Born term of the πN scattering amplitude in the Skyrme model

Tomoaki Itoh and Masaki Arima*

Department of Physics, Faculty of Science, Osaka City University, Osaka 558-8585, Japan
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The Skyrme model is applied to the study of the πN scattering amplitude. The useful expression of amplitude given by the chiral reduction formula is employed. The calculation is performed in the lowest order of $1/N_c$: a source of interaction is a classical soliton taking a hedgehog configuration. It is important to consider the zero modes both for the translation invariance and for the isospin symmetry simultaneously. Despite the former negative impression, the Skyrme model correctly produces the Born term in the scattering amplitude.

I. INTRODUCTION

Skyrme's inventive idea of baryon structure was revived by Adkins, Nappi and Witten, by whom a baryon is described as a soliton of the nonlinear pseudoscalar meson field. The Skyrme model reproduces the static properties of nonstrange baryons (N and Δ) with about "30% accuracy" [1].

If the Skyrme model were also effective for the dynamical properties of baryons, we would accept it as a reliable model for low-energy QCD [2]. Among the many people who have attempted to apply this model to scattering problems, Hayashi *et al.* obtained negative results regarding its applicability [3,4]. Although the general canonical quantization method for the nonlinear field theory has been used [5–7], the Born term was not derived from the classical soliton solution and its zero modes in the Skyrme model.

The Born term is actually suppressed in the lowest order calculation when the isoscalar field is considered. The mechanism of this suppression is relevant to the zero mode for the spatial translation invariance of the soliton. This argument is not, however, applicable to the Skyrme model straightforwardly: there is another zero mode related to the isospin rotation of the soliton.

The claim in Ref. [3] regarding the Born term is not decisive regarding the Skyrme model, because the isospin rotation is completely excluded from consideration. It is not quite evident whether the Born term disappears from the Skyrme model as explained in Ref. [3]. Another method of analysis using the Feynman path integral was also tried by Dorey *et al.* [8,9], but the Born term problem in the Skyrme model remained to be solved.

In this paper we show that the Born term actually exists in the πN amplitude if all the zero modes are properly taken into account in the classical soliton solution. Thus the Skyrme model is treated consistently with the general quantization method for the nonlinear field theory. Because the recent progress of the string theory reinforces the argument of a large N_c world of QCD, it is worth examining the Skyrme model in more depth as an effective model of hadron phenomenology.

Our approach to the Born term problem is different from that of Hayashi *et al.* They used the popular LSZ formula. We apply the chiral reduction formula developed by Yamagishi and Zahed [10] to examine the scattering problem in the Skyrme model.

The S matrix is determined by the field operator and its time evolution, which usually takes the form of a differential operator on the time-ordered product of fields in the LSZ formula. There exists another specific method for the theory with chiral symmetry: The time evolution of the chiral field is subjected to the Veltman-Bell equation along with the chiral currents given by Schwinger's action principle and the external field method. This equation reduces the S matrix as the sum of responses to external sources. This is the chiral reduction formula.

The dynamical aspect of the chiral field is determined by the currents and their algebraic relations [11], so that the chiral reduction formula is essentially composed of chiral currents. Although the spontaneous breaking of symmetry gives a nonlinear character to the field and makes its definition arbitrary, this does not matter for this formula because the chiral currents are uniquely defined everywhere. The Nambu-Goldstone boson is taken into account as an asymptotic form of the axial vector current. Its validity is not limited to the πN threshold region; the chiral dynamics of hadron resonances is also within the scope of this formula [12,13].

A practical advantage is that the chiral reduction formula is free of the differential operator, and each term in this formula respects all symmetry. In the popular LSZ formula, in contrast, the differential operator produces terms that apparently break the Lorentz invariance, such as the Schwinger term. Careful treatment is required to make sure that they are finally canceled out.

Crucial to our attempt is the proper treatment of the zero mode for the isospin rotation in the lowest order calculation with respect to $1/N_c$. The collective coordinate method of quantization is used to consider the spin-isospin property in the Skyrme model [1]. If this property is overlooked in the analysis, we will not find the Born term in the πN scattering amplitude.

We note that because the soliton is regarded as a heavy object with the mass of order N_c, the nonrelativistic approximation is always assumed. The relativistic collection is included if the higher order contributions with respect to $1/N_c$ is considered.

*arima@sci.osaka-cu.ac.jp

This paper is organized as follows. Section II gives a brief introduction of the Skyrme model. In Sec. III, we calculate the Born term using the Skyrme model with the chiral reduction formula. The summary is given in Sec. IV.

II. SKYRME MODEL: A MODEL OF NUCLEON

A. Effective Lagrangian

A starting point of the Skyrme model is the nonlinear realization of chiral symmetry. For nonstrange hadrons this symmetry takes the form of $SU_R(2) \times SU_L(2)$, which is broken spontaneously to the diagonal vector (isospin) $SU_V(2)$ symmetry. The pion field appears as the Nambu-Goldstone boson field, which is parametrized by the representative ξ for the left coset $SU_R(2) \times SU_L(2)/SU_V(2)$.

Any group element $g \in SU_R(2) \times SU_L(2)$ is decomposed as

$$g = \xi h = \exp\left(\frac{i}{f_\pi} \phi \cdot A\right) h, \tag{1}$$

where $h \in SU_V(2)$, the generator A_i ($i = 1, 2, 3$) spans the tangent space of the left coset, ϕ_i defines the isovector pion field, and f_π is the pion decay constant [14]. The fundamental quantity for the effective Lagrangian is the projection of the Maurer-Cartan one-form, $\xi^\dagger \partial_\mu \xi$, on this tangent space [15,16];

$$\alpha_\mu \equiv (\xi^\dagger \partial_\mu \xi)_\perp = \sum_{i=1}^{3} \alpha_{\mu i} A_i. \tag{2}$$

The group element $g \in SU_R(2) \times SU_L(2)$ transforms this one-form as $\xi^\dagger \partial_\mu \xi \to \xi'^\dagger \partial_\mu \xi' = h\xi^\dagger \partial_\mu \xi h^\dagger + h\partial_\mu h^\dagger$, where $\xi' = g\xi h^\dagger$ and $h \in SU_V(2)$ (h depends on both g and ξ). The pion field ϕ_i is transformed nonlinearly by g. However, if g belongs to $SU_V(2)$, the transformation becomes linear.

The one-form α_μ is h invariant because $h\partial_\mu h^\dagger$ belongs to the algebra of $SU_V(2)$. Thus α_μ is a basic building block to compose an h-invariant effective Lagrangian. In terms of ϕ_i, α_μ is written explicitly as

$$\alpha_{\mu i} = \frac{i}{f_\pi}(j_0(|\phi|)\partial_\mu \phi_i + (1 - j_0(|\phi|))\hat{\phi} \cdot \partial_\mu \phi \hat{\phi}_i), \tag{3}$$

where $\hat{\phi}_i = \phi_i/|\phi|$, $|\phi| = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2}$, and j_0 is the spherical Bessel function.

Among many possible forms of the effective Lagrangian, the Skyrme model chooses

$$\mathcal{L} = -\frac{f_\pi^2}{4} \text{tr}\alpha_\mu \alpha^\mu + \frac{1}{32e^2}\text{tr}[\alpha_\mu, \alpha_\nu][\alpha^\mu, \alpha^\nu]$$
$$+ \frac{m_\pi^2 f_\pi^2}{4}\text{tr}(\xi^2 + (\xi^2)^\dagger), \tag{4}$$

where two-dimensional representation is assumed for A_i, and the dimensionless parameter e determines the strength of the so-called the Skyrme term. The last term, which explicitly breaks the chiral symmetry, is required by the PCAC condition [17], where m_π is the pion mass.

B. Hedgehog configuration

The parametrization, Eq. (1), shows that ϕ_i is defined on the manifold S^3. The homotopy relation $\pi_3(S^3) = Z$ suggests the existence of a classical nontrivial solution that fulfills the least-action principle for the effective Lagrangian, Eq. (4). According to Coleman and Palais [18,19], this solution is static and takes a "hedgehog" configuration,

$$U_0(\vec{r}) \equiv (\xi_0(\vec{r}))^2 = e^{iF(r)\hat{r}\cdot\tau}, \tag{5}$$

where the Pauli matrix τ_i is used instead of A_i. The function $F(r)$ satisfies the Eular-Lagrange equation derived from the Lagrangian, Eq. (4), with the boundary conditions $F(0) = \pi$ and $F(\infty) = 0$, so that the soliton has winding number 1.

The hedgehog configuration, Eq. (5), constrains the direction of the isovector field ϕ_i to be in parallel with the spatial position vector. While this configuration is not compatible with the isospin symmetry, because the system itself must preserve this symmetry, the isospin rotation produces the zero mode. We quantize this zero mode so as to provide a suitable isospin property for the Skyrme model using the collective coordinate method of Ref. [1].

The hedgehog soliton $U_0(\vec{r})$ is rotated by a time-dependent matrix $A(t)$ [$A(t) \in SU_V(2)$], which is the two-dimensional representation of isospin rotation,

$$U(t, \vec{r}) = A(t)U_0(\vec{r})A^\dagger(t). \tag{6}$$

The Eular angle parametrizes $A(t)$ and is treated as the collective coordinate for this zero mode [20]. The isospin $\hat{I}$ is defined by means of $A(t)$ as

$$\hat{I}_i = i\mathcal{I}\text{tr}\tau_i \dot{A}(t)A^\dagger(t), \tag{7}$$

where $\mathcal{I}$ is the moment of inertia. This isospin, Eq. (7), is quantized and then satisfies the commutation relation of the Lie algebra of $SU_V(2)$. Owing to the hedgehog configuration, the spin $\hat{J}$ is related to the isospin $\hat{I}$ through

$$\hat{I}_i = -\sum_{j=1}^{3} \mathcal{D}_{ij}^1 \hat{J}_j, \tag{8}$$

where $\mathcal{D}_{ij}^1$ is the D function of $SU_V(2)$ in the three-dimensional representation:

$$\mathcal{D}_{ij}^1(\alpha, \beta, \gamma) = \tfrac{1}{2}\text{tr}\tau_i A(t)\tau_j A^\dagger(t). \tag{9}$$

As far as the zero mode is considered only for the isospin rotation, the spin of the baryon is always equal to its isospin: $\hat{J}^2 = \hat{I}^2$.

The baryon state is the simultaneous eigenstate of $\hat{I}$ and $\hat{J}$. The nucleon state is, for example, written as a function of the Eular angle (α, β, γ),

$$\langle \alpha\beta\gamma | 1/2, I_3, J_3 \rangle = \frac{(-1)^{I_3}}{2\pi}\mathcal{D}_{-I_3 J_3}^{1/2}(\alpha, \beta, \gamma), \tag{10}$$

which is normalized on S^3.

C. Axial vector current

The external field method is a convenient way to define the axial vector current for the chiral field. When the minimal scheme is assumed, the external vector and axial vector fields (ρ_μ and a_μ, respectively) are introduced into the effective Lagrangian through the covariant derivative [21]

$$\Delta_\mu U = \partial_\mu U - i \left[\sum_{a=1}^{3} \rho_{\mu a} V_a, U \right] - i \left\{ \sum_{a=1}^{3} a_{\mu a} A_a, U \right\}, \tag{11}$$

where the generator V_a is for $SU_V(2)$. The linear response of the Lagrangian to the variation of a_a^μ gives the axial vector current $J_{A,\mu a}$ composed of the chiral field,

$$\begin{aligned}
J_{A,\mu a} &= \left. \frac{\partial \mathcal{L}}{\partial a_a^\mu} \right|_{\rho_\mu = a_\mu = 0} \\
&= i f_\pi^2 \left(\cos|\phi| \alpha_{\mu a} + 2 \sin^2 \frac{|\phi|}{2} \hat{\phi} \cdot \alpha_\mu \hat{\phi}_a \right) \\
&\quad - \frac{i}{e^2} \left(\cos|\phi| ((\alpha_\mu \times \alpha_\nu) \times \alpha^\nu)_a \right. \\
&\quad \left. + 2 \sin^2 \frac{|\phi|}{2} (\alpha_\mu \times \alpha_\nu) \times \alpha^\nu \cdot \hat{\phi} \, \hat{\phi}_a \right), \tag{12}
\end{aligned}$$

where the parametrization, Eq. (1), is used to write $J_{A,\mu a}$ explicitly in terms of ϕ_a and $\alpha_{\mu a}$. The power expansion of $J_{A,\mu a}$ on ϕ_a shows that this current has a pion pole contribution in the asymptotic region:

$$J_{A,\mu a} \to -f_\pi \partial_\mu \phi_a. \tag{13}$$

The formal definition of the scalar and pseudoscalar currents is given by introducing an additional coupling term,

$$\frac{f_\pi^2}{4} \text{tr}((s - i\tau \cdot p)U + U^\dagger(s + i\tau \cdot p)), \tag{14}$$

in the effective Lagrangian, Eq. (4). The external scalar and pseudoscalar fields (s and p, respectively) compose the four-dimensional representation of $SO(4) \sim SU_R(2) \times SU_L(2)$. The pseudoscalar current and its asymptotic form are

$$\pi_a = \frac{1}{f_\pi} \frac{\partial \mathcal{L}}{\partial p_a} = \frac{i}{4} \text{tr}\tau_a(U^\dagger - U) \to \phi_a. \tag{15}$$

III. SKYRME MODEL AND BORN TERM

According to the chiral reduction formula, the scattering amplitude consists of responses to the external sources. The Born term in the πN scattering amplitude is relevant to the axial vector current,

$$\begin{aligned}
&\langle \pi_b(k')N(p')|\hat{S}|\pi_a(k)N(p)\rangle_A \\
&= -\frac{1}{f_\pi^2} k^\mu k'^\nu \int d^4x \, d^4x' e^{-ikx+ik'x'} \\
&\quad \times \langle N(p')|T^*(j_{A,\mu a}(x) j_{A,\nu b}(x'))|N(p)\rangle, \tag{16}
\end{aligned}$$

where $\hat{S}$ is the S matrix operator, $j_{A,\mu a}$ is the axial vector current, $k(k')$ is the initial(final) pion momentum, and μ, $\nu(a, b)$ are the Lorentz(isospin) indices. The initial(final) nucleon state with momentum $p(p')$ is represented by $|N(p)\rangle(|N'(p)\rangle)$, which is the spin-isospin eigenstate.

Because the spontaneous breaking of chiral symmetry leads to mixing between the axial vector current and the gradient of the pseudoscalar field [see Eq. (13)], the axial vector current does not vanish in the asymptotic region in general. In the chiral reduction formula this mixing is removed by redefining the current as

$$j_{A,\mu a} = J_{A,\mu a} + f_\pi \partial_\mu \pi_a, \tag{17}$$

which is free from the pion pole contribution in the asymptotic region.

The current $j_{A,\mu a}$ consists of the classical soliton solution in the lowest order of $1/N_c$. The zero mode of the soliton and its quantization add the operator character and the time dependence to this current.

First, we consider the zero mode for the translation invariance. The spatial translation of the localized soliton does not change the energy and produces the zero mode of the system. We quantize the position of the classical soliton as a quantum mechanical operator and write it as $\hat{X}(t)$ in the Heisenberg picture. The axial vector current depends on $\hat{X}(t)$ as $j_{A,\mu a}(\vec{x} - \hat{X}(t))$, and its coordinate representation becomes

$$\langle \vec{X}, t | j_{A,\mu a}(\vec{x} - \hat{X}(t)) | \vec{X}', t \rangle = j_{A,\mu a}(\vec{x} - \vec{X}) \delta(\vec{X} - \vec{X}'), \tag{18}$$

where $|\vec{X}, t\rangle$ is the eigenstate of $\hat{X}(t)$ with eigenvalue $\vec{X}$.

The nucleon state is $\langle \vec{X}, t | N(\vec{p}) \rangle = e^{i\vec{p}\cdot\vec{X} - ip_0 t}|N\rangle$, where $p_0 = \sqrt{m_N^2 + |\vec{p}|^2}$ is the energy of a nucleon with momentum $\vec{p}$ and mass m_N, and $|N\rangle$ stands for the nucleon-state vector in the spin-isospin space. The nucleon matrix element of $j_{A,\mu a}$ becomes

$$\begin{aligned}
&\langle N'(\vec{p}')|j_{A,\mu a}(\vec{x} - \hat{X}(x_0))|N''(\vec{p}'')\rangle \\
&= e^{i(p'-p'')x} \langle N'|\tilde{j}_{A,\mu a}(\vec{p}'' - \vec{p}')|N''\rangle, \tag{19}
\end{aligned}$$

where $\tilde{j}_{A,\mu a}(\vec{k})$ is defined as

$$\langle N'|\tilde{j}_{A,\mu a}(\vec{k})|N''\rangle = \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \langle N'|j_{A,\mu a}(\vec{x})|N''\rangle. \tag{20}$$

The nucleon pole contribution to the πN scattering amplitude, Eq. (16), is

$$\begin{aligned}
&-\frac{1}{f_\pi^2} \int d^4x \, d^4x' e^{-ikx+ik'x'} \langle N'(\vec{p}')|T^*(k^\mu j_{A,\mu a}(\vec{x} - \hat{X}(x_0)) k'^\nu j_{A,\nu b}(\vec{x}' - \hat{X}(x_0')))|N(\vec{p})\rangle \\
&= -\frac{1}{f_\pi^2} \left\{ \frac{\langle N'|k^\mu \tilde{j}_{A,\mu a}(-\vec{k})|N''\rangle \langle N''|k'^\nu \tilde{j}_{A,\nu b}(\vec{k}')|N\rangle}{p_0' - p_0'' - k_0 + i\epsilon} + \frac{\langle N'|k'^\nu \tilde{j}_{A,\nu b}(\vec{k}')|N''\rangle \langle N''|k^\mu \tilde{j}_{A,\mu a}(-\vec{k})|N\rangle}{p_0 - p_0'' + k_0 + i\epsilon} \right\}
\end{aligned}$$

$$\times\, i(2\pi)^4\delta(k'+p'-k-p) \sim \frac{1}{f_\pi^2 k_0}\{\langle N|\vec{k}\cdot\tilde{\vec{j}}_{A,a}(-\vec{k})|N''\rangle\langle N''|\vec{k}'\cdot\tilde{\vec{j}}_{A,b}(\vec{k}')|N\rangle$$
$$-\, \langle N'|\vec{k}'\cdot\tilde{j}_{A,b}(\vec{k}')|N''\rangle\langle N''|\vec{k}\cdot\tilde{\vec{j}}_{A,a}(-\vec{k})|N\rangle\}i(2\pi)^4\delta(k'+p'-k-p),\tag{21}$$

where the single-nucleon state (N'') is assumed to dominate the intermediate state; its momentum and the state vector in the spin-isospin space are denoted p'' and $|N''\rangle$, respectively. Each matrix element of $\vec{k}\cdot\tilde{\vec{j}}_A$ in Eq. (21) corresponds to the π-nucleon vertex. The approximations $p_0 \sim p_0' \sim p_0'' \sim m_N$, $m_N \gg k_0$, are used in the last step. The soliton mass (the nucleom mass m_N) is so heavy that the nonrelativistic approximation is applicable here.

Next we calculate the π-nucleon vertex by considering the zero mode for the isospin rotation. The current $j_{A,\mu a}$ is still an operator in the isospin space and depends on the Eular angle introduced in the isospin rotation, Eq. (6). This operator property is important for finding the Born term; otherwise, Eq. (21) will vanish. The spatial ith component of the axial vector current in the lowest order of $1/N_c$ is explicitly written as

$$j_{A,ia}(\vec{x}) = \left(f_\pi^2\frac{\sin 2F}{2r} + \frac{1}{e^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\frac{\sin 2F}{2r}\right)$$
$$\times\, \delta_{ac}\mathcal{D}_{ic}^1(\alpha,\beta,\gamma)$$
$$+ \left[f_\pi^2\left(F' - \frac{\sin 2F}{2r}\right) - \frac{1}{e^2}\left(\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right.\right.$$
$$\left.\left.\times\, \frac{\sin 2F}{2r} - \frac{2F'\sin^2 F}{r^2}\right)\right]\hat{r}_a\hat{r}_c\mathcal{D}_{ic}^1(\alpha,\beta,\gamma)$$
$$= \left(j_{A1}(r)\delta_{ac} + j_{A2}(r)\hat{r}_a\hat{r}_c\right)\mathcal{D}_{ic}^1(\alpha,\beta,\gamma),\tag{22}$$

where F is defined in Eq. (5), $r = |\vec{x}|$, $\hat{r} = \vec{x}/r$, and a, c stand for the isospin indices. The dependence on the Eular angle appears through the D function $\mathcal{D}_{ic}^1(\alpha,\beta,\gamma)$.

Using Eqs. (10) and (22) and the integration for the Eular angle, we obtain

$$\langle N'|\vec{k}\cdot\tilde{\vec{j}}_{A,a}(-\vec{k})|N''\rangle = -\tfrac{1}{3}J(k)\langle N'|\tau\cdot\varphi\sigma\cdot\vec{k}|N''\pi_a\rangle,\tag{23}$$

where

$$J(k) = 4\pi\int_0^\infty dr\, r^2\left(j_0(kr)\left(j_{A1}(r) + \frac{1}{3}j_{A2}(r)\right)\right.$$
$$\left. - \frac{2}{3}j_2(kr)j_{A2}(r)\right),\tag{24}$$

with the spherical Bessel functions j_0 and j_2, and $\langle 0|\varphi_c|\pi_a\rangle = \delta_{ca}$.

Finally, we find the Born term in the πN scattering amplitude with the nonrelativistic approximation. When we properly evaluate the spin-isospin structure of the π-nucleon vertex, the Born term becomes

$$\langle \pi_b(k')N'(p')|\hat{S}|\pi_a(k)N(p)\rangle$$
$$= -i(2\pi)^4\delta(k'+p'-k-p)\frac{J(k')J(k)}{9f_\pi^2}\frac{1}{\omega}$$

$$\times\, (\langle N'\pi_b|\tau\cdot\varphi\sigma\cdot k'|N''\rangle\langle N''|\tau\cdot\varphi\sigma\cdot k|N\pi_a\rangle$$
$$- \langle N'|\tau\cdot\varphi\sigma\cdot k|N''\pi_a\rangle\langle N''\pi_b|\tau\cdot\varphi\sigma\cdot k'|N\rangle),\quad (25)$$

where $\omega = k_0$ is the pion energy.

The Born term actually exists in the πN scattering amplitude in the Skyrme model. We arrive at this result in the lowest order calculation with respect to $1/N_c$ using the classical soliton solution. We stress here that it is quite important to take account of the zero mode both for the spatial translation and for the isospin rotation simultaneously. The argument regarding the missing Born term is caused by the lack of consideration of the zero mode for the isospin rotation in the lowest order calculation.

Before closing our discussion, we compare our results with the standard expression of the πN scattering amplitude. The nucleon matrix element of the axial vector current, in the nonrelativistic limit, is generally written as

$$\langle N'(\vec{p}')|A_{ia}(x)|N(\vec{p})\rangle$$
$$= \langle N'|\frac{\tau_a}{2}(\delta_{ij}g_A(q^2) + q_i q_j h_A(q^2))\sigma_j|N\rangle e^{-iqx},\tag{26}$$

where $q = p - p'$. The axial vector coupling constant is defined by $g_A = g_A(0)$. In the Skyrme model the left-hand side corresponds to

$$\langle N'(\vec{p}')|j_{A,ia}(\vec{x} - \hat{X}(x_0))|N(\vec{p})\rangle = \langle N'|\tilde{j}_{A,ia}(\vec{q})|N\rangle.\tag{27}$$

This relation gives g_A in the $|\vec{q}| \to 0$ limit as $g_A = \frac{2}{3}J(0)$.

We note that the spatial integral of J converges in our calculation, but the corresponding integral in Ref. [1] does not. Thus the value is well defined for the axial vector coupling constant obtained in our calculation. This is due to the definition, Eq. (17), in which the pion pole contribution is extracted from the axial vector current. The numerical integration gives 0.6 for g_A when the value from Ref. [1] is used for f_π and e. This result shows that our calculation is qualitatively consistent with that in Ref. [1].

The πNN coupling constant $g_{\pi NN}$ is given by a residue of the S matrix, and the strength of the Born term is written as $g_{\pi NN}^2/4m_N^2$. Then we obtain the Goldberger-Treiman relation, $g_{\pi NN} = m_N g_A/f_\pi$.

IV. SUMMARY

We can determine the Born term in the πN scattering amplitude in the Skyrme model. Not only the zero mode for the spatial translation, but also that for the isospin rotation must be included in the lowest order calcuation with the classical soliton solution. The chiral reduction formula holds promise as a way to study the πN interaction in

the Skyrme model. Despite the former negative argument, we find that it is worthwhile to examine the Skyrme model as an effective model of baryon and meson-baryon interaction.

We are now going to include the higher order contributions with respect to $1/N_c$. The fluctuation of the classical soliton, which is not dealt with in this paper, is the same-order contribution as the zero mode's. We expect that the inclusion of the fluctuation will improve the accuracy of the calculation (e.g., the axial vector coupling constant).

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