

Measurement and analysis of the $pp \rightarrow pp\gamma$ reaction at 310 MeVA. Johansson,^{1,*} H. Habertzettl,^{2,†} K. Nakayama,^{3,4,‡} and C. Wilkin^{5,§}¹*Department of Physics and Astronomy, Uppsala University, Box 535, S-751 21 Uppsala, Sweden*²*Center for Nuclear Studies, Department of Physics, The George Washington University, Washington, DC 20052, USA*³*Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA*⁴*Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*⁵*Physics and Astronomy Department, UCL, London WC1E 6BT, United Kingdom*

(Received 28 January 2011; published 12 May 2011)

The $pp \rightarrow pp\gamma$ reaction has been studied at a beam energy of 310 MeV by detecting both final protons at the PROMICE-WASA facility and identifying the photon through the resulting missing-mass peak. The photon angular distribution in the center-of-mass system and those of the proton-proton relative momentum with respect to the beam direction and to that of the recoil photon were determined reliably up to a final pp excitation energy of $E_{pp} \sim 30$ MeV. Except for very small E_{pp} values, the behavior of these distributions with excitation energy is well reproduced by a new refined model of the hard bremsstrahlung process. The model reproduces absolutely the total cross section and its energy dependence to within the experimental and theoretical uncertainties.

DOI: [10.1103/PhysRevC.83.054001](https://doi.org/10.1103/PhysRevC.83.054001)

PACS number(s): 25.40.Ep, 25.20.-x, 13.60.-r

I. INTRODUCTION

The classic motivation for measuring the emission of bremsstrahlung in nucleon-nucleon collisions was the study of the off-shell behavior of the associated elastic scattering amplitude, although it is now known that off-shell effects cannot be measured, even in principle. Nevertheless, the bremsstrahlung reaction can provide a window into the underlying dynamical mechanisms that govern the NN interaction and the electromagnetic currents of nucleons and mesons alike. This is especially true for the hard part of the bremsstrahlung spectrum, where the photon takes a large fraction of the available center-of-mass energy. In this region the $\Delta(1232)$ isobar may also play some role and the reaction has then much in common with meson production.

Hard bremsstrahlung has been studied in the radiative capture in neutron-proton scattering, $np \rightarrow d\gamma$, to quite high energies through the measurement of the inverse reaction of deuteron photodisintegration [1,2]. The energy dependence of the cross section provides direct evidence that one of the main driving terms is the excitation of an S -wave ΔN pair that de-excites through an $M1$ transition into $d\gamma$.

The situation is radically different in proton-proton collisions. The analogous $M1$ transition is forbidden for $pp \rightarrow pp\gamma$ by angular momentum and parity conservation when the two protons emerge with very low excitation energy $E_{pp} = W_{pp} - 2m_p$, where W_{pp} is the total energy of the final pp subsystem in its rest frame [3]. There are therefore significant cancellations among the large amplitudes in the pure S -wave diproton limit, so that any Δ effect must enter in more subtle ways. Furthermore, the $E1$ transition is generally suppressed by the vanishing of an electric dipole operator for the proton

pair. It is therefore to be expected that the bremsstrahlung production rate should be much lower in pp collisions than in np .

One technique used to investigate the emission of hard bremsstrahlung in proton-proton collisions is the photodisintegration of a pp pair in ${}^3\text{He}$. Events where two fast protons emerge from $\gamma {}^3\text{He} \rightarrow ppn$ are interpreted in terms of an interaction on a diproton, with the neutron merely appearing as a *spectator* [4–6]. Such data show little evidence for the presence of an intermediate ΔN pair, certainly much less than for those with fast pn pairs [5,6]. However, because the capture on pn spin-triplet pairs is so much stronger, the pp data extracted in this way may be contaminated by final-state interactions, possibly involving np charge exchange. This can only be checked through direct $pp \rightarrow pp\gamma$ measurements.

Proton-proton bremsstrahlung has been studied in several experiments but, in general, these were undertaken by detecting the emerging protons in pairs of small counters, often placed on either side of the beam direction [7,8], which has led to the low E_{pp} region being especially poorly sampled. The geometric acceptance was much increased in a series of refined KVI experiments at 190 MeV [9,10], but even here the low E_{pp} region was not favored. Whereas the COSY time-of-flight spectrometer also has wider acceptance, the data obtained at 293 MeV have only limited statistics and no attempt was made to evaluate the cross section as a function of E_{pp} [11].

Data on the hardest part of the pp bremsstrahlung spectrum were also obtained at the COSY-ANKE magnetic spectrometer by selecting the two final protons with $E_{pp} < 3$ MeV [12,13]. A proton beam energy range from 353 to 800 MeV was investigated but only for c.m. photon angles θ_γ , where $\cos \theta_\gamma > 0.95$. The results reveal a broad peak in the cross section at an energy around 650 MeV with a full width at half maximum (FWHM) ≈ 220 MeV. This suggests the possible influence of intermediate ΔN pairs, though not necessarily in a relative S wave.

Much higher statistics were obtained over a wider range of pp excitation energies and photon angles at the PROMICE-WASA facility in Uppsala. The experiment was carried out

* ritaj@telia.com

† helmut@gwu.edu

‡ nakayama@uga.edu

§ cw@hep.ucl.ac.uk

at a single beam energy of $T_p = 310$ MeV and results were recently published for $E_{pp} < 3$ MeV [14]. These data are completely consistent with those from ANKE at 353 MeV over the small-angle domain covered by the ANKE experiment [12,13]. However, it is clear from this comparison that a reliable decomposition into multipoles requires data over a wide angular range. The low E_{pp} data from Uppsala were interpreted as indicating the dominance of the $E1$ and $M2$ multipoles [14] with no evidence for any important $E2$ contribution, in contrast to theoretical expectations [15]. The purpose of the present paper is to extend the analysis up to $E_{pp} \approx 30$ MeV to test theoretical models over a wider range of excess energies.

A state-of-the-art model has recently been developed that for the first time describes successfully proton-proton bremsstrahlung in the hundred-MeV range [16,17]. This model, which is hereinafter denoted as HN, is summarized in Sec. II. In this approach the photon is coupled everywhere to a relativistic pp scattering amplitude in a way that ensures consistency with gauge invariance. Although this reproduces very well the detailed KVI $pp \rightarrow pp\gamma$ measurements at 190 MeV [9], it is possible that at 310 MeV the tail of the Δ might have some influence. In this context it should be noted that the minimal inclusion of the Δ isobar [18] (see also Ref. [16]) improves the theoretical description of the 280-MeV TRIUMF data [8].

The experimental approach used in this work is identical to that employed at PROMICE-WASA for pion production [19,20] and so Secs. III and IV merely provide outlines of the salient points of the method and the data analysis, respectively. The results given in Sec. V show that, away from the region of small E_{pp} values, where there can be significant cancellations between different contributions, the theory of Sec. II works remarkably well. It describes the photon angular distribution and those of the diproton relative momentum in different E_{pp} intervals as well as the energy dependence of the $pp \rightarrow pp\gamma$ total cross section. The fact that the theory reproduces the absolute normalization of these high-momentum-transfer data to within the experimental and theoretical uncertainties is striking. However, the theoretical predictions of the photon angular distributions obtained without intermediate ΔN contributions are better at low E_{pp} than those that include them. This brings into question whether the present simplified treatment of these isobar contributions is acceptable. Our conclusions and suggestions for further work are to be found in Sec. VI.

II. THEORETICAL MODEL

The experimental data presented in this paper represent the most complete measurement of bremsstrahlung in proton-proton collisions at an energy so far above threshold. We therefore compare them with a refined model [16] that has recently been successfully applied [16,17] to describe both the TRIUMF data at 280 MeV [8] and the high-precision KVI data [9] at the lower proton energy of $T_p = 190$ MeV. This solved a long-standing discrepancy between experiment and the then existing theory. The novel approach is derived within

a quantum field-theory formalism by coupling the photon everywhere possible to an underlying two-nucleon T matrix that is derived from a relativistic NN scattering equation. The basic idea of the method is that introduced by Habermatzl, Nakayama, and Krewald [21] for pion photoproduction, based on the field-theoretical approach of Habermatzl [22]. The model accounts for the important interaction current in the NN bremsstrahlung reaction in a manner that is consistent with the generalized Ward-Takahashi identity (WTI), which ensures gauge invariance at the microscopic level. This feature is absent from all earlier models.

Following Ref. [16], one starts from the nucleon-nucleon T matrix determined by the relativistic Bethe-Salpeter (BS) equation

$$T = V + VG_0T = V + TG_0V, \quad (1)$$

where V represents the driving two-nucleon potential. The two-nucleon propagator, $G_0 = S_1S_2$, describes the intermediate propagation of two free noninteracting nucleons (with individual Feynman propagators S_i , $i = 1, 2$) sharing the given fixed reaction energy. This relativistic four-dimensional equation is then reduced in a covariant manner to the three-dimensional Blankenbecler-Sugar (BbS) equation [23,24] by replacing the propagator G_0 by $G_0 \rightarrow g_0$ where g_0 restricts the intermediate two nucleons to be on their mass shells in a manner that preserves the (relativistic) unitarity of the equation.

The driving potential V used here is based on the one-boson-exchange model developed by the Bonn group [24], which contains nucleonic and mesonic degrees of freedom. In addition to reproducing the low-energy pp scattering data and the deuteron properties, the resulting NN interaction fits the NN phase shifts up to the threshold for pion production. This version is used, rather than a more modern potential, because the necessary interaction current that is fully consistent with this potential is already available from the work of Ref. [17], where it was shown to be crucial in resolving longstanding theoretical issues with the KVI data [9].

By coupling the photon to the system of two interacting nucleons, it can be shown, again following Ref. [16], that the resulting bremsstrahlung amplitude may be written as

$$M^\mu = (Tg_0 + 1)J^\mu(1 + g_0T), \quad (2)$$

where the final-state interaction (FSI) and initial-state interaction (ISI) are included through the NN T matrices on the left and right, respectively.

The basic photon production current from the two nucleons,

$$J^\mu = d^\mu G_0V + VG_0d^\mu + V^\mu, \quad (3)$$

contains nucleonic and mesonic terms, as well as a four-point contact-type term, as illustrated in Fig. 1(a).

The two disconnected nucleonic terms, shown in Fig. 1(b) and subsumed in the dual current d^μ , are given by

$$d^\mu \equiv \Gamma_1^\mu(\delta_2 S_2^{-1}) + (\delta_1 S_1^{-1})\Gamma_2^\mu. \quad (4)$$

Here Γ_i^μ is the $NN\gamma$ vertex for nucleon i ($=1, 2$), S_i denotes the propagator of the nucleon i , and δ_i represents an implied δ function that ensures that the incoming and outgoing momenta of the intermediate nucleon i are identical.

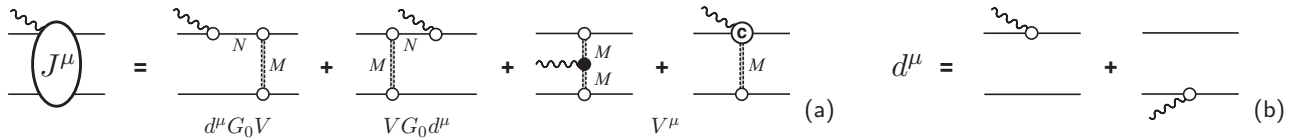


FIG. 1. (a) Basic photon production current J^μ used to describe the $pp \rightarrow pp\gamma$ reaction in the present work; analogous photon couplings along the lower nucleon line are omitted. N denotes an intermediate nucleon, and M incorporates all the meson exchanges. As indicated by the symbols below the diagrams, the first two describe the nucleonic current, while the meson-exchange current is depicted by the third. The fourth diagram contains the $NM \rightarrow N\gamma$ four-point contact current which, together with the meson-exchange current, constitutes the interaction current V^μ of Eq. (3). Diagram (b) shows the photon coupling to both intermediate nucleons as subsumed in the dual current d^μ .

We mention that the dynamical structure of this formulation takes care of the fact that the translation of the three-dimensional BbS reduction to the bremsstrahlung reaction must be implemented such that a physical photon cannot couple to a nucleon that is on shell before *and* after the coupling takes place. For more details regarding this nontrivial issue, see Ref. [16].

The V^μ of Eq. (3) describes the photon coupling to the internal mechanisms of the interaction V ; that is, it corresponds to the interaction current. For a one-boson-exchange model of the NN interaction, such as that employed here, V^μ consists of mesonic and four-point contact currents. Unlike the case of proton-neutron bremsstrahlung, where there is a large mesonic current contribution [25,26], this is to a large extent suppressed for proton-proton bremsstrahlung because only neutral mesons can then be exchanged. The dominant mesonic current contributions that we include arise from the anomalous $v\pi\gamma$ couplings ($v = \rho, \omega$). These transitions are transverse and thus cannot be obtained by simply coupling the photon to the underlying NN T matrix; they must be inserted by hand into J^μ .

The four-point contact current appears as a consequence of imposing gauge invariance in the form of the generalized WTI on the resulting amplitude. Note that the $v\pi\gamma$ meson-exchange currents have no influence on this because they are purely transverse. In general, contact-type currents have very complicated microscopic dynamical structures [22] that cannot be taken into account explicitly at present. Instead, one must revert to employing generalized *phenomenological* contact currents, constructed such that the full reaction amplitude satisfies the generalized WTI, which is necessary to ensure full gauge invariance at the microscopic level. As a consequence, no unique determination of the reaction amplitude is possible because the WTI does not constrain the transverse part of the amplitude. In the present case, the dynamics of the hadron interactions is described in terms of *phenomenological* form factors. The resulting phenomenological four-point interaction currents that describe the interaction of the photon with this hadronic three-point function, therefore, are constructed purely in terms of these hadron form factors [16,17].

In this paper we use our dynamical model in the analysis of the $pp \rightarrow pp\gamma$ reaction data at a proton incident energy of 310 MeV. Although this is well below the maximum of Δ production at about 650 MeV, earlier analyses [18] (see also Ref. [16]) of the TRIUMF data at 280 MeV [8] show that its inclusion can improve the agreement with data in certain geometries. We therefore investigate the effect of introducing

the Δ in a minimal fashion, following the application section of Ref. [16], by implementing the Δ contributions in J^μ at the tree level. For this purpose two more terms, analogous to the first two on the right-hand side of Fig. 1(a), are added, with the Δ resonance replacing the intermediate nucleon N . This Δ resonance current has no bearing on gauge invariance because the $\Delta N\gamma$ transition vertex is purely transverse. However, in a full $NN \rightleftharpoons \Delta N$ coupled-channels approach, in addition to the tree-level Δ resonance current considered here, there will also be additional box-type contributions with intermediate ΔN and $\Delta\Delta$ pairs that produce purely transverse five-point contact-type contributions to the interaction current V^μ [16]. At this stage, therefore, the present minimal tree-level inclusion of the Δ currents should be considered only exploratory.

The Bonn potential employed in the present study for generating the nucleon-nucleon T matrix is given in momentum space; it is therefore nontrivial to include the Coulomb interaction. Coulomb effects have been investigated in pp bremsstrahlung in the past [27]. However, the associated distortions are mainly relevant at very small pp invariant masses, a regime which has not been well sampled in most of the earlier experiments.

To test the influence of the Coulomb interaction over the wider acceptance of the present experiment, we also consider the NN interaction based on the Paris potential [28]. The Paris work was carried out in coordinate space and includes fully the Coulomb interaction [27] but only within the framework of the nonrelativistic Lippman-Schwinger equation. We have therefore formally transformed this into the relativistic BbS equation by a proper redefinition of the potential, through the so-called minimal relativity factor [29], to be able to use this interaction consistently within the present relativistic approach. The transformed interaction reproduces the same nucleon-nucleon observables for relativistic kinematics as the original one for nonrelativistic kinematics.

One shortcoming in the present approach for incorporating the Paris potential is that, for simplicity, we have retained the production current J^μ calculated from the Bonn potential. As a result, the consistency of the ISIs and FSIs with the production current J^μ is lost but, for the purpose of checking the Coulomb effects, this inconsistency is not of major concern.

III. EXPERIMENT

The data of the present experiment were obtained at the The Svedberg Laboratory in Uppsala, where a 48-MeV proton

beam from the cyclotron was injected into the CELSIUS ring [30], accelerated to 310 MeV, and then stored. An average beam-on intensity of 3 mA was achieved during an experimental data-taking period of approximately 100 h.

The measurements were carried out at the PROMICE-WASA facility [31] and results at small proton-proton excitation energies have already been published [14]. Furthermore, the $pp \rightarrow pp\gamma$ data were obtained simultaneously with those on $pp \rightarrow pp\pi^0$ [19], whose results were reported in greater detail in Ref. [20]. Because the detector assembly and the measurement techniques were identical in the two experiments, and the experimental procedures and data analysis differed only in minor details, the description here is kept quite brief.

An internal gas-jet hydrogen target, with a density of about $2 \times 10^{14} \text{ cm}^{-2}$, was used in conjunction with the stored proton beam. By operating an electron cooler throughout the experiment, the background was reduced significantly and the counting rate increased through an improvement in the beam-target overlap.

Even though the PROMICE-WASA facility was equipped to detect high-energy photons, in the bremsstrahlung study reported here, only protons were measured in the final state. After exiting the scattering chamber, the protons passed through a forward window counter (FWC), a tracker, and a forward trigger hodoscope (FTH) and usually stopped in a forward range hodoscope (FRH). The four-quadrant scintillator of the FWC eliminated most of the beam-halo background. To be accepted by the main trigger, coincident protons must appear in different quadrants. Events with protons in the same quadrant were allowed by a secondary trigger but, in accord with Monte Carlo expectations, these were very few in number and were not considered in the subsequent analysis.

Information on the proton angles was extracted from the FTH and, even more precisely, from the tracker. Events with polar angles between about 3° and 22° were recorded. As described fully in Ref. [20], the energy associated with a proton track was deduced from a combination of the calculated angle-dependent range up to the entrance of the stopping scintillator and the measured light output of that detector. A few protons stopped in one of the thin dead regions between the scintillator planes and these were then assigned the energy corresponding to the midpoint of the dead layer.

As an extra check on the particle identification, it was further required that both protons of an accepted event penetrate at least into the second layer of the FTH, which consists of 24 spiral scintillator segments. The minimum energy of each proton was therefore 38 MeV. This condition meant that all coincident pairs of protons stopped in the second FRH scintillator or earlier so that there was effectively no high-energy limitation imposed by the design of the apparatus.

In the missing-mass distribution of the $pp \rightarrow ppX^0$ reaction shown in Fig. 2, there are two clear peaks corresponding to $X^0 = \gamma$ and $X^0 = \pi^0$, with very little overlap. Before making any detailed cuts, these peaks contained in total 66 521 $pp\gamma$ and 861 449 $pp\pi^0$ candidates. The exclusion of events affected by the detector gaps, and those where the proton time difference fell outside a 65-ns band, eliminated 7.3% and 1.5% of these, respectively. There is only a small ($\approx 5\%$) background

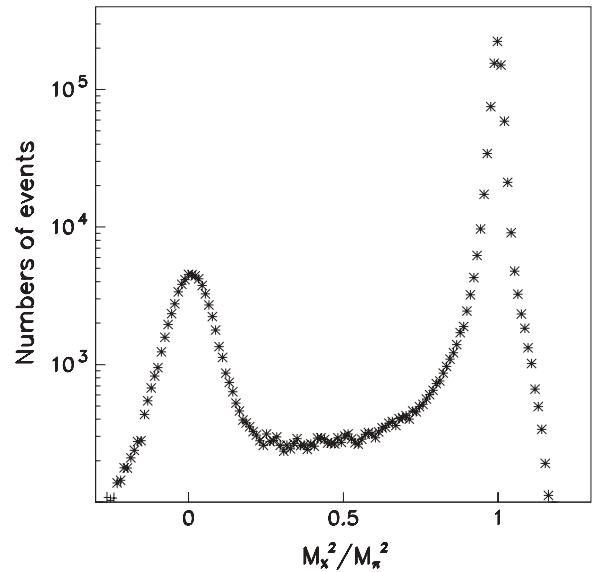


FIG. 2. Distribution in the square of the missing mass in the $pp \rightarrow ppX$ reaction presented in units of the neutral pion mass. Clear peaks are seen, arising from the $pp \rightarrow pp\pi^0$ and $pp \rightarrow pp\gamma$ reactions, sitting on a slowly varying background.

under the γ peak that arises mainly from the rescattering of one of the protons from a pion-production reaction. The maximum polar angle of protons from π^0 production depends sensitively upon the proton beam energy T_p . A measurement of this angle, which was close to 18° , showed that $T_p = 309.7 \pm 0.3 \text{ MeV}$.

The width of the γ peak is $\sigma(M_X^2) = 0.056M_{\pi^0}^2$. By retaining only events at a little over the two FWHM level, namely, $|M_X^2/M_{\pi^0}^2| < 0.137$, to a good approximation this cut compensates for the neglect of the small background contribution [14].

The angular distribution of the protons in the laboratory system for the selected $pp \rightarrow pp\gamma$ events is shown in Fig. 3. In spite of a slight but significant misalignment between the beam and the detector axes, the angular cutoffs at both small and large angles are quite sharp and very well reproduced by the Monte Carlo simulation that used the Bonn pp potential in the model described in Sec. II. The phase-space simulation gives a marginally poorer representation, especially at large angles. In both cases the predictions have been normalized to the total number of events.

For each of the emerging protons, a timing signal was extracted from the first of the FTH spiral detectors. The time at the target position was then estimated using the information on the particle energy, the hit position in the scintillators, and the time of flight. The calibration, which was improved over that used in Ref. [20], led to a distribution for the time difference between the two protons with a peak width of 1.1 ns FWHM. This was essentially the same for both the forward-going and the backward-going photons, though a correction was introduced to compensate for a slight offset of 0.2 ns in the forward case. Cuts at $\pm 1.8 \text{ ns}$ applied to the data of Fig. 4 reduced the number of accidental coincidences to less than 1% so that it was then justified to employ a kinematic fitting.

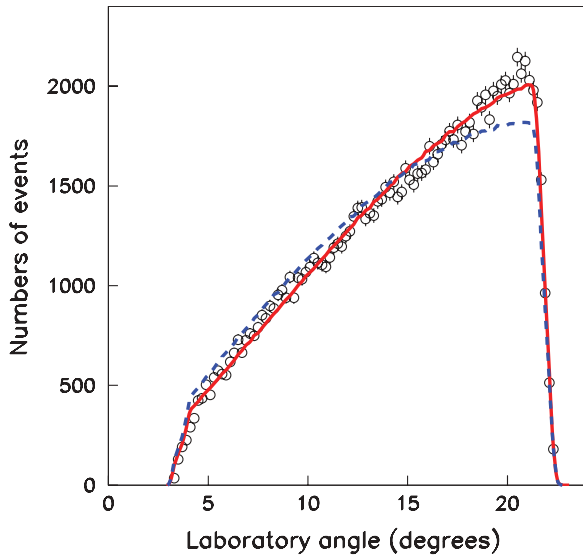


FIG. 3. (Color online) Angular distribution of the final protons from the $pp \rightarrow pp\gamma$ reaction in the laboratory (open circles) compared to Monte Carlo simulations based on the HN model [16], using the Bonn potential as input (red solid curve). Similar predictions obtained with a phase-space model (blue dashed curve) are also shown. Both sets of predictions were normalized to the total number of events.

This was achieved by adjusting the energies of the two protons to give zero missing mass. Taken together with the sharper time difference cut, this reduced the number of candidates to the 58 026 $pp\gamma$ events that were used in the subsequent analysis.

IV. ANALYSIS

In the case of a production reaction such as $pp \rightarrow pp\gamma$, the unpolarized cross section is a function of four independent variables. The standard set chosen for the analysis consists of

- (i) E_{pp} , the excitation energy in the final pp system;
- (ii) θ_γ , the c.m. production angle of the photon;
- (iii) θ_q , the c.m. polar angle of the pp relative momentum \vec{q} with respect to the beam direction;
- (iv) φ_q , the azimuthal angle between \vec{q} and the photon momentum.

Other variables, such as the laboratory proton angle that was used in the construction of Fig. 3, can be expressed in terms of these four quantities.

To convert the observed numbers of events into cross sections, knowledge of the detector acceptance is needed in the four-dimensional space. This was achieved using a Monte Carlo simulation, where the detector system was described in great geometric detail. Identical cuts were then placed on the simulated and experimental events. Frequently only phase-space was used in the simulations but, in principle, the acceptances might depend significantly on the actual reaction probability. Estimates were therefore made not only for simple phase space, but also for a realistic reaction matrix, assumed to be represented by the HN model [16]. In the latter case

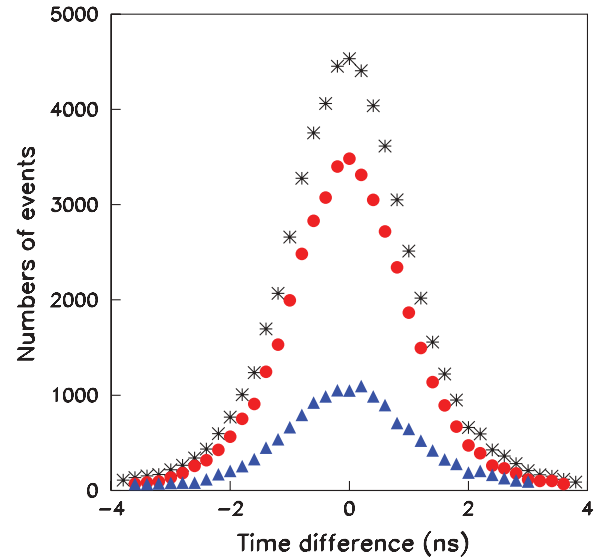


FIG. 4. (Color online) Time difference between the two protons emerging from the target. The peaks have similar widths for photons in the forward (blue triangles) and backward (red circles) c.m. hemisphere, as well as their sum (black crosses). Events in the interval ± 1.8 ns were retained for the subsequent analysis.

the program interpolated within a lookup table of the reaction matrix yielded by this model in the four standard variables. Values of the acceptance in the (E_{pp}, θ_γ) space are shown in Fig. 5.

It should first be noted that the PROMICE-WASA detector only registered protons with laboratory angles less than 22° and

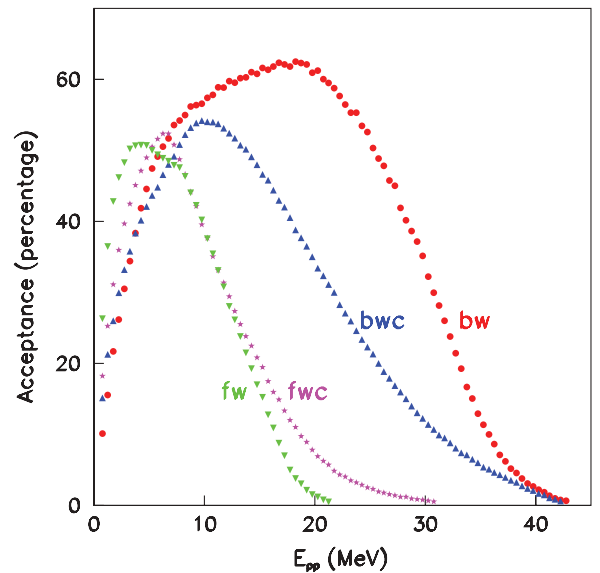


FIG. 5. (Color online) Monte Carlo estimates of the percentage acceptance for $pp \rightarrow pp\gamma$ at $T_p = 310$ MeV obtained using the HN model [16] with the Bonn potential. The results are divided into four regions in the photon CM angle θ_γ , defined in the text, namely, the backward (red circles), the backward central (blue triangles), the forward central (magenta stars), and the forward (green inverted triangles).

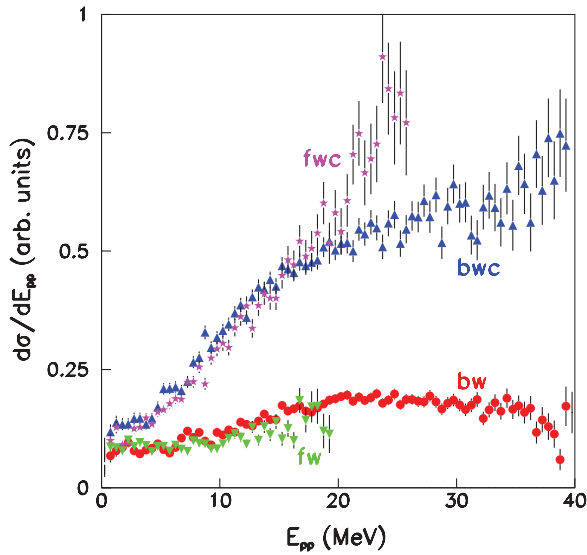


FIG. 6. (Color online) Cross section for $pp \rightarrow pp\gamma$ at $T_p = 310$ MeV in arbitrary units, obtained using the HN model [16] with the Bonn potential to evaluate the acceptance. The experimental data are divided into the same four angular regions as in Fig. 5. Data were only plotted when the acceptance was estimated to be above 2%.

at 310 MeV this means that only the region $E_{pp} < 42$ MeV was sampled. Although at low E_{pp} the acceptance could be quite large, being up to 60%, this decreased to much lower values at higher E_{pp} and small θ_γ . This is illustrated in Fig. 5 by showing the acceptance as a function of E_{pp} in four ranges of the photon (c.m.) angle. These are, respectively, the backward $-1 < \cos\theta_\gamma < -0.8$ (bw), the backward central $-0.8 < \cos\theta_\gamma < 0$ (bwc), the forward central $0 < \cos\theta_\gamma < 0.8$ (fwc), and the forward $0.8 < \cos\theta_\gamma < 1$ (fw) regions. For photons emitted in the forward hemisphere, the recoiling protons are slower and a greater fraction emerge at larger angles than allowed for in design of the PROMICE-WASA detector, and this leads to a more severe cut at high E_{pp} . Protons from these events are also more likely to be distorted by secondary interactions. However, it also means that the beam-pipe effect kicks in at lower E_{pp} , which is also clearly seen in Fig. 5.

Owing to the identical nature of the protons in the entrance channel, the $pp \rightarrow pp\gamma$ cross section is symmetric in the c.m. system around $\theta_\gamma = 90^\circ$. The effects of the variation of the acceptance with θ_γ at large E_{pp} can also be seen in Fig. 6, which shows the cross sections extracted as functions of E_{pp} in the same four regions of $\cos\theta_\gamma$, used in Fig. 5. In all cases the data were terminated when the estimated acceptance dropped below 2%, and this occurred much earlier for small values of θ_γ . Except at the edges of the acceptance, the cross sections deduced using a phase space model to evaluate the acceptance differed only marginally from those obtained on the basis of the dynamical model.

The crucial forward/backward symmetry is clearly respected to within the uncertainties for $E_{pp} < 20$ MeV but between 22 and 26 MeV there is some deviation, which is more apparent in the angular distributions to be presented in Sec. V. On general grounds one would expect the data from the forward photon hemisphere to be less reliable because the

associated protons are less energetic and can emerge at larger angles. The statistics in the backward hemisphere are also much larger.

The integrated luminosity of 340 nb^{-1} was derived from a comparison of elastic proton-proton scattering results measured in parallel with tabulated cross sections, as described in Ref. [20]. Owing to the large prescaling factor associated with the pp trigger used, an error bar of about 10% must be associated with this value. This includes also effects connected with the evaluation of the proton acceptance in the apparatus and any uncertainty in the pp database used in the comparison.

Of the other systematic uncertainties discussed in Ref. [20], proton rescattering in the detector material might contribute 2%, as might the treatment of the background under the γ peak. Although the PROMICE-WASA geometric acceptance is very good, the extrapolation to unexplored regions and its dependence upon reaction models can give up to 3%, though this depends upon the value of E_{pp} . The known systematic uncertainty is therefore judged to be $\approx 15\%$ overall. However, despite the care taken with the calibrations and the evaluation of the acceptance, the forward/backward symmetry is not completely respected at high E_{pp} , as evidenced by the divergence between the fwc and bwc data in Fig. 6. We therefore cannot exclude larger systematic uncertainties even in the backward photon hemisphere for $E_{pp} \gtrsim 30$ MeV.

V. RESULTS

More than 58 000 kinematically well-defined $pp \rightarrow pp\gamma$ events are available for analysis in terms of the four-dimensional differential cross section, as described in Sec. IV. In the present paper only one single differential and three double differential distributions are presented. Data points are shown if the acceptance at this point is estimated to be larger than 2%. Only the statistical uncertainties are shown explicitly by error bars and these do not include the $\approx 15\%$ overall systematic effects. The azimuthal dependence of the data can be quite strong but, because this seems mainly to be a reflection of the acceptance, it is not further investigated. The data would allow explorations of higher dimensionality, but further guidance from theory would be necessary to exploit this fruitfully.

The c.m. differential cross section in the photon angle is presented in Fig. 7 averaged over 3-MeV bins in the pp excitation energy from 0–3 MeV to 39–42 MeV, with the upper end of each interval being indicated in the relevant panel. The angular cuts on the data clearly reflect the acceptance dependence presented in Fig. 5.

As already reported [14], the data for $E_{pp} < 3$ MeV show a strong minimum at $\theta_\gamma = 90^\circ$ and an almost pure $\cos^2\theta_\gamma$ behavior. The level of this minimum rises as E_{pp} increases. The data show that the $\cos^4\theta_\gamma$ term is generally small and its strength cannot be determined with precision.

The full dynamical model of Haberzettl and Nakayama [16] of Sec. II has been evaluated using the Bonn potential [24] without the Δ contribution. The curves are consistent with the shapes of the angular distributions, as measured in the backward hemisphere for all except the lowest E_{pp} bin. The

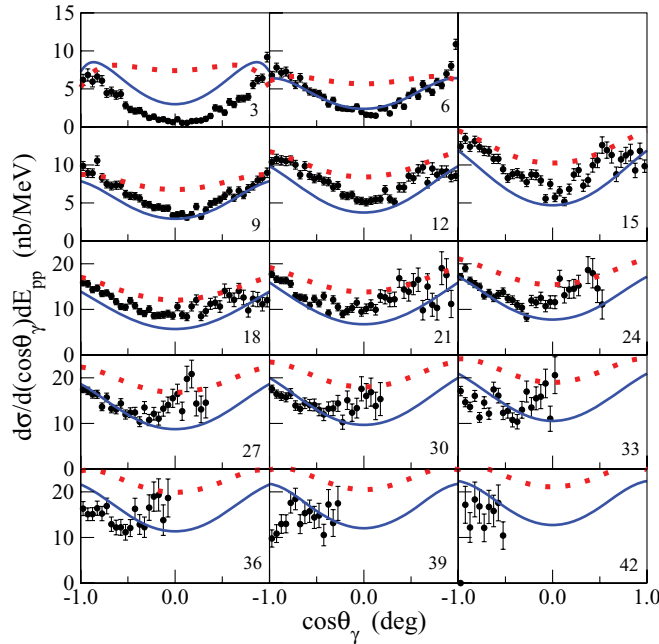


FIG. 7. (Color online) Differential cross section for $pp \rightarrow pp\gamma$ at 310 MeV. The data have been averaged over 3-MeV bins in E_{pp} and the upper end of each interval is indicated by the number at the bottom right of the corresponding panel. The curves correspond to the absolute predictions of the Habermehl and Nakayama model [16] of Sec. II, using the Bonn potential [24]. The solid (blue) one represents the model without any Δ contribution, whereas for the dashed (red) one the Δ has been switched on. The latter has been included in a minimal fashion, as explained in Sec. II.

strengths are also well described, especially in view of the uncertainties in the absolute scales of both the theory and experiment. A minimum is predicted at 90° for all energy bins but for $E_{pp} < 3$ MeV this is not sufficiently deep and there seems to be no sign there of the leveling off near the forward/backward directions expected from the theoretical model.

The theoretical predictions are, of course, sensitive to the assumptions in the model. Thus, when the “minimal” inclusion of the Δ contribution is switched on in the calculation, the effects are surprisingly large and the agreement with the data is much poorer, especially at low E_{pp} where the central minimum is largely absent. The shapes are far less changed at high E_{pp} and the data there can be well reproduced if the predictions are scaled by a factor of ≈ 0.8 .

The description of the angular distributions with the Paris potential [28] is very similar to that obtained with the Bonn potential. The Coulomb effects that are included here are only significant for very low E_{pp} but this is also the region where the theoretical model is least satisfactory.

The second angular distribution to be discussed is that of the pp relative momentum vector \vec{q} with respect to that of the proton beam in the overall c.m. frame. This is shown in Fig. 8 in the same 3-MeV bins that were used in Fig. 7. It is difficult to measure the angles of the vector \vec{q} when its magnitude is small so that any apparent deviation from isotropy for $E_{pp} < 3$ MeV may not be significant.

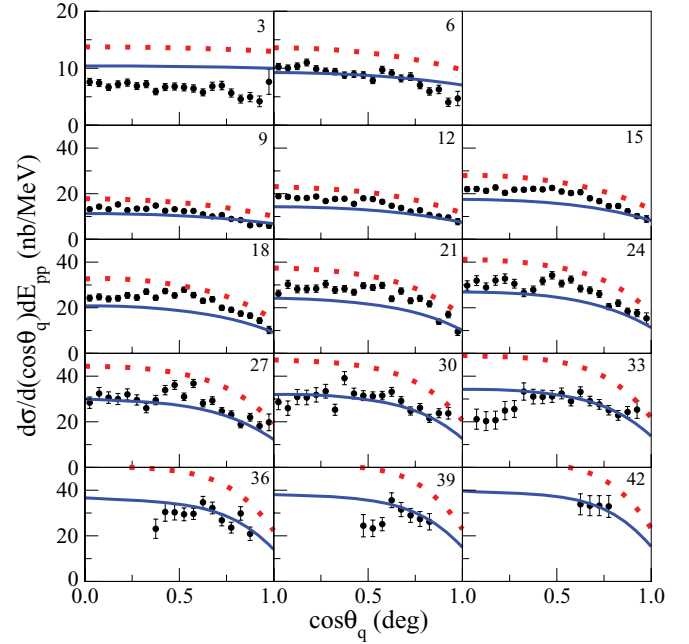


FIG. 8. (Color online) The distribution in the angle between the pp relative momentum vector \vec{q} and that of the incident beam direction in the overall c.m. frame. It should be noted that there may be significant systematic measurement uncertainties at low E_{pp} . Because the final protons are identical, data are only shown in one hemisphere and the total cross section is obtained by summing over this region. The curves correspond to the predictions of the Habermehl and Nakayama model [16] of Sec. II obtained using the Bonn potential [24]. The notation is the same as in Fig. 7.

The data for $E_{pp} > 3$ MeV show clear evidence of a forward dip and the predictions of the HN model [16] on the basis of the Bonn potential follow this trend quantitatively. As can be seen from Fig. 8, in this case there is very little difference in the shape of the predictions whether the Δ is included or not. The lack of data for small $|\cos\theta_q|$ at high E_{pp} is a consequence of the limited acceptance in these regions.

The corresponding distribution of \vec{q} with respect to the photon direction is shown in Fig. 9. Although this is evaluated in the frame of the recoiling pp pair (the helicity distribution), little change would be seen if this were replaced by the overall c.m. frame. Both the data and the models display fairly flat shapes, with the possible exception of the very low energy bins, where the drop in the data for small helicity angle θ_h may reflect the difficulty in measuring two protons when they emerge with similar angles and momenta.

Note that the data in the three angular distributions are different representations of the same 58 026 events, so that the cross sections integrated over angle must be identical. The “holes” seen at various places of phase space in the diagrams indicate possible sources of systematic errors. As a consequence, one must conclude that the integrated $pp \rightarrow pp\gamma$ cross section can only be safely extracted when $E_{pp} \lesssim 30$ MeV. The energy variation of this cross section is shown in Fig. 10 as a function of E_{pp} . For one of the sets of points, only results from the backward photon hemisphere (bw + bwc) are used. The other set uses in addition the fwc data and the

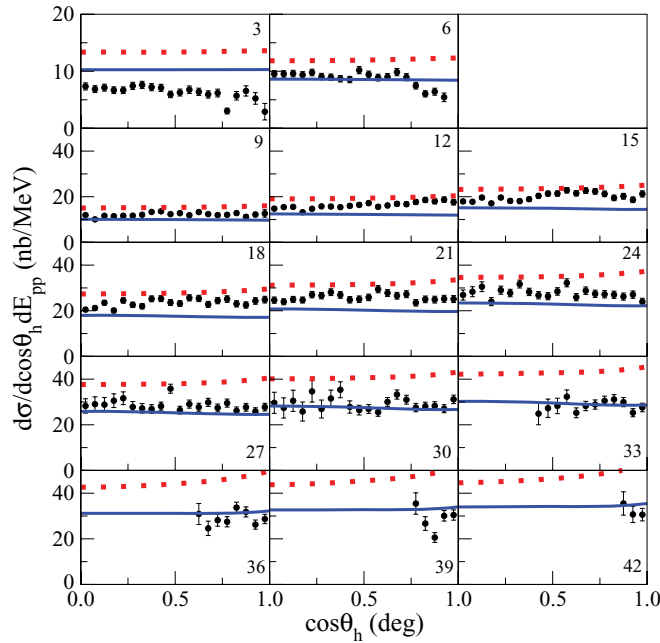


FIG. 9. (Color online) The distribution in the helicity angle between the pp relative momentum vector \vec{q} and that of the photon in the frame of the recoiling pp pair. The notation is the same as in Fig. 7.

forward/backward symmetry is only invoked to derive the data in the very forward region.

The reasonable description of the angular distributions by the theoretical model is translated into one of the integrated cross section, where the predictions have been smeared over the 0.5-MeV bins used in the data presentation. To ensure agreement with the data above 10 MeV, the theoretical results that included only the nucleonic and meson-exchange current terms were scaled by factors of 1.30 and 1.45 for the Bonn and Paris evaluations, respectively. When the ΔN intermediate states are included in the minimal way described in Sec. II, the corresponding factors are 0.80 and 0.90, respectively.

The drawback of using the Bonn potential is immediately apparent. The unrealistic spike at very low E_{pp} is significantly softened when this is replaced by the Paris potential, which includes the Coulomb repulsion in the pp system. Otherwise, there is little difference between the predictions based upon the two potentials. In either case the model seems to overestimate the cross section for the production of the 1S_0 state of the two final protons. It must again be stressed that in this region there can be delicate cancelations among the contributions [3]. If the 1S_0 prediction were reduced, the energy dependence might be reproduced, though with a slightly too-low overall normalization.

The inclusion of the effects of the Δ in an approximate way gives a rather similar description of the data in Fig. 10. However, as mentioned already, the Δ effects beyond the tree level that would arise in a consistent $NN \rightleftharpoons \Delta N$ coupled-channels approach [16], have been ignored in the present calculations. Using the Paris rather than the Bonn potential induces changes analogous to those seen in the non- Δ scenario.

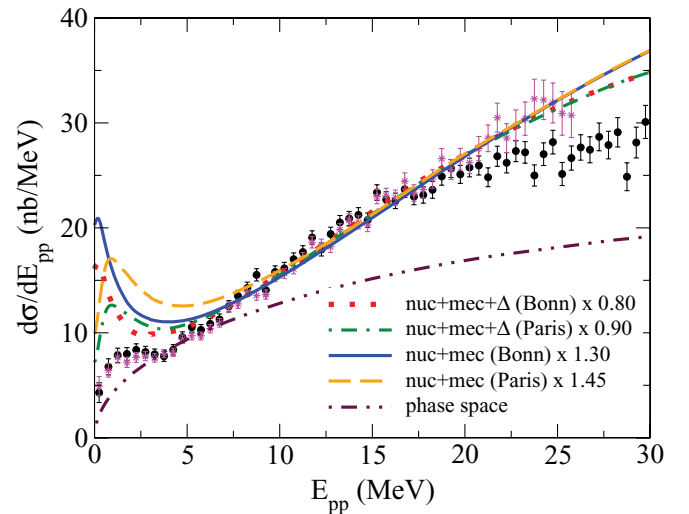


FIG. 10. (Color online) Cross section for the $pp \rightarrow pp\gamma$ reaction measured at $T_p = 310$ MeV integrated over all angles. The (black) circles were obtained by taking data from the backward photon hemisphere and invoking the forward/backward symmetry. For the (magenta) stars, data in the interval $-1 < \cos\theta_\gamma < 0.8$ were used in the determination. Overall systematic errors of $\pm 15\%$ are not shown. The solid (blue) curve represents the evaluation of the model described in Sec. II using the Bonn NN input without the inclusion of any Δ contribution, whereas the red dots represent an attempt to include this in a minimalist way. The (orange) dashed and (green) dash-dotted curves are the analogous estimates based upon using the Paris potential that includes the Coulomb repulsion. The curves are scaled by the factors shown in the figure. The (maroon) dot-dot-dashed curve illustrates the energy dependence to be expected from a pure phase-space model. It is arbitrarily normalized at around 5 MeV, that is, just above the region whose energy dependence is largely driven by the strong pp FSI.

Phase space does not provide an acceptable description of the E_{pp} dependence of the integrated cross section shown in Fig. 10. This is by no means unexpected because the spin-parity constraints associated with the 1S_0 final state are not built into such a naive approach.

The total $pp \rightarrow pp\gamma$ cross section integrated up to $E_{pp} = 30$ MeV is $\sigma(30) = (0.59 \pm 0.09) \mu\text{b}$, where the statistical error is negligible and the quoted uncertainty is purely systematic. In the COSY-TOF measurement at 293 MeV [11] there were very few events with $E_{pp} > 60$ MeV and a total cross section of $\sigma = (3.5 \pm 0.3 \pm 0.7) \mu\text{b}$ was obtained by extrapolating to the kinematic limit on the basis of a phase-space variation. If, instead, the total cross section below 30 MeV is estimated, a value of $\sigma(30) = (0.54 \pm 0.12) \mu\text{b}$ is found. The agreement between the COSY-TOF result and ours lies well within the error bars.

To illustrate the variation of the angular dependence of the photon with excitation energy, the differential cross section data of Fig. 7 have been fitted by

$$\frac{d\sigma}{d\cos\theta_\gamma} = \frac{1}{2} \sum_{n=0}^2 a_{2n} P_{2n}(\cos\theta_\gamma). \quad (5)$$

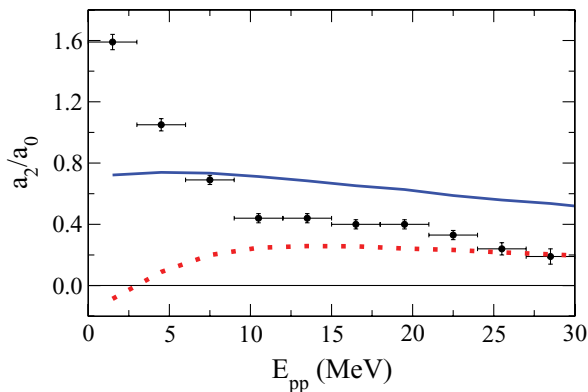


FIG. 11. (Color online) Ratio of the Legendre coefficients defined by Eq. (5). Values extracted from experimental data are compared to the predictions of the Haberzettl and Nakayama model [16] with and without the minimalist Δ inclusion (dotted red and solid blue lines, respectively).

Figure 11 shows the ratio a_2/a_0 as a function of E_{pp} compared to the predictions of the present calculation based on the HN model [16]. Neither the prediction with nor that without the Δ contribution can describe the rise in a_2/a_0 at low E_{pp} and the inclusion of the Δ is particularly disappointing in this region.

VI. CONCLUSIONS

We have presented here detailed measurements of bremsstrahlung production in proton-proton collisions at a beam energy of 310 MeV. The differences between the data extracted from the forward and backward photon hemispheres increases for excitation energies above about 20 MeV and, in such cases, the backward data are more reliable because of the faster protons and the much higher acceptance. Using the forward/backward symmetry of the reaction, full acceptance was achieved up to an $E_{pp} \approx 30$ MeV and some information obtained even close to the kinematic limit of 42 MeV imposed by design of the apparatus. The big advantage of this experiment compared to others undertaken above the pion-production threshold is the large acceptance coupled with good statistics. It is therefore not surprising that the values extracted for the cross sections depend very little on whether one uses phase space or the HN model to estimate the acceptance. However, to go to higher E_{pp} one would need larger counters than those provided by the PROMICE-WASA setup.

Away from the very small E_{pp} region, the dynamical model of Haberzettl and Nakayama [16], whose main points are summarized in Sec. II, is rather successful in describing all the experimental results as functions of E_{pp} provided an overall scaling factor close to unity is applied. As well as the integrated cross section, these include the angular distributions of the photon and those of the pp relative momentum with respect to the beam direction and to that of the recoil photon. It is very gratifying to note that any rescaling of the predictions required to achieve this success is well within the combined experimental and theoretical uncertainties. The latter are clearly very hard to quantify but they must at least

encompass the differences between the inclusion or not of the Δ contributions.

The situation at low E_{pp} is more uncertain because there are significant cancellations among the driving terms [3] and the theoretical results are therefore much more sensitive to small contributions. Using the Paris rather than the Bonn potential in the evaluation of the model allows the Coulomb interaction to be included and this does smooth the predictions slightly at low E_{pp} . However, it must be noted that the switch from Bonn to Paris was not done fully consistently.

The good agreement between theory and experiment was obtained without considering any effects that might arise from the virtual excitation of the Δ isobar. Although the basic model was tuned to describe the 190-MeV KVI data [9], by 310 MeV the influence of the Δ might start to be felt. In this context it should be noted that the introduction of the Δ isobar [16,18] seems to improve the theoretical description of the 280-MeV TRIUMF data [8]. However, we find that the inclusion of the Δ effects in the minimal way described in Sec. II actually makes the agreement with the shapes of the photon angular distribution worse for $E_{pp} \lesssim 12$ MeV and this difference is somewhat puzzling. We view it more as an indication that Δ effects are not very well understood and that minimal inclusion is not warranted in these instances, rather than as a measure of theoretical errors. Further theoretical work is clearly needed and the results might be improved by introducing a phenomenological five-point contact current to take account of the ΔN box diagrams, as explained in Ref. [16].

At higher E_{pp} the differences are less important and the four versions of the model give very similar integrated cross sections in Fig. 10 provided that they are scaled by factors that are all fairly close to unity. With the scaling factors as shown, the predictions start to differ from the more reliable backward-angle data above 20 MeV. However, in view of the 15% overall systematic error in the data and the arbitrariness in the scaling factors one cannot draw firm conclusions as to the significance of this.

The very small E_{pp} region has been studied near the forward direction with the COSY-ANKE spectrometer up to 800 MeV [12,13]. The results show an energy dependence that suggests some influence from intermediate ΔN pairs. However, for kinematic reasons, at these higher beam energies the γ and π^0 peaks, which are so prominent in the missing-mass plot of Fig. 2, merge and the extraction of a $pp \rightarrow pp\gamma$ signal is much more delicate. Under these conditions it may be necessary to measure the photon in coincidence and data of this type from the COSY-WASA facility are currently being analyzed at 500 and 550 MeV [32].

However, it should be stressed that data in the small E_{pp} region taken at well below the pion production threshold would also be very valuable because the uncertainties regarding the inclusion of the Δ contribution would then be minimized. Data with polarized beam and target, along with results on the differential cross section and analyzing power, would allow some of the electromagnetic multipoles to be determined explicitly and therefore allow one to identify defects in the models more clearly [33].

The bremsstrahlung reaction is the simplest of all the high-momentum-transfer reactions in proton-proton collisions at

intermediate energies. Unlike cases of meson production, there is no need to consider the FSI of the meson with one of the protons. The quality of the agreement between the predictions of a modern bremsstrahlung model with the high-statistics and high-acceptance data achieved at the PROMICE-WASA facility is striking and should encourage further experimental and theoretical work in the field.

ACKNOWLEDGMENTS

We are very grateful to the TSL/ISV personnel and the PROMICE-WASA collaboration for their support in this

experiment. We acknowledge in particular the great help provided by Jozef Złomańczuk in the early stages of this experiment and its analysis. We also thank the Research Computing Center at the University of Georgia and the Supercomputing Center at Forschungszentrum Jülich for providing computing resources that were necessary to obtain the theoretical results shown in the present paper. This work was supported by the European Community under the “Structuring the European Research Area” Specific Programme Research Infrastructures Action (Hadron Physics, Contact No. RII3-cT-204-506078), FFE Grant No. 41788390 (COSY-058), and by the Swedish Research Council.

-
- [1] R. Crawford *et al.*, *Nucl. Phys. A* **603**, 303 (1996).
 [2] P. Rossi *et al.*, *Phys. Rev. Lett.* **94**, 012301 (2005).
 [3] J. M. Laget *Nucl. Phys. A* **497**, 391 (1989).
 [4] G. Audit *et al.*, *Phys. Lett. B* **227**, 331 (1989); N. d’Hose *et al.*, *Phys. Rev. Lett.* **63**, 856 (1989); G. Audit *et al.*, *Phys. Rev. C* **44**, R575 (1991); *Phys. Lett. B* **312**, 57 (1993); A. J. Sarty *et al.*, *Phys. Rev. C* **47**, 459 (1993); S. Niccolai *et al.*, *ibid.* **70**, 064003 (2004); I. Pomerantz *et al.*, *Phys. Lett. B* **684**, 106 (2010).
 [5] T. Emura *et al.*, *Phys. Rev. Lett.* **73**, 404 (1994).
 [6] D. J. Tedeschi *et al.*, *Phys. Rev. Lett.* **73**, 408 (1994).
 [7] B. M. K. Nefkens, O. R. Sander, D. I. Sober, and H. W. Fearing, *Phys. Rev. C* **19**, 877 (1979); B. V. Przewoski *et al.*, *ibid.* **45**, 2001 (1992); K. Yasuda *et al.*, *Phys. Rev. Lett.* **82**, 4775 (1999).
 [8] K. Michaelian *et al.*, *Phys. Rev. D* **41**, 2689 (1990).
 [9] H. Huisman *et al.*, *Phys. Rev. C* **65**, 031001(R) (2002).
 [10] M. Mahjour-Shafiei *et al.*, *Phys. Rev. C* **70**, 024004 (2004); *Phys. Lett. B* **632**, 480 (2006); M. Mahjour-Shafiei *et al.*, *Eur. Phys. J. A* **41**, 25 (2009).
 [11] R. Bilger *et al.*, *Phys. Lett. B* **429**, 195 (1998).
 [12] V. Komarov *et al.*, *Phys. Rev. Lett.* **101**, 102501 (2008).
 [13] D. Tsirkov *et al.*, *J. Phys. G: Nucl. Part. Phys.* **37**, 105005 (2010).
 [14] A. Johansson and C. Wilkin, *Phys. Lett. B* **673**, 5 (2009); **680**, 111 (2009).
 [15] P. Wilhelm, J. A. Niskanen, and H. Arenhövel, *Phys. Rev. C* **51**, 2841 (1995); P. Wilhelm, H. Arenhövel, and J. A. Niskanen, *Phys. Rev. Lett.* **74**, 1034 (1995).
 [16] H. Haberzettl and K. Nakayama, [arXiv:1011.1927](https://arxiv.org/abs/1011.1927)[nucl-th].
 [17] K. Nakayama and H. Haberzettl, *Phys. Rev. C* **80**, 051001(R) (2009).
 [18] F. de Jong, K. Nakayama, and T.-S. H. Lee, *Phys. Rev. C* **51**, 2334 (1995); F. de Jong, K. Nakayama, V. Herrmann, and O. Scholten, *Phys. Lett. B* **333**, 1 (1994); M. Jetter and H. W. Fearing, *Phys. Rev. C* **51**, 1666 (1995).
 [19] J. Złomańczuk *et al.*, *Phys. Lett. B* **436**, 251 (1998).
 [20] R. Bilger *et al.*, *Nucl. Phys. A* **663**, 633 (2001).
 [21] H. Haberzettl, K. Nakayama, and S. Krewald, *Phys. Rev. C* **74**, 045202 (2006).
 [22] H. Haberzettl, *Phys. Rev. C* **56**, 2041 (1997).
 [23] R. Blankenbecler and R. Sugar, *Phys. Rev.* **142**, 1051 (1966); K. Erkelenz, *Phys. Rep. C* **13**, 191 (1974).
 [24] R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989); J. Haidenbauer (private communication); following J. Haidenbauer and K. Holinde, *Phys. Rev. C* **40**, 2465 (1989), the OBEP-B potential can be constrained by the *pp* scattering length with only a minor readjustment of its parameters.
 [25] V. R. Brown and J. Franklin, *Phys. Rev. C* **8**, 1706 (1973).
 [26] K. Nakayama, *Phys. Rev. C* **39**, 1475 (1989).
 [27] V. Hermann, K. Nakayama, O. Scholten, and H. Arellano, *Nucl. Phys. A* **582**, 568 (1995).
 [28] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, *Phys. Rev. C* **21**, 861 (1980).
 [29] V. Herrmann and K. Nakayama, *Phys. Rev. C* **45**, 1450 (1992); **46**, 2199 (1992).
 [30] C. Ekström *et al.*, *Phys. Scr., T* **22**, 256 (1988).
 [31] H. Calén *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* **379**, 57 (1996).
 [32] J. Złomańczuk (private communication).
 [33] K. Nakayama and F. Huang, *Phys. Rev. C* **82**, 065201 (2010).