

Radiative energy loss in an anisotropic quark-gluon plasma

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We calculate radiative energy loss of heavy and light quarks in an anisotropic medium (static) in a first-order opacity expansion. Such an anisotropy can result from the initial rapid longitudinal expansion of the matter created in relativistic heavy-ion collisions. Significant dependency of the energy loss on the anisotropy parameter ξ and the direction of propagation of the partons with respect to the anisotropy axis is found. It is shown that the introduction of early-time momentum-space anisotropy can enhance the fractional energy loss in the direction of the anisotropy, whereas it decreases when the parton propagates perpendicular to the direction of the anisotropy.

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I. INTRODUCTION

One of the goals for the ongoing relativistic heavy-ion collision experiments at the Relativistic Heavy Ion Collider (RHIC) and the upcoming experiments at the CERN Large Hadron Collider (LHC) is to produce a quark-gluon plasma (QGP) and study its properties. According to the prediction of lattice quantum chromodynamics, a QGP is expected to be formed when the temperature of nuclear matter is raised above its critical value, $T_c \sim 170$ MeV or, equivalently, the energy density of nuclear matter is raised above $1 \text{ GeV}/\text{fm}^3$ [1]. The possibility of QGP formation at the RHIC experiment, with an initial density of $5 \text{ GeV}/\text{fm}^3$, is supported by the observation of high p_T hadron suppression (jet quenching) in the central Au-Au collisions compared to the binary-scaled hadron-hadron collisions [2]. Apart from jet quenching, several possible probes have been studied in order to characterize the properties of QGPs.

However, many properties of QGPs are still poorly understood. The most debated question is whether the matter formed in the relativistic heavy-ion collisions is in thermal equilibrium or not. The measurement of the elliptic flow parameter and its theoretical explanation suggest that the matter quickly comes into thermal equilibrium (with $\tau_{\text{therm}} < 1 \text{ fm}/c$, where τ_{therm} is the time of thermalization) [3]. On the contrary, a perturbative estimation suggests a relatively slower thermalization of QGPs [4]. However, recent hydrodynamical studies [5] have shown that, due to the poor knowledge of the initial conditions, there is a sizable amount of uncertainty in the estimate of thermalization or isotropization time. It is suggested that (momentum) anisotropy-driven plasma instabilities may speed up the process of isotropization [6], in which case one is allowed to use hydrodynamics for the evolution of the matter. However, instability-driven isotropization is not yet proven at RHIC and LHC energies.

In the absence of a theoretical proof favoring the rapid thermalization and the uncertainties in the hydrodynamical fits of experimental data, it is very hard to assume hydrodynamical behavior of the system from the very beginning. Therefore, it

has been suggested to look for some observables which are sensitive to the early time after the collision. For example, jet quenching vis-à-vis energy loss of partons could be an observable where the initial-state momentum anisotropy can play important role. This is the issue that we address here.

It is known that the energy loss of partons (also dubbed “jet quenching”) in QCD plasma can proceed in two ways: by two-body scattering and also via gluon radiation. These are known as collisional and radiative energy loss, respectively. The phenomena of jet-quenching has been investigated by various authors [2]. More recently, the nonphotonic single-electron data show more suppression than expected, which cannot be explained by radiative loss alone. A substantial amount of work has been done to look into this issue in recent times [2].

Note that the existing calculations on energy loss have been performed in isotropic QGP, which is true immediately after its formation [7]. However, subsequent rapid expansion of the matter along the beam direction causes faster cooling in the longitudinal direction than in the transverse direction [4]. As a result, the system becomes anisotropic with $\langle p_L^2 \rangle \ll \langle p_T^2 \rangle$ in the local rest frame. At some later time when the effect of the parton interaction rate overcomes the plasma expansion rate, the system returns to the isotropic state again and remains isotropic for the rest of the period. Thus, during the early stage, the plasma remains anisotropic and any calculation of energy loss should, in principle, include this aspect. The collisional energy loss in anisotropic media for heavy fermions has been calculated in Refs. [8,9]. In these calculations it is found that the deviations from the isotropic results are of the order of 10% for $\xi = 1$ and of the order of 20% for $\xi = 10$. It is observed that the collisional energy loss varies with the angle of propagation by up to 50%.

Recently, in Ref. [10], the transport coefficient \hat{q} has been calculated in anisotropic media which, in turn, affects the radiative energy loss. Here, we attempt to provide a quantitative estimate of radiative energy loss by modifying the static-scatterer model [11] appropriate for anisotropic media.

The other interesting aspect which, in recent years, has attracted considerable attention, is the possibility of the growth of unstable modes in an anisotropic plasma [12]. For example, in Ref. [13] the authors calculate \hat{q} for a two-stream plasma and show that the momentum broadening grows exponentially in

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time as the spontaneously growing fields exert an exponentially growing influence on the propagating parton. This momentum broadening of a fast parton which radiates gluons due to scattering off the plasma constituents therefore controls the radiative energy loss [14]. In an evolving plasma this is an important component which, however, is not included in the present manuscript. Therefore the results we report here can be considered to be something like a zeroth-order approximation.

The plan of the paper is the following. In Sec. II we briefly mention how to calculate the two-body potential in an anisotropic medium along with the modified expression for the fractional energy loss. Section III will be devoted to discussing the results. Finally, we conclude in Sec. IV.

II. FORMALISM

In this section, we recapitulate the basic formalism of the radiative energy loss of a fast-moving parton in an infinitely extended static isotropic plasma [11,15,16]. As in Ref. [11] we restrict ourselves to the radiative energy loss quarks at first order in opacity involving three diagrams, as shown in Fig. 1, where we assume that an on-shell heavy quark produced in the remote past is propagating through an infinite QCD medium that consists of randomly distributed static scattering centers. In the original Gyulassy-Wang formalism [17] static interactions are modeled here as color-screened Yukawa potentials originally developed for the isotropic QCD medium and given by

$$V_n = V(q_n)e^{i\vec{q}_n \cdot \vec{x}_n} = 2\pi\delta(q^0)v(\vec{q}_n)e^{-iq_n x_n}T_{a_n}(R) \otimes T_{a_n}(n), \quad (1)$$

with $v(\vec{q}_n) = 4\pi\alpha_s/(\vec{q}_n^2 + \mu^2)$, where μ is the Debye mass. The quantity x_n is the location of the n th scattering center and T (summed over a_n) denotes the color matrices of the parton and the scattering center. Note that the potential has been derived by using a hard thermal loop (HTL) propagator in a QGP medium. In a plasma with momentum anisotropy, the two-body interaction, as expected, becomes direction dependent. It has been observed that, on a distance scale on the order of the inverse Debye mass, the attraction for the quarks aligned along the direction of the anisotropy is stronger than for transverse alignment [18]. Therefore, the radiative energy loss will also depend on the direction of momentum of the quarks emitting Bremsstrahlung gluons. This necessitates the introduction of an anisotropy-dependent potential to estimate the radiative energy loss in a plasma having an anisotropic momentum distribution.

The heavy-quark potential in an anisotropic plasma has recently been calculated in Ref. [18], for which one starts with

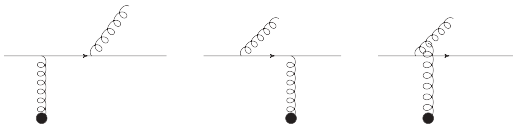


FIG. 1. Feynman diagrams contributing to the soft-gluon radiation in a static medium to first order in opacity.

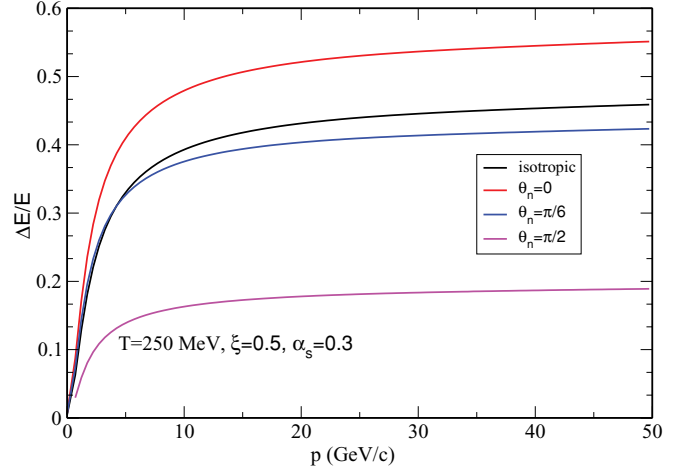


FIG. 2. (Color online) Fractional energy loss for light quark when mean free path is given by Eq. (19).

the retarded gluon self-energy expressed as [19]

$$\Pi^{\mu\nu}(P) = g^2 \int \frac{d^3k}{(2\pi)^3} v^\mu \frac{\partial f(\vec{k})}{\partial K^\beta} \left(g^{\nu\beta} - \frac{v^\nu P^\beta}{P \cdot v + i\epsilon} \right). \quad (2)$$

We have adopted the following notation for four vectors: $P^\mu = (p_0, \vec{p}) = (p_0, \mathbf{p}, p_z)$; that is, \vec{p} (with an explicit vector superscript) describes a three-vector while \mathbf{p} denotes the two-vector transverse to the z direction.

To include the local anisotropy in the plasma, one has to calculate the gluon polarization tensor incorporating anisotropic distribution functions of the medium. This subsequently can be used to construct an HTL-corrected gluon propagator which, in general, assumes a very complicated form. Such an HTL propagator was first derived in Ref. [20] in the time-axial gauge. A similar propagator has also been constructed in Ref. [18] to derive the heavy-quark potential in an anisotropic plasma, which, as we know, is given by the Fourier transform of the propagator in the static limit.

The self-energy, apart from momentum P^μ , also depends on a fixed anisotropy vector $n^\mu = (1, \vec{n})$ and $\Pi^{\mu\nu}$ can be cast in a suitable tensorial basis appropriate for an anisotropic plasma in a covariant gauge in the following way [18]:

$$\Pi^{\mu\nu} = \alpha A^{\mu\nu} + \beta B^{\mu\nu} + \gamma C^{\mu\nu} + \delta D^{\mu\nu}, \quad (3)$$

where the basis tensors are constructed out of p^μ , n^μ , and the four-velocity of the heat bath u^μ . The detailed expressions for the quantities that appear in Eq. (3) can be found in Ref. [18]. The anisotropy enters through the distribution function

$$f(\vec{p}) = f_{\text{iso}} \sqrt{\vec{p}^2 + \xi(\vec{p} \cdot \vec{n})^2}, \quad (4)$$

where the parameter ξ is the degree-of-anisotropy parameter ($-1 < \xi < \infty$) and is given by $\xi = \langle \mathbf{p}^2 \rangle / (2\langle p_z^2 \rangle) - 1$. Note that ξ can also be related to the shear viscosity [21].

Since the self-energy is symmetric and transverse, all the components are not independent. After a change of variables ($p' = \vec{p}^2 [1 + \xi(\hat{\mathbf{p}} \cdot \vec{n})^2]$), the spatial components can

be written as

$$\Pi^{ij} = \mu^2 \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\vec{v} \cdot \vec{n})n^l}{1 + \xi(\vec{v} \cdot \vec{n})^2} \left(\delta^{jl} + \frac{v^j p^l}{P \cdot v + i\epsilon} \right). \quad (5)$$

Now α , β , γ , and δ are determined by the following contractions:

$$\begin{aligned} p^i \Pi^{ij} p^j &= \vec{p}^2 \beta, \\ A^{il} n^l \Pi^{ij} p^j &= [\vec{p}^2 - (n \cdot P)^2] \delta, \\ A^{il} n^l \Pi^{ij} A^{jk} n^k &= \frac{\vec{p}^2 - (n \cdot P)^2}{\vec{p}^2} \alpha + \gamma, \\ \text{Tr} \Pi^{ij} &= 2\alpha + \beta + \gamma, \end{aligned} \quad (6)$$

where the expressions for α , β , γ , and δ are given in Ref. [20].

After knowing the gluon HTL self-energy in anisotropic media the propagator can be calculated. The result, after some

cumbersome algebra [10,18], is

$$\begin{aligned} \Delta^{\mu\nu} &= \frac{1}{(P^2 - \alpha)} [A^{\mu\nu} - C^{\mu\nu}] + \Delta_G \left[(P^2 - \alpha - \gamma) \frac{\omega^4}{P^4} B^{\mu\nu} \right. \\ &\quad \left. + (\omega^2 - \beta) C^{\mu\nu} + \delta \frac{\omega^2}{P^2} D^{\mu\nu} \right] - \frac{\lambda}{P^4} P^\mu P^\nu, \end{aligned} \quad (7)$$

where

$$\Delta_G^{-1} = (P^2 - \alpha - \gamma)(\omega^2 - \beta) - \delta^2 [P^2 - (n \cdot P)^2]. \quad (8)$$

Now the momentum space potential can be obtained from the static gluon propagator in the following way:

$$\begin{aligned} v(\mathbf{q}, q_z, \xi) &= g^2 \Delta^{00}(\omega = 0, \mathbf{q}, q_z, \xi) \\ &= g^2 \frac{\vec{q}^2 + m_\alpha^2 + m_\gamma^2}{(\vec{q}^2 + m_\alpha^2 + m_\gamma^2)(\vec{q}^2 + m_\beta^2) - m_\delta^2}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} m_\alpha^2 &= -\frac{\mu^2}{2\mathbf{q}^2 \sqrt{\xi}} \left[q_z^2 \tan^{-1}(\sqrt{\xi}) - \frac{q_z \vec{q}^2}{\sqrt{\vec{q}^2 + \xi \mathbf{q}^2}} \tan^{-1} \left(\frac{\sqrt{\xi} q_z}{\sqrt{\vec{q}^2 + \xi \mathbf{q}^2}} \right) \right], \\ m_\beta^2 &= \mu^2 \frac{[\sqrt{\xi} + (1 + \xi) \tan^{-1}(\sqrt{\xi})](\vec{q}^2 + \xi \mathbf{q}^2) + \frac{\vec{q}^2(1+\xi)}{\sqrt{\vec{q}^2 + \xi \mathbf{q}^2}} \tan^{-1} \left(\frac{\sqrt{\xi} q_z}{\sqrt{\vec{q}^2 + \xi \mathbf{q}^2}} \right)}{2\sqrt{\xi}(1 + \xi)(\vec{q}^2 + \xi \mathbf{q}^2)}, \\ m_\gamma^2 &= -\frac{\mu^2}{2} \left[\frac{\vec{q}^2}{\xi \mathbf{q}^2 + \vec{q}^2} - \frac{1 + \frac{2q_z^2}{\mathbf{q}^2}}{\sqrt{\xi}} \tan^{-1}(\sqrt{\xi}) + \frac{q_z \vec{q}^2(2\vec{q}^2 + 3\xi \mathbf{q}^2)}{\sqrt{\xi}(\xi \mathbf{q}^2 + \vec{q}^2)^{3/2} \mathbf{q}^2} \tan^{-1} \left(\frac{\sqrt{\xi} q_z}{\sqrt{\vec{q}^2 + \xi \mathbf{q}^2}} \right) \right], \\ m_\delta^2 &= -\frac{\pi \mu^2 \xi q_z \mathbf{q} |\vec{q}|}{4(\xi \vec{q}^2 + \vec{q}^2)^{3/2}}, \end{aligned} \quad (10)$$

with $\vec{q} = (\mathbf{q}, q_z)$. For the general anisotropy vector \vec{n} we have $\mathbf{q} = \vec{q} - (\vec{q} \cdot \vec{n})\vec{n}$ and $q_z = \vec{q} \cdot \vec{n}$.

For $q_z = 0$, the potential in an anisotropic medium simplifies to

$$v(\mathbf{q}, \xi) = \frac{4\pi\alpha_s}{\mathbf{q}^2 + R(\xi)\mu^2}, \quad (11)$$

$$R(\xi) = \frac{1}{2} \left[\frac{1}{1 + \xi} + \frac{\tan^{-1} \sqrt{\xi}}{\sqrt{\xi}} \right]. \quad (12)$$

For small anisotropy and $q_z = 0$, the two-body interaction can be written as

$$v(\mathbf{q}, \xi \ll 1) = 4\pi\alpha_s \left[\frac{1}{\mathbf{q}^2 + \mu^2} + \frac{2}{3} \frac{\mu^2 \xi}{(\mathbf{q}^2 + \mu^2)^2} \right]. \quad (13)$$

Now, in Fig. 1, the parton scatters with one of the color centers with the momentum $Q = (0, \mathbf{q}, q_z)$ and radiates a gluon with momentum $K = (\omega, \mathbf{k}, k_z)$. The method for calculating the amplitudes of the diagrams depicted in Fig. 1 is discussed in Refs. [15,16] and we shall quote the main results only. The quark energy loss is calculated by folding the rate of gluon radiation $\Gamma(E)$ with the gluon energy by

assuming $\omega + q_0 \approx \omega$. In this approximation one finds

$$\frac{dE}{dL} = \frac{E}{D_R} \int x dx \frac{d\Gamma}{dx}. \quad (14)$$

Here, D_R is defined as $[t_a, t_c][t_c, t_a] = C_2(G)C_R D_R$ where $C_2(G) = 3$, $D_R = 3$, and $[t_a, t_c]$ is a color commutator (see Ref. [11] for details). The quantity x is the longitudinal momentum fraction of the quark carried away by the emitted gluon.

where in anisotropic media we have

$$\begin{aligned} x \frac{d\Gamma}{dx} &= \frac{C_R \alpha_s}{\pi} \frac{L}{\lambda} \int \frac{d^2 \mathbf{k}}{\pi} \frac{d^2 \mathbf{q}}{\pi} |v(\mathbf{q}, q_z, \xi)|^2 \frac{\mu^2}{16\pi^2 \alpha_s^2} \\ &\quad \times \left[\frac{\mathbf{k} + \mathbf{q}}{(\mathbf{k} + \mathbf{q})^2 + \chi^2} - \frac{\mathbf{k}}{\mathbf{k}^2 + \chi} \right]^2. \end{aligned} \quad (15)$$

In the last expression, $v(\vec{q}, \xi)$ is the two-body quark-quark potential given by Eq. (9) and $\chi = m_q^2 x^2 + m_g^2$, where $m_q^2 = \mu^2/2$ and $m_g^2 = \mu^2/6$.

In the present case, we assume that the parton is propagating along the z direction and the anisotropy vector \vec{n} makes an angle θ_n with the z axis [i.e., $\vec{n} = (\sin \theta_n, 0, \cos \theta_n)$]. Thus, θ_n describes the direction of propagation of the parton with respect to the anisotropy axis. In such cases, we replace \mathbf{q}

and q_z in Eq. (9) by $\mathbf{q} \rightarrow \sqrt{\mathbf{q}^2 - q^2 \sin^2 \theta_n \cos^2 \phi}$ and $q_z \rightarrow |\mathbf{q}| \cos \phi \sin \theta_n$, where $\mathbf{q} = (|\mathbf{q}| \cos \phi, |\mathbf{q}| \sin \phi)$.

For arbitrary ξ the radiative energy loss can be written as

$$\frac{\Delta E}{E} = \frac{C_R \alpha_s L \mu^2}{\pi^2 \lambda} \int dx d^2 \mathbf{q} \frac{|v(\mathbf{q}, q_z, \xi)|^2}{16\pi^2 \alpha_s^2} \left[-\frac{1}{2} - \frac{k_m^2}{k_m^2 + \chi} + \frac{\mathbf{q}^2 - k_m^2 + \chi}{2\sqrt{\mathbf{q}^4 + 2\mathbf{q}^2(\chi - k_m^2) + (k_m^2 + \chi)^2}} \right. \\ \left. + \frac{\mathbf{q}^2 + 2\chi}{\mathbf{q}^2 \sqrt{1 + \frac{4\chi}{\mathbf{q}^2}}} \ln \left(\frac{k_m^2 + \chi}{\chi} \frac{(\mathbf{q}^2 + 3\chi) + \sqrt{1 + \frac{4\chi}{\mathbf{q}^2}}(\mathbf{q}^2 + \chi)}{(\mathbf{q}^2 - k_m^2 + 3\chi) + \sqrt{1 + \frac{4\chi}{\mathbf{q}^2}}\sqrt{\mathbf{q}^4 + 2\mathbf{q}^2(\chi - k_m^2) + (k_m^2 + \chi)^2}} \right) \right]. \quad (16)$$

In the above expression, λ denotes the average mean-free path of the quark given by

$$\frac{1}{\lambda} = \frac{1}{\lambda_g} + \frac{1}{\lambda_q}, \quad (17)$$

which in this case would be ξ -dependent. In the last expression λ_g and λ_q correspond to the contributions coming from q - g and q - q scatterings, respectively.

Explicitly, with Eq.(11), we have

$$\lambda_i^{-1} = \frac{C_R C_2(i) \rho_i}{d_A} \int d^2 \mathbf{q} \frac{4\alpha_s^2}{[\mathbf{q}^2 + R(\xi)\mu^2]^2}, \quad (18)$$

where $C_R = 4/3$, $C_2(i)$ is the cashimir for the d_i -dimensional representation and $C_2(i) = (N_c^2 - 1)/(2N_c)$ for quark and $C_2(i) = N_c$ for gluon scatterers. The quantity $d_A = N_c^2 - 1$ is the dimensionality of the adjoint representation and ρ_i is the density of the scatterers. Using $\rho_i = \rho_i^{\text{iso}}/\sqrt{1 + \xi}$ we obtain

$$\frac{1}{\lambda} = \frac{18\alpha_s T \zeta(3)}{\pi^2 \sqrt{1 + \xi}} \frac{1}{R(\xi)} \frac{1 + N_F/6}{1 + N_F/4}, \quad (19)$$

where N_F is the number of flavors. For $\xi \rightarrow 0$ Eq. (19) reduces to the well-known result [11]

$$\frac{1}{\lambda} = \frac{18\alpha_s T \zeta(3)}{\pi^2} \frac{1 + N_F/6}{1 + N_F/4}. \quad (20)$$

It is evident that the changes from the isotropic medium appear here as the coefficient $R(\xi)$ of the Debye mass and the coefficient $1/\sqrt{1 + \xi}$ of the number density. In the limit $\xi \rightarrow 0$ we recover all the previously known results, as may be checked from Ref. [11].

III. RESULTS

For the quantitative estimates of the fractional energy loss in an anisotropic medium, first we consider a plasma at a temperature $T = 250$ MeV with the effective number of degrees of freedom $N_F = 2.5$ with the strong coupling constant $\alpha_s = 0.3$ and $L = 5$ fm. We also note that the mean-free path of the propagating parton depends on the anisotropy parameter ξ [see Eq.(19)]. The fractional energy loss for nonzero ξ ($\xi = 0.5$) for light flavor is shown in Fig. 2. As is evident from Eq. (16), the energy loss in anisotropic media depends on the angle of propagation of the fast partons with respect to the anisotropy axis (\vec{n}). This is also illustrated in Fig. 2. It is observed that, for a nonzero value of the anisotropy parameter ξ , the fractional energy loss increases in the direction parallel to the anisotropy axis. However, away from the anisotropy axis, the fractional energy loss decreases because the quark-quark potential is stronger in the anisotropy direction.

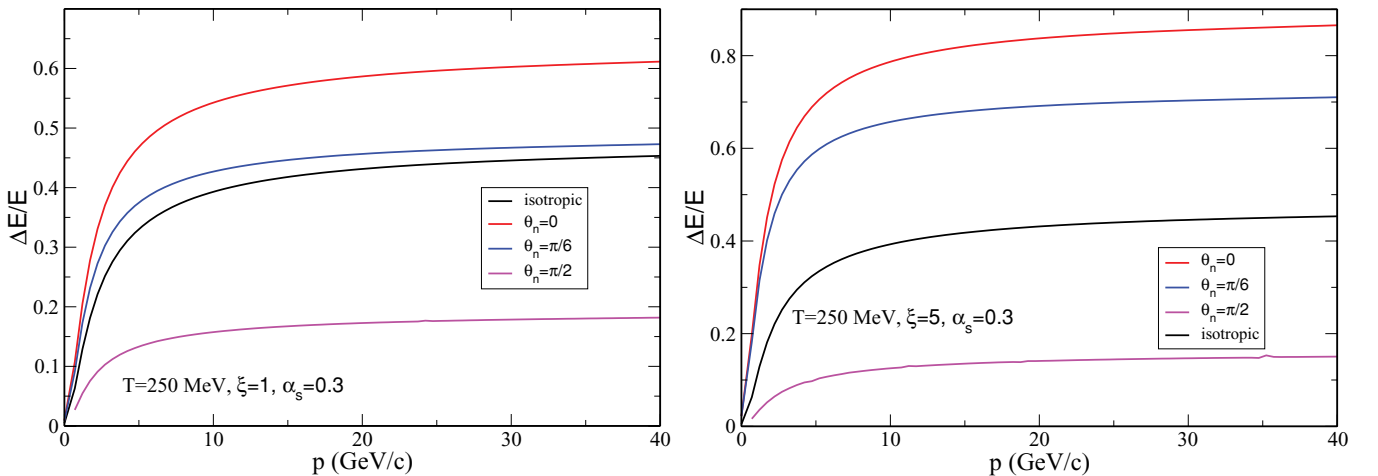


FIG. 3. (Color online) Same as Fig. 2 for $\xi = 1$ (left panel) and for $\xi = 5$ (right panel).

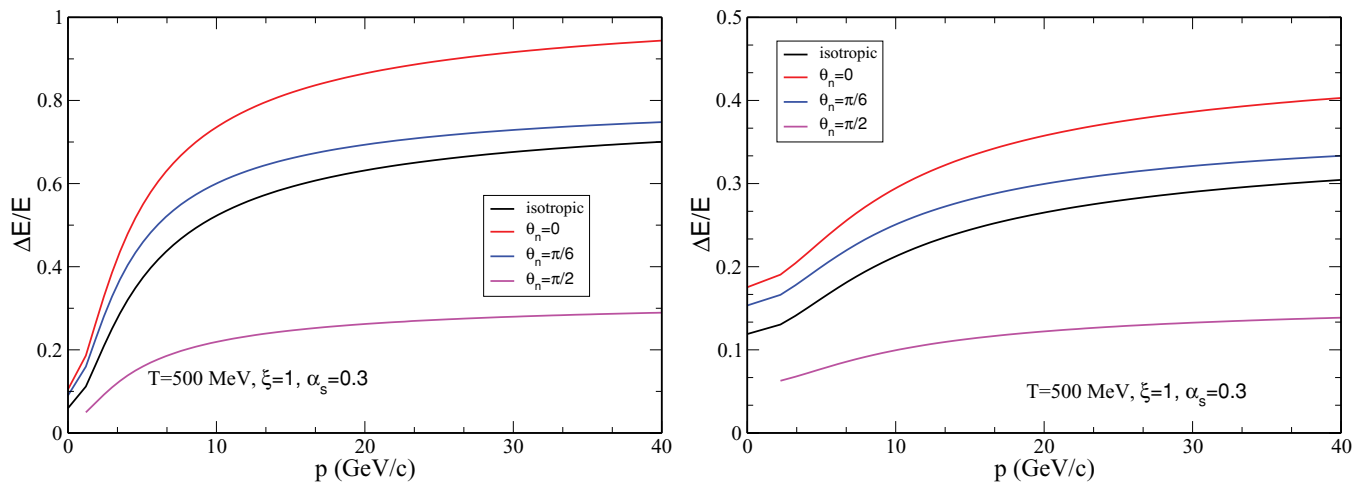


FIG. 4. (Color online) Same as Fig. 2 for charm (left panel) and bottom quarks (right panel) with $\xi = 1$ and $T = 500$ MeV.

For higher values of the anisotropy parameter ξ the results are shown in Fig. 3. It is seen that the fractional energy loss increases with ξ in the anisotropy direction. For $\xi = 1$ and $\xi = 0.5$, the fractional energy loss increases marginally for $\theta_n = \pi/6$ and it becomes larger for $\xi = 5$ for the same value of θ_n . However, in the perpendicular direction the fractional energy loss decreases substantially. Note that, for small anisotropy, the results are almost similar to the case when the mean-free path is independent of the anisotropy parameter. However, for larger values of ξ the result changes reasonably, as can be verified by calculating λ from Eq. (19) for larger anisotropy [see Fig. 2].

For the heavy quarks (i.e., for charm and bottom), the results are shown in Fig. 4 for $\xi = 1$. Similar to light quarks, we find enhancement in the anisotropy direction as well as for $\theta_n = \pi/6$. However, for $\theta_n = \pi/2$ the energy loss (fractional) decreases for the reasons mentioned earlier.

IV. SUMMARY

In this work, we have calculated the fractional energy loss due to gluon radiation in an infinite-size anisotropic medium treating the scatterer as providing a screened coulomb-like

potential. We have seen that the potential gets modified in anisotropic media. It is observed that the fractional energy loss depends on the direction of propagation of the fast partons with respect to the anisotropy axis as well as on the anisotropy parameter ξ . An enhancement is seen in the direction parallel to the anisotropy direction \vec{n} where, as in the transverse direction, it reduces due to weaker quark-quark interaction. It is also observed that, for higher values of ξ , the fractional energy loss increases for a given direction with respect to the anisotropy axis. We also note that, due to the dependency of the mean-free path on the anisotropy parameter, the energy loss increases as ξ increases.

We do not include the recoil of the scatterer in this work. However, this condition can be relaxed by incorporating the recoil corrections, which play an important role, as shown in Ref. [11]. This will be included in a future presentation. Furthermore, the finite-size effect on the radiative energy loss in anisotropic media would also be interesting to study.

The present calculation can be extended to include the effect of the growth of unstable modes to obtain results valid in a more realistic scenario, as mentioned in the introduction. Inclusion of such effects might modify the quantitative estimate of the nuclear modification factor at RHIC and LHC energies.

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