# **Superdeformed oblate superheavy nuclei**

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We study stability of superdeformed oblate (SDO) superheavy  $Z \geq 120$  nuclei predicted by systematic microscopic-macroscopic calculations in 12D deformation space and confirmed by the Hartree-Fock calculations with the SLy6 force. We include into consideration high-*K* isomers that very likely form at the SDO shape. Although half-lives  $T_{1/2} \lesssim 10^{-5}$  s are calculated or estimated for even-even spin-zero systems, decay hindrances known for high-*K* isomers suggest that some SDO superheavy nuclei may be detectable by the present experimental technique.

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# **I. INTRODUCTION**

The question of what is the largest possible atomic number *Z*max of an atomic nucleus is still unsettled. The recent experiments on heavy-ion fusion in Dubna claim  $Z_{\text{max}} \geq 118$ [\[1\]](#page-4-0), with a partial confirmation of hot-fusion cross sections coming from GSI [\[2\]](#page-4-0) and LBL Berkeley [\[3\]](#page-4-0). Predictions on the stability of superheavy nuclei are based either on the Hartree-Fock plus BCS (HFBCS) or HFB studies with some effective interaction chosen out of the existing set or on the more phenomenological microscopic-macroscopic method, more tested in the extensive multidimensional fission barrier studies. Although these models differ quantitatively, they consistently predict prolate deformed superheavy nuclei with  $Z = 100-112$ , which is confirmed experimentally for nuclei around  $254$ No [\[4\]](#page-5-0) and spherical or oblate deformed systems with  $Z \ge 114$  and  $N = 174-184$  (see, e.g., Refs. [\[5,6\]](#page-5-0)). In the present work we show that microscopic-macroscopic calculations predict superdeformed oblate (SDO) nuclei, with characteristic quadrupole deformations  $-0.4 \lesssim \beta_{20} \lesssim -0.5$ (spheroids with the axis ratio  $\approx$ 3:2), for  $Z \ge 120$ . Let us notice that such oblate deformations would be larger than any known through experiment at present.

To the best of our knowledge, the first theoretical hint about such huge oblate deformation came from the Woods-Saxon (WS) results in Ref. [\[5\]](#page-5-0). Unfortunately, in the same work [\[5\]](#page-5-0), prolate well-deformed ground states (g.s.) were obtained for nuclei  $Z \ge 120$ ,  $N \le 166$  in the self-consistent calculations with the Skyrme forces SLy7 and SkP, contradicting the WS results. Still other g.s. shapes of the nuclei in question were found in the later HFB study with the Skyrme SLy4 model [\[6\]](#page-5-0); for example, the self-consistently predicted shape of the nucleus  $Z = 122$  and  $N = 166$  was slightly oblate  $(\beta_2 \simeq -0.12)$  in Ref. [\[6\]](#page-5-0) and prolate ( $\beta_2 \simeq 0.2$ ) in Ref. [\[5\]](#page-5-0).

In our present study we use nine more deformations than in the WS calculations of Ref. [\[5\]](#page-5-0), including both triaxiality and reflection asymmetry. Therefore, we can extend discussion of the exotic oblate-deformed superheavy nuclei here. Relying on the calculated energy surfaces, nuclear masses, and cranking mass parameters, we calculate or estimate half-lives for selected even-even SDO systems. Then we consider an idea, advanced, for example, in Ref. [\[7\]](#page-5-0), of extra stable high-*K* shape isomers, also in odd systems, whose existence at the SDO

shape is very likely. Expected decay hindrances point to the possibility that some of these exotic-shaped superheavy nuclei, far from the conventionally expected "island of stability," live long enough to be detected. The additionally performed HF-BCS calculations with the Skyrme SLy6 force also predict the SDO minima in  $Z \ge 120$ ,  $N \le 166$  nuclei. Thus, there is no fundamental contradiction between microscopic-macroscopic and self-consistent predictions of SDO.

# **II. METHOD OF CALCULATION**

Within the microscopic-macroscopic method, energy of a deformed nucleus is calculated as a sum of two parts: the macroscopic one, being a smooth function of *Z*, *N* and deformation, and the fluctuating microscopic one, which is based on some phenomenological single-particle (s.p.) potential. A deformed Woods-Saxon potential model used here is defined in terms of the nuclear surface, as exposed in Ref. [\[8\]](#page-5-0). We admit shapes defined by the following equation of the nuclear surface:

$$
R(\theta, \varphi) = c(\{\beta\})R_0 \bigg\{ 1 + \sum_{\lambda > 1} \beta_{\lambda 0} Y_{\lambda 0}(\theta, \varphi) + \sum_{\lambda > 1, \mu > 0, \text{even}} \beta_{\lambda \mu c} Y_{\lambda \mu}^c(\theta, \varphi) \bigg\},
$$
 (1)

where  $c({\{\beta\}})$  is the volume-fixing factor. The real-valued spherical harmonics  $Y_{\lambda\mu}^c$ , with even  $\mu > 0$ , are defined in terms of the usual ones as  $Y_{\lambda\mu}^c = (Y_{\lambda\mu} + Y_{\lambda-\mu})/\sqrt{2}$ . In other words, we consider shapes with two symmetry planes. The  $n_p = 450$ lowest proton levels and  $n_n = 550$  lowest neutron levels from  $N_{\text{max}} = 19$  lowest shells of the deformed harmonic oscillator were taken into account in the diagonalization procedure. We have determined the s.p. spectra for every investigated nucleus, so that no scaling to a *central* nucleus was needed. The Strutinsky smoothing was performed with the sixth-order polynomial and the smoothing parameter equal to  $1.2\hbar\omega_0$ . For the macroscopic part we used the Yukawa plus exponential model [\[9\]](#page-5-0).

All parameters used in the present work, determining the s.p. potential, the pairing strength, and the macroscopic energy,

are equal to those used previously in the calculations of masses [\[10\]](#page-5-0) and fission barriers [\[11\]](#page-5-0) of heaviest nuclei. We took the "universal set" of potential parameters and the pairing strengths  $G_n = (17.67 - 13.11 \cdot I)/A$  for neutrons and  $G_p =$  $(13.40 + 44.89 \cdot I)/A$  for protons  $[I = (N - Z)/A]$ . These pairing constants have been adjusted to odd-even mass differences for nuclei beyond lead which could be calculated from the known masses of atomic nuclei in 2001 [\[12\]](#page-5-0). As always within this model, *N* neutron and*Z* proton s.p. levels have been included when solving BCS equations. Systematic calculations have shown that this microscopic-macroscopic model leads to a very good agreement with experimental masses in the superheavy region: The root-mean-square deviation for masses of 238 nuclei  $Z \ge 84$ ,  $N \ge 126$  [\[13\]](#page-5-0) equals 0.37 MeV [\[14\]](#page-5-0).

We used a rich variety of shapes, with possible nonaxiality and mass asymmetry, to reliably determine energy landscapes of the heaviest nuclei. A deformation set included both traditional quadrupole deformations  $β$  and  $γ$ , where  $β_{20}$  = *β* cos *γ* and  $β_{22c} = −β \sin γ$  (for  $γ = n × 60°$ , with *n* integer, a quadrupole shape is axially symmetric); three hexadecapole distortions  $\beta_{40}$ ,  $\beta_{42c}$ , and  $\beta_{44c}$ ; the higher-rank even axial multipoles  $\beta_{60}$  and  $\beta_{80}$ ; and the following odd-multipole deformations  $\beta_{30}$ ,  $\beta_{32c}$ ,  $\beta_{50}$ ,  $\beta_{52c}$ , and  $\beta_{70}$ —altogether 12 parameters. The range of deformation parameters covered a region of shapes up to, and little behind, the fission barrier, where the shape parametrization [\(1\)](#page-0-0) may be hoped sufficient.

Energy landscapes were obtained by a multidimensional energy minimization on a map of equidistant mesh points ( $\beta$  cos  $\gamma$ ,  $\beta$  sin  $\gamma$ ) with respect to ten other deformations. We used a rather large mesh spacing of 0.05 to make timeconsuming calculations feasible. A subsequent interpolation served to visualize results. To check the results we monitored the continuity of the resulting ten deformation parameters with respect to  $\beta$  cos  $\gamma$  and  $\beta$  sin  $\gamma$  and their stability with respect to the choice of their starting values. To assess the latter, we repeated the minimization for the whole map for selected nuclei by choosing random starting values. We have found that the results agreed with the ones obtained previously. Additional minimizations have been done to further verify the found minima, in particular, the axially symmetric minima were reproduced by the minimization over the axially symmetric deformations *βλ*0.

## **III. RESULTS**

*Equilibria and SDO minima; fission barriers*. Quadrupole deformations *β*<sup>20</sup> of the ground state (global) minima, calculated for ∼300 even-even nuclei are shown in Fig. 1. In addition to spherical and well- or weakly deformed prolate and oblate equilibrium shapes, there is a region of SDO nuclei for  $Z \ge 120$ ,  $N \le 168$ , of particular interest here. SDO global minima occur also for large  $N = 190,192$  and  $Z = 118 - 122$  (one such minimum for  $N > 184$ , resulting from microscopic-macroscopic 3D calculations, was shown in Ref. [\[15\]](#page-5-0)). Although some weakly deformed minima have nonaxial distortions, energies of  $Z = 120$  isotopes plotted vs  $\beta_{20}$  $\beta_{20}$  $\beta_{20}$  for axially symmetric shapes in Fig. 2 fairly illustrate the shape competition and coexistence in the  $Z \ge 120$  region.



FIG. 1. (Color online) Calculated ground-state quadrupole deformations  $\beta_{20}$ .

The secondary SDO minima exist there for  $168 \le N \le 172$ and  $N \geq 184$ . They appear also in  $Z \leq 118$  nuclei. Typically, they lie ≈2 MeV above the g.s. This has an effect on the *α* decay of the SDO  $Z = 120$  isotopes (see below). In the whole  $Z \geq 114$  region, the deepest minima, spherical or oblate, occur for  $N = 174-184$ ; for  $Z = 124,126$  they are predominantly oblate. In general, one can conclude that g.s. shapes of heaviest nuclei obtained here qualitatively agree with the previous microscopic-macroscopic results [\[5,16\]](#page-5-0), as well as with some self-consistent results (see, e.g.,Refs. [\[6,17\]](#page-5-0)). This conclusion should be treated only as a general remark, especially for very heavy ( $Z \ge 118$ ) systems. On the borders of  $\beta$  stability theoretical evaluations of g.s. deformations, if available, can differ qualitatively and quantitatively.

Energy maps in the  $(\beta \cos \gamma, \beta \sin \gamma)$  plane are necessary to appreciate fission barriers (Fig. [3\)](#page-2-0). The conspicuous result of our calculations is that triaxial saddles are found in many of the studied nuclei. They may lower the axial fission barrier by up to 2.5 MeV. This lowering increases with *N* and is larger for bigger *Z*. The role of triaxiallity on static fission barriers has been shown before in many publications (see, e.g., Refs. [\[5,6,11,18–23\]](#page-5-0) and recently in Ref. [\[24\]](#page-5-0)). The oddmultipole deformations do not change the barriers as much, but they lower some oblate minima and modify the energy maps around and beyond the saddles.

Crucial for stability is that barriers diminish with *N decreasing* below 174–176 and with *Z* approaching 126. The first feature is common also to the self-consistent HFBCS results (e.g., Ref. [\[25\]](#page-5-0)), while the second is very distinctive for the microscopic-macroscopic model used here [\[11\]](#page-5-0). Hence, the largest barriers of ≈3*.*4 MeV predicted for SDO nuclei  $286120$  and  $288122$  are rather small as compared to the 5.6 MeV barrier for <sup>296</sup>120 [\[11\]](#page-5-0). The barriers for  $N \ge 190$ SDO nuclei are still smaller, so we do not consider them further. As the  $\alpha$ -decay rates increase with  $Z$ , we concentrate on the SDO nuclei around  $Z \approx 120$  and  $N \approx 166$ .

*Other models.* To convince ourselves that the SDO minima are not a strange twist of the particular model we have looked for minima in the interesting nuclei by using (1) the same microscopic model and another version of the macroscopic energy, the LSD liquid drop model of Ref. [\[26\]](#page-5-0), (2) the selfconsistent HFBCS method with the Skyrme force SLy6 [\[27\]](#page-5-0)

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FIG. 2. (Color online) Energy relative to the spherical macroscopic contribution,  $E(\beta_{20}) - E_{\text{macro}}$ (sphere), for the  $Z = 120$  isotopic chain; each point results from the minimization over  $\beta_{\lambda 0}$ ,  $\lambda = 3-8$ .

and the state-dependent pairing induced by the *δ* interaction with the strengths  $V_n = 316$  MeV fm<sup>3</sup> for neutrons and  $V_p =$  $322$  MeV fm<sup>3</sup> for protons. The HF equations were solved on the 3D spatial mesh and the symmetry of solutions was the same as in the microscopic-macroscopic model, allowing for both triaxial and reflection-asymmetric shapes (for other details of the HFBCS calculations, see Ref. [\[28\]](#page-5-0)).

Both additional calculations support the prediction of the global SDO minima. These minima are even by  $\approx$ 1 MeV deeper with the LSD variant of the macroscopic energy. In the HFBCS calculations, the energy competition between prolate, oblate, and SDO minima and fission barriers come out similar



FIG. 3. (Color online) Energy surface of <sup>288</sup>122, normalized as in Fig. 2. Crosses mark the saddles.

as in the microscopic-macroscopic study. In particular, for  $Z =$ 120, the SDO minima are the lowest ones for  $N = 164, 166$ while for  $Z = 118$  they are excited by about 2–2.5 MeV. Moreover, the fission barrier height in  $^{286}$ 120 is again about 3 MeV.

*Stability against fission.* We checked fission half-lives  $T_{sf}$ by calculating WKB action with cranking mass parameters for selected nuclei. We assumed the zero-point energy of 0.5 MeV. To handle fission paths in 12D deformation space we calculate, instead of the mass parameter tensor, the effective mass parameter along a prescribed 12D path. Technically, this is done by replacing analytic derivatives with respect to deformations by the finite differences [\[29\]](#page-5-0).

Two possible classes of fission paths and barriers along them may be read from Fig. 3. The barriers along the axial saddle (at  $\beta \approx 0.3$ ,  $\gamma = 0$ ) are longer and have thinner peaks. They can compete with the triaxial path only when there is a deep normal oblate minimum, that is, for  $N = 166$  or  $N = 168$ . Triaxial barriers and the related WKB action change smoothly from isotope to isotope. At present, we did not attempt a minimization over paths in the 12D space; we checked WKB actions along a few chosen short trajectories along the barrier. These are known to provide the smallest action from the previous studies [\[18,30\]](#page-5-0). The smallest action we found along triaxial, nearly straight paths. They give half-lives  $10^{-6}$  s for <sup>286</sup>120 and  $10^{-5}$  s for <sup>288</sup>122. Because these barriers are rather short and simple, we estimate an error from the path being nonoptimal of about 1 order of magnitude on the basis of calculated actions and our experience.

*Stability against α* decay. From the calculated masses and the improved formula a la Viola-Seaborg [\[31\]](#page-5-0), we obtain for the g.s.  $\rightarrow$  g.s. transitions  $log_{10}(T_{1/2}^{\alpha}[s]) = -9.1$  for <sup>286</sup>120, and *longer*  $T_{\alpha}$  for the lighter  $Z = 120$  isotopes (Fig. [4\)](#page-3-0). This follows from the dependence of  $Q_{\alpha}$  values on the neutron number (see Fig. [5\)](#page-3-0). These SDO  $\rightarrow$  prolate transitions (Fig. [6\)](#page-3-0)

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FIG. 4. (Color online) Calculated  $log_{10}(T_{1/2}^{\alpha}[\text{s}])$  values for g.s. to g.s. (the lower line) and g.s to SDO (the upper line) *α* decays of *Z* = 120 neutron-deficient nuclei.

must be strongly hindered by a very different structure of both configurations, in particular, the occupation of intruder states at SDO shape (see below). If the hindrance would be complete, only  $SDO \rightarrow SDO$  transitions would remain. As already mentioned, SDO configurations in the  $Z = 118$  daughters are excited by  $\approx$ 2 MeV (2.5 MeV in HFBCS). This leads to a considerable decrease of  $Q_{\alpha}$  (see Fig. 5) and consequently increase in half-life:  $log_{10}(T_{1/2}^{\alpha}[\text{s}])$  becomes equal to  $-5.5$  for <sup>286</sup>120 and  $T_{\alpha}$  are *shorter* for lighter isotopes (see Fig. 4). Clearly, this result holds as long as the configuration hindrance factors are smaller than 10<sup>−</sup>3*.*6.

*Kisomers at SDO deformation; odd systems.* With halflives  $T_{1/2}$  < 10<sup>-5</sup> s—the present limit for detection of synthesized superheavy nuclei—superheavy SDO systems might be considered merely as a theoretical curiosity. A fascinating possibility for their longer life-times is related to *K* isomerism (see Refs. [\[7,32\]](#page-5-0)). Indeed, high-*K* configurations at the SDO shape are very likely (see Fig. [7\)](#page-4-0). Owing to large deformation, the neutron  $k_{17/2}$  and proton  $j_{15/2}$  intruder states with large



FIG. 5. (Color online) Calculated  $Q_{\alpha}$  values for g.s. to g.s. (upper line) and g.s. to SDO (lower line)  $\alpha$  decays of  $Z = 120$  neutrondeficient nuclei.



FIG. 6. (Color online) Mechanism of the *α*-decay hindrance of the SDO <sup>286</sup>120; energy normalized as in Fig. [2.](#page-2-0)

angular momentum projections on the symmetry axis  $\Omega$ are close to the Fermi level for  $Z = 120$ ,  $N = 166$ . Of unique structure and parity, they provide identity to high-*K* 2(4)-quasiparticle configurations. Candidates for low-lying *K* isomers are the so called "optimal" configurations [\[33\]](#page-5-0), with singly occupied large- $\Omega$  orbitals close to the Fermi level. In <sup>286</sup>120, the candidates are the proton  $(13/2^-,7/2^+)10^$ and neutron (15/2<sup>+</sup>,9/2<sup>−</sup>)12<sup>−</sup> configurations. The possible low-lying or ground states in odd nuclei are the neutron 15/2<sup>+</sup> state in 285120 and the proton 13/2<sup>−</sup> state in 285119; the low-lying 14<sup>−</sup> state could be expected in the odd-odd 284119. Detailed predictions would require energy minimization at fixed configuration with blocking.

In assessing stability of high-*K* isomers or odd nuclei we rely on estimates and analogies with well established experimental facts, as we cannot precisely calculate their decay rates. Let us notice that the considered SDO nuclei are proton-unstable, but in view of the large Coulomb barrier the related lifetimes may not concern us, at least for even-*Z* nuclei. Indeed, one can find in Fig. 21 of Ref. [\[5\]](#page-5-0), that the one-proton emission half-life for <sup>286</sup>120 is predicted to be larger than 1000 years; it becomes 1/100–1/10 s for <sup>290</sup>124; half-lifes for odd-Z are much smaller, for example,  $10^{-5}$ – $10^{-4}$  s for <sup>285</sup>119. Owing to centrifugal barrier, the half-life will be larger than for  $l = 0$ , which was assumed in Ref. [\[5\]](#page-5-0). One can also notice that the odd-*Z*, high-*K* states are protected by the centrifugal barrier if the high- $\Omega$  protons are blocked.

*Fission hindrance*. As is well known,  $T_{sf}$  for odd and oddodd heavy and superheavy nuclei are by 3–5 orders longer than for their even-even neighbors. Similar increase was found for high-*K* isomers, with respect to (prolate) shape isomers on which they are built, in even  $240-244$ Cm [\[34\]](#page-5-0). For SDO superheavy *K* isomers two factors combine to increase fission half-life: (1) The axial fission path is closed by the conservation of the *K* quantum number and (2) triaxial barriers increase owing to a decrease in pairing caused by the blocking of two neutrons or protons. Additional hindrance of fission is expected for configurations involving blocked high- $\Omega$  intruder states.

In typical nuclei, the moment of inertia at the saddle is larger than at the g.s., so the fission barrier for higher spins

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FIG. 7. (Color online) Calculated s.p. energies vs projection of the angular momentum on the symmetry axis in <sup>286</sup>120 at the SDO g.s.:  $\beta_{20} = -0.449, \beta_{40} = 0.054, \beta_{60} = 0.017, \beta_{80} = -0.017.$ 

is lower than for the g.s. Considering the effect of the saddle deformation on the fission barrier at high spins for SDO nuclei we prefer to rely on our HFBCS results, as the Woods-Saxon moments of inertia have to be renormalized owing to toolarge radii [\[35\]](#page-5-0). The geometrical moments of inertia from the HFBCS calculation in <sup>288</sup>120 are  $\mathcal{J}_\perp$  = 71 b,  $\mathcal{J}_\parallel$  = 109 b at the SDO shape (the subscript refers to the relative orientation to the rotation and symmetry axes), and  $\mathcal{J}_{\perp}^{b} = 107$  b at the triaxial barrier (i.e., the larger one of the two). The actual moment of inertia at the barrier is reduced by pairing to  $f^b \mathcal{J}_{\perp}^b$ , with  $f^b$  substantially smaller than 1. Without pairing,  $\mathcal{J}_{\parallel}$  is an *average* moment of inertia of yrast noncollective high-*K* states [\[33\]](#page-5-0). As  $\mathcal{J}_{\parallel} > f^b \mathcal{J}_{\perp}^b$  and pairing at SDO g.s. is *weaker* than at the barrier, we infer that there should be *no* decrease with *K* in the fission barrier for SDO *K*-isomers.

*α-decay hindrance*. Although this seems the least certain of our arguments, *K*-isomerism *may* substantially increase *α* half-lives: The high- $K$  isomer in <sup>270</sup>Ds has a longer (partial) half-life  $T_{\alpha} = 6.0^{+8.2}_{-2.2}$  ms than the g.s.,  $T_{\alpha}$ (g.s.) =  $100^{+140}_{-40}$   $\mu$ s [\[36\]](#page-5-0). For SDO nuclei, an additional hindrance may result from a difference between the parent and daughter high-*K* configuration, or, for the same configuration, from its extra excitation in the daughter, leading to a smaller  $Q_\alpha$ .

*Stability against β decay*. The  $β^+$ -decay rates  $λ_β$  for neutron-deficient candidates for the SDO *K* isomers can be estimated by neglecting the emitted electron energy  $m_e c^2$  in the decay energy:  $Q_{\beta} = [M(A, Z) - M(A, Z - 1) - m_e]c^2$ . Then one has  $\lambda_{\beta} \sim |M|^2 G_F^2 Q_{\beta}^5$ , where  $|M|$  is the transition matrix element and  $G_F$  is the Fermi constant. Even for a perfect overlap,  $|M|^2 \sim 1$ , using our calculated masses we obtain half-lives  $T_\beta = \ln 2/\lambda_\beta$  of the order of 0.1–1 s for even and odd SDO nuclei, consistent with the results by Möller *et al.* [\[37\]](#page-5-0). Because for high-*K* isomers |*M*| is reduced, their  $\beta^+$  decay is even slower.

Although the production of SDO nuclei is another subject, one may notice here that the SDO shape is much closer to the sticking-point configuration of the prolate and spherical heavy ions in the side collision than the sphere.

### **IV. CONCLUSIONS**

Summarizing, within both microscopic-macroscopic and Skyrme HF methods, one obtains SDO shapes of the ground or low excited states of superheavy  $Z \ge 120$  nuclei. Although even-even, spin-zero nuclei decay by a quick  $\sim 10^{-5}$ – $10^{-6}$  s fission or *α* decay, longer half-lives are expected for high-*K* isomers which very likely exist in some even or odd systems. In this work we connected the predictions of the SDO stability, the idea of both *K* and shape isomerism and the estimates of their influence on half-lives. Through calculations or estimates it follows, somewhat paradoxically, that such extremely exotic configurations may live long enough to be detected. Especially the retardation of the  $\alpha$  decay of the  $Z = 120$ SDO configuration and the unique interplay of axial vs triaxial fission paths (including values of cranking mass parameters) provide important hints. The very fact that superdeformed oblate minima occur in WS and self-consistent calculations and their geometrical sense point to their universality, as a transitional form between (close to) spherical and toroidal configurations for still heavier hypothetical high-*Z* systems.

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