1}{2} M\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right) + v_{ls}\mathbf{l}\cdot\mathbf{s} + v_{ll}\left(\mathbf{l}^2 - \langle \mathbf{l}^2\rangle_{N_{\rm osc}}\right). \tag{3}$$

In Eq. (2), P_τ is the pair annihilation operator, $\tau = 1$ and 2 denote a neutron or a proton, respectively, and the chemical potentials λ_τ are determined so as to give the correct average particle numbers $\langle N_\tau\rangle$. The oscillator frequencies in Eq. (3) are related to the quadrupole deformation parameters ϵ_2 and γ in the usual way. They, along with the pairing gaps Δ_τ, are determined from experimental information. The orbital angular momentum $\mathbf{l}$ in Eq. (3) is defined in the singly stretched coordinates $x'_k = \sqrt{\frac{\omega_k}{\omega_0}} x_k$ and the corresponding momenta, with $k = 1$, 2, and 3 denoting x, y, and z, respectively. We apply the RPA to the residual pairing plus the doubly stretched quadrupole-quadrupole ($Q''\cdot Q''$) interaction between quasiparticles. The interaction Hamiltonian is given by

$$H_{\rm int} = -\sum_{\tau=1,2} G_\tau \tilde{P}^\dagger_\tau \tilde{P}_\tau - \frac{1}{2}\sum_{K=0,1,2} \kappa^{(+)}_K Q''^{(+)\dagger}_K Q''^{(+)}_K - \frac{1}{2}\sum_{K=1,2} \kappa^{(-)}_K Q''^{(-)\dagger}_K Q''^{(-)}_K, \tag{4}$$

where the doubly stretched quadrupole operators are defined by

$$Q''_K = Q_K\left(x_k \to x''_k = \frac{\omega_k}{\omega_0} x_k\right), \tag{5}$$

and those with a good signature are

$$Q^{(\pm)}_K = \frac{1}{\sqrt{2(1+\delta_{K0})}}(Q_K \pm Q_{-K}), \tag{6}$$

and $\tilde{P}_\tau$ is defined by subtracting the vacuum expectation value from P_τ. Among RPA modes determined by the equation of motion,

$$[h' + H_{\rm int}, X^\dagger_n]_{\rm RPA} = \omega_n X^\dagger_n, \tag{7}$$

we choose the γ-vibrational phonons, $n = \gamma(\pm)$, which have outstandingly large $K = 2$ transition amplitudes:

$$|T^{(\pm)}_K| = |\langle [Q^{(\pm)}_K, X^\dagger_{\gamma(\pm)}]\rangle|. \tag{8}$$

The particle-vibration-coupling Hamiltonian takes the form

$$\begin{aligned} H_{\rm couple}(\gamma) = &\sum_{\mu\nu}\Lambda_{\gamma(+)}(\mu\nu)(X^\dagger_{\gamma(+)} a^\dagger_\mu a_\nu + X_{\gamma(+)} a^\dagger_\nu a_\mu) \\ &+ \sum_{\bar\mu\bar\nu}\Lambda_{\gamma(+)}(\bar\mu\bar\nu)(X^\dagger_{\gamma(+)} a^\dagger_{\bar\mu} a_{\bar\nu} + X_{\gamma(+)} a^\dagger_{\bar\nu} a_{\bar\mu}) \\ &+ \sum_{\mu\bar\nu}\Lambda_{\gamma(-)}(\mu\bar\nu)(X^\dagger_{\gamma(-)} a^\dagger_\mu a_{\bar\nu} + X_{\gamma(-)} a^\dagger_{\bar\nu} a_\mu) \\ &+ \sum_{\mu\bar\nu}\Lambda_{\gamma(-)}(\bar\nu\mu)(X^\dagger_{\gamma(-)} a^\dagger_{\bar\nu} a_\mu + X_{\gamma(-)} a^\dagger_\mu a_{\bar\nu}) \end{aligned} \tag{9}$$

where μ and $\bar\mu$ denote quasiparticles with $r = -i$ and $+i$, respectively. The coupling vertices are given by

$$\begin{aligned} \Lambda_{\gamma(+)}(\mu\nu) &= -\sum_{K=0,1,2} \kappa^{(+)}_K T''^{(+)}_K Q''^{(+)}_K(\mu\nu), \\ \Lambda_{\gamma(+)}(\bar\mu\bar\nu) &= -\sum_{K=0,1,2} \kappa^{(+)}_K T''^{(+)}_K Q''^{(+)}_K(\bar\mu\bar\nu), \\ \Lambda_{\gamma(-)}(\mu\bar\nu) &= -\sum_{K=1,2} \kappa^{(-)}_K T''^{(-)}_K Q''^{(-)}_K(\mu\bar\nu), \\ \Lambda_{\gamma(-)}(\bar\nu\mu) &= -\sum_{K=1,2} \kappa^{(-)}_K T''^{(-)}_K Q''^{(-)}_K(\bar\nu\mu), \end{aligned} \tag{10}$$

where $Q''^{(\pm)}_K(\alpha\beta)$ denote quasiparticle scattering matrix elements that do not contribute to RPA phonons. Eigenstates of the total Hamiltonian thus specified at each $\omega_{\rm rot}$ take the form

$$\begin{aligned}|\chi_j\rangle &= \sum_{\mu}\psi_j^{(1)}(\mu)a_\mu^\dagger|\phi\rangle + \sum_{\mu}\psi_j^{(3)}(\mu\gamma)a_\mu^\dagger X_\gamma^\dagger|\phi\rangle \\ &+ \sum_{\bar\mu}\psi_j^{(3)}(\bar\mu\bar\gamma)a_{\bar\mu}^\dagger X_{\bar\gamma}^\dagger|\phi\rangle + \sum_{\mu}\psi_j^{(5)}(\mu\gamma\gamma) \\ &\times \frac{1}{\sqrt{2}}a_\mu^\dagger X_\gamma^\dagger X_\gamma^\dagger|\phi\rangle + \sum_{\mu}\psi_j^{(5)}(\mu\bar\gamma\bar\gamma)\frac{1}{\sqrt{2}}a_\mu^\dagger X_{\bar\gamma}^\dagger X_{\bar\gamma}^\dagger|\phi\rangle \\ &+ \sum_{\bar\mu}\psi_j^{(5)}(\bar\mu\gamma\bar\gamma)a_{\bar\mu}^\dagger X_\gamma^\dagger X_{\bar\gamma}^\dagger|\phi\rangle, \end{aligned} \quad (11)$$

for the $r = -i$ sector, where γ and $\bar\gamma$ abbreviate $\gamma(+)$ and $\gamma(-)$, respectively, and $|\phi\rangle$ is the rotating-vacuum configuration. The eigenstates for the $r = +i$ sector take a form similar to the above, except that the suffixes μ are replaced by $\bar\mu$.

This model was first developed for studying the signature dependence of the level energies and the $E2$ and the $M1$ transition rates in one-quasiparticle (zero-phonon) bands [30,31], and then applied to study the $E2$ intensity relation [4,32] in the 1γ rotational bands [33]. The present study is the first application to the 2γ states in rotating odd-A nuclei. The construction of the model space means that this model is applicable up to the energy region $\lesssim E_{1\rm qp} + 2\Delta$; otherwise noncollective three-quasiparticle (3qp) states dominate over the collective states.

III. RESULTS AND DISCUSSION

Three new rotational bands that feed the $\pi[422\,5/2^+]$ ground band of ${}^{103}_{41}\mathrm{Nb}_{62}$ were observed by a very recent γ-ray study of spontaneous fission fragments from ^{252}Cf [27]. Among these, the one that is built on the $9/2^+$ bandhead was assigned to the $K = \Omega + 2$ sequence of the 1γ bands. One of the others, built on the $13/2^+$ bandhead, was assigned to the $K = \Omega + 4$ sequence of the 2γ bands. This is the second observation of the 2γ band in odd-A nuclei, and for which the first theoretical calculation [29] was reported. In the present cranked-shell-model calculation, diagonalization is performed for the five major shells, $N_{\rm osc} = 2$ to 6 for the neutron and 1 to 5 for the proton with the Nilsson parameters v_{ls} and v_{ll} taken from Ref. [34]. The three mean-field parameters, which are assumed to be $\omega_{\rm rot}$ independent for simplicity, the pairing gaps $\Delta_n = 1.05$ MeV and $\Delta_p = 0.85$ MeV, and the deformation $\epsilon_2 = 0.31$ are adopted from experimental analyses [22,27]. The triaxiality γ is chosen so as to reproduce the measured signature splitting of the ground band in the PVC calculation. The chosen value, $\gamma = -7^\circ$, gives the overall reproduction of the signature splitting in the rotating frame except near the bandhead, as shown in Fig. 1(a). Note here that it was reported that signature splitting can be reproduced without invoking γ deformation in the ancestral model [35] of the triaxial projected shell model. This suggests that the appropriate value of the γ deformation is model dependent, as noticed in Ref. [36] and discussed below.

The strengths of the residual doubly stretched quadrupole interaction are determined as follows: in the reference configuration with $\omega_{\rm rot} = 0$ and $\gamma = 0$ in which K is a good quantum number, $\kappa_2^{(+)} = \kappa_2^{(-)}$ is determined to reproduce within the RPA the observed γ vibrational energy, 0.812 MeV, in the adjacent ^{104}Mo. If a fully collective β vibration exists, $\kappa_0^{(+)}$ can be determined so as to reproduce its energy but since the collective character of the observed 0_2^+ is not clear, $\kappa_0^{(+)}$ is set equal to $\kappa_2^{(\pm)}$. And $\kappa_1^{(+)} = \kappa_1^{(-)}$ is determined so as to make the energy of the Nambu-Goldstone mode zero. The values for the residual pairing interaction are determined so as to reproduce the adopted pairing gaps. Then the RPA calculation is performed with $\gamma = -7^\circ$. The obtained $\omega_{\rm rot}$ dependence and signature splitting [4,37] of the excitation energy of the γ vibration are weak as shown in Fig. 1(b).

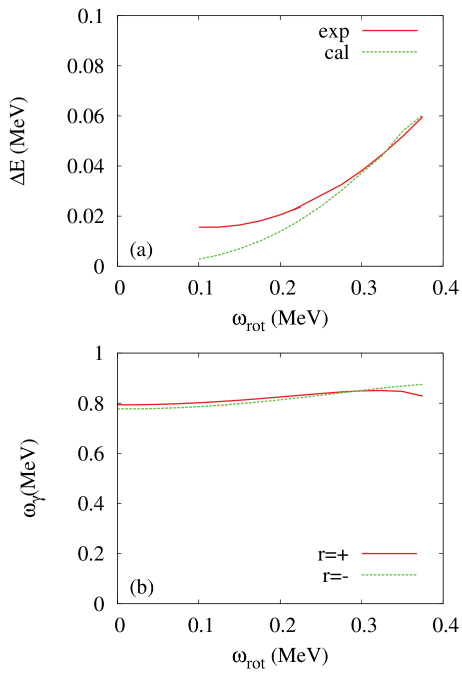

FIG. 1. (Color online) (a) Experimental and calculated signature splitting in the $\pi[422\,5/2^+]$ one-quasiparticle band as a function of the rotation frequency. The theoretical curve is the result of particle vibration coupling rather than the cranked shell model alone. The latter (not shown) is larger than the former. (b) Excitation energies of the two types of γ-vibrational RPA phonons as a function of the rotation frequency.

Using these quantities, $H_{\rm couple}(\gamma)$ is diagonalized in a space of dimension 15 (the number of quasiparticle states with $N_{\rm osc} = 4) \times 6$ [(1qp, 1qp$\otimes\gamma(+)$, $\overline{\rm 1qp}\otimes\gamma(-)$, 1qp$\otimes\gamma(+)\otimes\gamma(+)$, 1qp$\otimes\gamma(-)\otimes\gamma(-)$, and $\overline{\rm 1qp}\otimes\gamma(+)\otimes\gamma(-)$, with a bar denoting the opposite signature] for each signature sector. The distribution of the strength (the probability in the wave function) of the $\pi[422\,5/2^+]\otimes\gamma$ vibration(s), which shows the collectiveness of each eigenstate directly, is presented in Fig. 2 for the case of the favored signature $r = -i$. The result for the unfavored $r = +i$ is similar.

In the favored ($r = -i$) sector, two dominant 1γ eigenstates are obtained as a result of the interaction between the $f\otimes\gamma(+)$ and the $u\otimes\gamma(-)$ basis states; hereafter f and u denote the favored and unfavored 1qp states originating from $\pi[422\,5/2^+]$, respectively. There is no general rule for the correspondence between these two bands in the signature scheme and the $K = \Omega \pm 2$ bands in the K scheme. However, since states with a lower K have lower intrinsic energies than

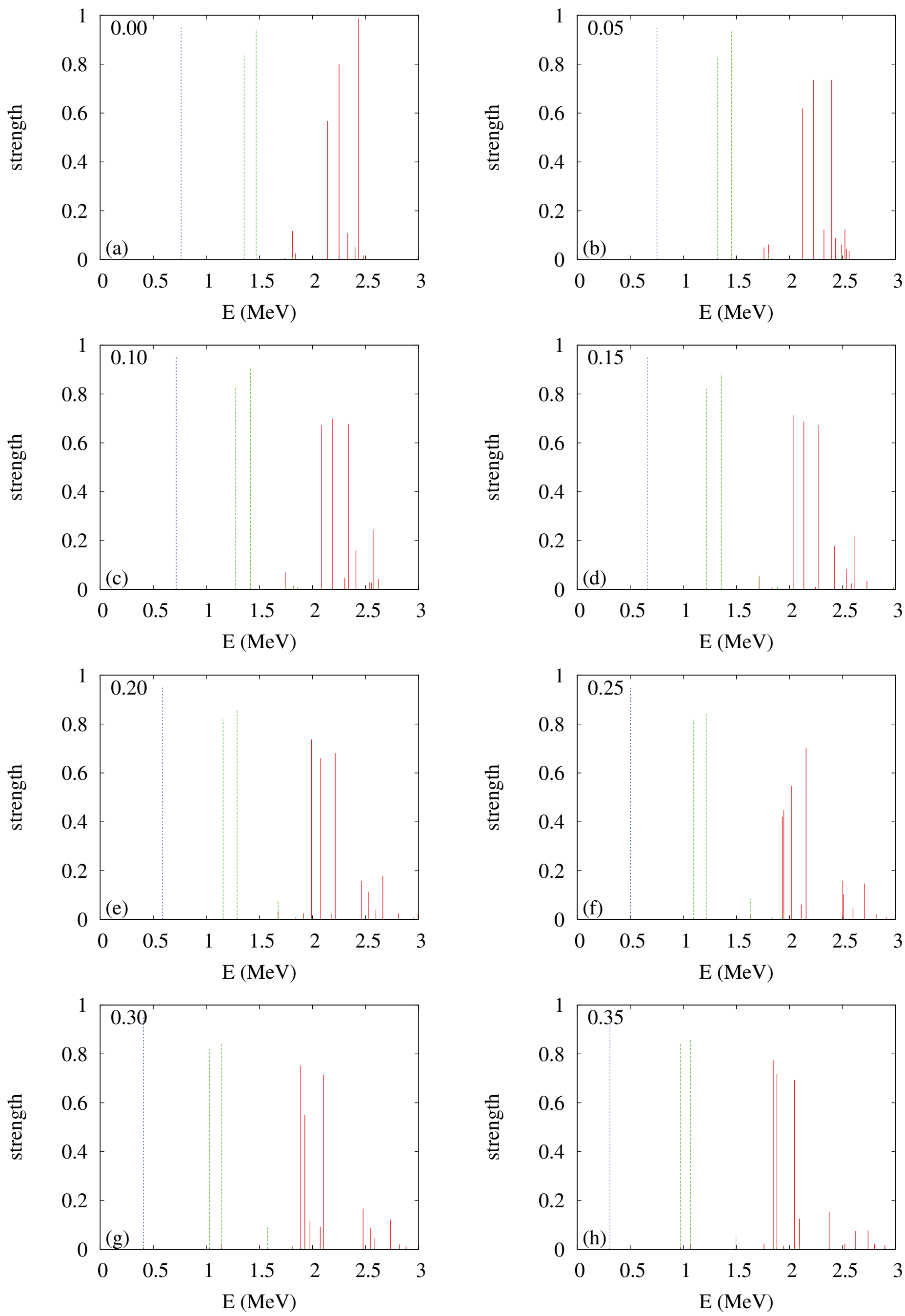

FIG. 2. (Color online) Distribution of the strength (the probability in the wave function) of the 1qp, 1qp$\otimes\gamma$, and 1qp$\otimes\gamma\otimes\gamma$ components in the favored signature ($r=-i$): $|\psi^{(1)}(\mu)|^2$ (blue dotted), $|\psi^{(3)}(\mu\gamma)|^2+|\psi^{(3)}(\bar{\mu}\bar{\gamma})|^2$ (green dashed), and $|\psi^{(5)}(\mu\gamma\gamma)|^2+|\psi^{(5)}(\mu\bar{\gamma}\bar{\gamma})|^2+|\psi^{(5)}(\bar{\mu}\gamma\bar{\gamma})|^2$ (red solid), respectively, at various rotation frequencies, $\omega_{\rm rot}=0$ to 0.35 MeV. Here 1qp (μ) means the one-quasiparticle state originating from $\pi[422\,5/2^+]$ at $\omega_{\rm rot}=0$.

those with higher K and the same I, the obtained lower band can be identified with the $K=\Omega-2$ band. Actually, in the previous analysis of the $E2$ intensity relation [33], a correspondence between the observed and the calculated states was established in this manner. The present result indicates that collectivity does not fragment much and the two bands are almost parallel as in the ^{165}Ho case [33]. But the higher band is slightly more collective and purer. This is consistent with the observation at $\omega_{\rm rot}=0$ that states with lower K are affected more by interactions with other states [25,38]. Actually only the $K=\Omega+2$ sequence was observed in many cases.

Fragmentation of the 2γ components is expected to depend sensitively on how other quasiparticle states distribute. In the present calculation, the 2γ strength concentrates mainly on three eigenstates that are located at $\lesssim E_{1{\rm qp}}+2\Delta_p$. They are obtained as a result of the interaction among the $f\otimes\gamma(+)\otimes\gamma(+)$, the $f\otimes\gamma(-)\otimes\gamma(-)$, and the $u\otimes\gamma(+)\otimes\gamma(-)$ basis states. With the same rule as for the 1γ case, they can be identified with $K=|\Omega-4|$, Ω, and $\Omega+4$ from the lowest. At $\omega_{\rm rot}=0$ the higher K states are more collective as expected. But a small rotation immediately mixes their collectivity. Then, the three collective bands maintain almost the same collectivity.

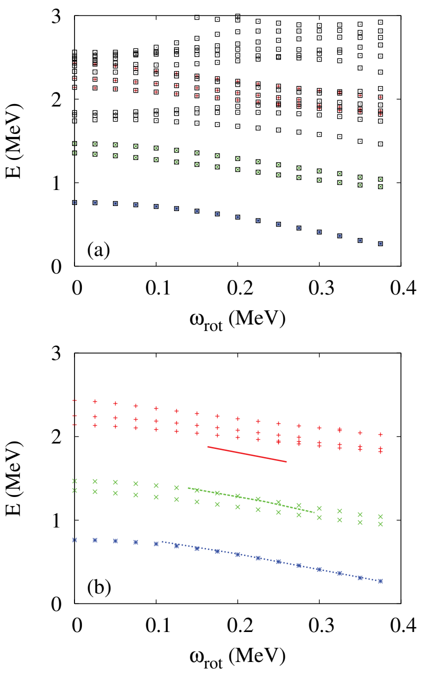

FIG. 3. (Color online) (a) Calculated eigenstates of $H_{\rm couple}(\gamma)$ at each rotation frequency. Corresponding to Fig. 2, those with large (> 40%) $\pi[422\,5/2^+] \otimes 0, 1, 2\gamma$ components are marked by symbols. At $\omega_{\rm rot} = 0.25$, 0.275, and 0.325 MeV, accidental fragmentation caused by interaction with a positively sloped 1qp state occurs. (b) Similar to (a) but the dominant $\pi[422\,5/2^+] \otimes 0\gamma$, 1γ, and 2γ states are compared with experimental data (curves) converted to the rotating frame by using the Harris parameters $\mathcal{J}_0 = 15.45$ MeV^{-1} and $\mathcal{J}_1 = 81.23$ MeV^{-3} that fit the yrast band of ^{104}Mo [23].

In order to see how these collective states interact with other noncollective states, all the eigenstates of $H_{\rm couple}(\gamma)$ that are located lower than 3 MeV are shown in Fig. 3(a). At $\omega_{\rm rot} = 0$, there are the second to fourth 1qp states with $N_{\rm osc} = 4$ between the 1γ states and 2γ states. These are expected to correspond to some of the levels that were known to be populated by the ^{103}Zr β decay but spin and parity have not been assigned [39]. One of them interacts with the 2γ states depending on $\omega_{\rm rot}$. Aside from this, the 2γ states keep their collective characters as shown in Fig. 2.

Next, the obtained collective states are compared with the observed ones in Fig. 3(b). This figure shows that the 1qp and the 1γ ($K = \Omega + 2$) states are reproduced almost perfectly. On the other hand, the calculated 2γ states are located obviously higher than the observed ones. The most probable reason for this is that the calculation ignores higher lying states, which push down the 2γ states. Although one may be afraid that inclusion of more states would lead to the fragmentation of collectivity, we expect this not to occur since the number of nearby states is still small.

Next we discuss the adopted triaxial deformation γ of Ref. [29] and the present calculation. It has two physical aspects: sign and absolute value. The former was originally considered in the rotating-mean-field model. Although the projection model in its original framework does not distinguish the sign of γ to our knowledge, in Ref. [36] a method of finding the main rotation axis was proposed. This made it possible to relate the result of the projection calculation to the sign of γ. In Ref. [29], the adopted $\epsilon = 0.3$ and $\epsilon' = 0.16$ correspond to $\gamma = \pm 28^\circ$. Although its sign is not described, odd-A nuclei with an $\Omega = 5/2$ odd nucleon show a $\gamma < 0$ rotation due to its shape-driving effect in addition to the approximately irrotational property of the even-even core in general. We regard the sign of Ref. [29] as consistent with ours. As for the absolute value, the origin of the difference is the generating mechanism of the γ band itself, as pointed out in Ref. [18]. It is constructed as a vibrational phonon excitation built on top of the yrast configuration in our model. In contrast, in the projection model, the γ band is generated as a rotational excitation as in an asymmetric rotor. Consequently, the appropriate γ deformation is not necessarily the same. The latter calls for a larger $|\gamma|$; for example, $|\gamma| = 25^\circ$ was adopted for ^{168}Er [18], which is regarded as a typical axially symmetric nucleus [4]. Note here that large fluctuations in the γ direction were shown for nuclei of that class, for example, in Ref. [40]. These results indicate that the model dependence of the adopted value of the γ deformation is not unphysical. Further study of the longstanding problem of how the vibrational and rotational descriptions are related would be interesting but is beyond the scope of the present paper.

Finally, we mention another type of collective motion where the two-phonon excitation was observed: the wobbling motion predicted by Bohr and Mottelson [4], Janssen and Mikhailov [41], and Marshalek [42] and first observed by Ødegård *et al.* [43]. This is a small-amplitude fluctuation of the rotation axis of triaxially deformed nuclei and has the same quantum number, $r = -1$, as the odd-spin members of the γ-vibrational bands. Two-phonon wobbling bands were observed in ^{163}Lu by Jensen *et al.* [44] and in ^{165}Lu by Schönwaßer *et al.* [45]. The excitation energies in the rotating frame exhibit anharmonicity such that the two-phonon states are located lower than twice the energies of the one-phonon states. The present author and Ohtsubo [46] argued that this anharmonicity indicates the softening of the collective potential surface, that is, the precursor of the second-order phase transition from the principal-axis rotation to the tilted-axis rotation. This is completely in accord with the situation where the softening of the γ vibration is the precursor of the second-order phase transition from the axial symmetric deformation to the static triaxial deformation. The new vacuum after the phase transition can accommodate anharmonic vibration either as $E_{2\,\rm phonon}/E_{1\,\rm phonon} > 2$ or < 2. Compare the potential energy surfaces in Fig. 3 in the first item of Ref. [8] and Fig. 3 in Ref. [46]; in general, stiff potentials, such as in the former, lead to high $E_{2\,\rm phonon}/E_{1\,\rm phonon}$ ratios while soft ones, such as in the latter, lead to low ratios.

To summarize, two-phonon γ-vibrational bands in rotating odd-A nuclei were observed recently in three nuclides [26–28]. After these observations, the first theoretical calculation using the triaxial projected shell model for one of them was reported [29]. In the present paper, we have applied to the same nuclide a completely different, mean-field-based model, which was previously developed for the signature-dependent properties of one-quasiparticle (zero-phonon) bands and later utilized for the $E2$ intensity relations in one-phonon bands. In the present calculation, after confirming the almost perfect reproduction of the zero- and one-phonon states, we concentrated on the effect of the odd quasiparticle on the distribution of the two-phonon collectivity. The obtained $K = \Omega + 4$ state is almost pure at $\omega_{\rm rot} = 0$, but a small rotation immediately delivers the strength to two other sequences; consequently, three collective sequences keep about 60% collectivity up to higher spins. This indicates future experimental observation is expected for more than one sequence of the 2γ bands as well as the other sequence of the 1γ. The restriction of the model space mentioned above is the reason for the result that the calculated 2γ states are located higher than observed.

There are some possibilities for improving the present calculation: First of all, the model space should be enlarged to more than two-phonon states. Further, extension of the microscopic approach by means of the self-consistent collective coordinate method [8] to rotating even- and odd-A systems is promising for the future.

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