

**$q\bar{q}$  potential at finite  $T$ , and weak coupling in  $\mathcal{N} = 4$  SUSY**

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We compute the potential between a  $q\bar{q}$  color-singlet state for  $\mathcal{N} = 4$  SUSY with gauge group  $SU(N)$  at finite temperature  $T$ , large distances  $rT \gg 1$ , and weak coupling  $g$ . As a first step, we only consider the electric modes and compute the Debye mass  $m_D$ , where we find that each of the  $8(N^2 - 1)$  bosonic degrees of freedom contributes to  $m_D^2$  (on average) with  $\frac{N}{N^2-1} \frac{1}{6} g^2 T^2$ , while each of the  $8(N^2 - 1)$  fermionic degrees of freedom contributes (on average) with  $\frac{N}{N^2-1} \frac{1}{12} g^2 T^2$ , yielding  $m_D^2 = 2Ng^2 T^2$ . Then, motivated by results obtained in the literature from both the weak-coupling results in QCD and the large-coupling investigations of  $\mathcal{N} = 4$  SUSY through AdS/CFT, we attempt to include magnetic mode corrections. Our results illustrate that, for this particular computation,  $\mathcal{N} = 4$  SUSY is in striking qualitative agreement with QCD.

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**I. INTRODUCTION**

Recently, the discovery of the AdS/CFT correspondence [1–4] has created great interest in the  $\mathcal{N} = 4$  SUSY  $SU(N)$  gauge theory because the duality allows for the investigation of several aspects of the gauge theory (see, for instance, [5–42] and [43] for a recent review on the field) at strong coupling through gravitational and generally classical computations.

In particular, one of the first computations using the gauge and gravity duality was the evaluation of the heavy-quark potential at large 't Hooft coupling (and large  $N$ ), both at zero temperature [6] and also at finite temperature [21,44–46]. In particular, in [21,46] it was found that, at large enough distances, the real part of the potential has a power-law falloff extending the results (obtained earlier) in the literature [44,45]. The method of [21,46] involved an analytic continuation of the solution of [44,45] applying the ideas of [47–49].

Amazingly enough, exactly the same power falloff was found in [50] for perturbative QCD (pQCD) at finite temperature. This fact partially motivated investigating the  $\mathcal{N} = 4$  SUSY  $SU(N)$  gauge theory at nonzero temperature in the framework of a perturbative, field theoretical approach.

Therefore, in this paper, we compute the heavy-quark color-singlet state potential of  $\mathcal{N} = 4$  SUSY at nonzero temperature and weak 't Hooft coupling. We compute the Debye mass and apply the technique of [50] that had been previously applied to QCD and investigate whether we may predict a similar power-law falloff of the potential. Such an investigation allows for a comparison with the result of [50] obtained in the framework of pQCD and with the result of [21,46] obtained in the framework of AdS/CFT. We note that the literature is rich with regard to perturbative calculations at finite temperature for both the heavy-quark potential for QCD [50–62] as well as for several (other) contexts of the  $\mathcal{N} = 4$  SUSY theory [5,63–68].

We organize this paper as follows. In Sec. II, we set up the problem arguing that the color-singlet potential is a gauge-invariant quantity and, hence, it makes sense to compute (perturbatively). We also give an elementary introduction of

the objects that we need in this work and that appear in thermal field theory. Section III deals with the relevant diagrams involved in the calculation. For a subset of the diagrams under consideration, we borrow results from the literature calculated for QCD and show how these results may be adapted for our case. In Sec. IV, we use the results of previous sections to evaluate the quark-pair potential. We initially compute the Debye mass giving rise to the expected Yukawa falloff and then we extend the computation in the spirit of [50]. Finally, in Sec. V, we summarize and discuss our results. In particular, we find that the potential has a power falloff (at sufficiently large distances) that agrees precisely with [50] and (the absolute value of) [21,46]. The notation and conventions that we use, as well as many useful formulas, may be found in Appendix A. In Appendix B, we begin from the Lagrangian for  $\mathcal{N} = 4$  SUSY in terms of superfields and write it in terms of component fields, exhibit the vanishing of the beta function, and derive the Feynman rules for both zero and nonzero temperature. In the rest of the Appendices, we show which diagrams contribute in the calculation that we are interested in and evaluate in great detail (one of) the relevant diagram(s) to exhibit the general idea behind these sorts of computations.

**II. SETTING UP THE PROBLEM****A.  $q\bar{q}$  potential and gauge invariance**

In QED, the magnetic and electric fields are gauge invariant. In non-Abelian gauge theories, on the other hand, gauge invariance (of the chromoelectromagnetic field) is not valid as a consequence of the presence of the nonlinear terms in the field strength tensor  $F_{\mu\nu}^a$ . However, a gauge-invariant quantity that one may construct is the free energy (potential) of a static, color-singlet (total color charge zero), quark-antiquark pair as a function of the separation  $r$  of the pair [69]. As the quantity that we wish to compute is gauge invariant, we choose to compute it in the temporal axial gauge (TAG).

**B. Elements of thermal field theory**

In this section, we present the minimum knowledge about field theory at finite temperature that one needs to perform

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the calculation in which we are interested. The self-energy of the gauge boson [two-point irreducible diagram, the precise definition of which is given below in Eq. (5)] at finite temperature may be decomposed as

$$\Pi^{\mu\nu} = G(q^0, |\mathbf{q}|)P_T^{\mu\nu} + F(q^0, |\mathbf{q}|)P_L^{\mu\nu}, \quad (1)$$

where  $q^\mu$  is the four-momentum of the gauge boson,  $G$  and  $F$  are scalar functions of  $q^0$  and  $|\mathbf{q}|$ , while  $P_T$  and  $P_L$  are the transverse and longitudinal projection tensors (to  $\mathbf{q}$ ), respectively, and their explicit form is given by

$$P_T^{\mu\nu} = \left( \delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2} \right) g^{i\mu} g^{j\nu}, \quad (2)$$

$$P_L^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q^\nu q^\mu}{q^2} \right) - P_T^{\mu\nu}, \quad \mu = 0, i.$$

These tensors satisfy

$$q_i P_T^{i\nu} = q_\mu P_T^{\mu\nu} = q_\mu P_L^{\mu\nu} = P_T^{\mu\kappa} P_{L\kappa}{}^\nu = 0. \quad (3)$$

Motivated by Eqs. (3) and (2), it is natural to associate  $G$  with the chromomagnetic modes and  $F$  with the chromoelectric ones. These factors may be expressed in terms of the diagonal components of  $\Pi^{\mu\nu}$  as

$$F = \frac{k^2}{\mathbf{k}^2} \Pi^{00}, \quad F + 2G = \Pi^{ii} - \Pi^{00}. \quad (4)$$

Taking into account the expression for the bare propagator of the gluon  $D_0^{\mu\nu}$  from Fig. 1 written in the TAG gauge, one may, formally at least, compute the exact (dressed) propagator to all orders in perturbation theory, the nonzero components of which are

$$D^{ij} = D_0^{ik} \sum_{N=0}^{\infty} [(-\Pi D_0)^N]_{k^j} = \frac{1}{G - k^2} \left( \delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2} \right) + \frac{1}{F - k^2} \frac{k^i k^j}{k_0^2} \quad (\text{TAG}). \quad (5)$$

This expression, in fact, provides the definition for  $\Pi^{\mu\nu}$ .

Now let us suppose that we place two oppositely (chromo)charged fermions at a distance  $r$  and seek for the

mutual force of the two charges treating them as small (static) perturbations in the thermal medium. Then, in the spirit of the linear response theory, one may show [69] that the potential of the  $q\bar{q}$  system as a function of the separation is given by

$$V(r) = Q_1 Q_2 \int \frac{d^3 q}{(2\pi)^3} \frac{e^{i\mathbf{q}\mathbf{r}}}{\mathbf{q}^2 + F(0, \mathbf{q})}$$

$$= Q_1 Q_2 \int \frac{d^3 q}{(2\pi)^3} \frac{e^{i\mathbf{q}\mathbf{r}}}{\mathbf{q}^2 - \Pi^{00}(0, \mathbf{q})}, \quad (6)$$

where the last equality is a consequence of the (left) equation (4) and (A1). Thus, we conclude that, in order to compute the potential, we only need the (00) component of  $\Pi^{\mu\nu}$ . In this paper, we compute it perturbatively to  $O(g^2 T^2)$  (see Sec. III).

### C. Approximations

As we will see,  $\Pi^{00}$  depends on the temperature  $T$  and the chemical potential  $\mu_i$  [the index  $i$  is associated with every global symmetry ( $i$ ) that gives rise to  $\mu_i$ ] through the ratio  $\mu_i/T$ , that is,

$$\Pi^{00} = \Pi^{00}(q^0, \mathbf{q}; T, \mu_i/T). \quad (7)$$

We will work in the region where

$$\frac{1}{\mu_i} \gg \frac{1}{T} \quad \forall i, \quad r \gg \frac{1}{T}. \quad (8)$$

In particular, the right approximation above implies that the integral in Eq. (6) receives its main contribution from  $\mathbf{q} \rightarrow 0$  and, as a result, (6) reduces to

$$V(r) = Q_1 Q_2 \int \frac{d^3 q}{(2\pi)^3} \frac{e^{i\mathbf{q}\mathbf{r}}}{\mathbf{q}^2 - \lim_{\mathbf{q} \rightarrow 0} \Pi^{00}(0, \mathbf{q})}, \quad (9)$$

where, in the color-singlet state, the product of charges is

$$Q_1 Q_2 = (gT_G^a) \otimes (-gT_G^a) = -C_2(G)g^2 = -Ng^2. \quad (10)$$

The expression  $T_G^a \otimes T_G^a$  is the tensor product of the adjoint times the (anti)adjoint representation, resulting in the Casimir

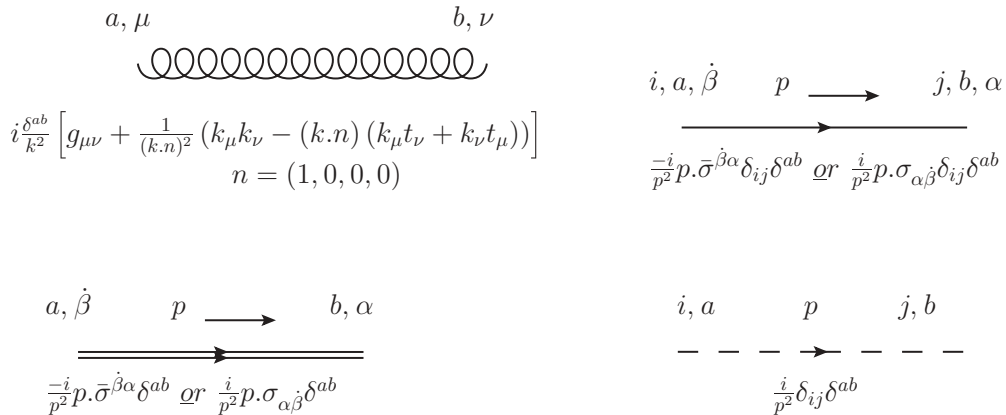


FIG. 1. Bare propagators for the gauge boson (upper left), which is in the TAG gauge, the fermions (upper right), the gaugino (lower left), and the scalars (lower right). The notation for the indices is explained in Appendix B, Eq. (B6). The choice of the appropriate rule for the fermions (and the gaugino) is determined by the the rest of the diagram to which the propagators are attached.

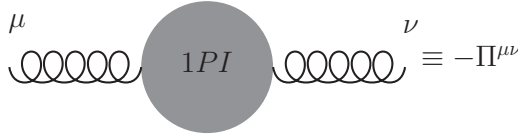


FIG. 2. Diagrammatic definition of (the colored)  $\Pi^{\mu\nu}$  suppressing color indices. There is no imaginary  $i$  involved in the definition.

operator and which, in turn, yields to the expected factor of  $N$ .

### III. EVALUATING THE RELEVANT DIAGRAMS

The gauge boson self-energy tensor is defined diagrammatically in Fig. 2. The diagrams that one has to calculate in order to compute  $V(r)$  to  $O(g^2)$  are given in Appendix C. Most of these have been calculated and may be found (for example) in [69]. The important point here is that there is not any scalar propagator involved in this computation because, according to Sec. II A, the potential we compute involves a color singlet and hence the fermion pair should be of the same flavor. However, according to the Lagrangian (B3) and the resulting Feynman rules for scalars (Fig. 3), the vertices change the flavor of the fermions (observe the presence  $\epsilon_{ijk}$  in the corresponding Feynman rules), which leads to the conclusion that scalar propagators do not contribute to the calculation under consideration. [In practice, one may imagine that the two interacting fermions are massive, charged under color, and couple (directly) only to the gauge bosons but not to the scalars. It is evident that these fermions are external particles to the  $\mathcal{N} = 4$  SUSY theory.]

#### A. Contribution of $\Pi_g^{00}$ due to the gauge loops

The contribution due to the gluons arises from the diagrams of Fig. 4. The result in the TAG is given by equation (8.65)

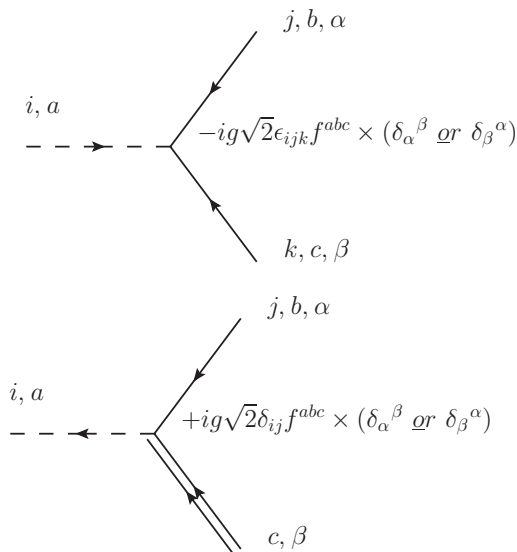


FIG. 3. Yukawa vertices for fermion-fermion and fermion-gaugino with a similar comment as in the caption of Fig. 1 regarding the choice of the rule.

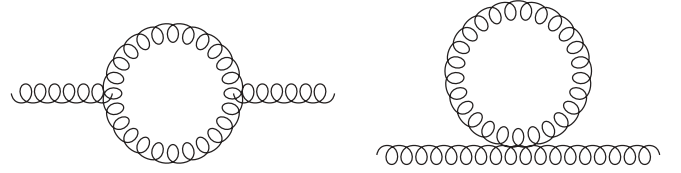


FIG. 4. Self-corrections of the gluon propagator.

of [69], and may also be found in [53], and is given by

$$\begin{aligned} \Pi_{\text{gmat}}^{00;ab}(q_0, |\mathbf{q}|) &= -\delta^{ab} \frac{g^2 N}{4\pi^2} \int_0^\infty dk k N_B(k) \\ &\times \text{Re} \left\{ 4 - \frac{(\mathbf{q}^2 - 2kq_0 - q_0^2)(2k + q_0)^2}{2k^2(k + q_0)^2} \right. \\ &+ \frac{(2k + q_0)^2}{2k|\mathbf{q}|} \left[ 1 + \frac{[k^2 + (k + q_0)^2 - \mathbf{q}^2]^2}{4k^2(k + q_0)^2} \right. \\ &\left. \left. \times \ln \left( \frac{R_{g+}}{R_{g-}} \right) \right] \right\}, \quad q_0 = i2\pi nT, k \equiv |\mathbf{k}| \end{aligned} \quad (11)$$

where  $N_B(k)$  is defined in (A10a),  $R_{g\pm} = \mathbf{q}^2 - 2kq_0 - q_0^2 \pm 2k|\mathbf{q}|$ , while the operator  $\text{Re}$  is defined in (A12). In the limits in which we are interested [see (8)], we eventually obtain that the matter (thermal) part contribution is

$$\begin{aligned} \Pi_{\text{gmat}}^{00;ab}(0, |\mathbf{q}| \rightarrow 0) &= -\delta^{ab} N g^2 \left( \frac{1}{3} T^2 - \frac{1}{4} T |\mathbf{q}| \right. \\ &\left. + O[\mathbf{q}^2 \log(\mathbf{q}^2/T^2)] \right), \quad q \ll T \end{aligned} \quad (12)$$

where we have used the left equation of (A13b) and (A14b) to extract the zeroth order and first order in  $|\mathbf{q}|$ , respectively.

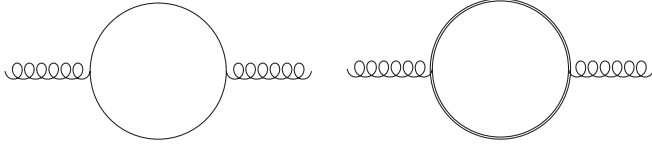


FIG. 5. Corrections due to the fermions (left) and the gauginos (right).

### B. Contribution of $\Pi_f^{00}$ due to the fermion loops

This is the contribution due to the diagrams of Fig. 5. For a Dirac fermion, this contribution has been calculated in the literature and has been reproduced by us. For instance, in [69], equation (5.51) gives the answer (matter contribution) for an Abelian gauge theory (QED). To adapt the answer to our case, we must multiply (5.51) by  $f^{acd}f^{acd} = \delta^{ab}N$  to account for the color factors times 4 to account for the four fermions [one gaugino and three adjoint fermions; see (B3)]. [Assume that they have the same chemical potentials  $\mu_i$  for simplicity; in any case, the  $\mu_i$ 's will not contribute (to zeroth order) in view of (8).] Finally, we should multiply by (1/2) to account for the fact that we deal with Weyl spinors with a corresponding trace that involves (four) sigma (instead of gamma) matrices; in view of (A9), the overall factor is 2 instead of 4, which occurs with the gamma matrices. The antisymmetric part in the trace cancels because  $\Pi^{00}$  is symmetric in the space-time indices. Therefore, we effectively multiply equation (5.51) of [69] by  $2N\delta^{ab}$  assuming massless fermions yielding to

$$\begin{aligned} \Pi_{f\text{mat}}^{00;ab}(q_0, |\mathbf{q}|) &= -2\delta^{ab} \frac{g^2 N}{\pi^2} \text{Re} \int_0^\infty dk k [N_F^+(k) + N_F^-(k)] \\ &\quad \times \left[ 1 + \frac{4kq_0 - 4k^2 - q_0^2 + q^2}{4k|\mathbf{q}|} \ln \left( \frac{R_{f+}}{R_{f-}} \right) \right], \\ q_0 &= i2\pi nT \end{aligned} \quad (13)$$

where  $R_{f\pm} = q_0^2 - \mathbf{q}^2 - 2q_0k \pm 2k|\mathbf{q}|$ . In the limits that we are concerned with [see (8) and (9)], we obtain that the matter (thermal) part is

$$\begin{aligned} \Pi_{f\text{mat}}^{00;ab}(0, |\mathbf{q}| \rightarrow 0) &= -4\delta^{ab} \frac{g^2 N}{\pi^2} \int_0^\infty dk k N_F(k) \\ &\quad \times \left[ 1 - \frac{k}{|\mathbf{q}|} \ln \left( \frac{R_{f+}(q^0=0)}{R_{f-}(q^0=0)} \right) \right] + O(\mathbf{q}^2) \\ &= -\frac{2}{3} \delta^{ab} N g^2 T^2 + O(\mathbf{q}^2), \end{aligned} \quad (14)$$

where, in extracting the zeroth contribution to  $|\mathbf{q}|$ , we have used (the right integral of) (A13b). We point out that there is no linear term in  $q^\mu$ , unlike in (12), and, that in extracting the leading terms in  $|\mathbf{q}|$ , one should exercise caution [for example, see proof of (A14a)].

### C. Contribution of $\Pi_{s_i}^{00}$ due to the scalar loops

The contributions of  $\Pi_{s_i}^{00}$  due to the scalar loops are shown in the two diagrams of Fig. 6. The first contribution ( $s_1$ ) is due to the diagram on the left of the figure and has been computed in the literature for the case of  $\phi^4$  theory [see equation (3.48)

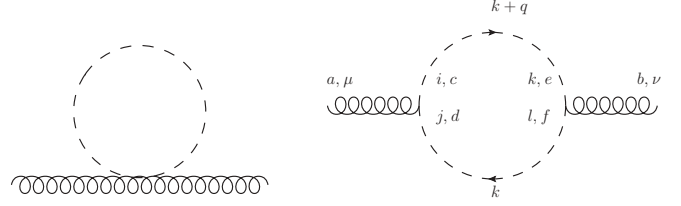


FIG. 6. Corrections due to the scalars.

of [69]). To adapt it for our case, we should, according to the Feynman rule of Fig. 7 (in the middle), replace  $\lambda \rightarrow g^2(f^{cad}f^{dbe} + f^{cbd}f^{dae})\delta^{ce}\delta_{ii}g^{00} = -6g^2N\delta^{ab}$  as expected. [A factor of 2 exists in scalar QED, while a factor 3 accounts for the three flavors of the complex (colored) scalars in the loop. The additional minus sign is traced simply to the fact that, instead of  $-\lambda$ ,  $g^2$  is used. The chemical potentials are again ignored.] We find that the contribution of this diagram is

$$\begin{aligned} \Pi_{s_1\text{mat}}^{00;ab}(q_0, |\mathbf{q}|) &= 6g^2N\delta^{ab}T \sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{(k_n^0)^2 - k^2} \\ &= -6g^2N\delta^{ab} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{k} N_B(k) \\ &= -\delta^{ab} \frac{1}{2} g^2 N T^2, \end{aligned} \quad (15)$$

where  $k_n^0 = i2\pi nT$ ,  $n \in \mathbb{Z}$ ,  $k \equiv |\mathbf{k}|$ , and  $N_B$  is given by (A10a). The first equality is a direct application of the Feynman rules for thermal field theory, the second equality follows by integrating in using (D2), and the third equality uses the left integral of (A13b). We note that this diagram is  $q^\mu$  independent.

The second contribution ( $s_2$ ) is due to the diagram on the right of Fig. 6 and which we evaluate in Appendix D [we evaluate this diagram in great detail (also ignoring the chemical potential contribution) as the rest of the five diagrams, see (12), (14), and (15), are evaluated in an analogous way] obtaining Eq. (D5). For  $q^0 = 0$ , which is what interests us, we have

$$\begin{aligned} \Pi_{s_2\text{mat}}^{00;ab}(q_0, |\mathbf{q}|) &= -3 \frac{g^2}{4\pi^2} \delta^{ab} \int_0^\infty dx \frac{1}{2} x^2 |\mathbf{q}|^2 \ln \left( \frac{1+x}{|1-x|} \right) \\ &\quad \times N_B \left( \frac{|\mathbf{q}|}{2} x \right), \end{aligned} \quad (16)$$

where we have made for the integral (D5) the substitution  $2k = |\mathbf{q}|x$ . Extracting the leading and subleading contribution for small  $|\mathbf{q}|$  is not straightforward, but it may be achieved by working in the same way as in proving (A14a) yielding

$$\Pi_{s_2\text{mat}}^{00;ab}(0, |\mathbf{q}| \rightarrow 0) = -\frac{1}{2} \delta^{ab} N g^2 T^2 + O(|\mathbf{q}|^2). \quad (17)$$

It is crucial to highlight that, as in the fermion case, the scalars have no linear contribution in  $|\mathbf{q}|$ . This completes the set of contributions to the order in  $g$  and  $|\mathbf{q}|$  that we are interested in.

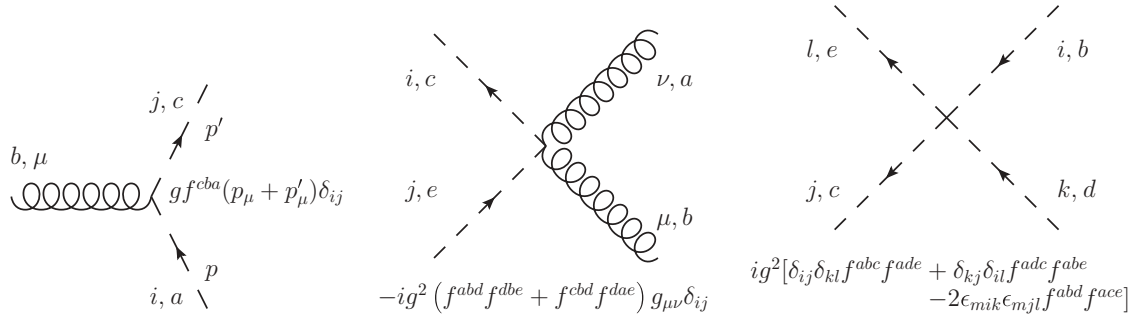


FIG. 7. Scalar-gluon and scalar-scalar interactions.

**IV. CALCULATING THE POTENTIAL**

Collecting the results from (12), (14), (15), and (17), the self-energy tensor to  $O(g^2)$  is

$$\Pi_{\text{mat}}^{00}(q_0 = 0, |\mathbf{q} \rightarrow 0|) = -m_D^2 + 2m_D t |\mathbf{q}| + O[\mathbf{q}^2 \log(\mathbf{q}^2/T^2)], \quad (18)$$

where  $m_D$  is the Debye mass and is given by

$$m_D^2 = 2Ng^2T^2. \quad (19)$$

This is one of the main results in this paper. On the other hand,  $t$  is positive and is given by

$$t = \frac{1}{16} \frac{m_D}{T}. \quad (20)$$

**A.  $V(r)$  from pure electric mode corrections**

Performing the integral of (9) using just the  $m_D^2$  for  $\Pi^{00}$ , we obtain

$$V(r) = -\frac{g^2 N}{4\pi} \frac{1}{r} e^{-m_D r}, \quad (21)$$

which is the expected Yukawa potential.

**B.  $V(r)$  from magnetic mode corrections**

One may push the calculation a little further and include corrections to the potential due to the linear contribution of  $|\mathbf{q}|$  in  $\Pi(0, \mathbf{q})$ . As this term is gauge invariant [50,70], it is tempting to try to include it in the evaluation of the potential extending the result of (21). Although it has not been rigorously proved that this expansion in  $|\mathbf{q}|$  is well defined, it is believed [50] that it is very likely to be the case. Assuming this, and applying the analysis of [50] for our case, we find that

$$V(r)|_{rT \gg 1} = \frac{2g^2 N}{\pi^2} \frac{t}{m_D^3 r^4} = \frac{1}{(4\pi)^2} \frac{1}{T^3 r^4}, \quad (22)$$

which shows that, at sufficiently large distances, the potential falls off as  $1/r^4$  and is also repulsive and  $N$  and  $g$  independent. This is the second main part of our investigation.

**V. DISCUSSION**

In this paper, we perform the calculation of the  $q\bar{q}$  potential in a thermal medium for the  $\mathcal{N} = 4$  SUSY theory at weak coupling. By considering the purely Yukawa modes at high temperatures, we find the expected Yukawa potential [Eq. (21)] with the Debye mass given by Eq. (19). In particular, we observe that each of the  $[8 \times (N^2 - 1)]$  bosonic degrees of freedom contribute to  $m_D^2$  (on average) with  $N/(N^2 - 1) \times 1/6 g^2 T^2$  [see (12), (15), and (17)], while each of the  $[8 \times (N^2 - 1)]$  fermionic degrees of freedom contributes (on average) with  $N/(N^2 - 1) \times 1/12 g^2 T^2$  [see (14)] leading to (19).

Next, and following [50], we include (a subset of the) magnetic corrections obtaining the potential of Eq. (22), which applies at large enough distances. In this approximation, the potential is independent from the coupling and the number of colors and falls off as  $1/r^4$  but it is repulsive (as in [50]). On the other hand, from AdS/CFT calculations, it was found [21] that the same power-law falloff at large distances but with an attractive force between the  $q\bar{q}$ . Motivated by that result, we were hoping, by including the scalar contributions for the  $\mathcal{N} = 4$  SUSY theory at weak coupling, to obtain the result of [50] with an additional overall negative sign agreeing with [21] and also with our intuition. We find that both the fermions and the scalars do not contribute to the magnetic modes (linearly in  $|\mathbf{q}|$ ) and, hence, the analysis of [50] proceeds (up to an overall factor) unaltered yielding to the repulsive power-law falloff potential of Eq. (22), which is  $N$  and  $g^2$  independent. We note that a power-law falloff potential for a singlet state in the QCD plasma was also found in [55]. We would like to point out the work of [60], in which a different definition for the Debye mass is given. By using this definition for  $m_D$ , the author shows that the prefactor of the Debye falloff of the potential may be gauge dependent. However, the screening mass itself, Eq. (19), is indeed gauge invariant.

This paper provides a concrete example of an observable of  $\mathcal{N} = 4$  SUSY, the behavior of which is qualitatively the same as the corresponding behavior of QCD. Consequently, one may conclude that studying nonperturbative phenomena of QCD by applying the AdS/CFT correspondence may not be far from reality and has the potential to yield to qualitatively correct results (see [71] for a related discussion).

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## APPENDIX A: CONVENTIONS, NOTATION, AND USEFUL FORMULAS

The metric tensor and the four-vectors follow the standard conventions

$$g_{\mu\nu} = (1, -1, -1, -1), \quad k^\mu = (k^0, \mathbf{k}) = (k^0, k^i),$$

$$\mu = 0; 1, 2, 3 = 0; i. \quad (\text{A1})$$

We define the  $\beta$  factor to be inversely proportional to the temperature  $T$ :

$$\beta = \frac{1}{T}. \quad (\text{A2})$$

The generators of  $SU(N)$ ,  $T^a$ , in the fundamental representation are normalized as

$$\text{Tr}[T^a T^b] = \frac{1}{2} \delta^{ab}, \quad a, b = 1, 2, \dots, N \quad (\text{A3})$$

and they obey the algebra

$$[T^a, T^b] = i f^{abc} T^c \quad \text{where } f^{abc} \text{ are the structure constants.} \quad (\text{A4})$$

The adjoint representation is defined by

$$(T^b)_{ac} = i f^{abc}. \quad (\text{A5})$$

The following summation formula for the structure constants is valid:

$$f^{acd} f^{bcd} = N \delta^{ab}. \quad (\text{A6})$$

The covariant derivative in the fundamental and the adjoint representation for **negative charge** {this convention agrees with [72] [see Eq. (2.96)], [73] [see Eq. (4.3.10)], and [74] (see exercise 7, Chap. VII)}, that is, for charge  $-g = -|g|$ , are defined by

$$D_\mu = \mathbb{I}_N \partial_\mu + i g A_\mu^a T^a, \quad \delta^{ac} D_\mu = \delta^{ac} \partial_\mu - g A_\mu^b f^{abc}. \quad (\text{A7})$$

We define the Pauli matrices in a four-vector form with dotted and undotted indices as in [73]:

$$\sigma_{\alpha\dot{\alpha}}^\mu = (\mathbb{I}_2, \sigma), \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} = (\mathbb{I}_2, -\sigma). \quad (\text{A8})$$

The trace formulas for four (alternate) sigma matrices are

$$\text{Tr}[\sigma^\mu \bar{\sigma}^\nu \sigma^\rho \bar{\sigma}^\kappa] = 2(g^{\mu\nu} g^{\rho\kappa} - g^{\mu\rho} g^{\nu\kappa} + g^{\mu\kappa} g^{\nu\rho} + i \epsilon^{\mu\nu\rho\kappa}), \quad (\text{A9a})$$

$$\text{Tr}[\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\rho \sigma^\kappa] = 2(g^{\mu\nu} g^{\rho\kappa} - g^{\mu\rho} g^{\nu\kappa} + g^{\mu\kappa} g^{\nu\rho} - i \epsilon^{\mu\nu\rho\kappa}). \quad (\text{A9b})$$

We define the fermion and the boson distributions in the presence of a chemical potential (which is a consequence of some global symmetry) by

$$N_F^\pm(p) = \frac{1}{\exp[\beta(E_p \pm \mu_F)] + 1}, \quad (\text{A10a})$$

$$N_B^\pm(p) = \frac{1}{\exp[\beta(E_p \pm \mu_B)] - 1}, \quad \beta = 1/T$$

$$N_F(p) = N_F^\pm(\mu_F = 0, p), \quad N_B(p) = N_B^\pm(\mu_B = 0, p). \quad (\text{A10b})$$

We will also need the summation formulas

$$T \sum_n f(p^0 = i\omega_n + \mu)$$

$$= \pm \frac{1}{2\pi i} \int_{-i\infty+\mu+\epsilon}^{i\infty+\mu+\epsilon} dp^0 \frac{f(p^0)}{e^{\beta(p^0-\mu)} \mp 1}$$

$$\pm \frac{1}{2\pi i} \int_{-i\infty+\mu-\epsilon}^{i\infty+\mu-\epsilon} dp^0 \frac{f(p^0)}{e^{-\beta(p^0-\mu)} \mp 1}$$

$$+ \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dp^0 f(p^0) + \frac{1}{2\pi i} \oint_C dp^0 f(p^0),$$

$$\epsilon > 0, \quad n \in \mathbb{Z} \quad (\text{A11a})$$

$$\omega_n = \frac{2\pi}{\beta} n \quad \text{for bosonic modes,} \quad (\text{A11b})$$

$$\omega_n = \frac{2\pi}{\beta} (2n + 1) \quad \text{for fermionic modes,}$$

and where  $C$  is a closed rectangular contour with long sides along the real axis at  $p^0 = 0$  and  $p^0 = \mu$ , which extend from  $i\infty$  to  $-i\infty$  and from  $-i\infty + \mu$  to  $+i\infty + \mu$ , respectively. The contour closes at infinity and it evidently has a counterclockwise direction. We note that the chemical potential  $\mu$  is not generally the same for bosons and fermions and, hence, an extra index is (generally) required. The first two terms of (A11a) show the thermal contribution of the particle and antiparticle and they vanish at  $T = 0$ , the third term is the vacuum piece at  $T = 0$  just as in ordinary field theory, while the last term is the contribution of the matter at finite density but at  $T = 0$ .

It is useful to define

$$\text{Re}[f(k_0, \mathbf{k}, x)] \equiv \frac{1}{2} [f(k_0, \mathbf{k}, x) + f(-k_0, \mathbf{k}, x)]$$

for any set of parameters  $x$ . (A12)

Finally, we will need the integrals

$$\int_0^\infty dp \frac{p}{\exp[\beta(p-\mu)] \mp 1} = \pm T^2 Li_2(\pm e^{\mu/T}), \quad (\text{A13a})$$

$$\begin{aligned} \int_0^\infty dp \frac{p}{\exp(\beta p) - 1} &= \frac{\pi^2 T^2}{6}, \\ \int_0^\infty dp \frac{p}{\exp(\beta p) + 1} &= \frac{\pi^2 T^2}{12}, \end{aligned} \quad (\text{A13b})$$

where  $Li_2$  is the dilogarithm function. We also need the asymptotic expansions

$$\int_0^\infty dx \frac{1}{e^{ax} - 1} \ln\left(\frac{1+x}{|1-x|}\right) = \frac{\pi^2}{2a} + \ln\left(\frac{a}{2\pi e}\right) + \gamma_E + O(a), \quad (\text{A14a})$$

$$\begin{aligned} \int_0^\infty dk \frac{1}{e^{k/T} - 1} \ln\left(\frac{q+2k}{|q-2k|}\right) \\ = \frac{1}{2}\pi^2 T + \frac{1}{2}q \ln\left(\frac{q}{4\pi e T}\right) + \frac{1}{2}\gamma_E q + O(q^2), \end{aligned} \quad (\text{A14b})$$

where  $\gamma_E$  is the Euler constant while (A14b) follows directly from (A14a). We sketch the proof of (A14a) below.

*Proof of (A14a):* In order to prove (A14a), one has to perform the following steps. Break the interval of integration into the subintervals  $I_1 = (0, 1)$  and  $I_2 = (1, \infty)$ . For the interval  $I_1$ , expand the logarithm in powers of  $x$  and also expand  $(e^{ax} - 1)^{-1} = 1/ax - 1/2 + O(ax)$  and multiply it with the series resulting from the logarithm. Then, perform the integrations recognizing singular and zeroth terms in  $a$ . Resum the (two resulting) series to obtain  $I_1 = \pi^2/4a - \ln(2)$ . For  $I_2$ , expand the logarithm in inverse powers of  $x$  and make the transformation  $ax \rightarrow x$ . This makes the integration range  $(1, \infty) \rightarrow (a, \infty)$ . Break this new integration region from  $I_2^b = (a, 1)$  to  $I_2^a = (1, \infty)$ . The  $I_2^a$  range gives only a zeroth-order contribution, which is given by  $I_2^a = 2 \int_1^\infty \frac{dx}{x} \frac{1}{e^x - 1}$ . For the  $I_2^b$  range, expand  $(e^{ax} - 1)^{-1} = c_{-1}/ax + c_0 + \sum_{n \geq 1} c_n x^n$  for appropriate coefficients  $c_n$  with  $c_{-1} = 1$  and  $c_0 = -1/2$ . The inverse powers of  $a$  are in the  $c_{-1}$ , which, when the corresponding series (due to the logarithm) is resummed, gives  $\pi^2/4a - 2$ . The  $\ln(a)$  term comes from  $c_0$ , where there is no series to be resummed. The zeroth orders in  $a$  come either from  $c_0$  and an appropriate series, which yields to  $\ln(2) - 1$  or from  $2 \sum_{n \geq 1} \int_0^1 c_n x^{n-1} = 2 \sum_{n \geq 1} c_n/n$ . This series may be written as  $2 \int_0^1 \frac{dx}{x} (\frac{1}{e^x - 1} - 1/x + 1/2)$ . Add the pieces to obtain  $I = I_1 + I_2 = I_1 + (I_2^a + I_2^b)$ , which yields to  $I = \pi^2/2a + \ln(a) - 1 + 2[-1 + \int_0^1 \frac{dx}{x} (\frac{1}{e^x - 1} - \frac{1}{x} + \frac{1}{2}) + \int_1^\infty \frac{dx}{x} \frac{1}{e^x - 1}] + O(a)$ . Now, the bracket is evaluated by noting that it has well-behaved integrals and it equals to the zeroth-order term of  $\zeta(s)\Gamma(s) = \int_0^\infty dx \frac{x^{s-1}}{e^x - 1}$  as  $s \rightarrow 0$ . But, this expansion equals  $-1/2s + 1/2[\gamma_E - \ln(2\pi)] + O(s)$  and this completes the proof.

## APPENDIX B: A BRIEF INTRODUCTION TO $\mathcal{N} = 4$ SUSY

### 1. The Lagrangian: From $\mathcal{N} = 1$ superfields to component fields

We fix our notation by assigning  $\lambda^a$  to the gaugino,  $f^{abc}$  are the structure constants of the gauge group  $SU(N)$ , and so  $a, b, c = 1, 2, \dots, N^2 - 1$  while  $i, j, k = 1, 2, 3$  are the three flavors of the fermions  $\psi_j^a$ . The fermions and the gaugino are left Weyl spinors. There is one gauge field and three colored complex scalars  $\phi_i^a$  that match the fermionic degrees of freedom.

The  $\mathcal{N} = 4$  SUSY Lagrangian with a non-Abelian group may be written compactly as

$$\mathcal{L}_{\mathcal{N}_4} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_{\text{WZ}_{YM}} + \mathcal{W}_{\mathcal{N}_4}, \quad (\text{B1})$$

where the first two terms may be found in any supersymmetry textbook [72,74]. These two terms always exist in any  $\mathcal{N} = 1$  SYM, they respect  $\mathcal{N} = 4$  SYM trivially and may be written easily in terms of  $\mathcal{N} = 1$  superfields [see first two terms of (7.6) and exercise 7 of Chap. VII in [74] or (2.134)–(2.136) of [72]], while the last term is a particular superpotential (given below) which respects  $\mathcal{N} = 4$  in a nontrivial way. In particular,  $\mathcal{L}_{\text{SYM}}$  contains the first two terms of (B3) (see below) and, in addition, the bosonic auxiliary field  $D^a$  in the form  $1/2 D^a D^a$ . The  $\mathcal{L}_{\text{WZ}_{YM}}$  (Wess-Zumino gauged terms) contains the gauge interactions of the  $\psi$ 's and  $\phi$ 's, the  $\lambda\psi\phi$  interactions [see third, fourth, and fifth terms of (B3), respectively], the term  $ig f^{abc} D^b \phi_i^{\dagger a} \phi_j^c$  and also the combination  $F_i^a F_i^{\dagger a}$ , where  $F_i^a$  is another auxiliary field. Finally,  $\mathcal{W}_{\mathcal{N}_4}$  written in terms of  $\mathcal{N} = 1$  superfields  $\Phi_i^a$  [see Eq. (14.32) of [72]] yields

$$\begin{aligned} \mathcal{W}_{\mathcal{N}_4} &= \frac{g}{3\sqrt{2}} \epsilon_{ijk} f^{abc} \Phi_i^a \Phi_j^b \Phi_k^c|_{\theta\theta} + \text{H.c.} \\ &= \frac{g}{\sqrt{2}} \epsilon_{ijk} f^{abc} (F_i^a \phi_j^b \phi_k^c - \psi_i^a \psi_j^b \phi_k^c) + \text{H.c.}, \end{aligned} \quad (\text{B2})$$

where, in the second equality, we have expanded out the  $\Phi$ 's according to Eqs. (5.3) and (5.8) of [74]. Thus, the  $\mathcal{W}_{\mathcal{N}_4}$  contribution gives the sixth term of (B3) and one additional term containing the auxiliary field  $F_i^a$ . The remaining  $\phi^4$  terms of (B3) are obtained by eliminating the two auxiliary fields in terms of the scalars setting the action on shell and yielding to (14.33) of [72] [we choose the standard conventions for the fermions as in [73,74] rather than the convention of [72] used for (14.33)]:

$$\begin{aligned} \mathcal{L}_{\mathcal{N}_4} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a - i\psi_i^{\dagger a} \bar{\sigma}^\mu D_\mu \psi_i^a \\ &\quad + D^\mu \phi_i^{\dagger a} D_\mu \phi_i^a - g\sqrt{2} f^{abc} (\phi_i^{\dagger c} \lambda^a \psi_i^b - \psi_i^{\dagger c} \lambda^{\dagger a} \phi_i^b) \\ &\quad - \frac{g}{\sqrt{2}} \epsilon_{ijk} f^{abc} (\phi_i^c \psi_j^a \psi_k^b + \psi_i^{\dagger c} \psi_j^{\dagger a} \phi_k^{\dagger b}) \\ &\quad + \frac{g^2}{2} (f^{abc} \phi_i^b \phi_i^{\dagger c}) (f^{ade} \phi_j^d \phi_j^{\dagger e}) \\ &\quad - \frac{g^2}{2} \epsilon_{ijk} \epsilon_{ilm} (f^{abc} \phi_j^b \phi_k^c) (f^{ade} \phi_l^{\dagger d} \phi_m^{\dagger e}). \end{aligned} \quad (\text{B3})$$

## 2. The vanishing of the beta function

It is known that the beta function for an SU( $N$ ) gauge theory (for positive charge  $g$ ) is given by

$$\beta(g) = -\frac{1}{(4\pi)^2} g^3 \left( \frac{11}{3} N - \frac{4}{3} \frac{1}{2} n_f \right), \quad (\text{B4})$$

where  $n_f$  is the number of Dirac fermions while the factor of  $1/2$  comes from the trace of two generators of the group. In the  $\mathcal{N} = 4$  SUSY case, one should replace the  $1/2$  by  $N$  as the trace takes place in the adjoint representation and set  $n_f = 1/2 \times (3 + 1) = 2$  for the four (three matter fermions and one gaugino) Weyl spinors. On the other hand, the contribution of the scalars to the gauge boson self-energy should be included as well. {We note that there are no additional corrections to the self-energy of the fermions as the additional interactions that exist in the  $\mathcal{N} = 4$  theory concerning the fermions would completely change the fermion [see fourth term of (B3)] or the flavor and the color [see fifth term of (B3)].}

This contribution is due to the diagrams of Fig. 6 where the complex scalar QED case yields to a combined contribution of  $1/(48\pi^2)g^3$  [75]. Hence, for our case, one should replace  $g^2 \rightarrow 3f^{acd}f^{bcd}g^2 = 3Ng^2\delta^{ab}$  in order to account for the trace of the three adjoint (complex) scalars inside the loops yielding to  $1/(4\pi)^2 3/3 g^3 N$ . Therefore, by adding all the contributions, one gets

$$\beta(g) = -\frac{1}{(4\pi)^2} g^3 \left( \frac{11}{3} N - \frac{4}{3} 2N - \frac{3}{3} N \right) = 0, \quad (\text{B5})$$

that is the vanishing of the beta function.

## 3. Feynman rules at $T = 0$

The Feynman rules for two-dimensional spinors may be found in [73], while the rest may be either derived from the Lagrangian (B3) or they are known. As we work in TAG, there are no ghost propagators and vertices. The notation for

the indices we use is

$$\begin{aligned} i, j, k &= 1, 2, 3 \quad \text{for flavor,} \\ a, b, c &= 1, 2, \dots, N^2 - 1 \quad \text{for color,} \\ \alpha, \dot{\alpha} &= 1, 2 \quad \text{for spinors.} \end{aligned} \quad (\text{B6})$$

## 4. Feynman rules at finite $T$

The Feynman rules are obtained from those of  $T = 0$  using the following empirical rule: by multiplying all of the rules for  $T = 0$  by  $(-i)$  and by including an additional minus sign to the gluon propagator (Fig. 1, upper left), and the gluon-gluon interactions (left and middle diagrams of Fig. 8). In addition, the following modifications are required:

$$\begin{aligned} \int \frac{d^4 p}{(2\pi)^4} &\rightarrow T \sum_n \int \frac{d^3 \mathbf{p}}{(2\pi)^3}, \quad p_0 \rightarrow i2\pi T \times n \text{ (bosons)} \\ &\text{or } \times (2n + 1) \text{ (fermions), } \quad n \in \mathbb{Z}. \end{aligned} \quad (\text{B7})$$

## APPENDIX C: DIAGRAMS FOR THE DRESSED PROPAGATOR TO $O(g^2)$

There are six diagrams to  $O(g^2)$  that contribute, and they all involve the gluon propagator as, according to (B3), the scalar vertices change the flavor of the outgoing fermions and, hence, the related diagrams do not contribute.

## APPENDIX D: EVALUATING $\Pi_{s_2}^{00}$

Using the appropriate Feynman Rules of Figs. 1 and 7, the right diagram of Fig. 6 reads as

$$\begin{aligned} \Pi_{s_2}^{00ab} &= g^2 \delta_{ii} f^{cad} f^{fbe} \delta^{ce} \delta^{df} T \sum_n \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \\ &\times \frac{(2k^0 + q^0)(-2k^0 - q^0)}{[(k^0)^2 - \mathbf{k}^2][(k^0 + q^0)^2 - (\mathbf{k} + \mathbf{q})^2]}, \end{aligned} \quad (\text{D1a})$$

$$k^0 = i2\pi T n, \quad q^0 = i2\pi T m, \quad n, m \in \mathbb{Z}. \quad (\text{D1b})$$

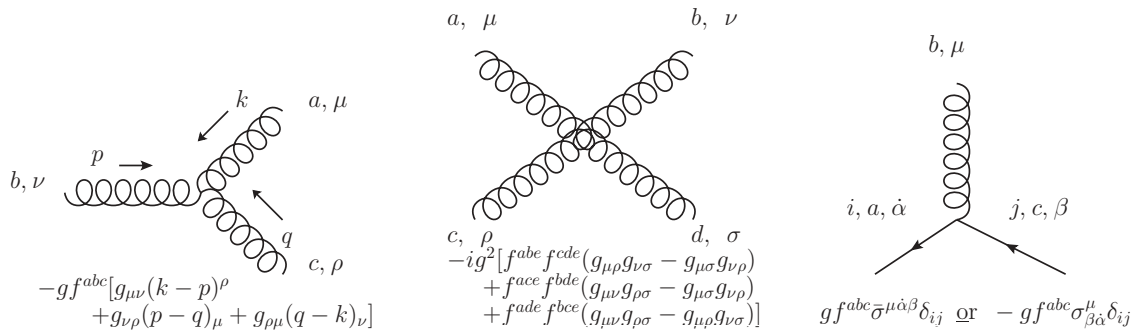


FIG. 8. Gluon-gluon and gluon-fermion interactions (with negative charge  $g$ ). For the gluon-fermion vertex, a similar comment as the one in the caption of Fig. 1 applies regarding the choice of the right rule. There also exists the analog rule for the gaugino-gluon vertex without the  $\delta_{ij}$ .



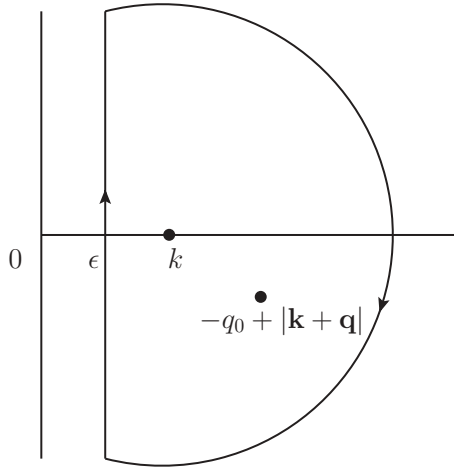


FIG. 9. Contour of integration in the  $\tilde{k}^0$  complex plane.

The thermal part of the summation formula (A11a) for bosons at zero chemical potential may take the form

$$T \sum_n f(i2\pi Tn) = \frac{1}{2\pi i} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} dk^0 [f(k^0) + f(-k^0)] N_B(k^0). \quad (\text{D2})$$

However, the corresponding  $f(k^0)$  in the case of (D1a) may be taken to be the integrand, which has the property that the operation  $k^0 \rightarrow -k^0$  is equivalent to  $q^0 \rightarrow -q^0$ . This observation will reduce our operations by a factor of 2 as we will only deal with the  $+k^0$  piece and then just add the  $-q^0$  contribution. In order to perform the integration, we use the the

contour of Fig. 9, where there exist two simple poles. Using (A6) and performing the sum (in practice the integrations), (D1a) becomes

$$\begin{aligned} \Pi_{s_2}^{00ab}|_{+q^0} = & 3g^2 N \delta^{ab} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left( \frac{(2k + q^0)^2 N_B(k)}{2k[(k + q^0)^2 - (\mathbf{k} + \mathbf{q})^2]} \right. \\ & \left. + \frac{(2|\mathbf{k} + \mathbf{q}| - q^0)^2 N_B(-q^0 + |\mathbf{k} + \mathbf{q}|)}{2[(-q^0 + |\mathbf{k} + \mathbf{q}|)^2 - k^2]|\mathbf{k} + \mathbf{q}|} \right), \end{aligned} \quad (\text{D3})$$

where  $k \equiv |\mathbf{k}|$ . Now, using the property that  $N_B(x + i2\pi n) = N_B(x) \forall x$  and changing variables in the second piece of the integrand from  $\mathbf{k}$  to  $-\mathbf{k} - \mathbf{q}$  results in the symmetrization of the two integrands under  $q^0 \leftrightarrow -q^0$ :

$$\begin{aligned} \Pi_{s_2}^{00ab}|_{+q^0} = & 3g^2 N \delta^{ab} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{N_B(k)}{2k} \\ & \times \left( \frac{(2k + q^0)^2}{[(k + q^0)^2 - (\mathbf{k} + \mathbf{q})^2]} + \frac{(2k - q^0)^2}{[(k - q^0)^2 - (-\mathbf{k} - \mathbf{q})^2]} \right). \end{aligned} \quad (\text{D4})$$

Finally, taking into account the  $-q^0$  contribution [effectively multiplying (D4) by a factor of 2] and performing the angular integrations yields

$$\begin{aligned} \Pi_{s_2}^{00ab} = & 3 \frac{g^2}{4\pi^2} N \delta^{ab} \text{Re} \int_0^\infty dk \frac{1}{|\mathbf{q}|} (2k + q^0)^2 \ln \left( \frac{R_{+s}(q^0)}{R_{-s}(q^0)} \right) \\ & N_B(k), \end{aligned} \quad (\text{D5})$$

where the operator Re is defined in (A12) while  $R_{\pm s} = (q^0)^2 - \mathbf{q}^2 + 2kq^0 \pm 2k|\mathbf{q}|$  and  $k \equiv |\mathbf{k}|$ .

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